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# Population synthesis of old neutron stars in the Galaxy

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**Abstract.** The paucity of old isolated accreting neutron stars in ROSAT observations is used to derive a lower limit on the mean velocity of neutron stars at birth. The secular evolution of the population is simulated following the paths of a statistical sample of stars for different values of the initial kick velocity, drawn from an isotropic Gaussian distribution with mean velocity $0 \leq \langle V \rangle \leq 550\ \mathrm{km\,s^{-1}}$. The spin–down, induced by dipole losses and the interaction with the ambient medium, is tracked together with the dynamical evolution in the Galactic potential, allowing for the determination of the fraction of stars which are, at present, in each of the four possible stages: Ejector, Propeller, Accretor, and Georotator. Taking from the ROSAT All Sky Survey an upper limit of $\sim 10$ accreting neutron stars within $\sim 140$ pc from the Sun, we infer a lower bound for the mean kick velocity, $\langle V \rangle \gtrsim 200 - 300\ \mathrm{km\,s^{-1}}$. The same conclusion is reached for both a constant ($B \sim 10^{12}$ G) and a magnetic field decaying exponentially with a timescale $\sim 10^9$ yr. Present results, moreover, constrain the fraction of low velocity stars, which could have escaped pulsar statistics, to $\lesssim 1\%$.

## 1. Introduction

Isolated neutron stars (NSs) are expected to be as many as $10^8$–$10^9$, a non–negligible fraction of the total stellar content of the Galaxy. The number of observed radio pulsars is now $\sim$ 1,000. Since the pulsar lifetime is $\sim 10^7$ yr, this implies that the bulk of the NS population, mainly formed of old objects, remains undetected as yet. Despite intensive searches at all wavelengths, only a few (putative) isolated NSs which are not radio pulsars (or soft $\gamma$ repeaters) have been recently discovered in the X–rays with ROSAT (Walter, Wolk & Neuhauser 1996; Haberl et al. 1998; Neuhauser & Trumper 1999. The extreme X–ray to optical flux ratio ($> 10^3$) makes the NS option rather robust, but the exact nature of their emission is still controversial. Up to now, two main possibilities have been suggested, either relatively young NSs radiating away their residual internal energy or much aged NSs accreting the interstellar gas. Both options have advantages and drawbacks. Standard cooling atmosphere models fail to predict in a natural way the spectrum the best studied object, RX J1856-3754 (see Walter et al. this volume). If the Bondi–Hoyle scenario applies, accretion is reduced for increasing star velocity and for $v > 20$ km s$^{-1}$ it fails to produce the luminosities inferred from ROSAT data (see Walter et al. this volume). Meanwhile, we feel that a more thorough analysis of the statistical properties of NSs is of interest and can be useful in providing indirect evidence in favor or against the accretion scenario.

As discussed by Lipunov (1992), isolated NSs can be classified into four main types: Ejectors, Propellers, Accretors and Georotators. In Ejectors the relativistic outflowing momentum flux is always larger than the ram pressure of the surrounding material so they never accrete and are either active or dead pulsars, still spun down by dipole losses. In Propellers the incoming matter can penetrate down to the Alfven radius, $R_A$, but no further because of the centrifugal barrier, and stationary inflow can not occur, but the piling up of the material at the Alfven radius may give rise to (supposedly short) episodes of accretion (Treves, Colpi & Lipunov 1993; Popov 1994). Steady accretion is also impossible in Georotators where (similarly to the Earth) the Alfven radius exceeds the accretion radius, so that magnetic pressure dominates everywhere over the gravitational pull. It is the combination of the star period, magnetic field and velocity that decides which type a given isolated NS belongs to and, since both $P$, $B$ and $V$ change during the star evolution, a NS can go through different stages in its lifetime.

While the dynamical evolution of NSs in the Galactic potential was studied by several authors (sse e.g. Madau & Blaes 1994; Zane et al. 1995), little attention was paid to the NSs magneto–rotational evolution. Recently, this issue was discussed in some detail by Livio, Xu & Frank (1998) and Colpi et al. (1998). Goal of this investigation is to consider these two issues simultaneously, coupling the dynamical and the magneto–rotational evolution for the isolated NS population.

