Trans-sonic propeller stage

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Abstract

We follow the approach used by Davies and Pringle [1] and discuss the trans-sonic substage of the propeller regime. This substage is intermediate between the supersonic and subsonic propeller substages. At the trans-sonic regime an envelope around a magnetosphere of a neutron star passes through a kind of a reorganization process. The envelope in this regime consists of two parts. In the bottom one turbulent motions are subsonic. Then at some distance, $r_{\rm s}$, the turbulent velocity becomes equal to the sound velocity. During this substage the boundary $r_{\rm s}$ propagates outwards till it reaches the outer boundary, and so the subsonic regime starts.

We found that the trans-sonic substage is unstable, so the transition between supersonic and subsonic substages goes on at the dynamical time scale. For realistic parameters this time is in the range from weeks to years.

Keywords: compact objects, isolated neutron stars, evolution

1 Introduction

Observational appearances of neutron stars (NSs) are mainly determined by their interactions with the surrounding plasma. The main stages are: *Ejector*, *Propeller* and *Accretor* (see [2]). Normally a NS is born at the stage of ejection, then as the spin period increases the NS passes the propeller and accretor stages. For NSs with large spatial velocities another stage (*Georotator*) can appear.

In a simplified model it is possible to define transitions between stages by comparing external and internal pressure. The external one can be approximated as the ram pressure of a flow of the interstellar medium or as the pressure of matter falling down onto the NS in its gravitational field. The internal one inside r_ℓ (the light cylinder) can be estimated as a pressure of the magnetic field of a NS.

2 Propeller stage with an intermediate regime

In [3, 1] the authors distinguish three substages of the propeller regime. (1) *Very rapid rotator* ($c_{\rm s}(r_{\rm in}) \simeq r_{\rm in}\Omega \gg v_{\it ff}$). (2) *Supersonic propeller* ($r_{\rm in}\Omega \gg c_{\rm s}(r_{\rm in})$). (3)

Subsonic propeller ($r_{\rm in}\Omega \ll c_{\rm s}(r_{\rm in})$ and $v_{\rm t}(r) < c_{\rm s}(r)$ in $r_{\rm in} < r < r_{\rm out} = r_{\rm G}$). Here $r_{\rm in}$ and $r_{\rm out}$ – are internal and external radii of the envelope, $c_{\rm s}$ – a sound velocity, $v_{\rm t}$ – a turbulent velocity, $v_{\it ff}$ – a free-fall velocity, $r_{\rm G} = 2GM/v^2$ – gravitational capture radius.

We will not discuss the stage of the very rapid rotator here. In this note we focus on the super- and subsonic substages and on a transition between them. At the supersonic stage (a classical propeller) accretion is not possible due to a centrifugal barrier. At the subsonic stage the magnetospheric (Alfven) radius, $r_{\rm M}$, is smaller than the corotation radius, $r_{\rm c}$, but the accretion does not start because temperature in the envelope is too high. Why does an intermediate (transitional) regime between the supersonic and subsonic substages should exist?

In general supersonic and subsonic regimes do cover all possible values of the rotational velocity, $\Omega = 2\pi/P$. The supersonic propeller formally operates till $r_{\rm in}^{super}\Omega \geq c_{\rm s}(r_{\rm in}^{super})$. Here $r_{\rm in}^{super} = r_{\rm G}^{2/9} r_{\rm M}^{7/9}$ [1]. The subsonic regime is on when

$$r_{\rm in}^{sub}\Omega \leq c_{\rm s}(r_{\rm in}^{sub}) \text{ and } v_{\rm t}(r) \leq c_{\rm s}(r) \qquad \text{for} \quad r_{\rm in}^{sub} \leq r \leq r_{\rm out} = r_{\rm G}\,, \tag{1}$$

here $r_{\rm in}^{sub} = r_{\rm M}$ [1]. As for the subsonic stage $v_{\rm t}(r_{\rm in}^{sub}) \simeq r_{\rm in}^{sub}\Omega$ and $v_{\rm t}/c_{\rm s} \propto r^{1/3}$, then this regime is valid for $r_{\rm in}^{sub}\Omega \leq c_{\rm s}(r_{\rm in}^{sub}) \left(r_{\rm in}^{sub}/r_{\rm G}\right)^{1/3}$.

