Radial Velocity (Bozza et al.)

$$P^2 = \frac{4\pi^2 a^3}{G(M_s + M_p)}$$

$$M_p \cdot a_p = M_s \cdot a_s$$

$$V[m/s] = 28{,}4 \left(\frac{P}{1yr}\right)^{-1/3} \left(\frac{M_p \cdot \sin i}{M_{Jup}}\right) \left(\frac{M_s}{M_\odot}\right)^{-2/3}$$

Jupiter ($a_p = 5.2$ au) => 11.2 m/s   P = 12 yrs

Earth => 0,1 m/s

Mass function $M_1 \gg M_2$ (M_1 - star)

$$\frac{G}{4\pi^2}(M_1 + M_2)P^2 = (a_1 + a_2)^3$$

$$M_1 a_1 = M_2 a_2$$

$$\langle \sin i \rangle = \frac{\pi}{4} = 0{,}79$$

$$\langle \sin^3 i \rangle = \frac{3\pi}{16} = 0{,}59$$

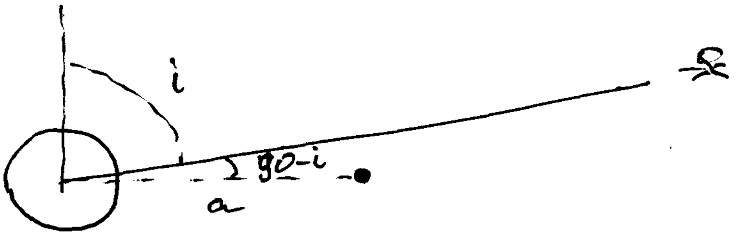
$$f(m) = \frac{M_2^3 \sin^3 i}{(M_1 + M_2)^2} = \frac{K_1^3 P (1-e^2)^{3/2}}{2\pi G} \approx \frac{M_2^3 \sin^3 i}{M_1^2}$$

$$\langle \sin i \rangle = \frac{\int_0^\pi p(i) \sin i \, di}{\int_0^\pi p(i)\, di} = \frac{\pi}{4} = 0{,}79$$

$$p(i < \theta) = \frac{2\int_0^\theta p(i)\, di}{\int_0^\pi p(i)\, di} = 1 - \cos\theta$$

$$p(i) = 2\pi \sin i \, di$$

Planetary transits (Bozza et al.)



$$b = \frac{\cancel{R_*}\, a \cdot \cos i}{R_*}$$

$a \cdot \cos i < R_* + R_p \approx R_*$ – transit condition

$$\frac{d\Omega}{4\pi} = \frac{2\pi \sin i \, di}{4\pi} = \frac{d(\cos i)}{2}$$

$$Pr\left(\cos i < \frac{R_* + R_p}{a}\right) = \frac{1}{2}\int_{-\frac{R_*+R_p}{a}}^{\frac{R_*+R_p}{a}} d(\cos i) = \frac{R_*+R_p}{a}$$

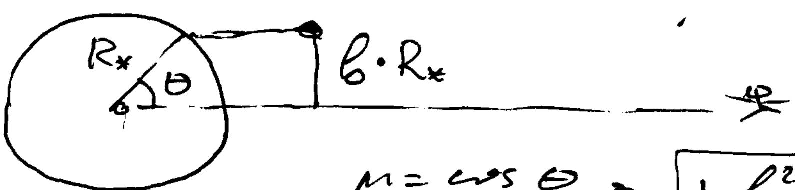
$R_* \gg R_p$

$$Pr\left(\cos i < \frac{R_*}{a}\right) \simeq \underline{\underline{0.0046}} \left(\frac{R_*}{R_\odot}\right)\left(\frac{1\,au}{a}\right)$$

$$\frac{\Delta f}{f} \approx \left(\frac{R_p}{R_*}\right)^2 = 0.0105 \left(\frac{R_p}{R_{Jup}}\right)^2 \left(\frac{R_*}{R_\odot}\right)^{-2}$$

limb darkening

$$I = I_0\left(1 - u(1-\mu)\right)$$



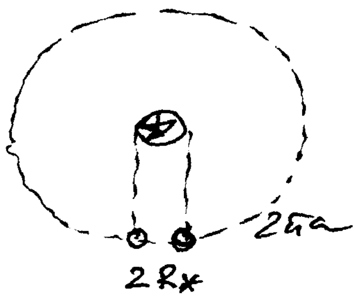
$$R_*^2 = b^2 R_*^2 + R_*^2 \cdot \cos^2\theta$$

$$\mu = \cos\theta = \sqrt{1-b^2}$$

$$\frac{\Delta f}{f} = \frac{\pi R_p^2 I_0 (1 - u + u \cdot \cos\theta)}{2\pi R_*^2 I_0 \int_0^{\pi/2} (1 - u + u\cos\theta)\sin\theta\cos\theta\, d\theta} =$$