The possibility that the low–velocity tail is underpopulated with respect to what was previously assumed should be seriously taken into account. It is our aim to revise the estimates on the number of old accreting neutron stars in the Galaxy in the light of these new data, in the attempt to reconcile theoretical predictions with present ROSAT limits (Neühauser & Trümper 1999).

## 2. The Model

In this section we summarize the main hypothesis introduced to track the evolution of single stars and describe shortly the technique used to explore their statistical properties, referring to Popov & Prokhorov (1998) for details on spatial evolution calculations and to Konenkov & Popov (1997) and Lipunov & Popov (1995) for details of magneto-rotational evolution.

### 2.1. Dynamical evolution

The dynamical evolution of each single star in the Galactic potential (taken in the form proposed by Miyamoto & Nagai 1975) is followed solving its equations of motion.

The period evolution depends on both the star velocity and the local density of the interstellar medium, any attempt to investigate the statistical properties of the NS population should incorporate a detailed model of the ISM geography. Unfortunately the distribution of molecular and atomic hydrogen in the Galaxy is highly inhomogeneous. Here we use the analytical distributions from Bochkarev (1992) and Zane et al. (1995) for the hydrogen density $n(R, Z)$. Within a region of $\sim 140$ pc around the Sun, the ISM is underdense, and we take $n = 0.07\ \mathrm{cm}^{-3}$.

In our model we assume that the NS birthrate is constant in time and proportional in magnitude to the square of the local gas density.

Neutron stars at birth have a circular velocity determined by the Galactic potential. Superposed to this ordered motion a kick velocity is imparted in a random direction. We use here an isotropic Gaussian distribution with dispersion $\sigma_V$, simply as a mean to model the true pulsar distribution at birth (see e.g. Cordes 1998). The mean velocity $\langle V \rangle = (8/\pi)^{1/2}\sigma_V$ is varied in the interval 0–550 km s$^{-1}$.

### 2.2. Accretion physics and period evolution

The accretion rate was calculated according to the Bondi formula

$$\dot{M} = \frac{2\pi (GM)^2 m_p n(R,Z)}{(V^2 + V_s^2)^{3/2}} \simeq 10^{11}\, n\, v_{10}^{-3}\ \mathrm{g\,s^{-1}} \qquad (1)$$

where $m_p$ is proton mass, the sound speed $V_s$ is always 10 $\mathrm{km\,s^{-1}}$ and $v_{10} = (V^2 + V_s^2)^{1/2}$ in units of 10 $\mathrm{km\,s^{-1}}$. $M$ and $R$ denote the NS mass and radius, which we take equal to $1.4\,M_\odot$ and 10 km, respectively, for all stars.

All neutron stars are assumed to be born with a period $P(0) = 0.02$ s, and a magnetic moment either $\mu_{30} = 1$ or $\mu_{30} = 0.5$, where $\mu_{30} = \mu/10^{30}\mathrm{G\,cm^3}$.

In the ejector phase the energy losses are due to magnetic dipole radiation. When the gravitational energy density of the incoming interstellar gas exceeds the outward momentum flux at the accretion radius, $R_{ac} \simeq 2GM/v^2$, matter starts to fall in. This happens when the period reaches the critical value

$$P_E(E \to P) \simeq 10\, \mu_{30}^{1/2}\, n^{-1/4}\, v_{10}^{1/2}\ \mathrm{s}. \qquad (2)$$

When $P > P_E(E \to P)$ the NS is in the propeller phase, rotational energy is lost and the period keeps increasing at a rate taken from Shakura (1975).

