

Planets in binaries

SERGEI POPOV

Catalogue of planets in binaries

CATALOGUE OF EXOPLANETS IN BINARY STAR SYSTEMS

Exoplanets in binary star systems

Number of planets: 132

Number of systems: 91

Exoplanets in multiple star systems

Number of planets: 34

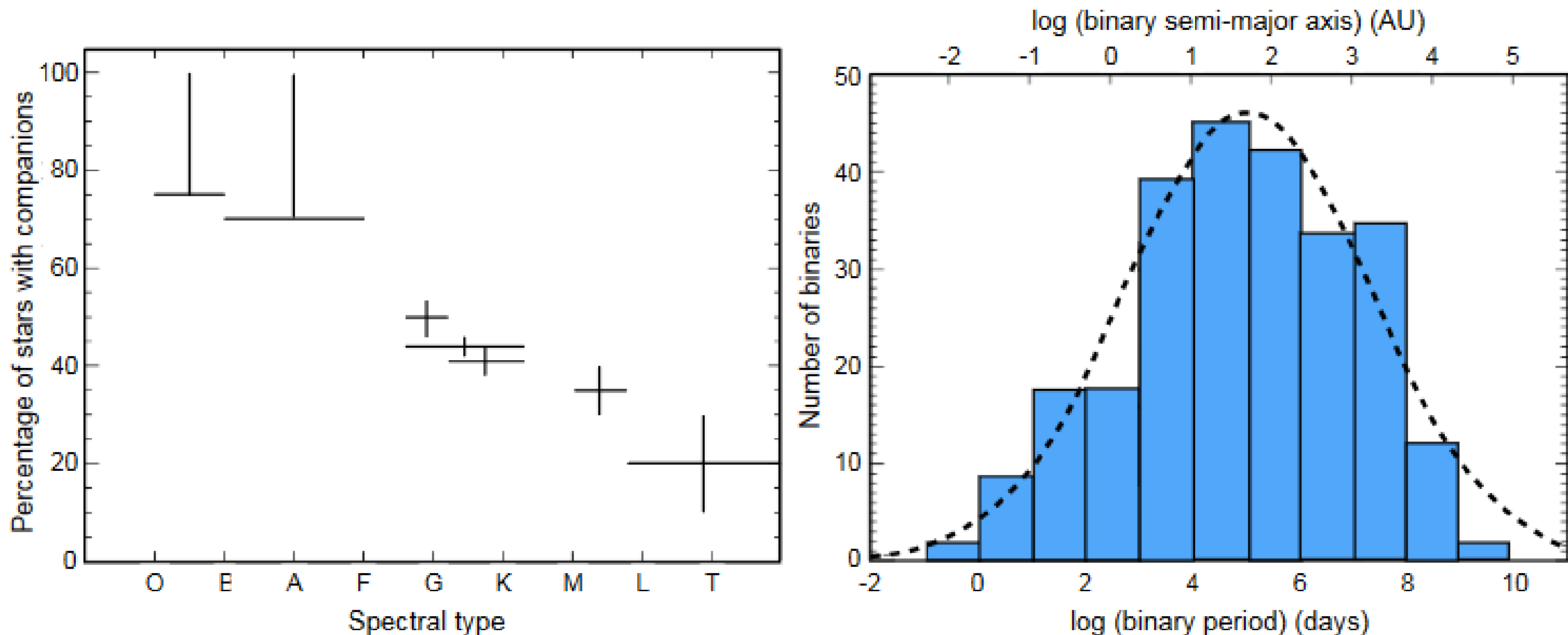
Number of systems: 24

Candidate systems

Number of planets: 30

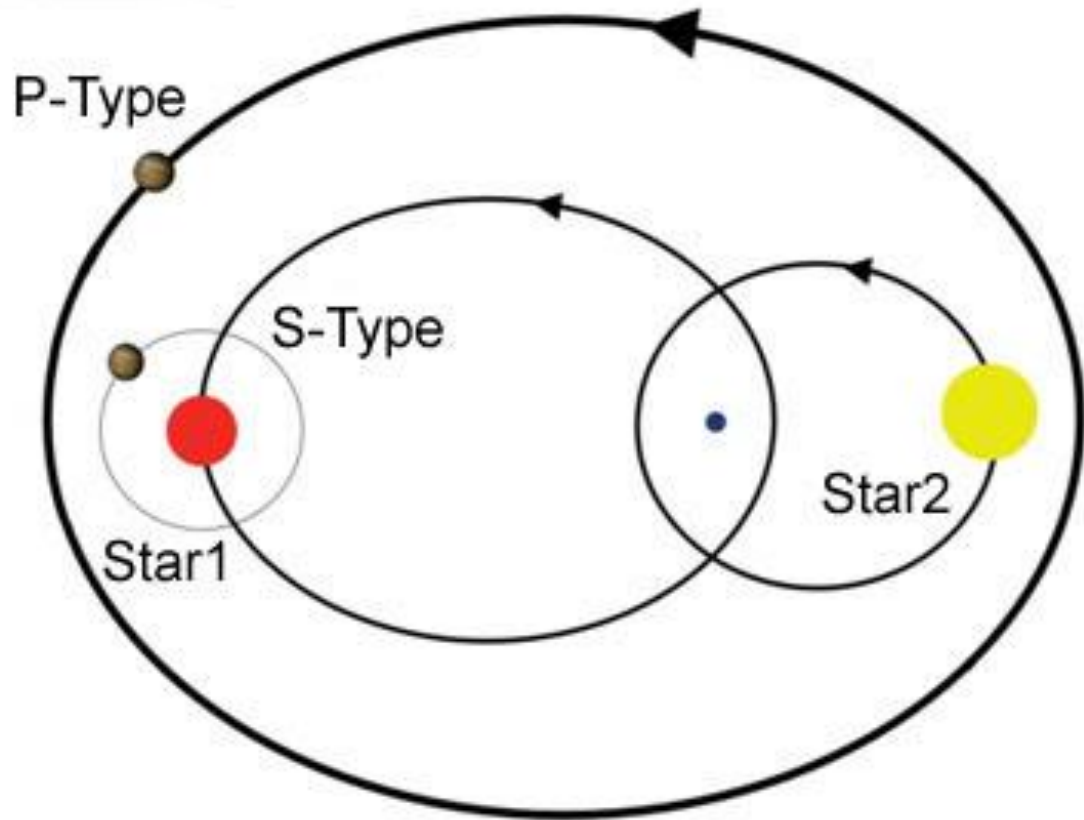
Number of systems: 23

Binary systems statistics



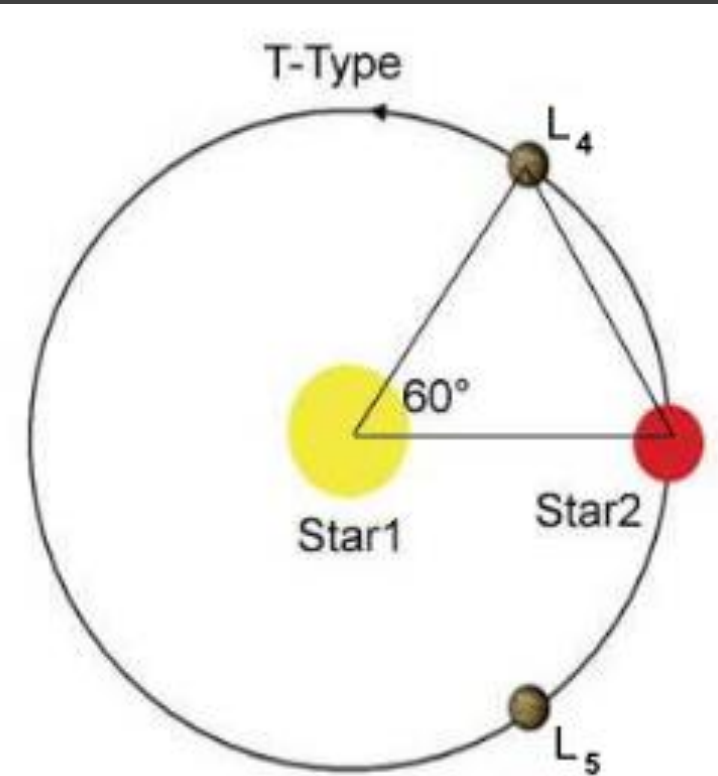
From Raghavan et al. 2010

S-type and P-type. And T-type!



The central stars strongly perturb the region around them, clearing out orbits to distances of 2–5 times the binary separation.

In addition to known S-type and P-type planetary orbits, also so-called T-type planets, similar to Trojans, can exist.



Orbit stability

1204.2014

$$R_p = \left(0.464 - 0.38 \frac{m_2}{m_1 + m_2} \right) a_*$$

S-type

Valid for the primary (more massive star)

a_* - binary semimajor axis

$$a_c = (1.60 + 4.12\mu - 5.09\mu^2)a_b$$

P-type

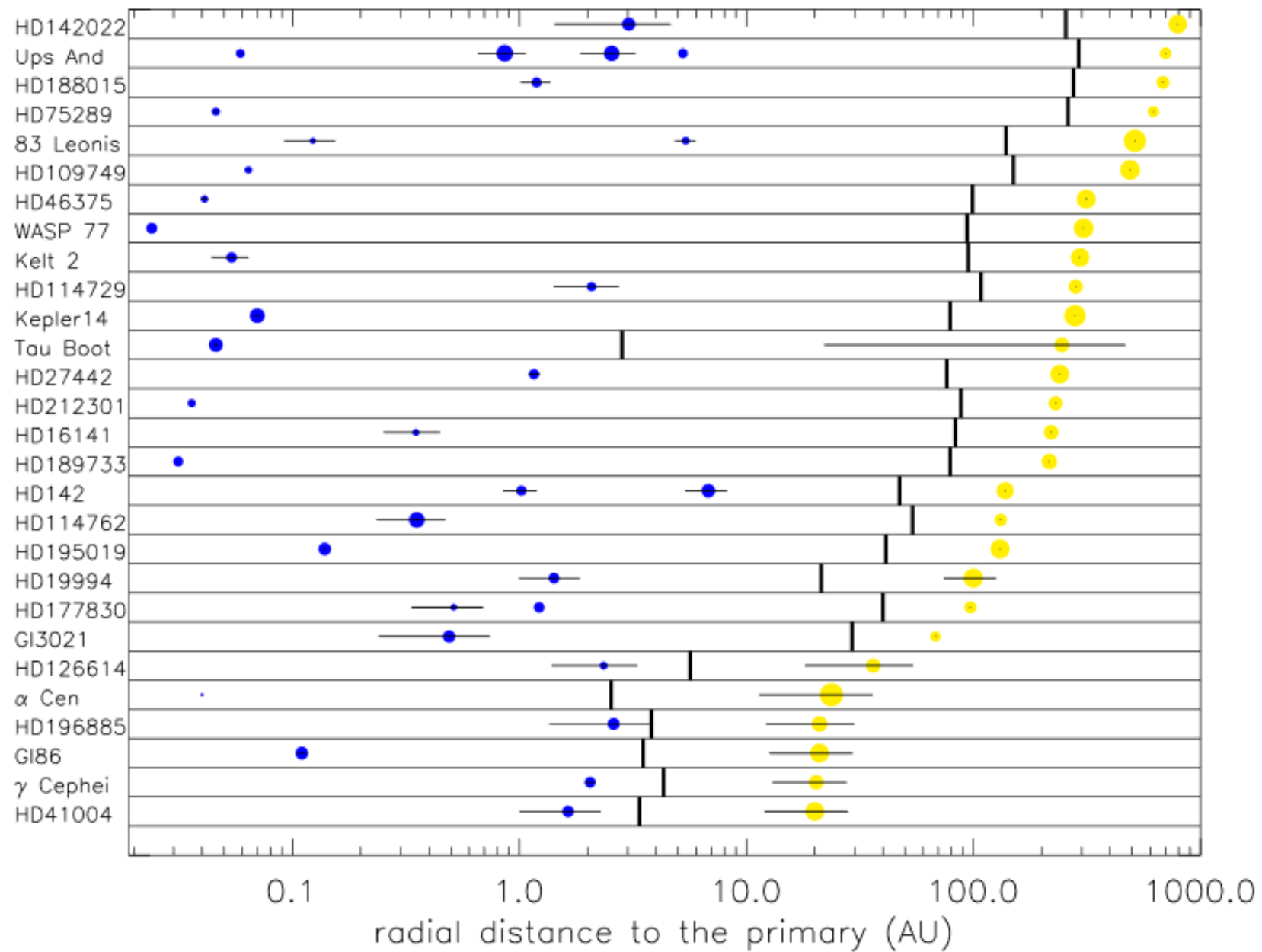
$$\mu = m_2 / (m_1 + m_2)$$

Both estimates are given for $e=0$.

Plus:
islands of stability
and instability
due to resonances, etc.

$$a_{\text{crit}} \approx 1.60 + 5.10 e_{\text{bin}} - 2.22 e_{\text{bin}}^2 + 4.12 \frac{M_s}{M_p + M_s} - 4.27 e_{\text{bin}} \frac{M_s}{M_p + M_s} \\ - 5.09 \frac{M_s^2}{(M_p + M_s)^2} + 4.61 e_{\text{bin}}^2 \frac{M_s^2}{(M_p + M_s)^2},$$