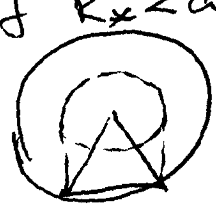
$$= \frac{3\left(1 - u + u\sqrt{1-b^2}\right)}{3-u}\left(R_p/R_*\right)^2$$

Transit duration



$$\frac{T}{P} = \frac{1}{\pi}\frac{R_*}{a}, \quad R_* \ll a \quad \Big| \Rightarrow T \sim P^{1/3}\rho_*^{-1/3}$$

If $R_* < a$



$$\frac{2\arcsin(R_*/a)}{2\pi} = \frac{1}{\pi}\arcsin\left(\frac{R_*}{a}\right)$$

$$P^2 = \frac{4\pi^2}{GM}a^3 \Rightarrow \frac{T}{P} = \frac{1}{\pi}\arcsin\left[R_*\left(\frac{4\pi^2}{GM_*P^2}\right)^{1/3}\right]$$

If $i \neq 90°$: $\quad \frac{t_T}{P} = \frac{1}{\pi}\arcsin\left[\frac{R_*}{a}\left\{\frac{[1+(R_p/R_*)]^2 - [\frac{a}{R_*}\cdot\cos i]^2}{1-\cos^2 i}\right\}^{1/2}\right]$

for $\cos i \ll 1$ $\quad \frac{t_T}{P} \simeq \frac{R_*}{\pi a}\sqrt{\left(1+\frac{R_p}{R_*}\right)^2 - b^2}$

$$\frac{T}{P} = \frac{1}{\pi}R_*\left(\frac{4\pi^2}{GM_*P^2}\right)^{1/3} \Rightarrow T = P^{1/3}\frac{1}{\pi}\left(\frac{3\pi}{G}\right)^{1/3}\rho_*^{-1/3}$$

$$T \cong 3^h\left(\frac{P}{4^d}\right)^{1/3}\left(\frac{\rho_*}{\rho_\odot}\right)^{-1/3}$$

$$\frac{dv_r}{dt} = \frac{2\pi K}{P} = \frac{GM_p}{a^2} = g_p\frac{R_p^2}{a^2} = g_p\frac{R_p^2}{R_*^2}\frac{R_*^2}{a^2}$$

velocity

$$g_p = \frac{2\pi K}{P}\left(\frac{R_*}{R_p}\right)^2\left(\frac{a}{R_*}\right)^2$$

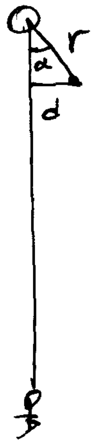
transit depth

transit duration

$$\rho_p = \frac{3g_p}{4\pi GR_p} = \frac{3g_p}{4\pi GR_*}\left(\frac{R_*}{R_p}\right)$$

$$R_* = \theta d = \theta/\hat{\pi}$$ параллакс

$$\rho_p = \frac{3g_p\hat{\pi}}{4\pi G\theta}(R_*/R_p)$$

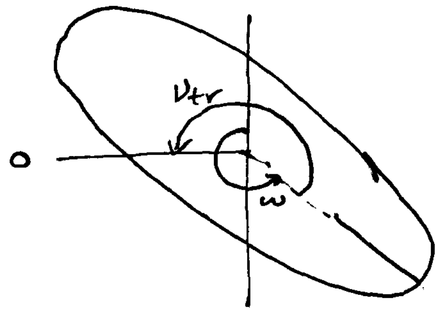


$d = r \cdot \sin\alpha$

$z \equiv d / R_*$

$p = R_p / R_*$

e - eccentricity
ω - argument of periastron
E - eccentric anomaly
ν - true anomaly