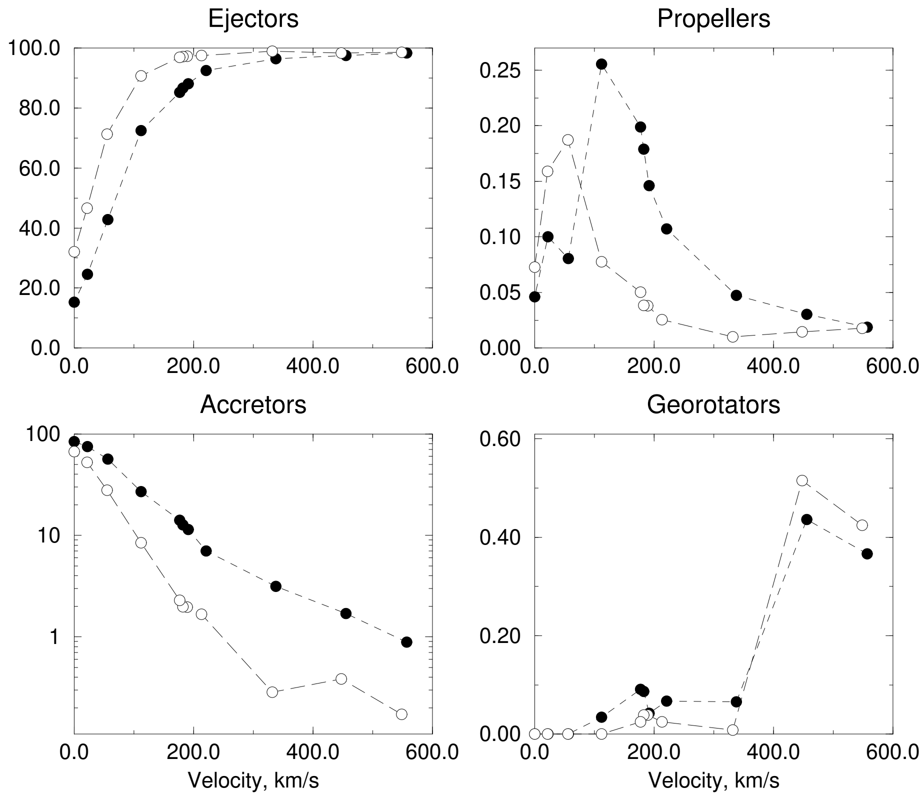


Figure 1. Fractions of NSs in the different stages vs. the mean kick velocity for $\mu_{30} = 0.5$ (open circles) and $\mu_{30} = 1$ (filled circles); typical statistical uncertainty for ejectors and accretors is $\sim$ 1-2%.

As the star moves through the inhomogeneous ISM a transition from the propeller back to the ejector phase may occur if the period attains the critical value

$$P_E(P \rightarrow E) \simeq 3\,\mu_{30}^{4/5} v_{10}^{6/7} n^{-2/7} \mathrm{s}\,. \tag{3}$$

Note that the transitions $P \rightarrow E$ and $E \rightarrow P$ are not symmetric as first discussed by Shvartsman in the early '70s.

Accretion onto the star surface occurs when the corotation radius $R_{co} = (GM\,P^2/4\pi^2)^{1/3}$ becomes larger than the Alfven radius (and $R_A < R_{ac}$, see below). This implies that braking torques have increased the period up to

$$P_A(P \rightarrow A) \simeq 420\,\mu_{30}^{6/7}\, n^{-3/7}\, v_{10}^{9/7}\ \mathrm{s}\,. \tag{4}$$

As soon as the NS enters the accretor phase, torques produced by stochastic angular momentum exchanges in the ISM slow down the star rotation at the equilibrium period

$$P_{eq} = 2.6 \times 10^3\, v_{(t)10}^{-2/3}\, \mu_{30}^{2/3}\, n^{-2/3}\, v_{10}^{13/3}\ \mathrm{s} \tag{5}$$