1503.03876

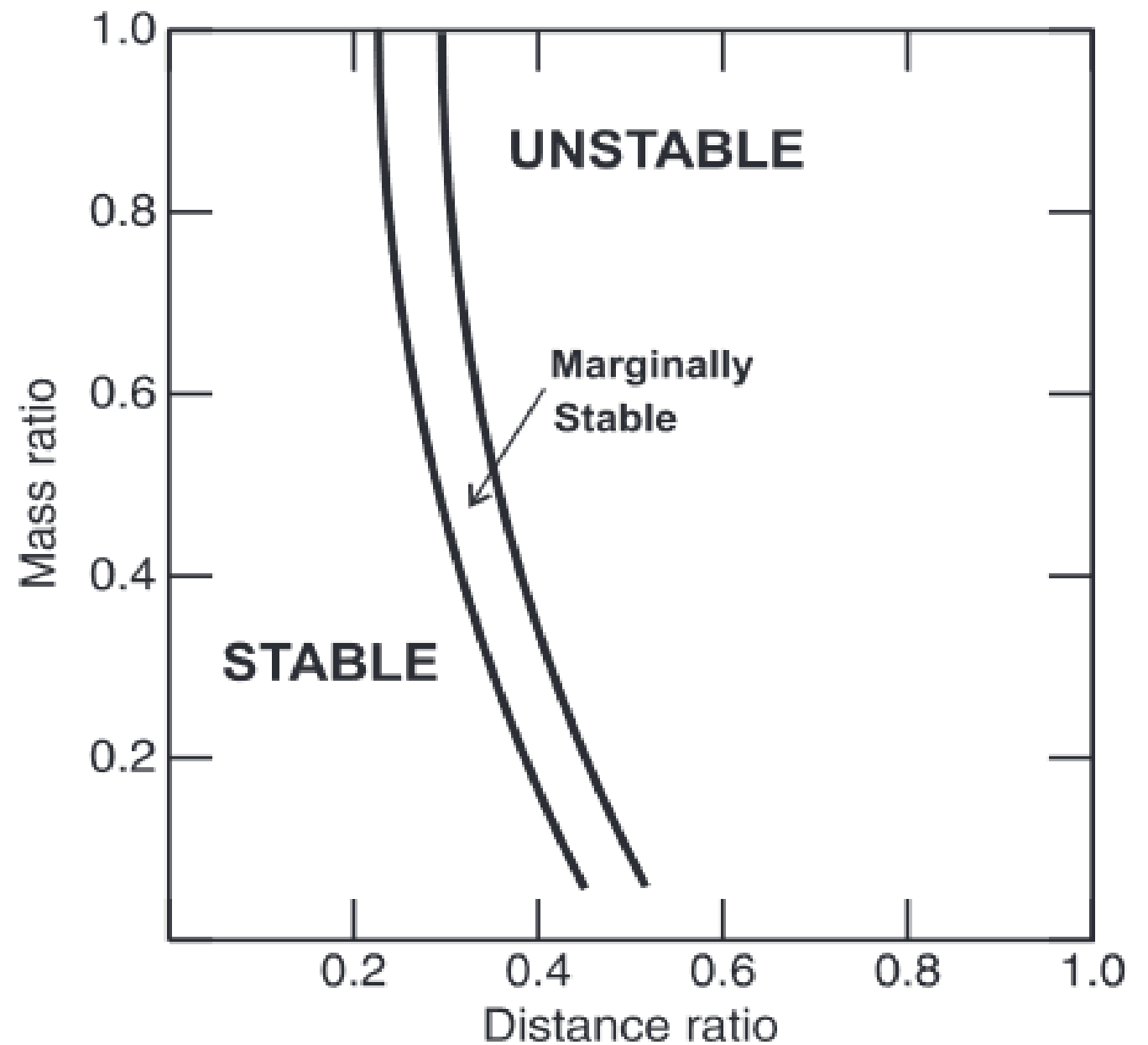


$$R_p = \left(0.464 - 0.38 \frac{m_2}{m_1 + m_2} \right) a_*$$

S-type

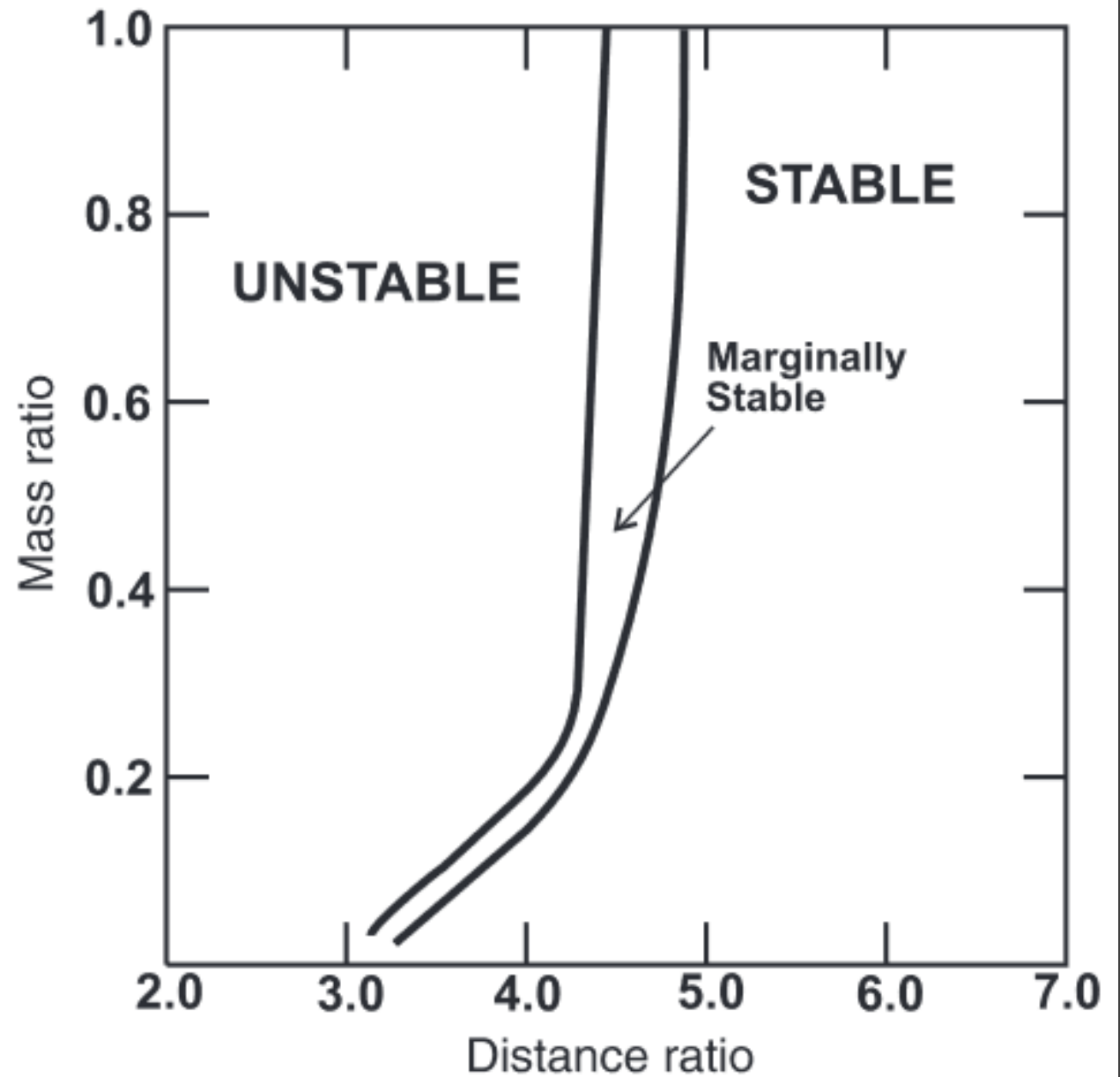
Distance ratio: $R_{\text{planet-A}}/R_{\text{AB}}$

Mass ratio: $M_{\text{B}}/M_{\text{A}}$

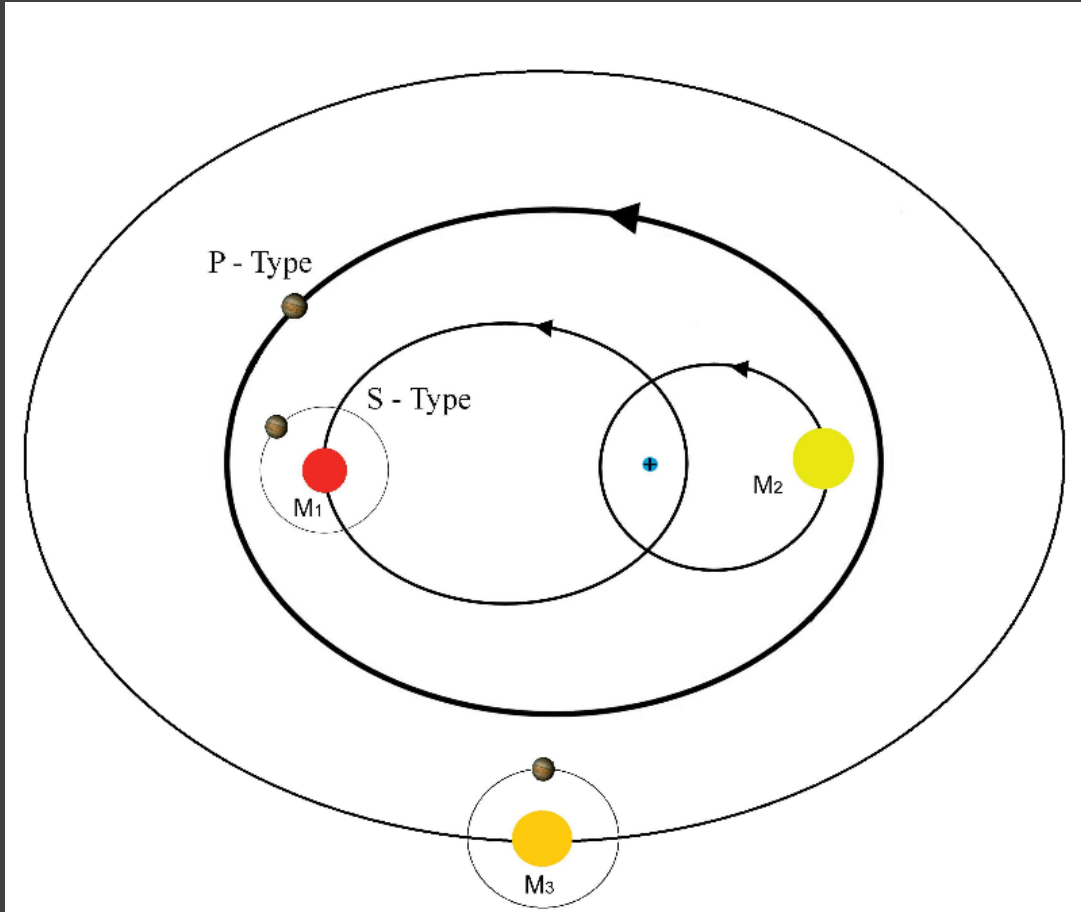


P-type

See a more detailed study
for non-zero eccentricities
in 1802.08868

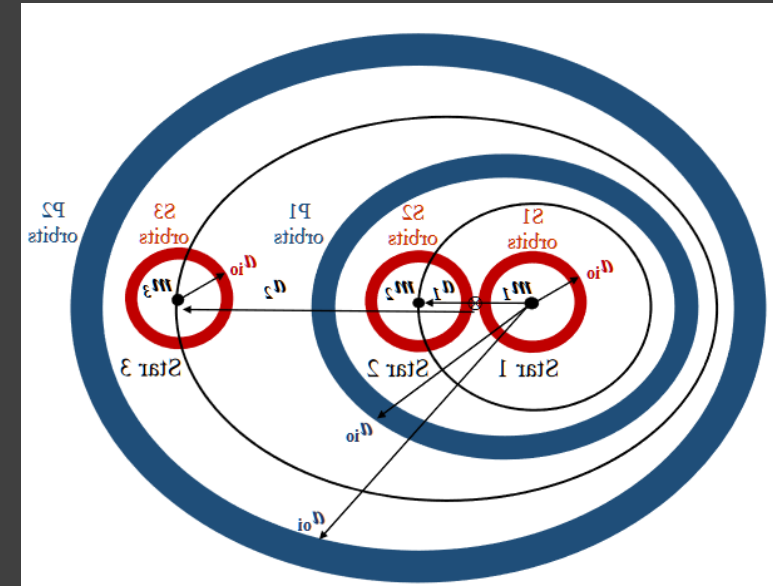


Planets in triple-star systems



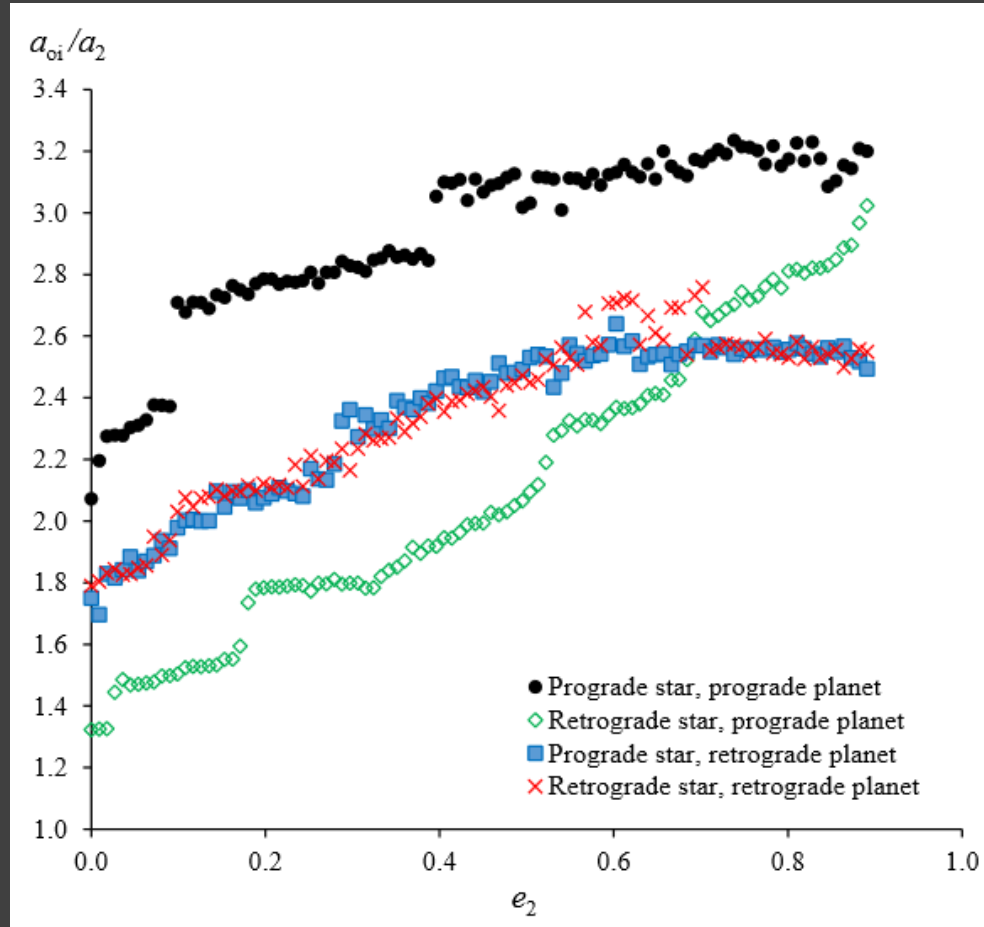
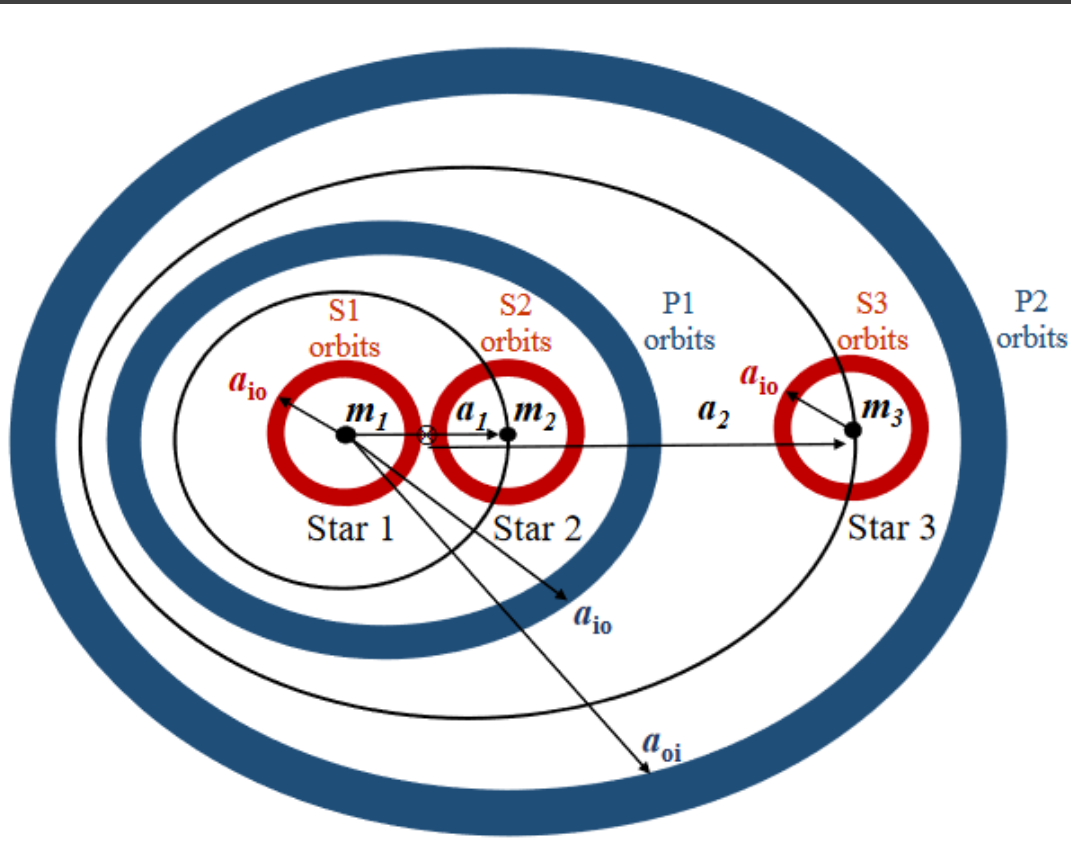
Stable orbits of S-, P-, and T-types are possible in different kinds of multiple systems (see [arXiv:1811.08221](https://arxiv.org/abs/1811.08221)).

32 triple systems with planets (Nov. 2018):
29 – at S3, 2 – S1, 1 – P1.



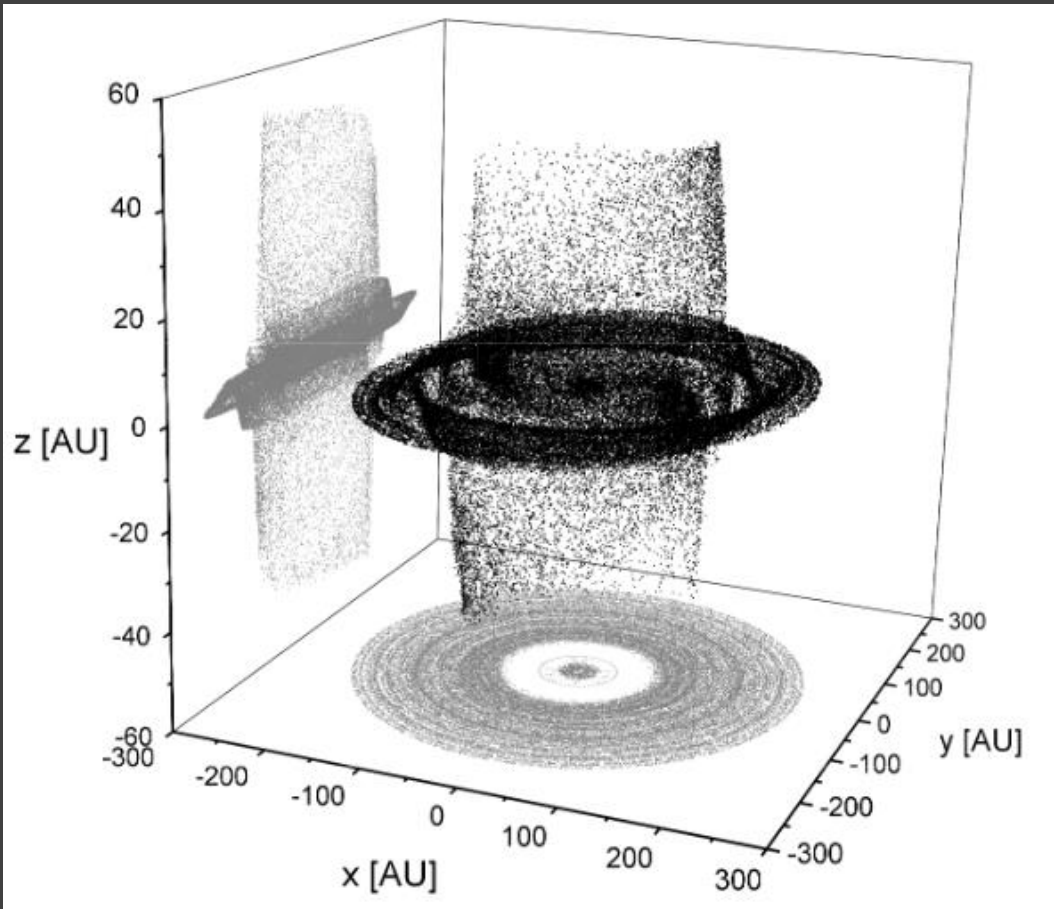
Stability of planets in triples

Eccentricity of the outer stellar orbit is the most crucial parameter for stability of P-type planetary orbits.



