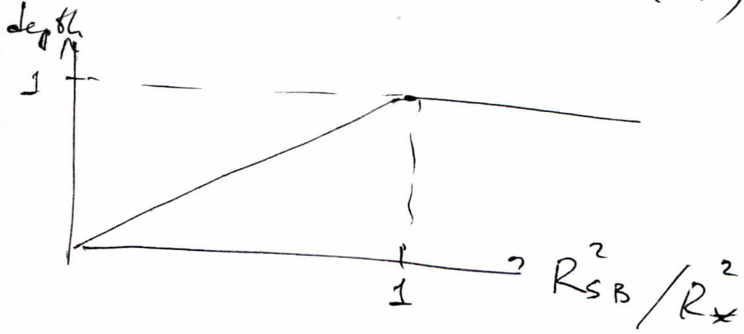
① Transit in WD-planet system.

Planet > WD (!)



$$\text{probability} = \left(\frac{R_* + R_{SB}}{a}\right)\left(\frac{1+e\sin\omega}{1-e^2}\right) \sim 10^{-2} a_6^{-1} \left(\frac{R_* + R_{SB}}{10^4}\right)$$

$$\text{duration} = 2\left(\frac{\sqrt{1-e^2}}{1+e\sin\omega}\right)\sqrt{\frac{a^3}{G(M_*+M_{SB})}} \arcsin\left[\frac{R_*}{a\sin i} \times \left\{\left(1+\frac{R_{SB}}{R_*}\right)^2 - \frac{a^2\cos^2 i\,(1-e^2)^2}{R_*^2(1+e\sin\omega)^2}\right\}^{1/2}\right]$$

$i = 90°$ for edge-on

Due to small R_{WD} it is easier to find small bodies by transit observ.

Most distant planet 2500 au

WD 0806-661b $7 M_{Jup}$

Direct imaging

Specific angular momentum

$J^2 = G M_{*0} a_0 \eta \;, \; \eta = 1-e^2 < 1$

$$\boxed{\frac{d^2\xi}{dt^2} = \frac{\eta}{\xi^3} - \frac{m(t)}{\xi^2}} \;, \quad \xi = r/a_0 \;; \; t = \frac{t}{\Omega^{-1}}$$

$$\Omega = \left(\frac{a_0^3}{G M_{*0}}\right)^{-1/2}$$

$M_p \ll M_*$

$$m(t) = M(t)/M_{*0}$$

$1-e \le \xi_0 \le 1+e$

Initial energy, $\mathcal{E}_0 = -1/2$. It is growing

$$\dot{\xi}_0 = \frac{2\xi_0 - \eta - \xi_0^2}{\xi_0^2} = \frac{[(1+e)-\xi_0][\xi_0-(1-e)]}{\xi_0^2}$$

Sign of $\dot{\xi}_0$ depends on the initial position.

Change of variables

$$u \equiv 1/m \Rightarrow \boxed{\dot{u}^2 \frac{d^2\xi}{du^2} + \ddot{u}\frac{d\xi}{du} = \frac{\eta}{\xi^3} - \frac{1}{u\xi^2}}$$

$$\left(\text{Rem: } \frac{d^2\xi}{dt^2} = \frac{d}{dt}\left[\frac{d\xi}{dt}\right] = \frac{du}{dt}\frac{d}{du}\left[\underbrace{\frac{du}{dt}}_{\dot{u}}\frac{d\xi}{du}\right] = \dot{u}\frac{d}{du}\left[\dot{u}\frac{d\xi}{du}\right] = \right.$$

$$\left. = \dot{u}^2\frac{d^2\xi}{du^2} + \underbrace{\dot{u}\frac{d\xi}{du}\frac{d\dot{u}}{du}}_{\dot{u}\frac{d\xi}{du}\frac{d\dot{u}}{dt}\underbrace{\frac{dt}{du}}_{1/\dot{u}}} = \ddot{u}\frac{d\xi}{du}\right)$$

$$f \equiv \frac{\xi}{u} = \xi \cdot m \Rightarrow \boxed{\dot{u}^2 u^2 f^3 \left[(u^2 f'' + 2uf') + \frac{u\ddot{u}}{\dot{u}^2}(uf' + f)\right] = \eta - f}$$

$\lambda^2 \equiv \dot{u}^2 u^2 f^3 \qquad \lambda \sim \dfrac{\text{orbital time scale}}{\text{mass-loss time scale}}$

(λ equiv Ψ in Veras (2016) p. 20)

$\beta = \dfrac{u\ddot{u}}{\dot{u}^2}$ - mass-loss rate parameter

$\beta = \frac{u\ddot{u}}{\dot{u}^2}$ - defines the mass loss

orbital energy $\mathcal{E} = \frac{1}{2}\dot{u}^2(uf' + f)^2 + \frac{\eta}{2u^2f^2} - \frac{1}{u^2 f}$

initially $\mathcal{E} = -\frac{1}{2}$ and then grows.