It is easy to check, that the end of the supersonic substage and the beginning of the subsonic one both correspond to:

$$\Omega_0 = \sqrt{2GM} r_{\rm M}^{-7/6} r_{\rm G}^{-1/3}. \tag{2}$$

However, the structure of the envelope on two substages is different, and an intermediate regime during which the structure of the envelope is reorganized in inevitable. We call this intermediate stage *trans-sonic propeller.*

3 Trans-sonic propeller

We assume that for all substages we can write $c_{\rm s}(r) \simeq v_{\it ff}$ [1]. At the trans-sonic substage the envelope is divided into two parts with a boundary at $r_{\rm s}$. Processes in the lower part of the atmosphere are similar to the ones on the subsonic stage: $v_{\rm t}(r_{\rm in}) \simeq r_{\rm in}\Omega < c_{\rm s}$. In the outer part of the envelope physical conditions are similar to the ones on the supersonic substage.

If the envelope is adiabatic, then for its bottom part the politropic index is equal to $n = 3/2$ and $\rho(r) \propto r^{-3/2}$, $p(r) \propto r^{-5/2}$. Following [1] we assume that the rotational energy of the NS is dissipated at the magnetospheric boundary, and that this energy is transported outwards by a turbulence. For such assumptions we have: $v_{\rm t}(r) \propto r^{-1/6}$ and $\mathcal{M}_{\rm t}(r) \equiv v_{\rm t}(r)/c_{\rm s}(r) \propto r^{1/3}$.

Till $\mathcal{M}_{\rm t} < 1$, i.e. while to $r < r_{\rm s}$ ($r_{\rm in} < r_{\rm s} < r_{\rm G}$) the structure of the envelope is not changed. For large radii turbulence becomes supersonic. Small-scale shock waves are formed, and they quickly dissipate part of the energy, so that the turbulent velocity decreases down to the sound velocity. In the range from $r_{\rm s}$ up to $r_{\rm G}$ the envelope structure is different from the bottom part: $\mathcal{M}_{\rm t}(r) \simeq 1$, and $\rho(r) \propto r^{-1/2}$, $p(r) \propto r^{-3/2}$.

To determine parameters of the whole atmosphere it is necessary to calculate the position of the boundary between two parts of the envelope, $r_{\rm s}$, and position of the inner boundary of the bottom part, $r_{\rm in}$ (during the transition it decreases from $r_{\rm G}^{2/9} r_{\rm M}^{7/9}$ to $r_{\rm M}$). To do it is necessary to solve the following system of equations:

$$\left\{ \begin{array}{ll} \dfrac{\mu^2}{8\pi} \dfrac{1}{r_{\rm in}^6} & = \dfrac{1}{2} \dfrac{\dot{M} v_\infty}{4\pi r_{\rm G}^2} \left(\dfrac{r_{\rm G}}{r_{\rm s}}\right)^{3/2} \left(\dfrac{r_{\rm s}}{r_{\rm in}}\right)^{5/2} \\ \sqrt{\dfrac{2GM}{r_{\rm s}}} & = \Omega \, r_{\rm in}^{7/6} r_{\rm s}^{-1/6} \end{array} \right. \tag{3}$$

However, the system is degenerate, and each equation can be reduced to: $r_{\rm s} \propto r_{\rm in}^{-7/2}$. If the following equation is fulfilled:

$$\frac{\mu^2}{8\pi} = \left(\frac{1}{2} \frac{\dot{M} v_\infty}{4\pi r_{\rm G}^{1/2}} \right) \frac{(2GM)^{3/2}}{\Omega^3} \tag{4}$$

then the system is compatible, i.e. for $\forall \, r_{in}$ in $r_{\rm M} < r_{\rm in} < r_{\rm M}^{7/9} r_{\rm G}^{2/9}$ $\exists \, r_{\rm s}$, that is a solution of (3) (where $r_{\rm M} = (\mu^2/\dot{M}\sqrt{2GM})^{2/7}$ is the Alfven radius).

The compatibility condition is fulfilled at the end of the supersonic substage ($\Omega = \Omega_0$). Later (during the transition) at any given moment (for any $\Omega < \Omega_0$) left-hand side of eq.(4) is smaller than the right-hand one. It means that the magnetospheric pressure and the envelope pressure have the same dependences on $r_{\rm in}$, but the latter one is always larger (first equation of (3)).

An energy release during the transition stage is negligible. Estimates for realistic isolated NS parameters give a value $\Delta E \lesssim 10^{30}$ erg.

4 Conclusions

1. The intermediate trans-sonic propeller substage in unstable.
2. The duration of the transition can be roughly estimated as $\sim r_{\rm G}/v_{\it ff}$ (from weeks to years for realistic isolated NSs).
3. The spin frequency is nearly unchanged during this transition.
4. The energy release during the transition is small.

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