P2-orbit

Orbits in non-coplanar triples

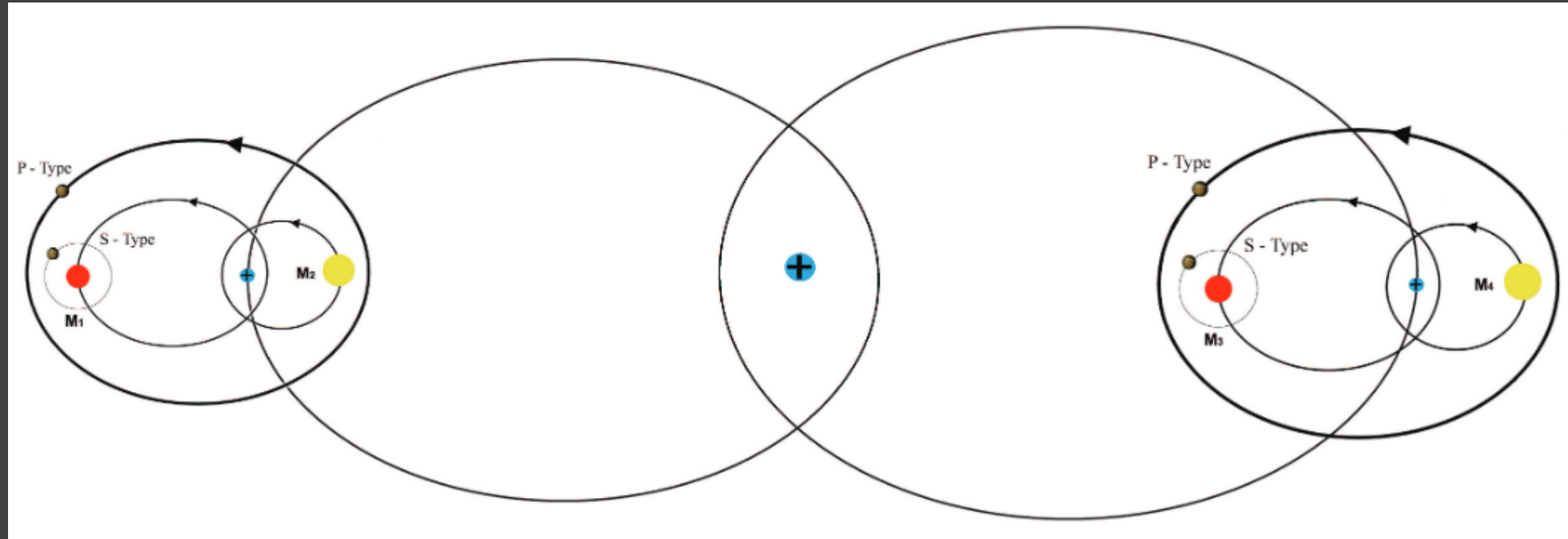


The typical annular vertical “chimney” consists of test particles in circular orbits of constant semi-major axis but varying inclination.

Some test particles develop Kozai resonance and some of these eventually undergo orbit flipping in to retrograde motion.

Fig. 4. Position in space of surviving test particles after 100 kyr. P1 and P2 prograde orbits, with $a = 40$ AU, $e_1 = e_2 = 0$, $i_1 = 0^\circ$, $i_2 = 65^\circ$, $\mu_1 = 0.5$, $\mu_2 = 2.0$.

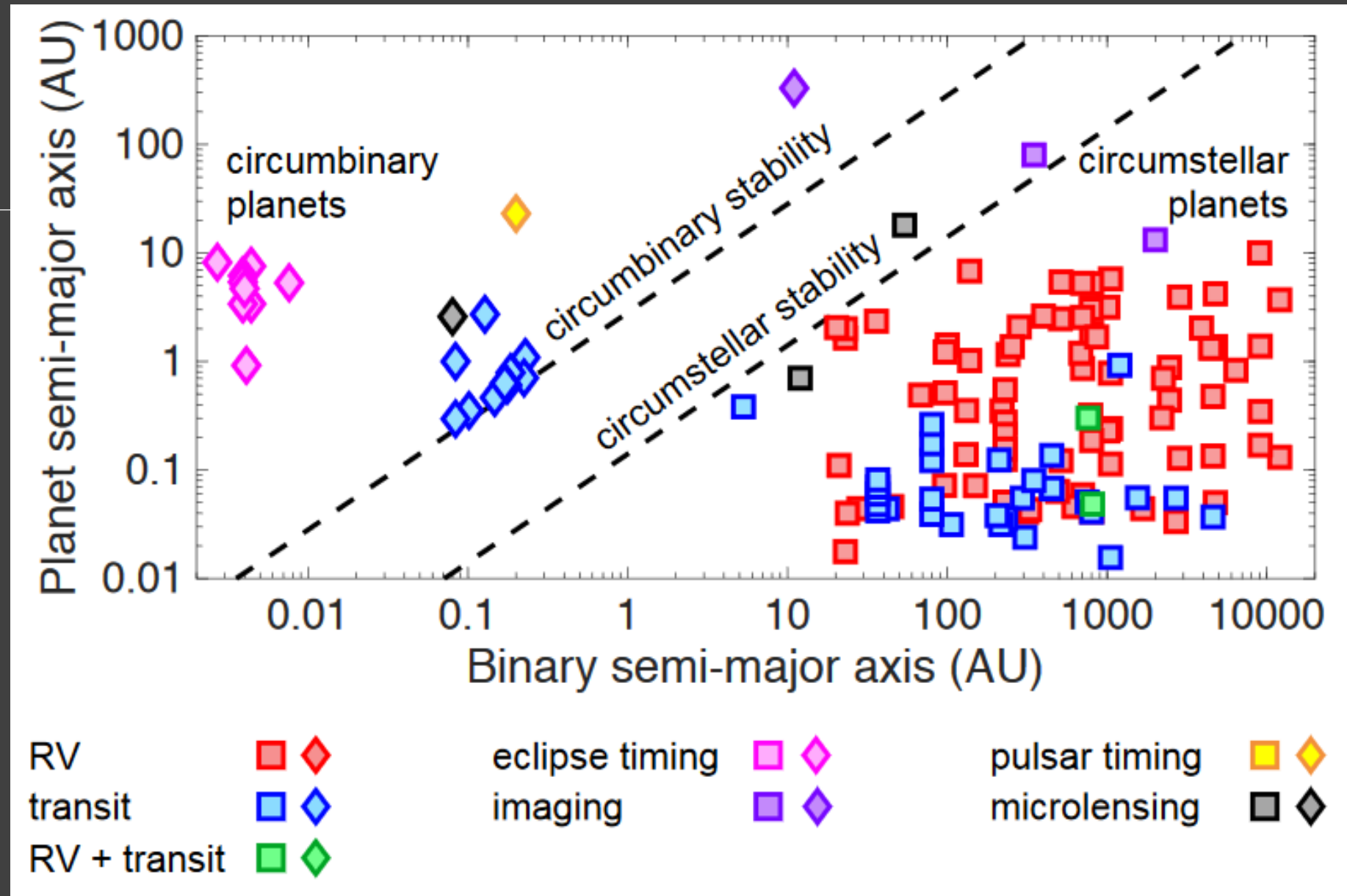
Quadruple star systems



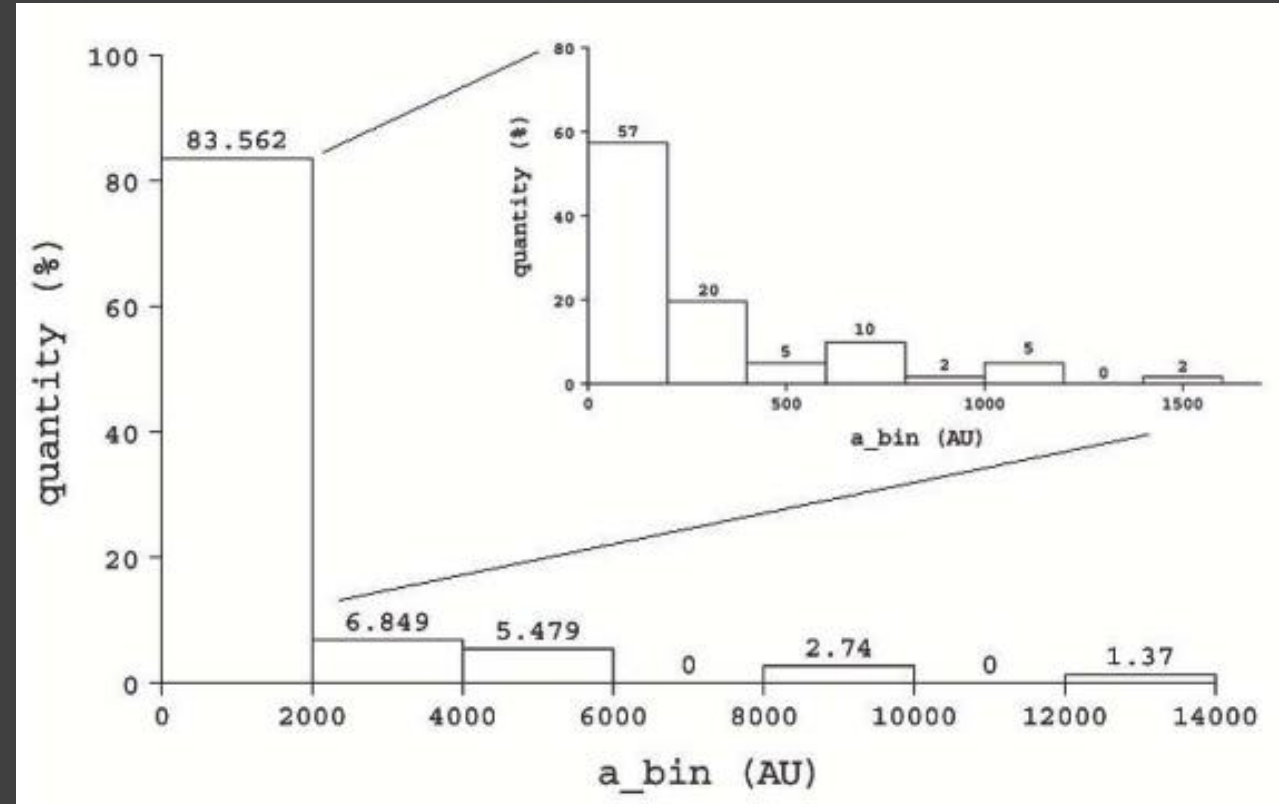
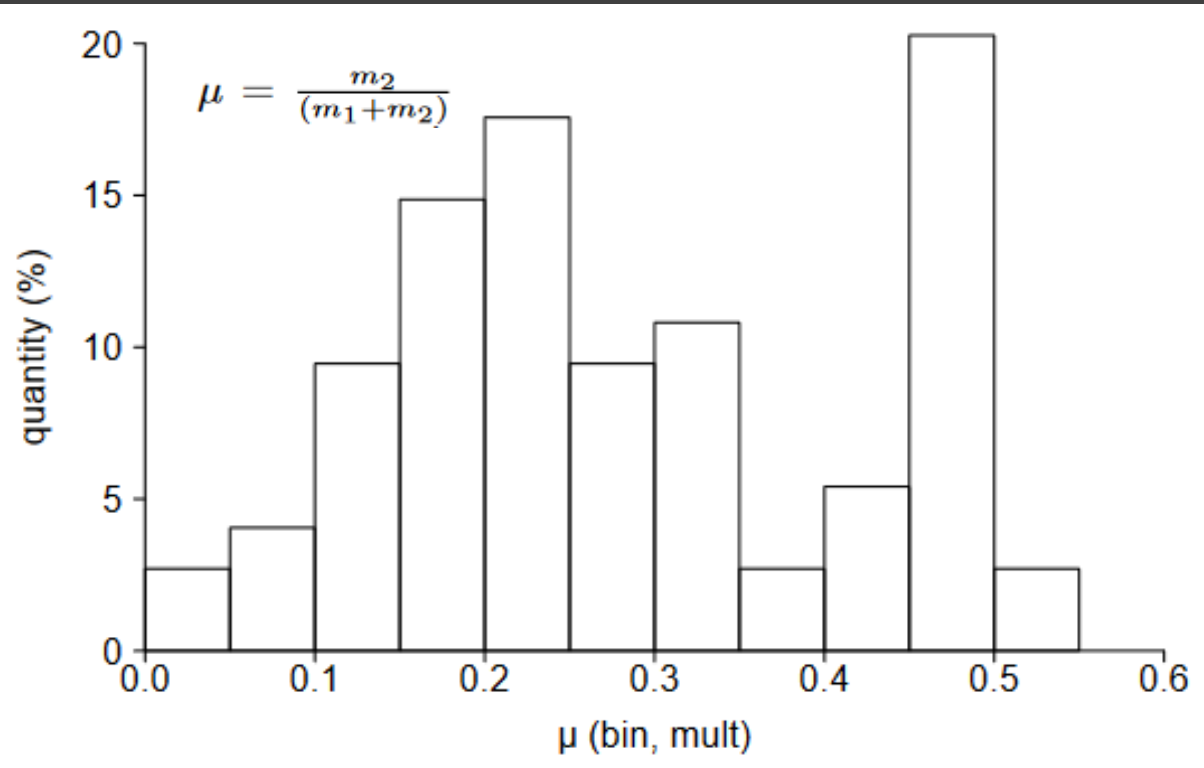
Statistics

Diamonds – circumbinary,
Squares – circumstellar

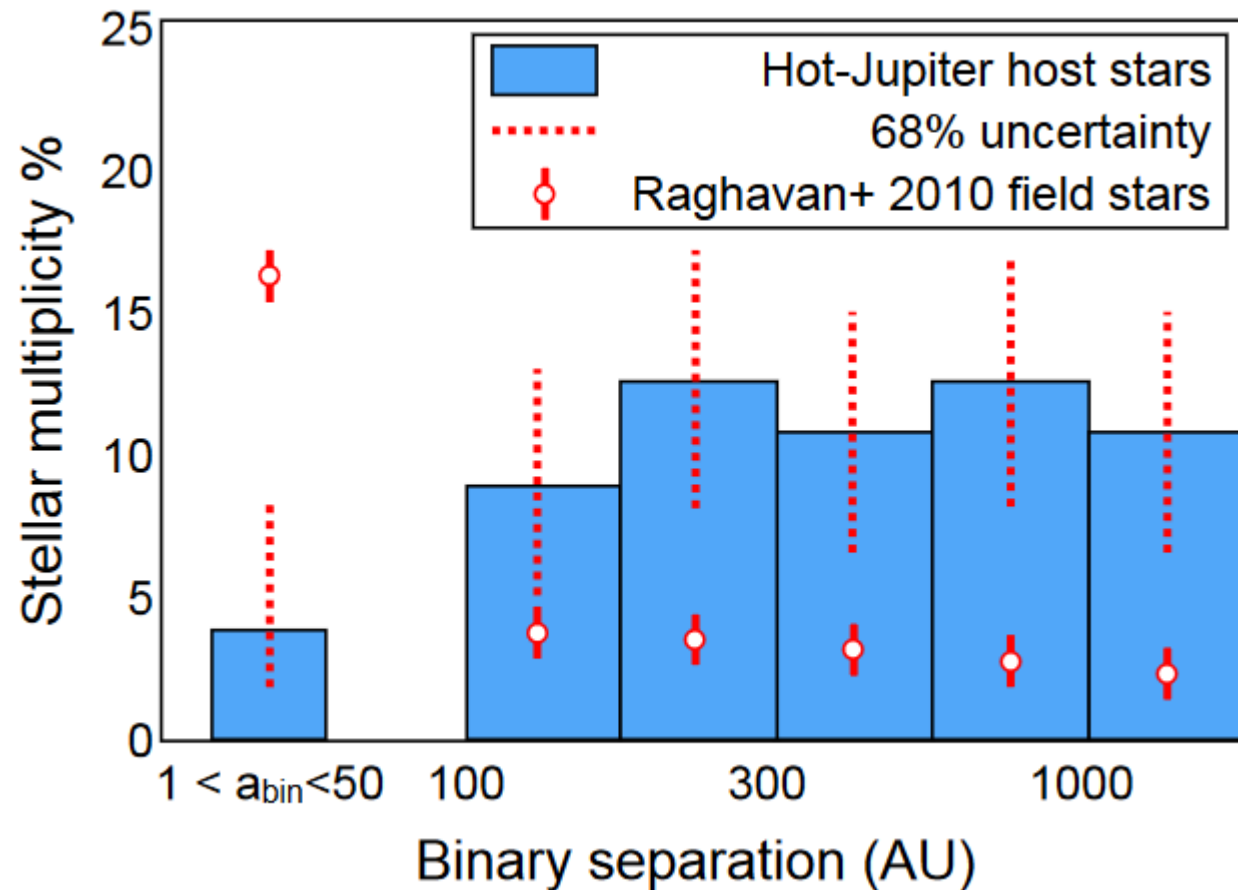
Analysis of selection effects
and biases for transits
can be found in 1904.04832.



Statistics



Statistics for hot jupiters hosts



Less close companions,
but more farther away.

First circumbinary planet found by microlensing

Only triple lensing model (star+2 planets or planet+ 2stars) can fit the light curve.