When $\mathcal{E} = 0$ - unbound.

$\frac{d\mathcal{E}}{du} = \frac{1}{u^3 f}$ $\qquad a \sim |\mathcal{E}|^{-1}$ $\quad$ a - grows.

$\dot{u} = \gamma u^\beta$, $\beta = \text{const.}$

γ defines mass-loss on the begining
($u = 1$ for $t = 0$)

$\dot{m} = -\gamma\, m^{(2-\beta)}$ $\qquad \gamma = \frac{\text{initial orbital time scale}}{\text{initial mass loss time scale}}$

$$\gamma = \frac{1}{\tau}\left(\frac{a_0^3}{G M_{*0}}\right)^{1/2} = 1,6 \cdot 10^{-7}\left(\frac{\tau}{1\,\mathrm{Myr}}\right)^{-1}\left(\frac{a_0}{1\,\mathrm{AU}}\right)^{3/2}\left(\frac{M_{*0}}{M_\odot}\right)^{-1/2}$$

$\tau = (M_* / \dot{M}_*)_0$

γ is always small.

$$\dot{M} = -\dot{M}_c \left(\frac{L_*}{L_\odot}\right)\left(\frac{R_*}{R_\odot}\right)\left(\frac{M_*}{M_\odot}\right)^{-1}$$

For MS: $L \sim m^p$ $\quad R \sim m^q$ $\Rightarrow$ $\dot{M} \sim M^{p+q-1}$ $\quad p+q-1 \simeq 2,5 \div 3$

$-1 \lesssim \beta \lesssim -\frac{1}{2}$

For AGB: $R \sim m^{-1/3}$ $\quad L$ depends on the core (i.e. $p \sim 0$)

$\dot{m} \sim m^{-4/3}$ $\qquad \beta = 10/3$

In general, if $\dot{M} \sim M^{\alpha_m}$ $\qquad \beta = 2 - \alpha_m$

a) constant $\dot{m}$

$\beta = 2$ $\qquad m(t) = 1-\gamma t$

$$u(t) = \frac{1}{1-\gamma t}$$

For $\beta > 2$ $\quad \dot{m}$ grows with time

For $\beta < 2$ $\quad \dot{m}$ decreases with time

$$\beta = 2 - \frac{m\ddot{m}}{\dot{m}^2}$$

b) Stellar mass decays exponentially $\beta = 1$

$m(t) = e^{-\gamma t}$ $\qquad u(t) = e^{\gamma t}$

For $\beta > 1$ $\quad m \to 0$ at finite time

c) $\beta = 0$

$m(t) = \frac{1}{1+\gamma t}$ $\qquad u(t) = 1+\gamma t$

d) $\beta = -1$

$m(t) = (1+2\gamma t)^{-1/2}$ $\qquad u(t) = (1+2\gamma t)^{1/2}$

For $\beta > -1$ planet unbound at $m \to 0$

$$m(t) = \frac{1}{u(t)} = [1-(\beta-1)\gamma t]^{\frac{1}{\beta-1}} \qquad \beta \neq 1$$

For $\beta = \text{const}$

$$\lambda^2 [u^2 f'' + (2+\beta) u f' + \beta f] = \eta - f$$

$$\lambda^2 = \gamma^2 u^{2\beta+2} \cdot f^3 \qquad [\text{as } \dot{u} = \gamma u^\beta] \text{ and } \lambda^2 = \dot{u}^2 u^2 f^3$$

$\lambda \ll 1$ adiabatic regime ($\Psi \ll 1$ in Veras (2016) p. 20).

Then $f \simeq \eta \equiv \text{const}$, f-oscillates around mean value

For $\lambda \gg 1$ $\quad u^2 f'' + (2+\beta) u f' + \beta f = 0$ (approxim.)