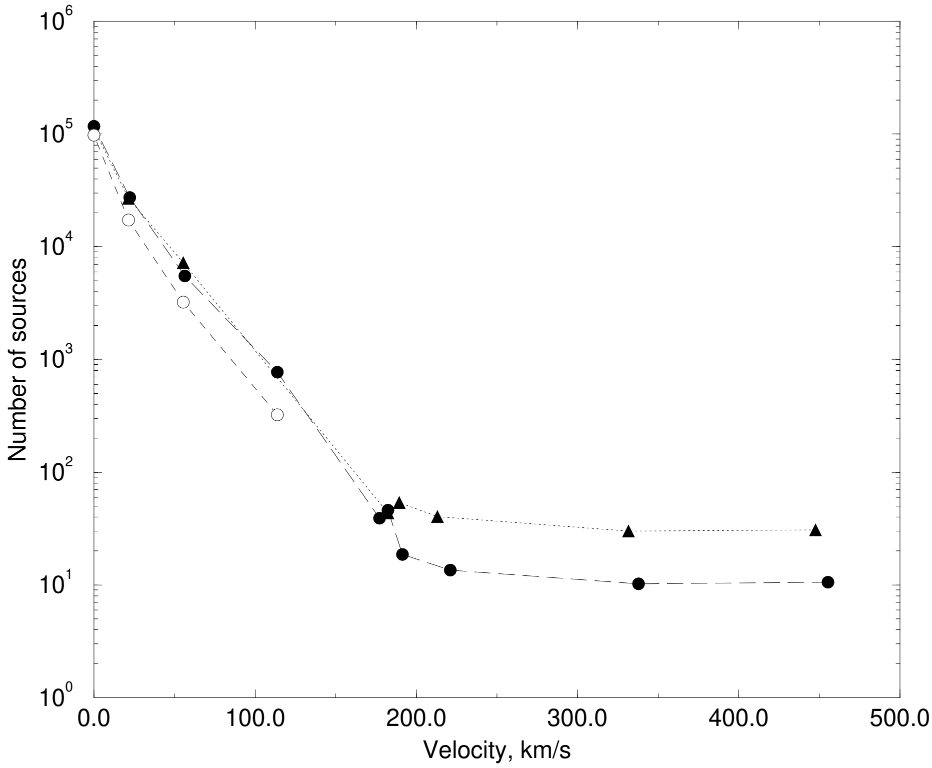


Figure 2. Number of accreting NSs in the Solar vicinity for a constant ($\mu_{30} = 1$, filled circles; $\mu_{30} = 0.5$, opened circles) and decaying field ($t_d = 2.2 \times 10^9$ yr, triangles).

where $v_{(t)}$ the turbulent velocity of the ISM (Lipunov & Popov 1995; Konenkov & Popov 1997).

At the very low accretion rates expected for fast, isolated NSs, it could be that the Alfven radius is larger than the accretion radius. The condition $R_A < R_{ac}$ translates into a limit for the star velocity

$$v < 410\, n^{1/10} \mu_{30}^{-1/5} \ \mathrm{km\,s^{-1}}\,. \tag{6}$$

## 3. Results and discussion

### 3.1. The NS census for a non–decaying field

We consider two representative values for the (costant) magnetic dipole moment, $\mu_{30} = 0.5$ and $\mu_{30} = 1$. The present fraction of NSs in the Ejector and Accretor stages as a function of the mean kick velocity is shown in figure 1.