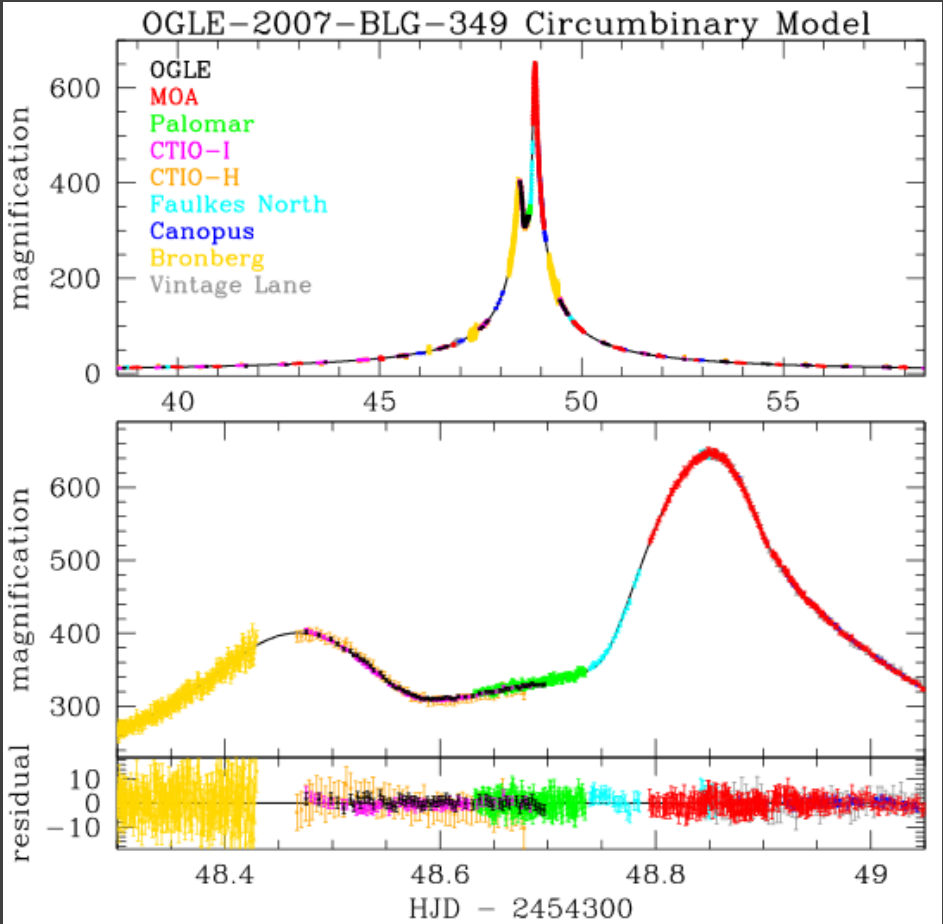
Subsequent HST observations favour the circumbinary model.

Parameter	units	average value	2- σ range
D_L	kpc	2.76 ± 0.38	2.06-3.56
M_{A+B}	M_\odot	0.71 ± 0.12	0.50-0.95
M_A	M_\odot	0.41 ± 0.07	0.28-0.54
M_B	M_\odot	0.30 ± 0.07	0.15-0.45
m_c	M_\oplus	80 ± 13	56-107
$a_{\perp AB}$	AU	0.061 ± 0.007	0.048-0.074
a_{AB}	AU	$0.080^{+0.027}_{-0.015}$	0.054-0.162
P_{AB}	days	$9.7^{+5.4}_{-2.5}$	5.7-28.1
$a_{\perp CMc}$	AU	$2.59^{+0.43}_{-0.34}$	1.97-3.89

Planetary orbit ~ 3.2 AU
Orbital period ~ 7 years

$80 \pm 13 M_\oplus$

Planetary orbit
→



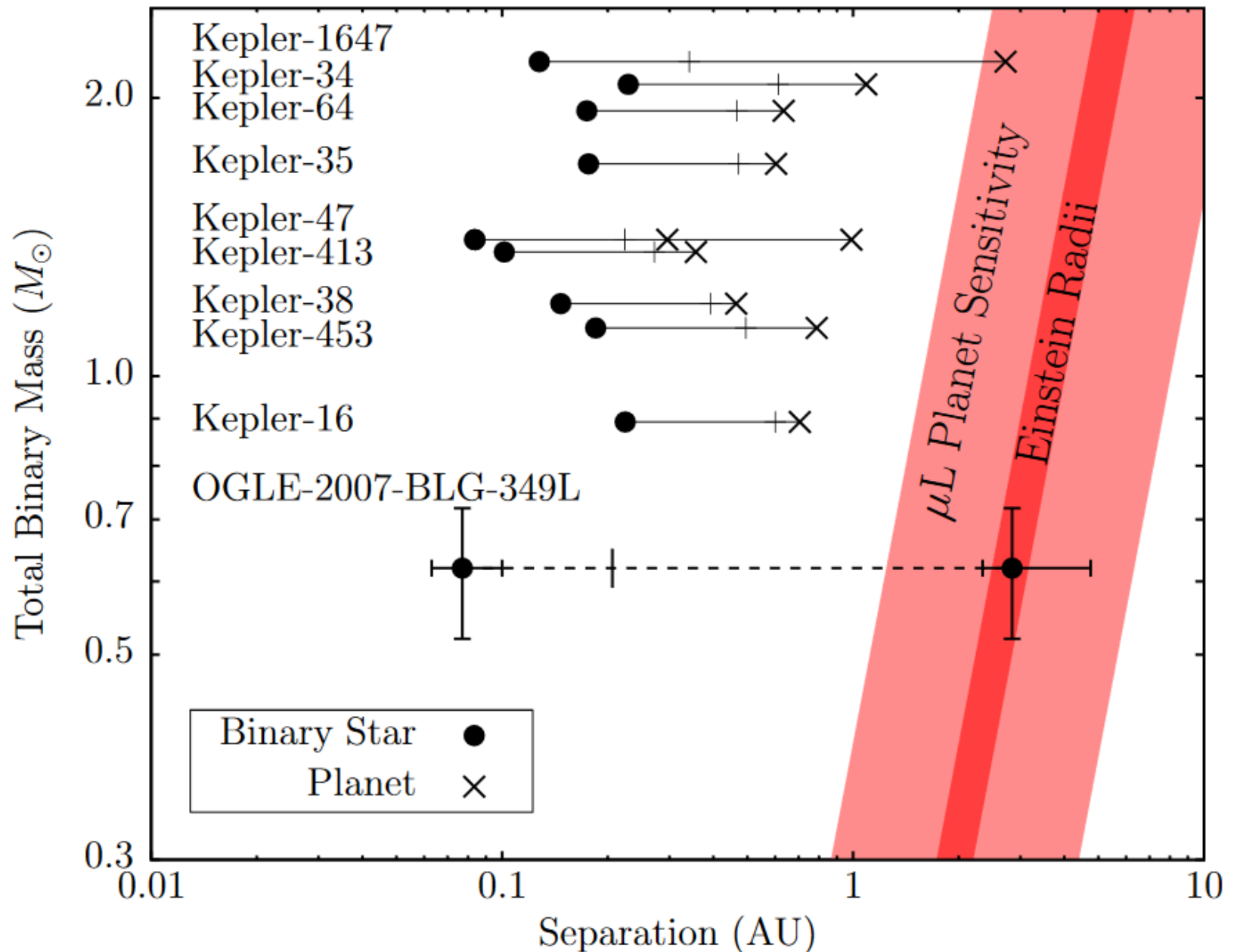
Comparison

Kepler planets have tight orbits, as if they moved close to their stars after formation. They are close to the stability limit.

$$a_c \simeq (2.28 \pm 0.01) + (3.8 \pm 0.3)e + (1.7 \pm 0.1)e^2,$$

a_c is measured in binary semi-major axis

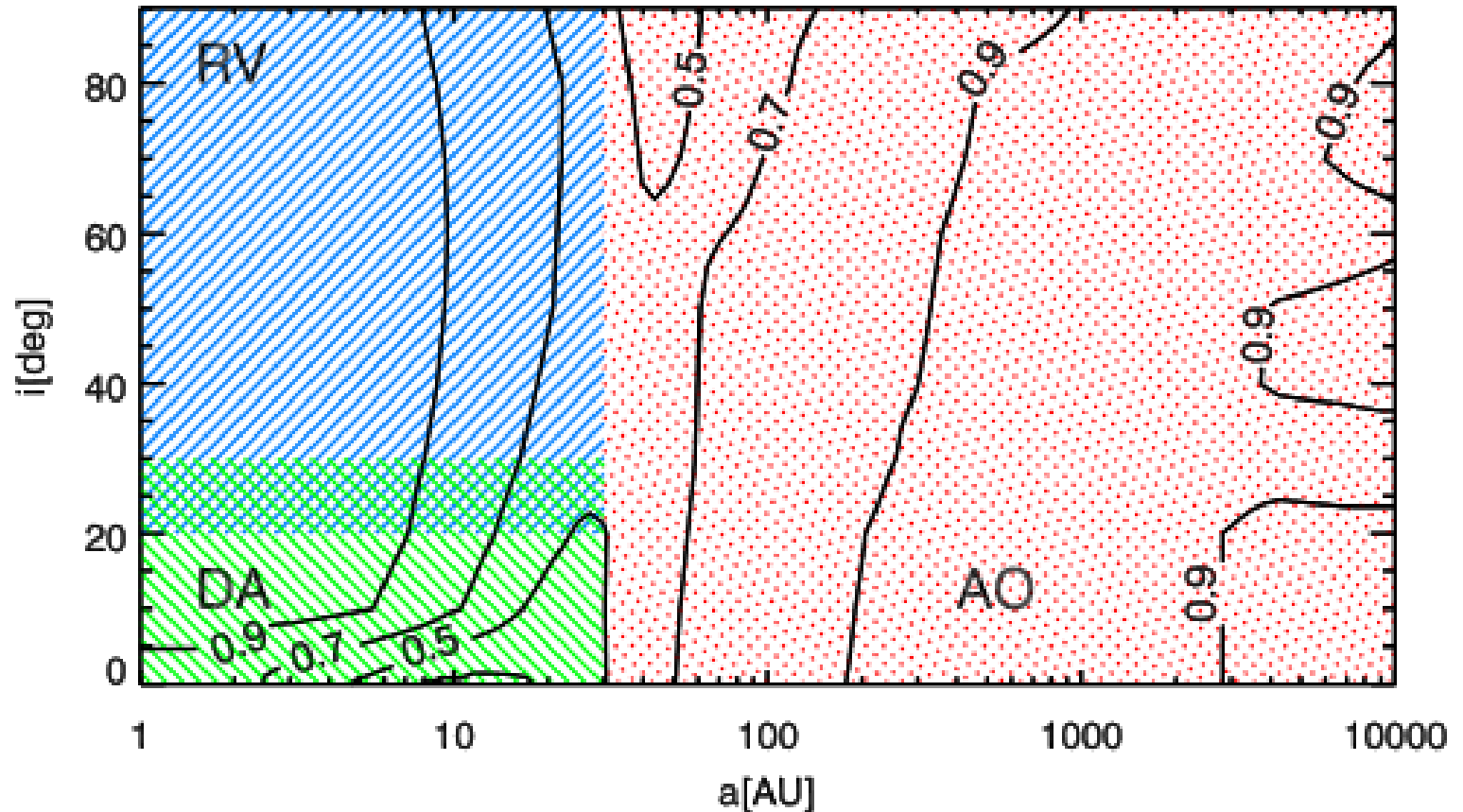
Circles show binary separation,
And crosses – planetary.
Vertical ticks mark the stability limit.



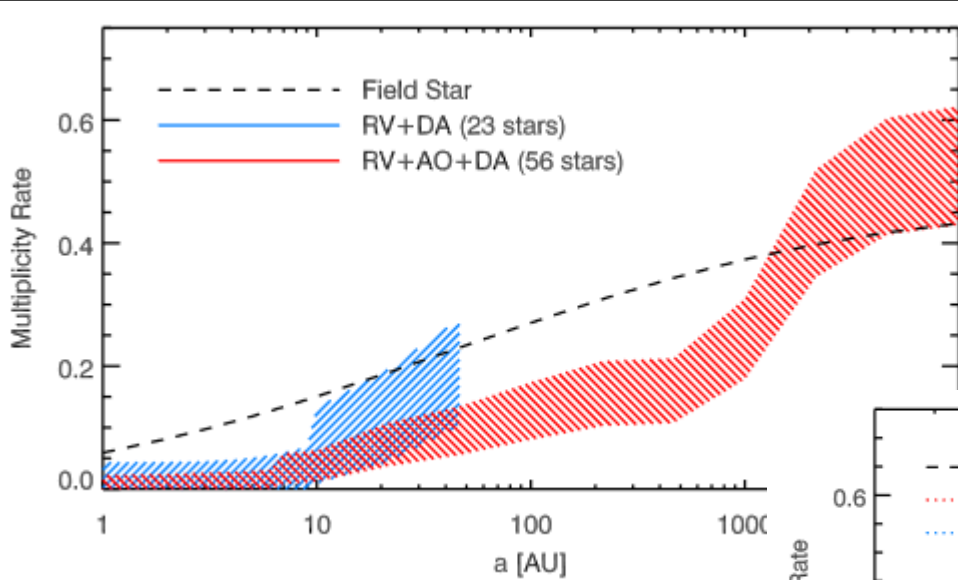
Search for binary component around planets hosts

Three techniques:

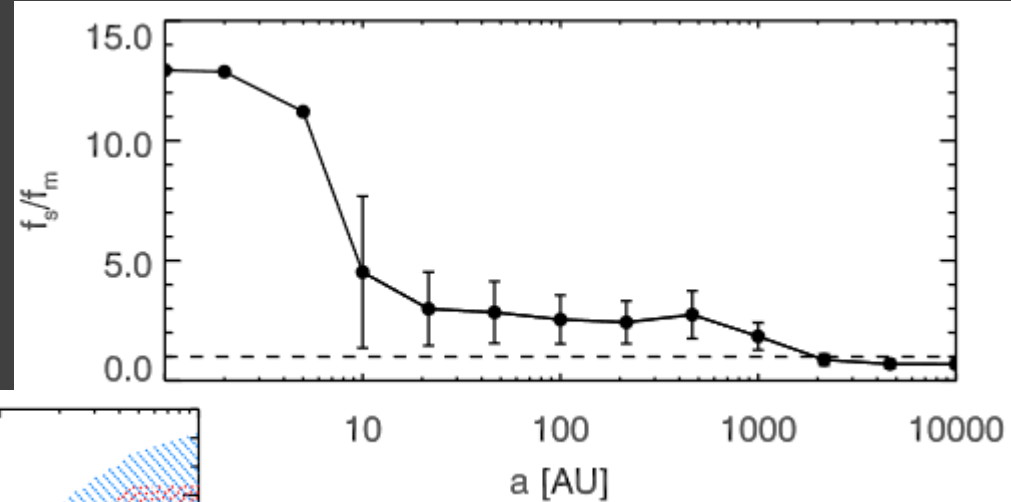
- RV
- AO
- DA



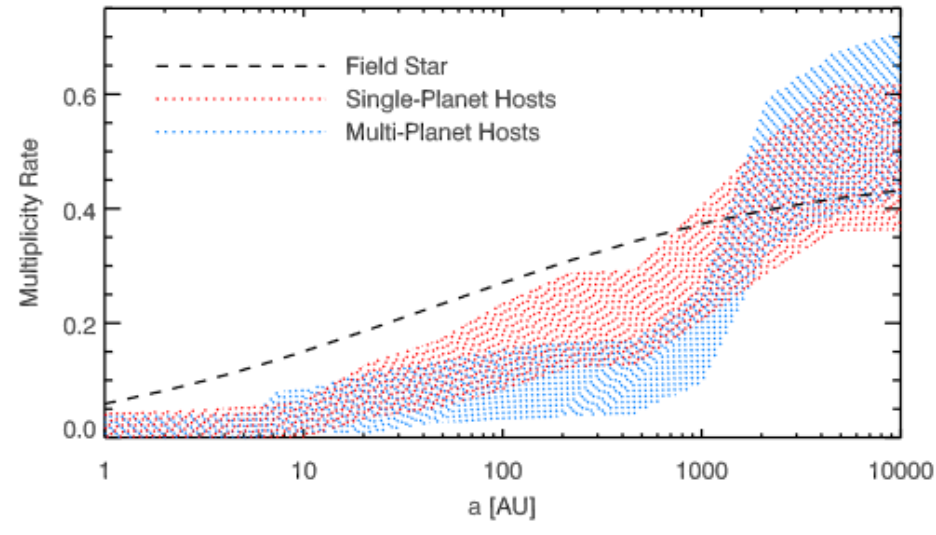
Statistics of planets in close binaries



Multiplicity rate of planet hosting stars is smaller than in the field for binary separations $< \sim 1000$ -1500 AU.



At binary separations $> \sim 1000$ -1500 AU S-type planets are as frequent as around single stars.
For smaller separations S-type planets are less numerous.