Solution: $\underline{f(u) = \frac{A}{u} + \frac{B}{u^\beta}}$

For $\lambda \ll 1$ and $f \simeq 1$: $\underline{\mathcal{E} = -\frac{1}{2u^2}}$

$$\dot{m} = -\gamma \cdot m^{(2-\beta)}$$

1) $\beta = -1 \qquad \dot{m} \sim m^3$

$$\gamma^2 [u^2 f'' + u f' - f] = \frac{\eta}{f^3} - \frac{1}{f^2}$$

Change $w = \log u$

$$\gamma^2 \left[\frac{d^2 f}{dw^2} - f\right] = \frac{\eta}{f^3} - \frac{1}{f^2}$$

$$\gamma^2 \left(\frac{df}{dw}\right)^2 = \gamma^2 f^2 + \frac{2}{f} - \frac{\eta}{f^2} - E, \qquad \begin{matrix} E > 0 \\ E = 1 \text{ for } e_0 = 0 \end{matrix}$$

$f(w)$ is oscillating between two values > 0 if $\eta E \le \frac{9}{8}$ and $\gamma \le (E/3)^{3/4}$

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2) $\beta = 0 \qquad \dot{m} = -\gamma \cdot m^2$

$$\gamma^2 (u^2 f')^2 = -\frac{\eta}{f^2} + \frac{2}{f} - E$$

$E = \text{const} = 1 - \gamma^2 (1-e)^2$ if $\dot{\xi} = 0$ (periastron)

in general $E = 1 - \gamma^2 f_0^2 \pm 2\gamma \left[2 f_0 - \eta - f_0^2 \right]^{1/2}$

3) rapid mass loss

power-law regime $\qquad f(u) = \frac{A}{u} + \frac{B}{u^\beta}$

for $\gamma \gg 1 \qquad f'(1) \simeq -\xi_1 \qquad 1 - e \leq \xi_1 \leq 1 + e$

$t = 0, u = 1$

$\Downarrow$

$B = 0 \quad A = \xi_1$

$f(u) = \xi_1 / u$

$$\mathcal{E} = -\frac{1}{2} + \frac{1}{\xi_1}\left(1 - \frac{1}{u}\right) \qquad \left[\text{as } \frac{d\mathcal{E}}{du} = \frac{1}{u^3 f}\right]$$

for $\mathcal{E} = 0 \quad m_f = \frac{1}{u_f} = \underline{1 - \frac{\xi_1}{2}} \qquad \left(\text{for } e = 0 \quad m_f = \frac{1}{2}\right)$

$m(t) = m_\infty + (1 - m_\infty) H(-t)$

Heaviside step function

$$\dot{\xi}^2 = \frac{2 m_\infty}{\xi} - \frac{\eta}{\xi^2} - 1 + \frac{2(1 - m_\infty)}{\xi_0}$$

$$2\mathcal{E}_f = -1 + \frac{2(1 - m_\infty)}{\xi_0} < 0 \qquad a_f = \frac{m_\infty}{2|\mathcal{E}_f|} \qquad e_f = \sqrt{1 - 2|\mathcal{E}_f| \eta / m_\infty^2}$$

General case

$x = u^{\alpha} \qquad \alpha = \beta + 1$

$$\gamma^2\alpha^2 x^2 [x^2 f_{xx} + 2x f_x] + \gamma^2 \beta x^2 f = \frac{\eta}{f^3} - \frac{1}{f^2}$$

$$\lambda^2 = \gamma^2 x^2 f^3$$

$$\gamma^2\alpha^2 [x^2 f_x]^2 + 2\gamma^2\beta \int^x x^2 f \cdot f_x \, dx = -\frac{\eta}{f^2} + \frac{2}{f} - E$$

$$J \equiv 2\gamma^2 \beta \int^x x^2 f f_x dx$$

$$\gamma^2\alpha^2 [x^2 f_x]^2 + J = \frac{2f - \eta - Ef^2}{f^2} \qquad J = O(\lambda^2)$$

$$u^2 \mathcal{E} = \frac{1}{2}\gamma^2 x^2 (\alpha x f_x + f)^2 + \frac{\eta}{2f^2} - \frac{1}{f}$$

$$E = 1 - \gamma^2 f_0^2 \pm 2\gamma (2f_0 - \eta - f_0^2)^{1/2}$$

Often $J \to 0 \quad (\lambda^2 \to 0)$

$$2u^2\mathcal{E} = -E \pm 2\gamma x (2f - \eta - Ef^2)^{1/2} + \gamma^2 x^2 f^2 + O(J)$$

$$\mathcal{E} = -m/2a \implies (am)^{-1} = -2\mathcal{E}u^2 = E + O(\lambda)$$

1. Outer boundary of the Solar system $\boxed{8}$

$M_{*0} = 1M_{\odot}$

$\Delta t = 1\,\mathrm{Myr}$ => m_f remains AGB-like

$\beta \in [1;3]$

$$\gamma = \frac{1}{\beta-1}\left[1 - m_f^{\beta-1}\right]\frac{1}{\Delta t}\left(\frac{a_0^3}{GM_{*0}}\right)^{1/2} = 1.6 \cdot 10^{-7}\frac{1}{\beta-1}\left[1-m_f^{\beta-1}\right]\left(\frac{a_0}{1\,\mathrm{AU}}\right)^{3/2}$$