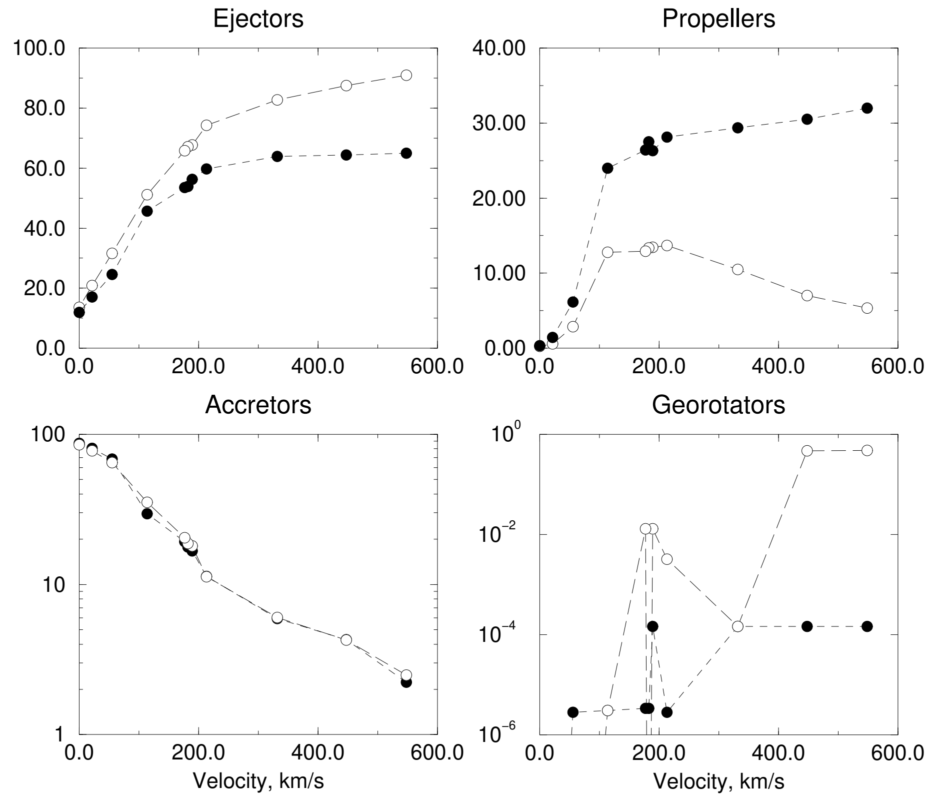


Figure 3. Fractions of NSs in the different stages vs. the average kick velocity for a decaying field with an e–folding time $t_d = 2.2 \times 10^9$ yrs (open circles) and $t_d = 1.1 \times 10^9$ yrs (filled circles).

Here, and in the following the total number of Galactic NSs was assumed to be $10^9$. A total number $\sim 10^9$ appears to be consistent with the nucleosynthesis and chemical evolution of the Galaxy, while $10^8$ is derived from radio pulsars observations. It is uncertain if all NSs experience an active radio pulsar phase, due to low initial magnetic fields or long periods, or to the fall–back in the aftermath of the supernova explosion. There is a serious possibility that the total number of NSs derived from radio pulsar statistics is only a lower limit.

In order to compare the expected number of accreting ONSs with the ROSAT All Sky Survey (RASS) results, we evaluated the number of those ONSs, within 140 pc from the Sun, producing an unabsorbed flux of $10^{-13}$ erg cm$^{-2}$ s$^{-1}$ or higher at energies $\sim$ 100 eV. The results are illustrated in figure 2. The main point is that for mean velocities below 200 km s$^{-1}$ the number of ONSs with a flux above the RASS detection limit would exceed 10. Most recent analysis on the number of isolated NSs in the RASS (Neühauser & Trümper 1999) indicate that the upper limit is below 10.

An important aspect is that our results exclude the possible presence of a consistent low–velocity population at birth, which exceeds that contained in the gaussian with $\langle V\rangle > 200$ km s$^{-1}$ ($\sim 1\%$ for $\langle V\rangle < 70$ km s$^{-1}$).

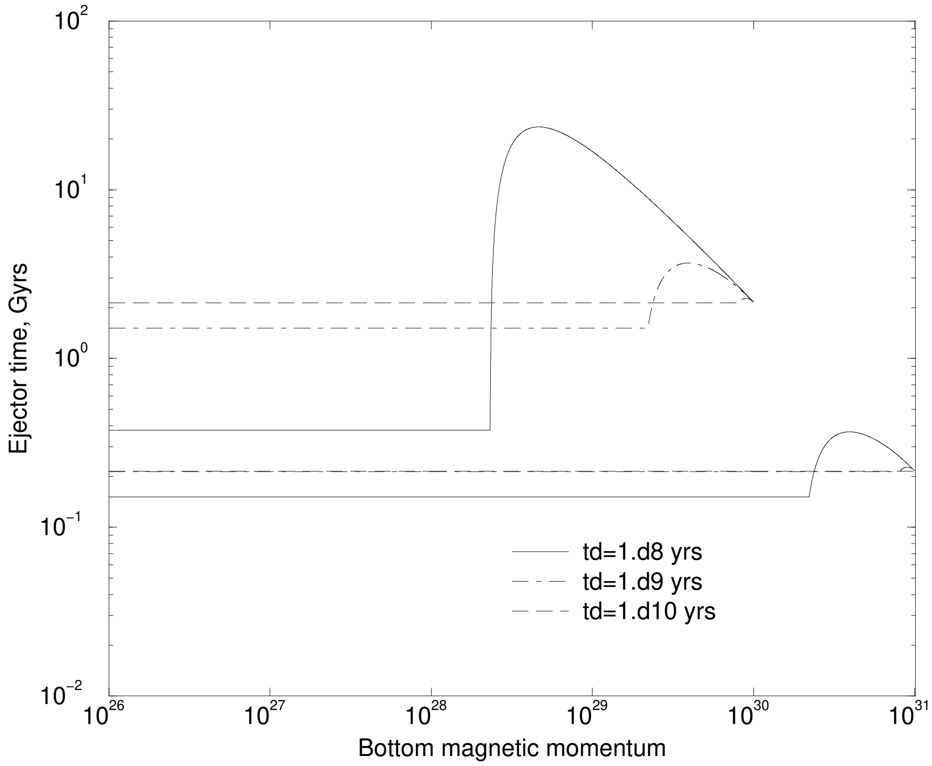


Figure 4. Ejector time vs. bottom magnetic momentum for two values of initial magnetic momentum and different decay timescales, $t_d = 10^8$, $10^9$, and $10^{10}$ yr.