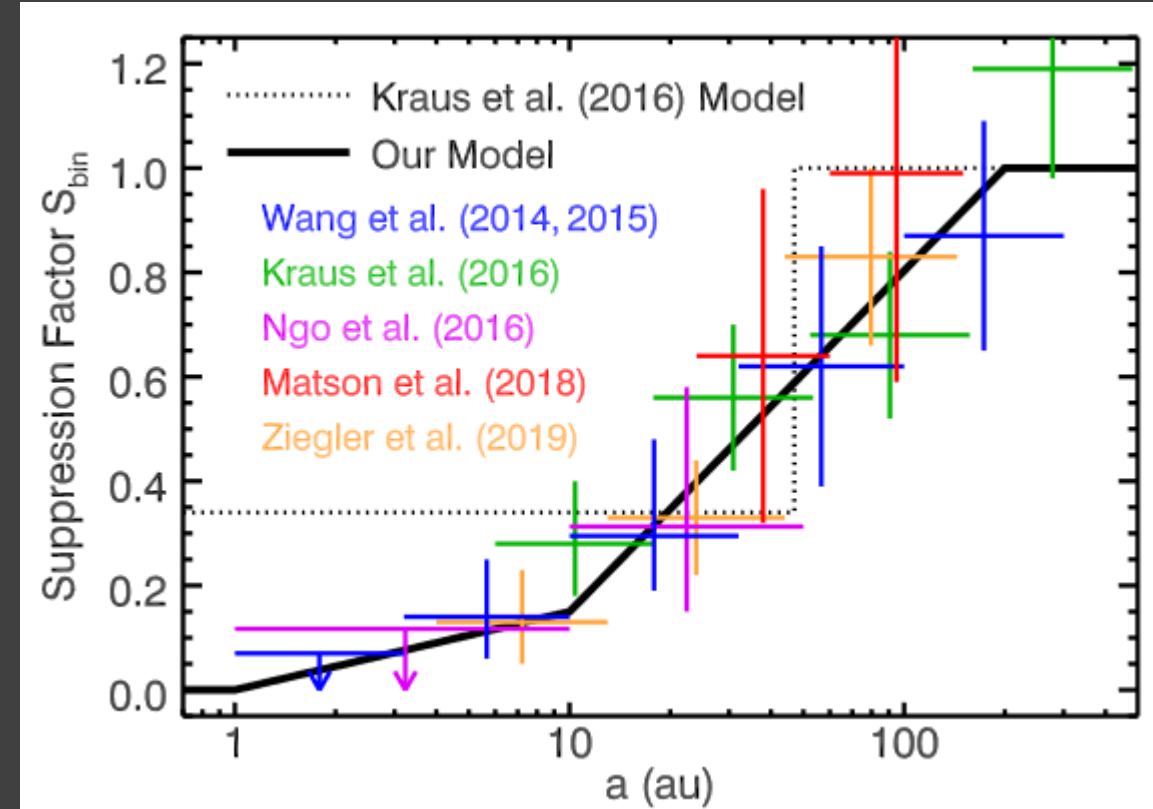


Close binaries suppress S-type planets

Binaries with $a \sim 1$ au fully suppress S-type planets.

Binaries with $a \sim 10$ au host close planets at $\sim 15\%$ the occurrence rate of single stars.

Wide binaries with $a \sim 200$ au have a negligible effect on planet formation.



Ratio of the stellar companion fraction in planet hosts versus field stars, as a function of binary separation.

ETV: Eclipse timing variations

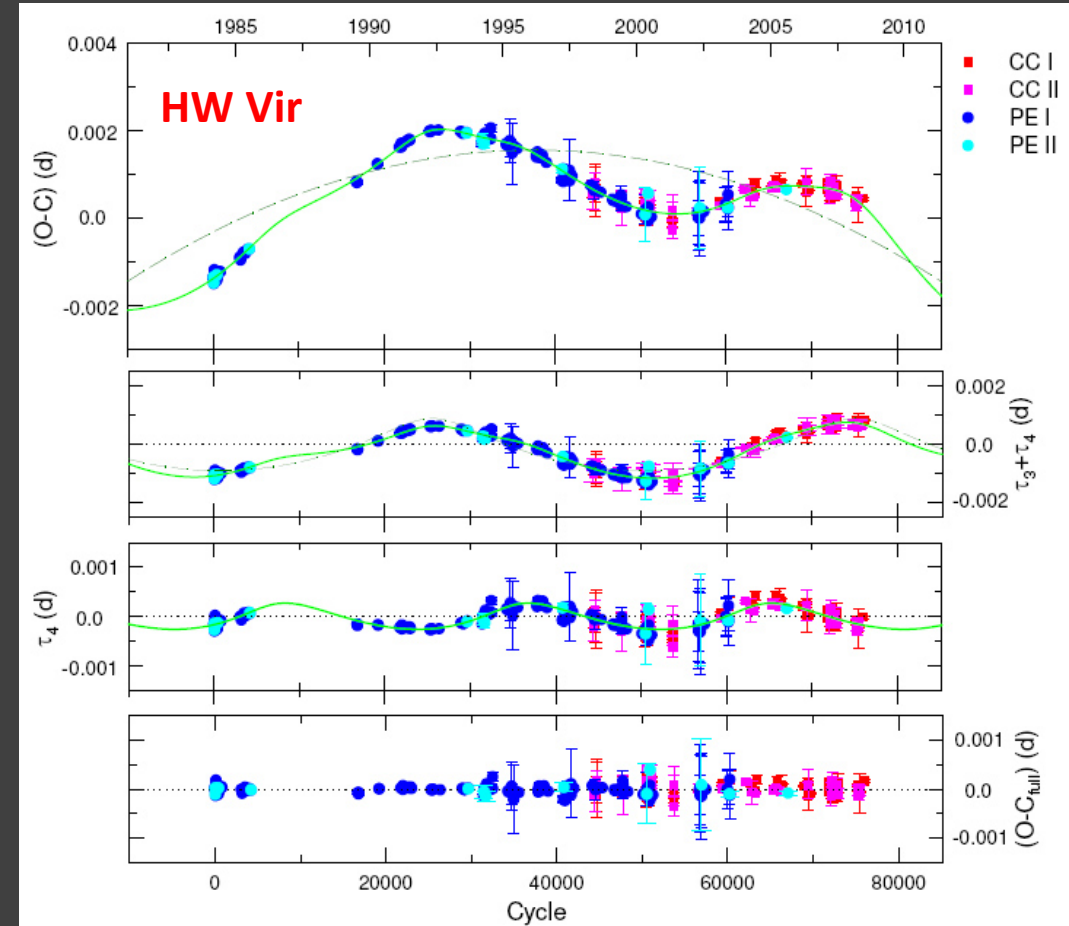
CoRoT: 4 sec – for bright stars (12 mag),
and 16 sec – for dim stars (15.2 mag).
Kepler: 0.5 sec – for bright stars (9 mag),
and 4 sec – for dim stars (14.5 mag).

HW Vir – short period (2.8 h) binary
P-type planets: 15.8 and 9.1 yrs

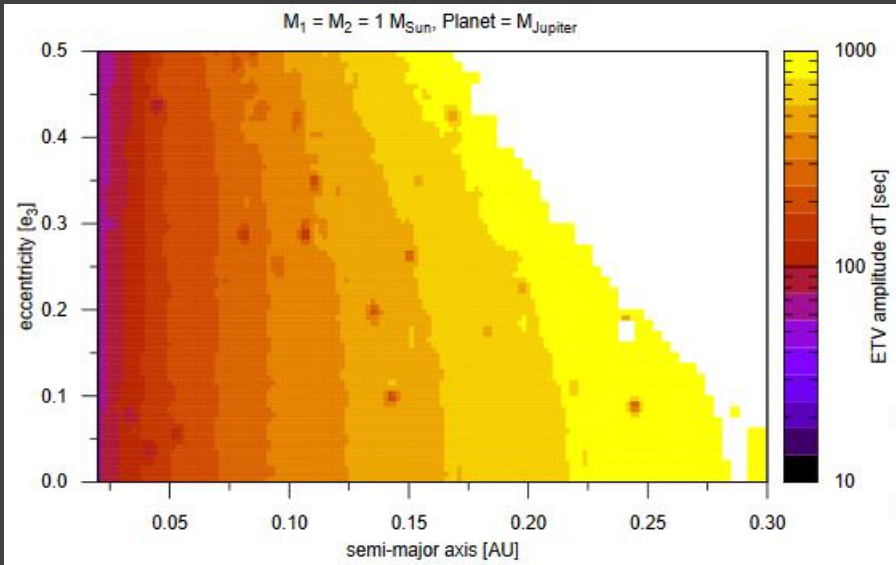
Residuals

One planet model

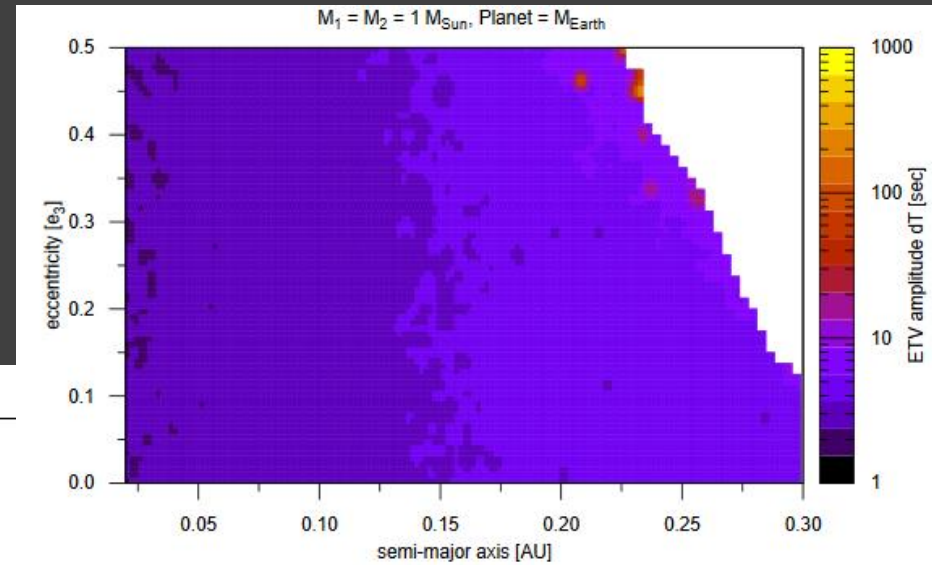
Two planets model



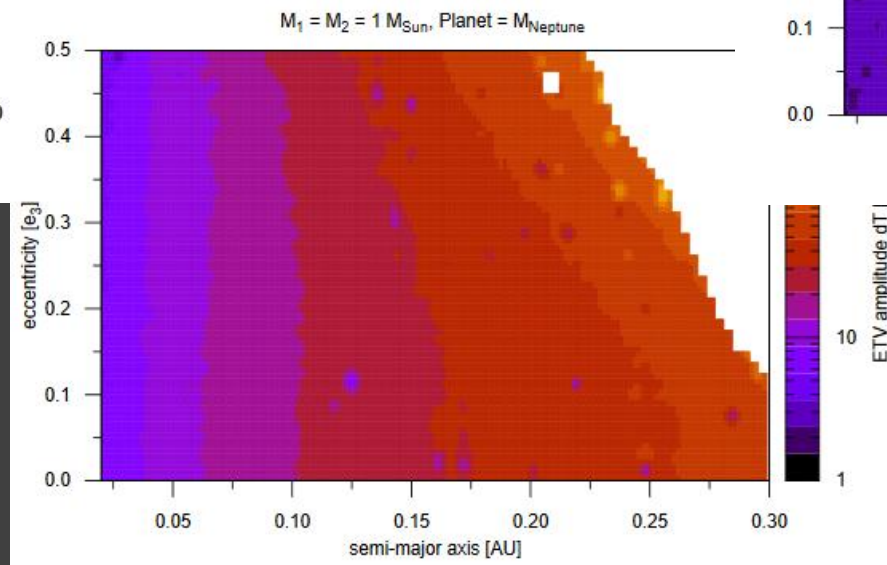
Modeling ETV. S-type systems



$a_{\text{bin}} = 1 \text{ AU}$
 $i = 0$ (planar orbit)

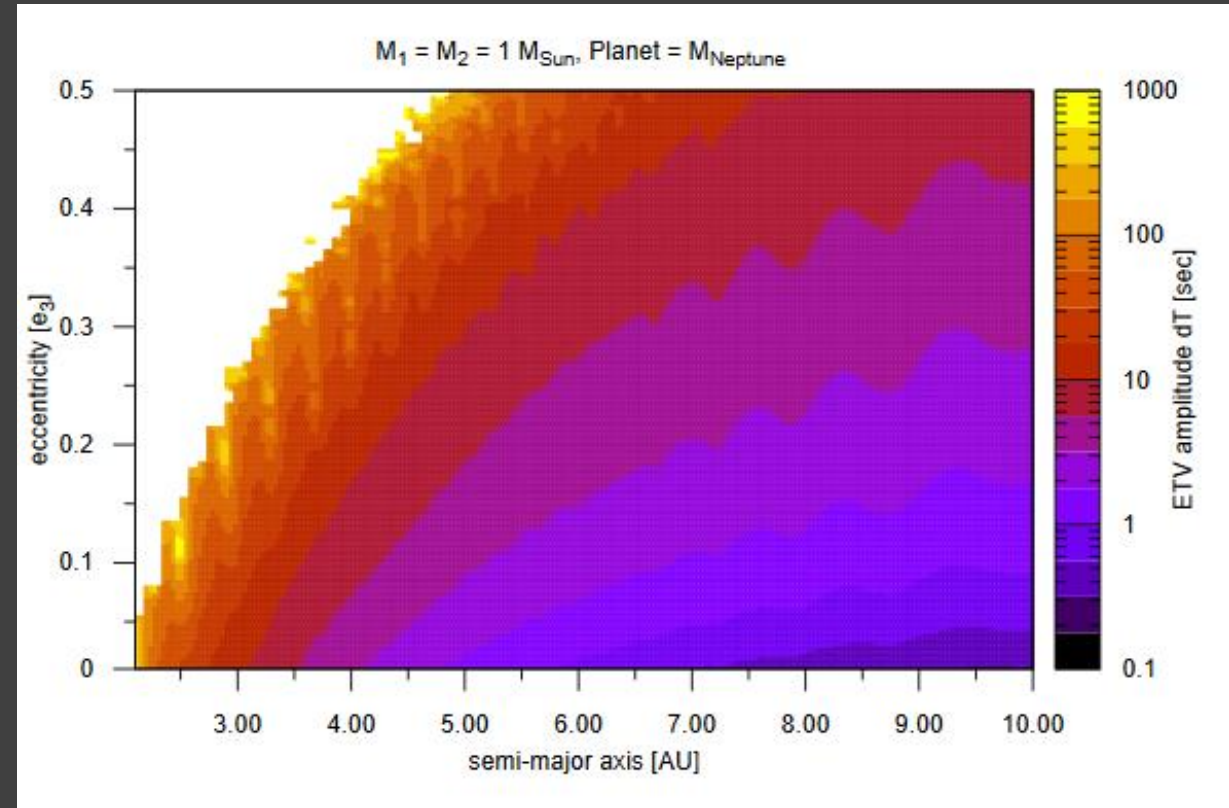
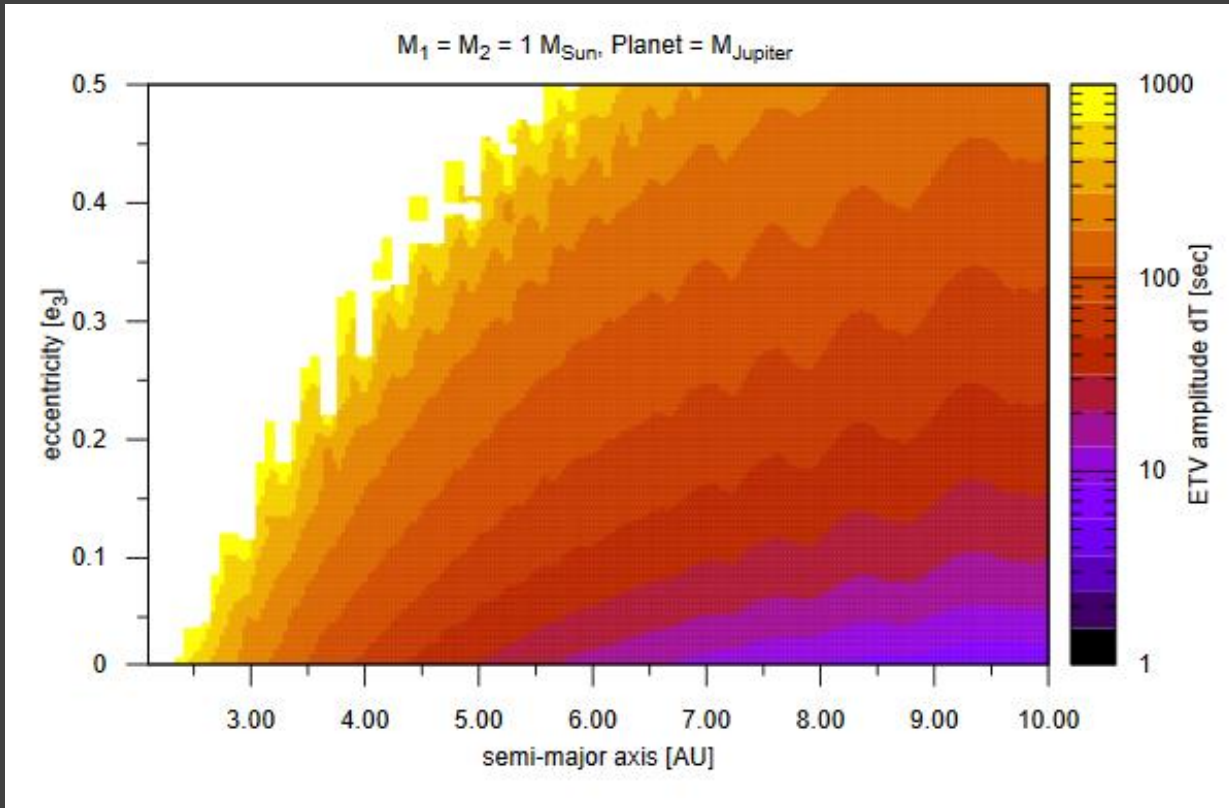