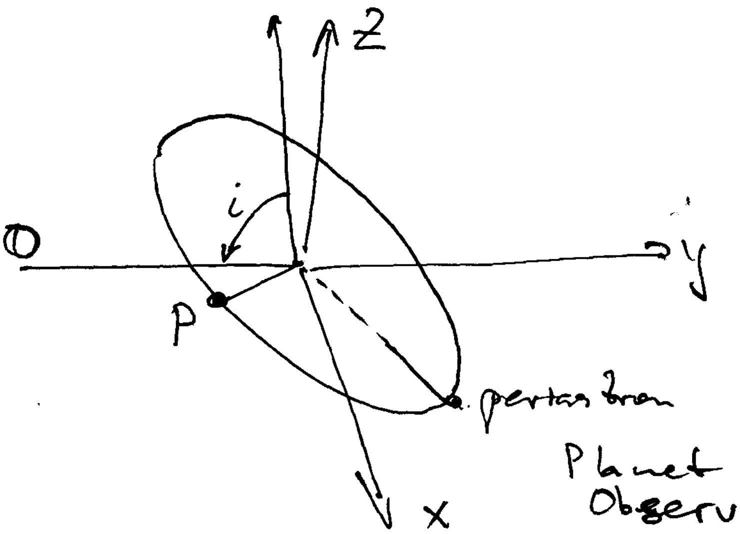
$$\nu_{tr} = \frac{\pi}{2} - \omega$$

$$E = 2\arctan\left[\sqrt{\frac{1-e}{1+e}}\tan\frac{\nu}{2}\right]$$

$$M = E - e\sin E$$

$$\nu = 2\arctan\left[\sqrt{\frac{1+e}{1-e}}\tan\frac{E}{2}\right]$$

$$r = a(1 - e\cdot\cos E)$$



$$x_p = r\cdot\sin(\nu + \omega - \pi/2)$$

$$z_p = -r\cdot\cos\left(\nu + \omega - \frac{\pi}{2}\right)\cdot\cos i$$

$$y_p = r\cdot\cos\left(\nu + \omega - \frac{\pi}{2}\right)\cdot\sin i$$

$$r\cdot\sin\alpha = \sqrt{x_p^2 + z_p^2}$$

$$z = r\cdot\sin\alpha / R_*$$

Planet velocity towards the Observer relative to the star

$$V_p = \frac{dy_p}{d\nu}\frac{d\nu}{dM}\frac{dM}{dt} = \frac{2\pi a}{P}\frac{\sin i}{\sqrt{1-e^2}}\left(e\cos\omega + \cos(\nu+\omega)\right)$$

the component of the star's reflex motion away from the observer → $V_r = K\cdot(e\cdot\cos\omega + \cos(\nu+\omega)) + \gamma$

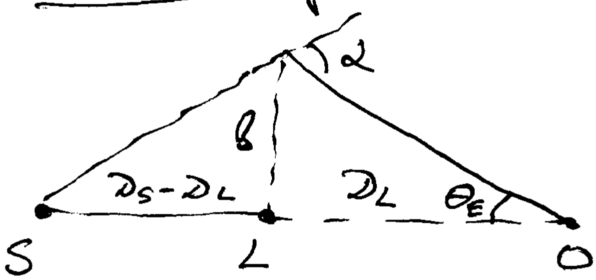
$$K = \frac{2\pi a}{P}\,\frac{m_p}{M_* + m_p}\,\frac{\sin i}{\sqrt{1-e^2}}$$

the transverse velocity of the planet at mid-transit

$$V_t = \frac{2\pi a}{P}\,\frac{e\sin\omega + 1}{\sqrt{1-e^2}}, \quad b = \frac{z_p}{R_*} = -\frac{a}{R_*}\,\frac{1-e^2}{1+e\sin\omega}$$

$$\frac{t_{tr}}{P} \simeq \frac{R_*}{a}\,\frac{\sqrt{(1+R_p/R_*)^2 - b^2}}{\pi}\,\frac{1+e\cdot\sin\omega}{1-e^2}$$

Microlensing



$$\alpha = \frac{4GM}{bc^2}$$

$$\theta_E = \sqrt{k M \pi_{rel}}$$

$$k = \frac{4G}{c^2 AU} \simeq 8.14 \frac{mas}{M_\odot}$$

$$\pi_{rel} = AU\left(D_L^{-1} - D_S^{-1}\right)$$

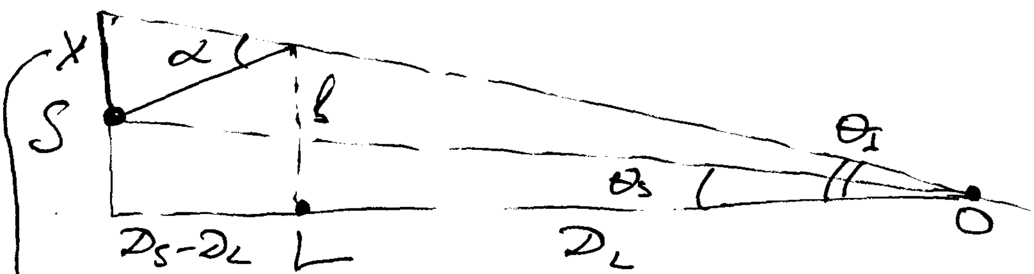
$$b = D_L \theta_E \qquad \alpha = b/D_L + b/(D_S - D_L)$$

$$\tau = \int dD_L \; \pi (D_L\theta_E)^2 n(D_L) \sim \frac{4\pi G M n}{c^2} D^2 = \frac{4\pi G\rho}{c^2} D^2 \sim \frac{G M_{tot}}{D_L^2} \sim \sim v^2/c^2$$

$$\left(\frac{v_{rot}}{c}\right)^2 \sim 10^{-6}$$ v_{rot} – rot. vel. in G.