$e_0 = 0$

power-law regime

$$A_\beta \equiv (\beta-1)^{2/3}\left[1 + \frac{C_0\beta}{1+\beta}\right]^{2(1+\beta)/3}$$

for $a > a_c$ unbound

$$a_c = (\Delta t)^{2/3}(GM_{*0})^{1/3} A_\beta \left\{\frac{m_f^{1+\beta}}{\left[1-m_f^{\beta-1}\right]}\right\}^{2/3} \simeq 34000\,\mathrm{AU}\, A_\beta \{...\}^{2/3}$$

for the Sun $m_f \simeq 1/2$ =>

$a_c \backslash \beta$	2	3	4
	8500	5350	3370

2. WD planets.

$M_{\star 0} = 5M_\odot \Rightarrow M_{WD} = 1 M_\odot$ (i.e. $m_f = 1/5$)
$u_f = 5$

$\Delta t = 1$ Myr $\quad \beta =$ const

$k^2 \ll 1$ (for $a_0 \sim 1 \div 100$ AU
and $\beta \simeq 3$)

$\gamma = 10^{-4} \div 10^{-7}$

$a_f = 5\left[1 - 390{,}625\gamma^2\right]^{-1} \quad \gamma \sim 10^{-7}$

~~(for a cycle)~~ $\quad$ i.e. $a_f \cong 5a_0$

Yarkovsky etc. (Veras 2016)

$$\frac{d^2\vec{r}}{dt^2} = -\frac{G(M_\star + M_{SB})}{r^3}\vec{r} +$$

$$+ \frac{A_{SB} L_\star}{4\pi M_{SB} c r^2} \left\{ \left(1 - \frac{\vec{v}\cdot\vec{r}}{cr}\right)\frac{\vec{r}}{r} - \frac{\vec{v}}{c} \right\} \cdot \left[Q_{abs}[\dots] + Q_{ref}[\dots] + \right.$$

$$\left. + \left\{ H\left(R_{SB} - \sqrt{\frac{K_{SB}\cdot P_{SB}^{spin}}{\pi \rho_{SB}\cdot C_{SB}}}\right)\right\} w \left(Q_{abs} - Q_{ref}\right) Y \right]$$

Heaviside $R_{SB} >$ few cm ÷ few m

$$\left\langle \frac{de}{dt} \right\rangle^{Yark} = \mathcal{O}\left(\frac{1}{c}\frac{A_{SB} L_\star}{8\pi M_{SB}\, n a^3}\right) \sim \frac{0{,}08}{Myr}\left(\frac{M_\star}{1M_\odot}\right)^{-1/2}\left(\frac{\rho_{SB}}{2g/cm^3}\right)^{-1} \cdot \left(\frac{R_{SB}}{1km}\right)^{-1}\left(\frac{a}{5AU}\right)^{-3/2}\left(\frac{L_\star}{10^3 L_\odot}\right)$$

$$\left\langle \frac{de}{dt} \right\rangle^{RP+rp} = \mathcal{O}\left(\frac{1}{c^2}\frac{5 A_{SB} L_\star}{8\pi M_{SB} a^2}\right) \sim \frac{1{,}8\cdot 10^{-5}}{Myr}\left(\frac{\rho_{SB}}{2g/cm^3}\right)^{-1}\left(\frac{R_{SB}}{1km}\right)^{-1}\left(\frac{a}{5AU}\right)^{-2}\frac{L}{10^3 L_\odot}$$

Note dependence on R_{SB}! RP+rp important for dust.
(Yarkovsky is not triggered for dust).

YORP Veras et al. 1409.4412

$a = 5$ a.e.

1) $L = L_\odot$

$$f = \frac{L}{4\pi r^2} = \frac{4\cdot 10^{33}}{4\pi\cdot 25\cdot (1.5)^2\cdot 10^{26}} \simeq 10^5 \frac{erg}{cm^2/s}$$

2) $L = 10^4 L_\odot$

$f = 10^5 \frac{erg}{cm^2/s}$ as at 0,05 a.e. !

$$\frac{ds}{dt} = \frac{Y}{2\pi\rho R^2}\left(\frac{f}{a^2\sqrt{1-e^2}}\right)$$

Y - defines an asteroid asymmetry

Usually $Y \sim 0.001 \div 0.01$

$$f = \frac{L}{L_\odot} f_\odot$$

f also depends on surface properties (albedo, etc.)

$$S_{crit} = \frac{2\pi}{\sqrt{3\pi/G\rho}} = 7.48\cdot 10^{-4}\left(\frac{\rho}{2\,g\,cm^{-3}}\right)^{\frac{1}{2}} \frac{rad}{s}$$