Jupiters are 100% detectable



Earth-like planets are undetectable

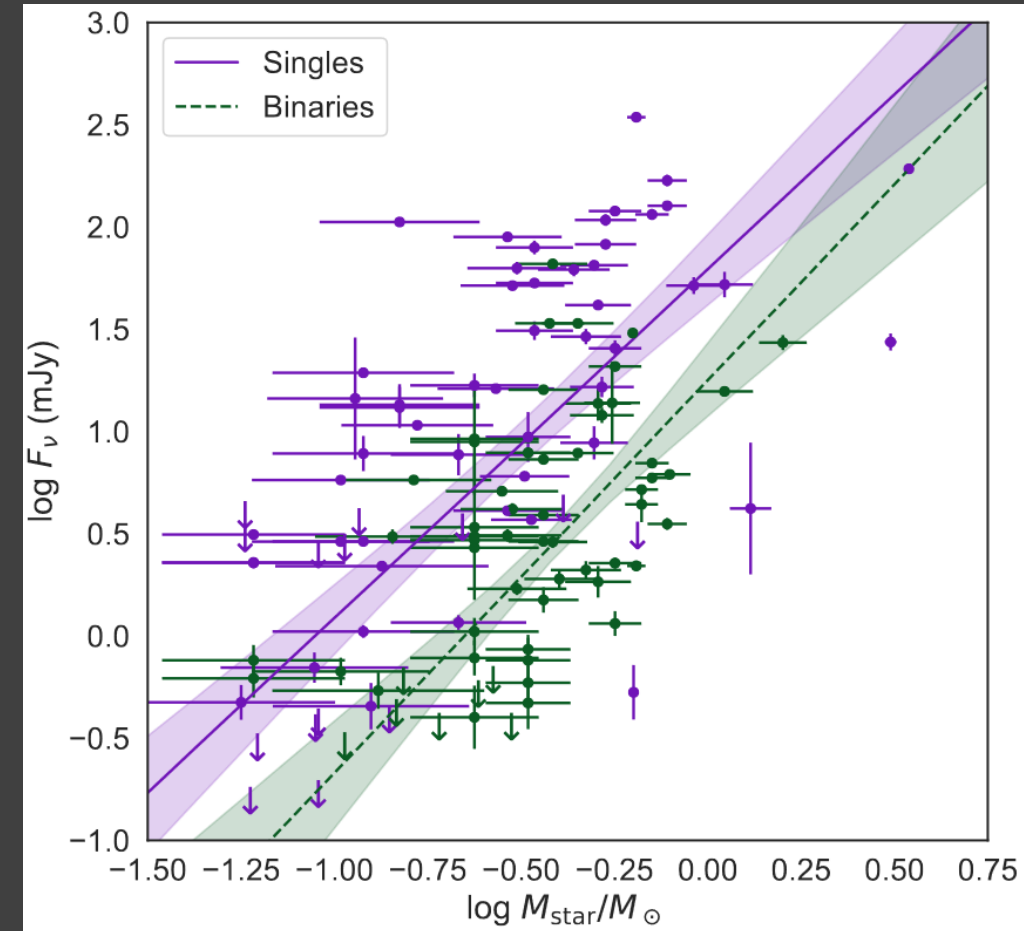
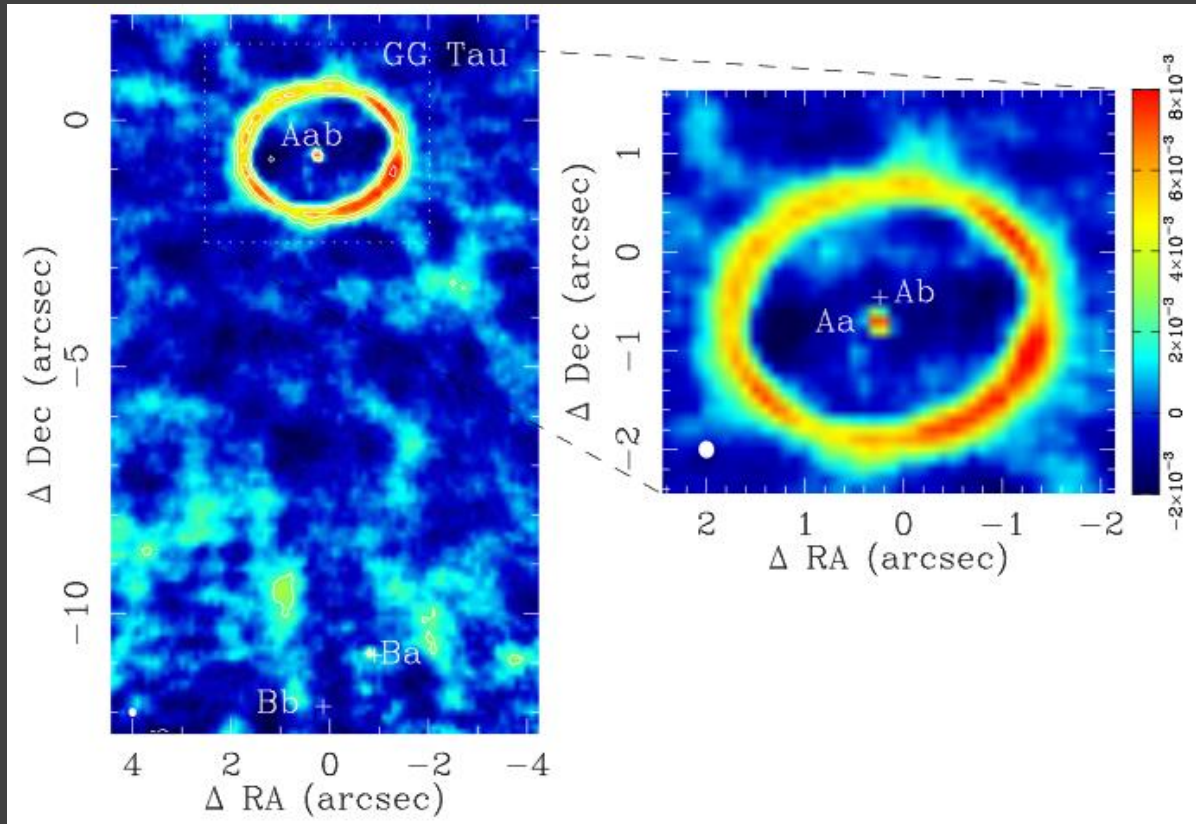
Modeling ETV. P-type systems



$a_{\text{bin}} = 1 \text{ AU}$; $e_{\text{bin}} = 0$

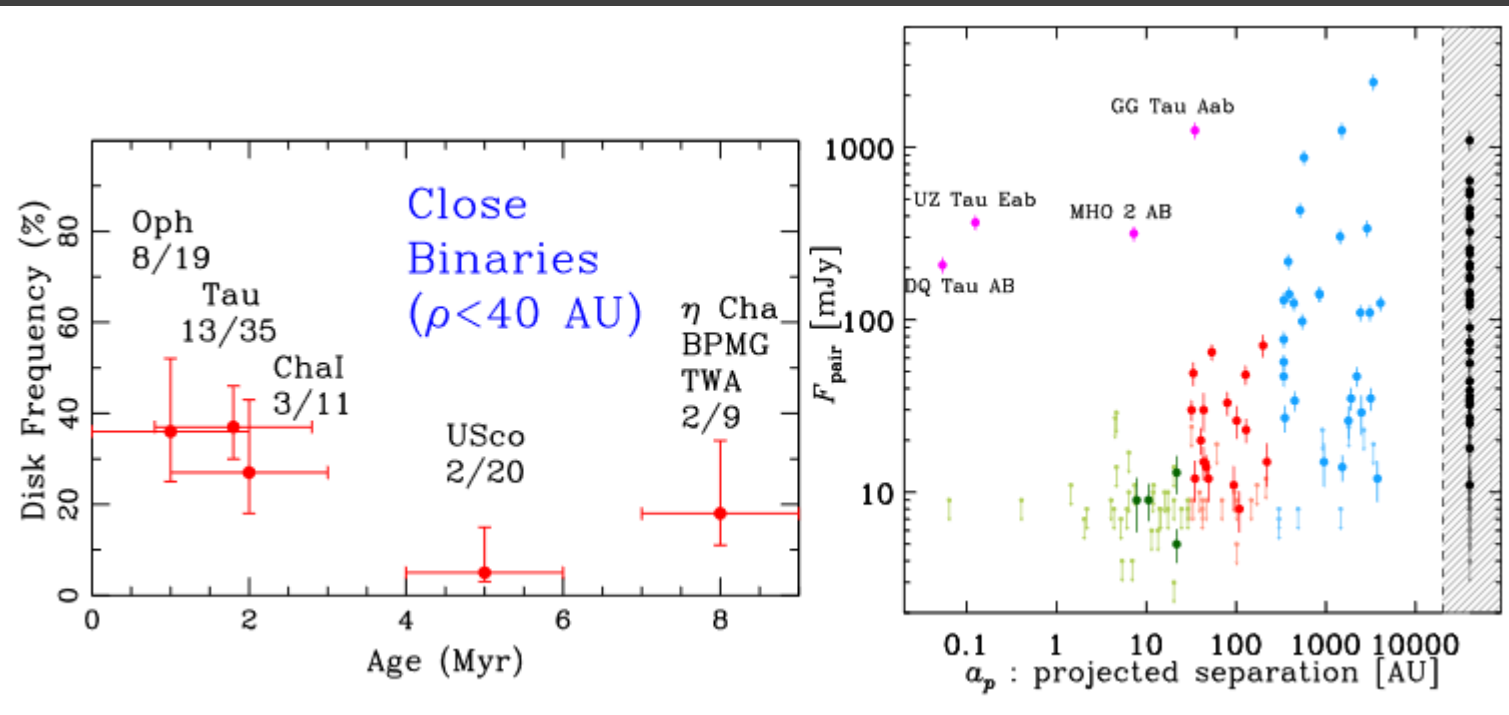
Protoplanetary discs in binaries

Many discs are resolved by ALMA in binary systems.
Discs in binaries are lighter (in close binaries lighter than in wide).



Protoplanetary discs in binaries. S-type

Discs in binaries might be truncated (at 1/3 - 1/4 of the orbital separation).



Perturbations in the disc also modifies planet growth.

Disc frequency for wide binaries is similar to that in single stars, but for close binaries it is lower.

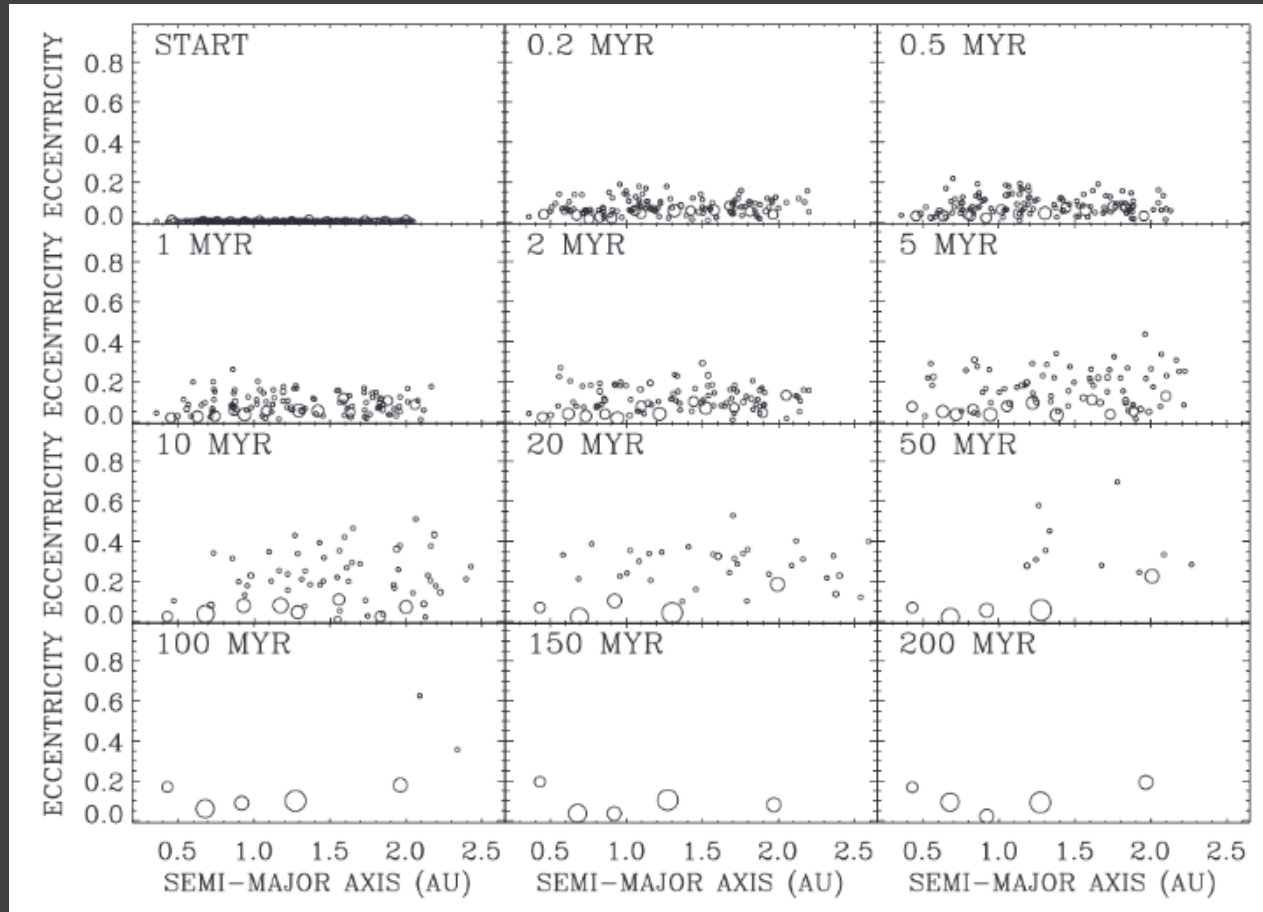
Dust mass is smaller for smaller binary separation.

Truncated discs with lower dust mass can be a bad place to form planets, especially massive.

Discs in close binaries are also short lived. Also bad for planet formation, especially for giants. In 2/3 of close binaries discs live for < 1 Myr.

Temperature is higher in such discs, so dust growth can be less efficient.

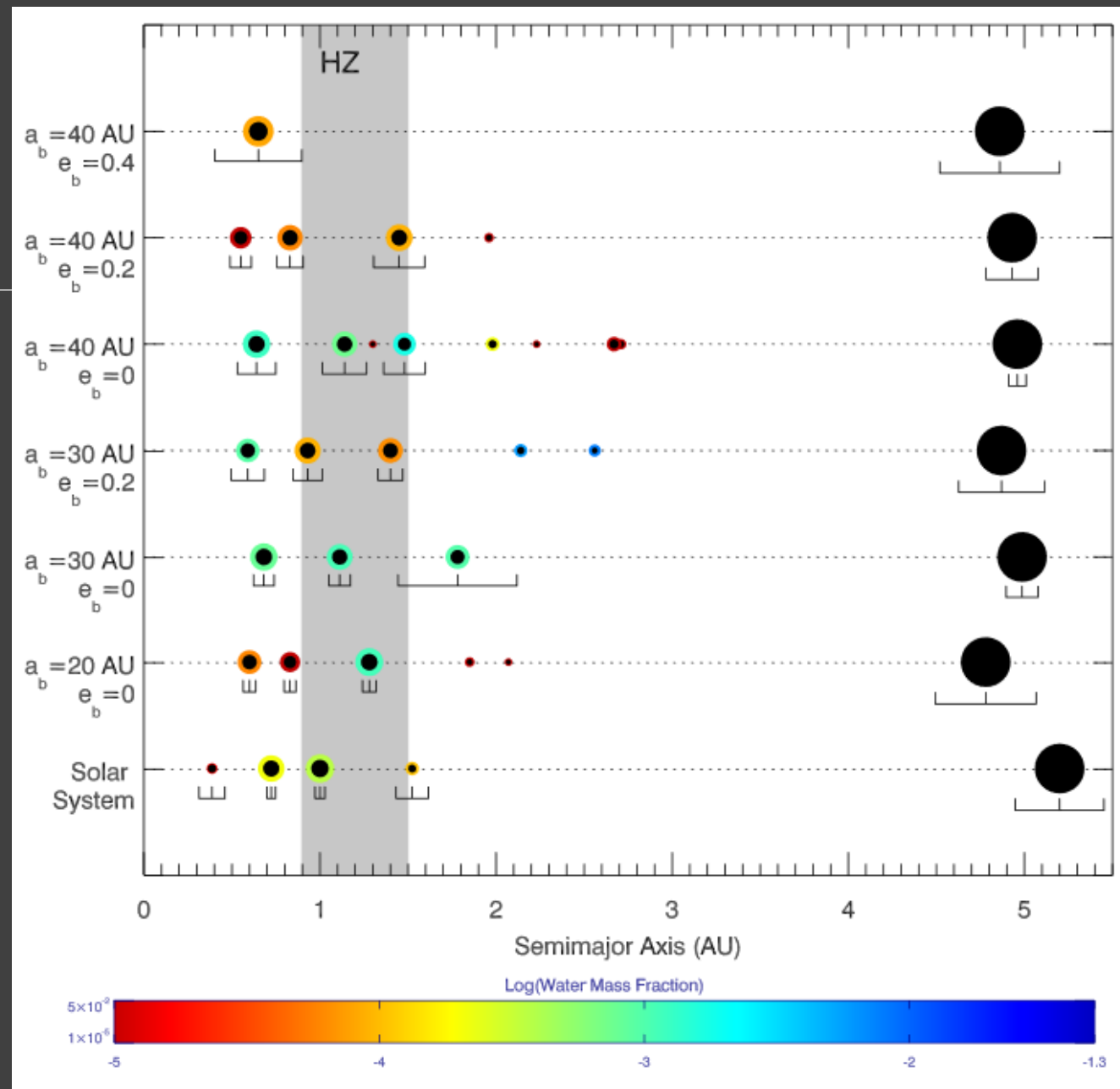
Alpha Centauri-like binary



In a compact binary system planet formation is possible only at small distances from the host star.