$$D_L \theta_E / v_{rot} \sim 30^d$$

$$\Gamma_* \sim 10^{-5}\, yr^{-1} \qquad M_{pl}/M_* \sim 10^{-4} \qquad \theta_E \sim M^{1/2} \Rightarrow \underline{\Gamma_{pl} \sim 10^{-7}\, yr^{-1}}$$



$(x=)$: $(\theta_I - \theta_S) D_S = \alpha (D_S - D_L)$ – for small angles.

$$\Downarrow$$

$$\theta_I(\theta_I - \theta_S) = \frac{4GM\pi_{rel}}{c^2 AU} \equiv \theta_E^2$$

$$u \equiv \theta_S/\theta_E \qquad u_\pm \equiv \frac{\theta_{I\pm}}{\theta_E} \qquad u_\pm^2 + u u_\pm - 1 = 0$$

$$u_\pm = \frac{u \pm \sqrt{u^2+4}}{2}$$

$$A_\pm = \pm \frac{u_\pm}{u}\frac{\partial u_\pm}{\partial u} = \frac{A \pm 1}{2}$$

$$A = \frac{u^2+2}{u\sqrt{u^2+4}} = \left(1 - Q^{-2}\right)^{-1/2}, \quad Q \equiv 1 + \frac{u^2}{2}$$

$$A_+ + A_- = A \qquad A_+ - A_- = 1 \qquad A \to 1/u \text{ for } u \ll 1$$

$$A(1) = 3/\sqrt{5} \simeq 1.34$$

Direct imaging

Sol. syst. at 10 pc.
Jupiter 0,5'' & 10^{-9} of the flux. (1505.06869)

Now it is possible to see self-luminous planets (10^{-5} in flux) at $\gtrsim 1$ arcsec

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Astrometry

Determines $m_2^3 / (m_1+m_2)^2$ (1505.06869)

Direct imaging (Bozza et al.)

$$\theta = \frac{a}{d}(1+e)$$

Timing (Perryman)

$$\tau_p = \frac{1}{c}\,\frac{a \cdot \sin i \; M_p}{M_*} \simeq 1{,}2\,\text{msec}\left(\frac{M_p}{M_\oplus}\right)\left(\frac{P}{1\,\text{yr}}\right)^{2/3}$$

for $M_* = 1.35\,M_\odot$

If $P_{orb} \gg \Delta t_{obs} \Rightarrow \dot{P} = \frac{P}{c}\,\frac{G M_p \sin i}{a^2}$

Timing (1708.00896)

$$\tau(t) = -\frac{1}{c}\int_0^t v_{rad}(t')\,dt' \qquad v_{rad}(t) = -c\,\frac{d\tau}{dt}$$

TTV (1706.09849) +TDV

$$\vec{F}_1 = -G\mu_1 M r_1^{-2}\hat{\vec{r}}_1 + \vec{F}_{12}$$

$$\mu = m_1 \cdot m_2 / M \qquad M = M_1 + M_2$$

$$\vec{F}_{12} = \mu_1 \ddot{r}_1 = G\mu_1 m_2 |r_2 - r_1|^{-3}(\vec{r}_2 - \vec{r}_1) - G\mu_1 m_2 r_2^{-2}\hat{\vec{r}}_2$$

O-C:

$$\delta t_1 = P_1 \frac{m_2}{m_0} \cdot f_{12}(\alpha_{12}, \theta_{12})$$

$$\delta t_2 = P_2 \frac{m_1}{m_0} f_{21}(\alpha_{21}, \theta_{21})$$

m_0 - star $\qquad \alpha_{ij} = \min\left(\frac{a_i}{a_j}, \frac{a_j}{a_i}\right)$

$$\delta t_i = P_i \sum_{j \neq i} \frac{m_j}{m_0} f_{ij}(\alpha_{ij}, \theta_{ij})$$

$$\delta P_2 = -\delta P_1 (m_1/m_2)(P_2/P_1)^{5/3}$$

$$\delta t_2 = -\delta t_1 (m_1/m_2)(P_2/P_1)^{2/3}$$

Main TTV variations come from resonances.

TDV

1. oblateness of the star
2. eccentricity variations due to a resonant interaction
3. secular precession of the orbital plane.