### 3.2. The NS census for a decaying field

We refer here only to a very simplified picture of the field decay in which $B(t) = B(0)\exp(-t/t_d)$. Calculations have been performed for $t_d = 1.1 \times 10^9$ yr, $t_d = 2.2 \times 10^9$ yr and $\mu_{30}(0) = 1$. Results are shown in figure 3.

For some values of $t_d$ and bottom field most of NS can stay at the Ejector stage, and the number of accretors and Propellers would not be increased. We show this analytical estimates graphically in figure 4, where the Ejector time, $T_E$, is plotted vs. bottom magnetic momentum for constant velocity and ISM density ($n = 1$ cm$^{-3}$, $v = 10$ km/s), different $t_d$ and two values of the initial magnetic momentum, $10^{30}$ and $10^{31}$ G cm$^3$ (see Popov & Prokhorov 1999).

Summarizing, we can conclude that, although both the initial distribution and the subsequent evolution of the magnetic field strongly influences the NS census and should be accounted for, the lower bound on the average kick derived from ROSAT surveys is not very sensitive to $B$, at least for not too extreme values of $t_d$ and $\mu(0)$, within this model.

## 4. Conclusions

In this paper we have investigated how the present distribution of neutron stars in the different stages (Ejector, Propeller, Accretor and Georotator) depends on the star mean velocity at birth. On the basis of a total of $\sim 10^9$ NSs, the fraction of Accretors was used to estimate the number of sources within 140 pc from the Sun which should have been detected by ROSAT. Most recent analysis of ROSAT data indicate that no more than $\sim 10$ non–optically identified sources can be accreting ONSs. This implies that the average velocity of the NS population at birth has to exceed $\sim 200\ \mathrm{km\,s^{-1}}$, a figure which is consistent with those derived from radio pulsars statistics. We have found that this lower limit on the mean kick velocity is substantially the same either for a constant or a decaying $B$–field, unless the decay timescale is shorter than $\sim 10^9$ yr. Since observable accretion–powered ONSs are slow objects, our results exclude also the possibility that the present velocity distribution of NSs is richer in low–velocity objects with respect to a Maxwellian. The paucity of accreting ONSs seem therefore to lend further support in favor of neutron stars as very fast objects.

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## References

Bochkarev, N.G. 1992, Basics of the ISM Physics (Moscow University Press)

Colpi, M., Turolla, R., Zane, S., & Treves, A. 1998, ApJ 501, 252

Cordes, J.M. 1998, in Neutron Stars and Pulsars, eds. Shibazaki, N. et al. (Tokyo: Universal Academy Press)

Haberl, F., Motch, C., & Pietsch, W. 1998, Astron. Nachr. 319, 97

Konenkov, D.Yu., & Popov, S.B. 1997, PAZh, 23, 569

Lipunov, V.M. 1992, Astrophysics of Neutron Stars (Springer & Verlag)

Lipunov, V.M., & Popov, S.B. 1995, AZh, 71, 711

Livio, M., Xu, C., & Frank, J. 1998 ApJ, 492, 298

Madau, P., & Blaes, O. 1994, ApJ, 423, 748

Miyamoto, M., & Nagai, R. 1975, Pub. Astr. Soc. Japan, 27, 533

Neühauser, R., & Trümper, J.E. 1999, A&A, 343, 151

Popov, S.B. 1994, Astron. Circ., N1556, 1

Popov, S.B., & Prokhorov, M.E. 1998, A&A, 331, 535

Popov, S.B., & Prokhorov, M.E. 1999, (in press)

Shakura, N.I. 1975, PAZh, 1, 23

Treves, A., Colpi, M., & Lipunov, V.M. 1993, A&A, 269, 319

Walter, F.M., Wolk, S.J., & Neühauser, R. 1996, Nature, 379, 233

Zane, S., Turolla, R., Zampieri, L., Colpi, M., & Treves, A. 1995, ApJ, 451, 739