Water delivery

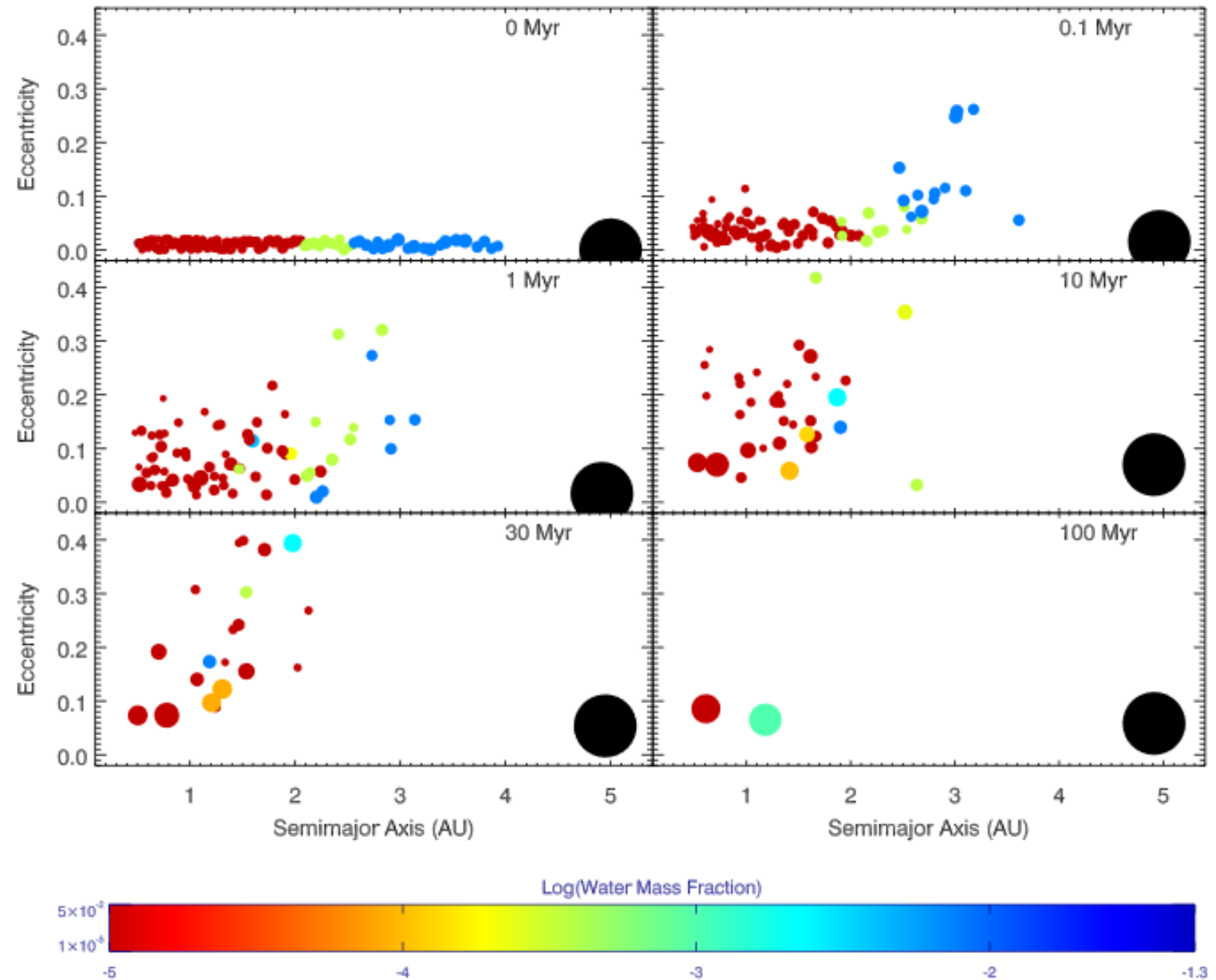
Earth-like planets can be formed in a habitable zone in a compact binary.



Earth-like

Snap shots of formation of Earth-like planets in a HZ.

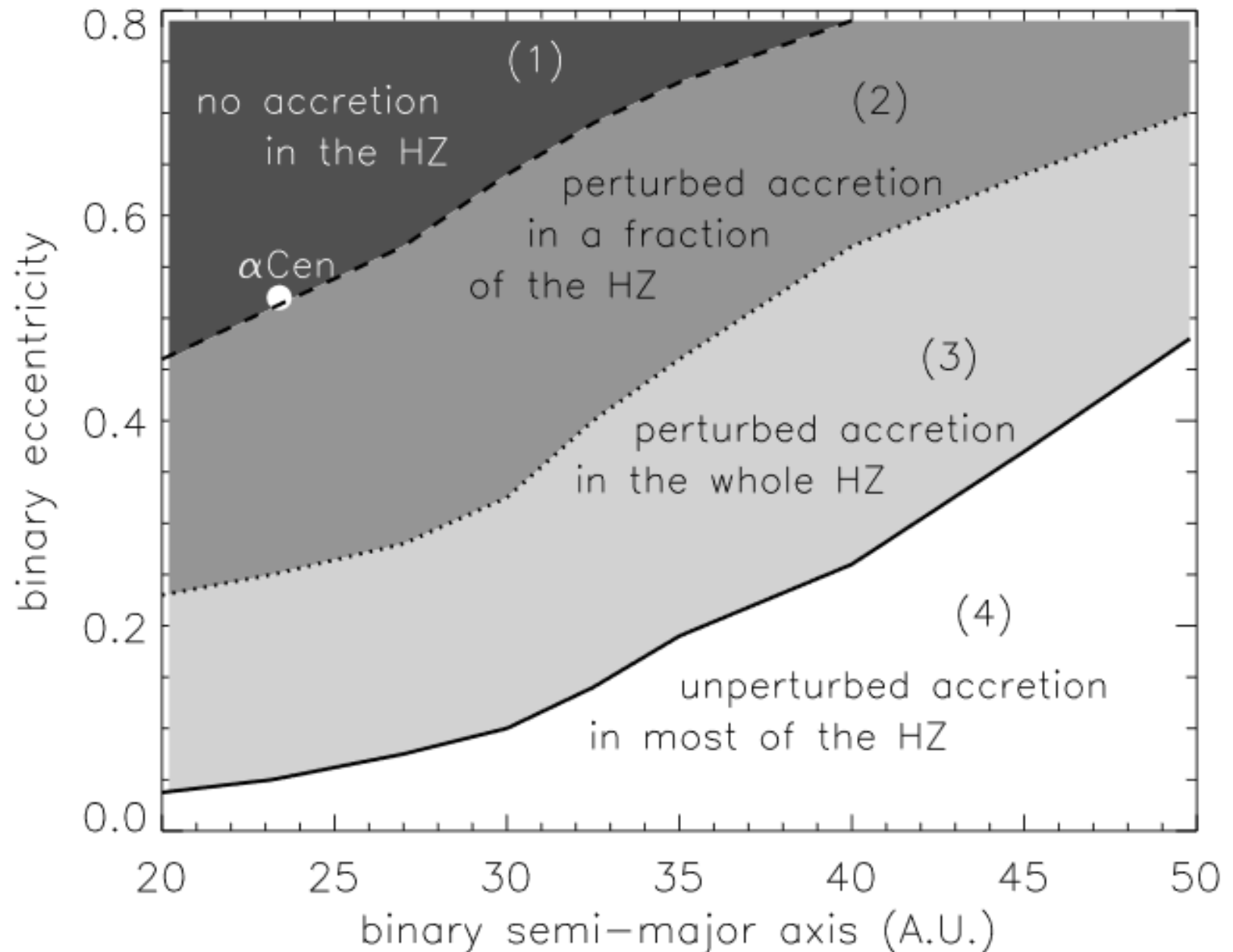
In this study it was assumed that the initial phases of planet formation have been successful.



Accretion

Accretion onto forming planets in a binary system is influenced dynamically by the companion.

High velocity collisions results in erosion, not in growth.



Other ideas for S-type

Several other ideas are discussed

- Planet migration
 - Changing of the binary separation
 - Gravitational instability
1. Planets could be formed at different orbits, and then migrate outwards. Seems problematic to increase the planetary orbit significantly.
 2. It is possible that some compact binaries have been wider, but then their orbits shrink due to interaction with stars in a cluster.
 3. Gravitational instability in a protoplanetary disc can help to form planets at larger distances avoiding problems with dust growth, accretion, etc.

Also, in multi-planets systems interaction between disk, planets and stars can result in retrograde orbits and other features (1911.00520).

Stability of orbits

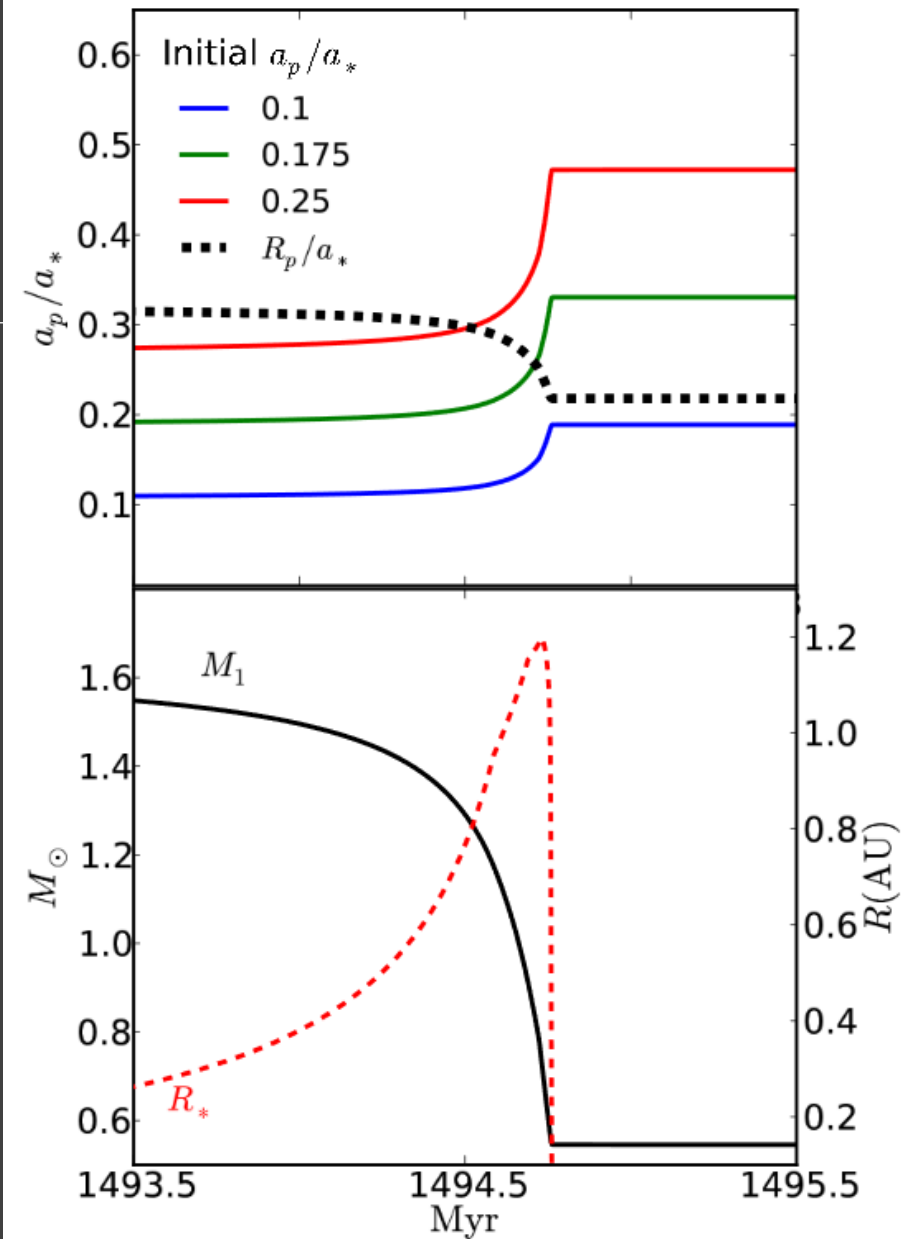
$$R_p = \left(0.464 - 0.38 \frac{m_2}{m_1 + m_2} \right) a_*$$

For slow (adiabatic) mass loss the orbit expands as:

$$a_f = \frac{M_i}{M_f} a_i,$$

If we consider the planetary mass as a very small value:

$$\left(\frac{a_{p,f}}{a_{*,f}} \right) / \left(\frac{a_{p,i}}{a_{*,i}} \right) = \left(\frac{m_{1,i}}{m_{1,f}} \right) \left(\frac{m_{1,f} + m_2}{m_{1,i} + m_2} \right)$$



Star-hoppers

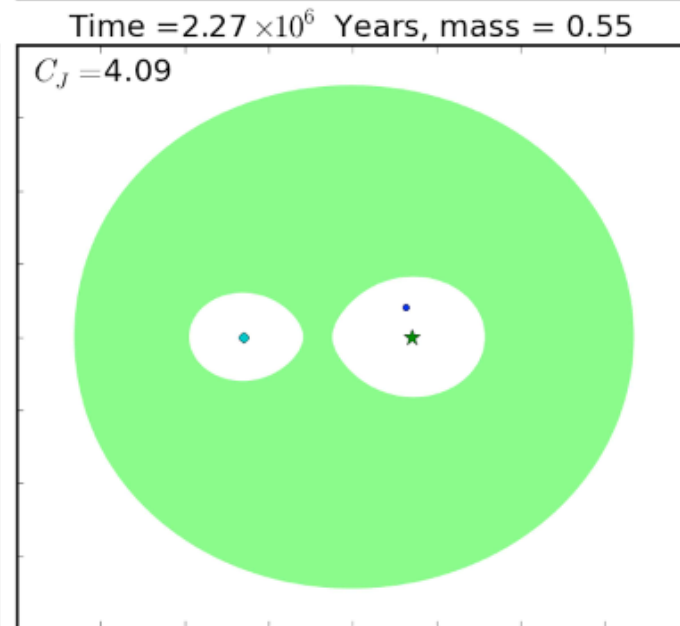
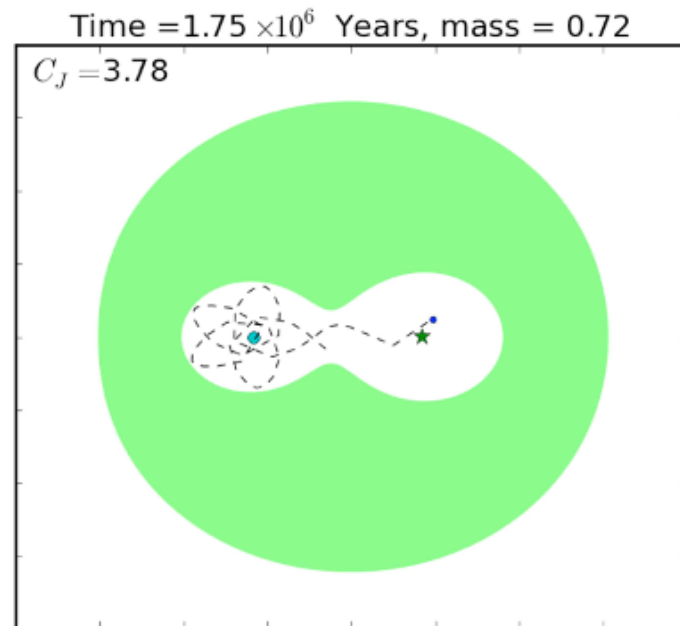
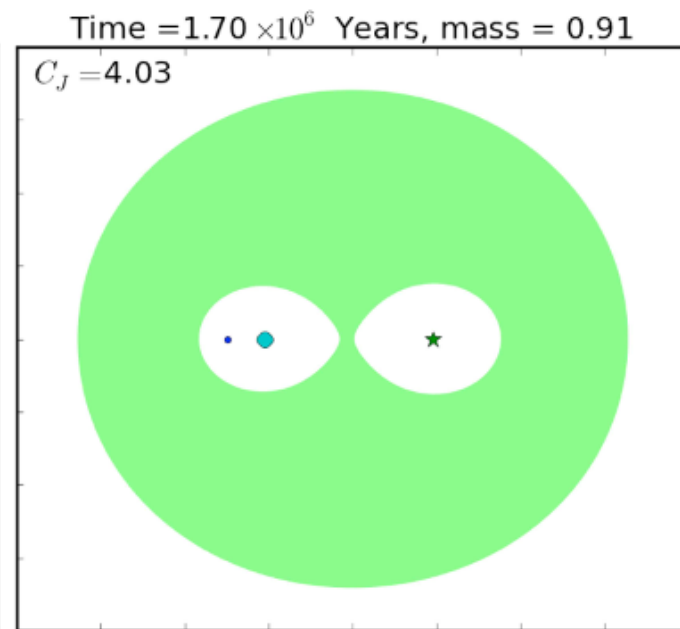
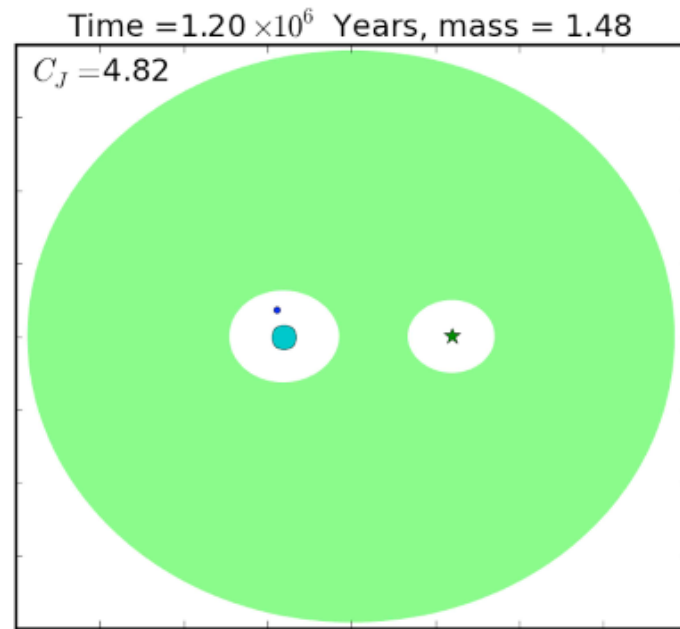
In a binary if the primary is loosing mass planets can change the host.

This is important to form some peculiar types of planets.

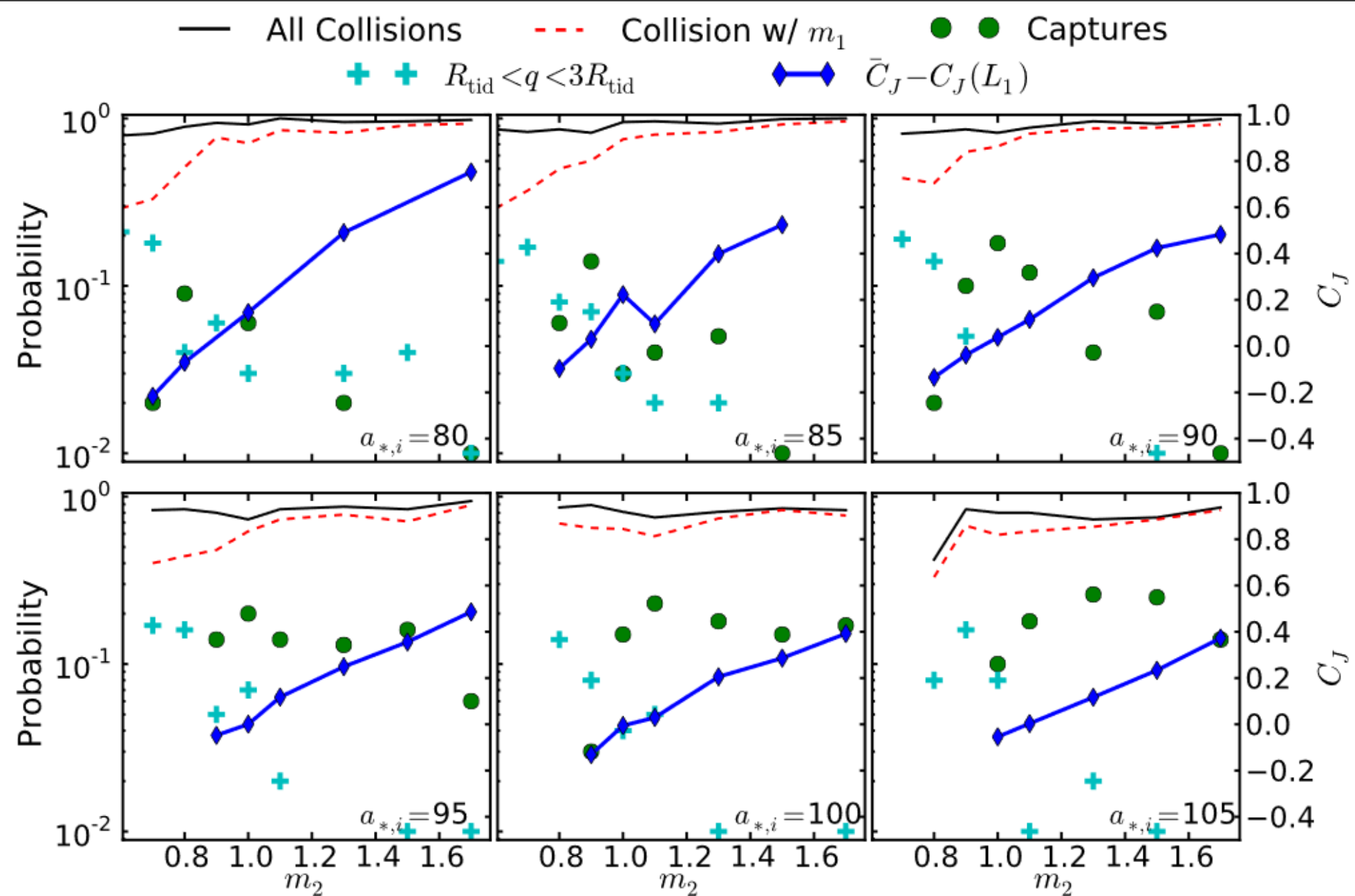
$$M_1 = 2 \rightarrow 0.55 M_{\text{solar}}$$

$$M_2 = 1 M_{\text{solar}}$$

$$a_{\text{ini}} = 90 \text{ AU}$$



Primary mass
2 solar masses.
It evolves
down to $M=0.55$.



Circumbinary (P-type) planets

$$a_{\text{crit}} \approx 1.60 + 5.10 e_{\text{bin}} - 2.22 e_{\text{bin}}^2 + 4.12 \frac{M_s}{M_p + M_s} - 4.27 e_{\text{bin}} \frac{M_s}{M_p + M_s} \\ - 5.09 \frac{M_s^2}{(M_p + M_s)^2} + 4.61 e_{\text{bin}}^2 \frac{M_s^2}{(M_p + M_s)^2},$$

Inside the critical radius
orbits of light satellites
are unstable.

A binary cleans out orbits around it up to 2-5 binary separations.

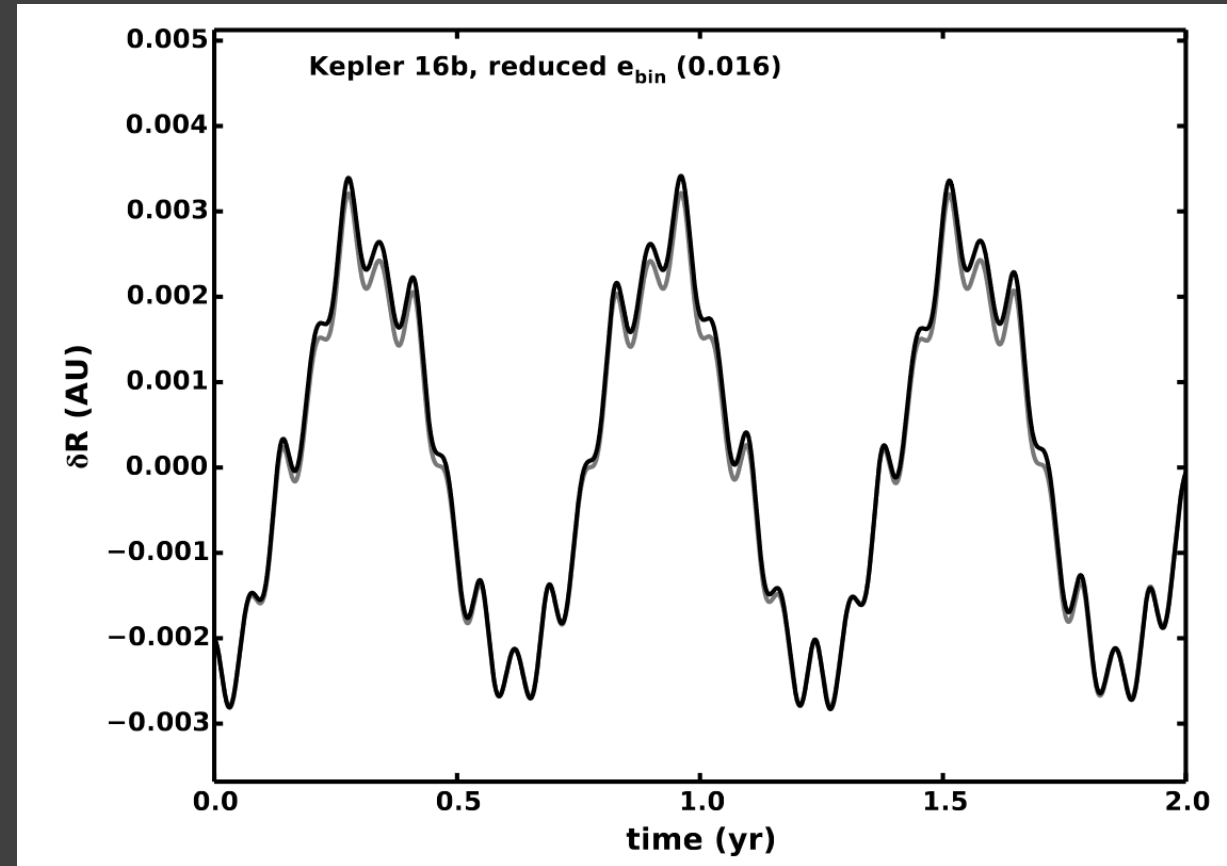
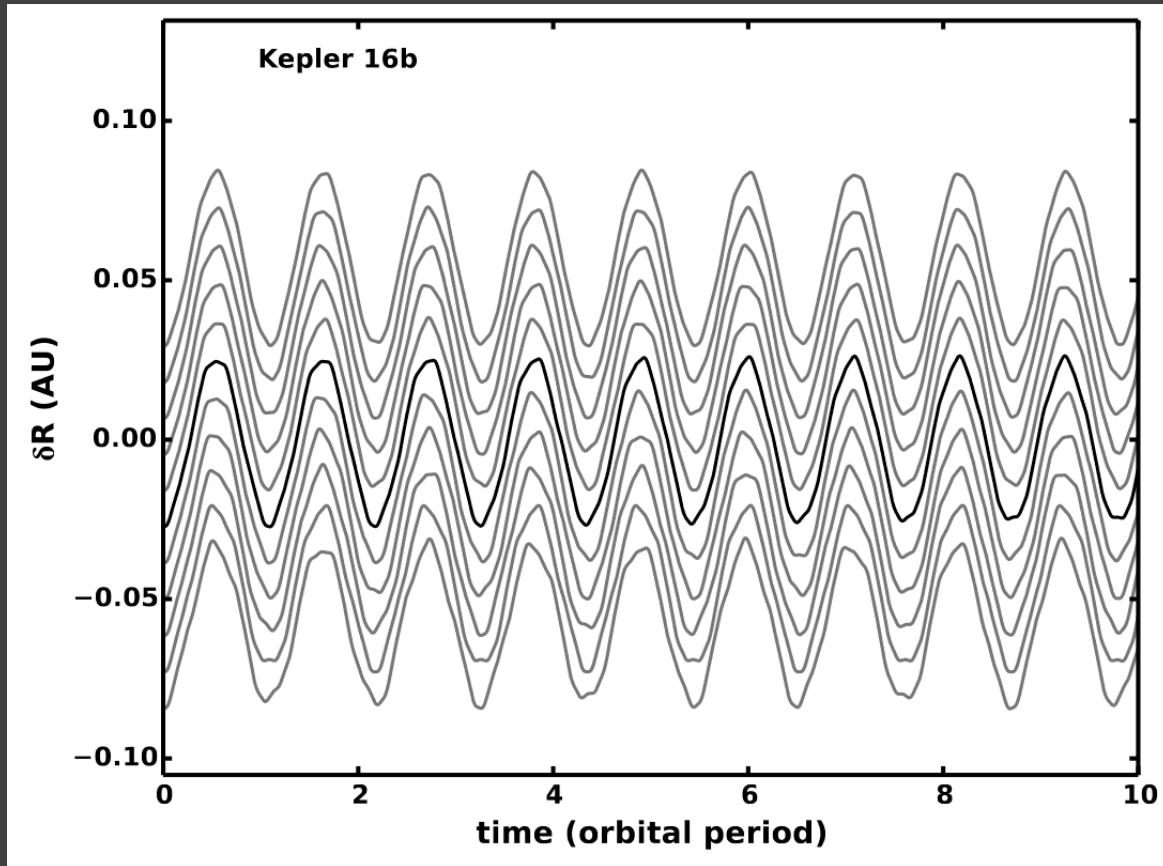
Outside critical distance (except some resonances) there is a family of nested quasi circular (most circular) orbits, which behave quite similar to orbits around single stars.

Beyond 6:1 resonance orbits are stable for small binary eccentricity.

This allows to form planets around binary stars (in the circumbinary regime) in a usual way.

Most circular orbits

Particles having these orbits make minimal radial excursions and never collide.



Planetary formation in P-type binaries

Gas and small particles quickly settle to most circular orbits.

$v_{\text{dest}} \gtrsim 0.1 \text{ km/s}$ [destructive collisions, $r = 1 \text{ km}$] In most circular orbits particles have low relative velocity.
So, collisions are not destructive.
And a set of lunar-size objects form planets as around single stars.

Still, there are problems with some known planets, as they are situated close to their hosts.

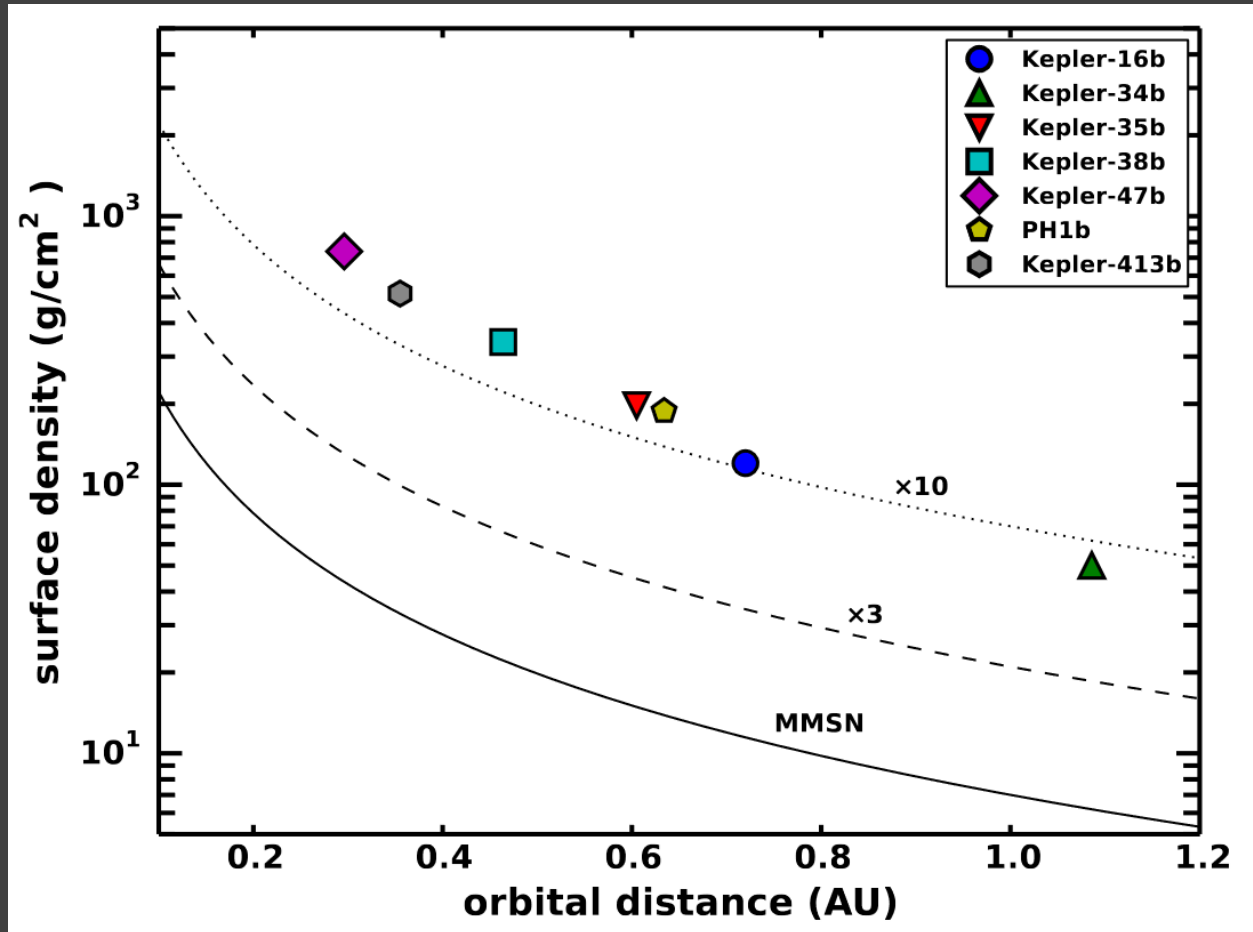
It requires too massive discs (>10 times more massive than in the classical MMSN scenario).

Four scenarios are discussed:

- *In situ* formation
- Migration – then assemble
- Migration through a gas disk
- Planet scattering

Analysis of six known planets favours
“migration –then assemble” or “disc migration”,
and in few cases – scattering, but not *in situ* formation.

In situ formation in massive discs



Minimal surface density necessary to build known planets.

Lower curve – MMSN (Hayashi 1981)

Upper – multiplied.

Typical disc masses in the MMSN 0.01 Msolar.

It is difficult to explain massive planets by *in situ* formation.

Light planets can form at small distances without migration.

Another circumbinary disc model

$$\frac{\partial \Sigma}{\partial t} = -\frac{1}{r} \frac{\partial}{\partial r} \left[\left(\frac{dl}{dr} \right)^{-1} \frac{\partial}{\partial r} \left(r^3 \nu \Sigma \frac{d\Omega}{dr} \right) + 2 \frac{\Sigma \Lambda}{\Omega} \right].$$

$$\Lambda(r) = \text{sgn}(r - a_b) f \frac{q^2 G M_c}{a_b} \left(\frac{a_b}{r - a_b} \right)^4,$$

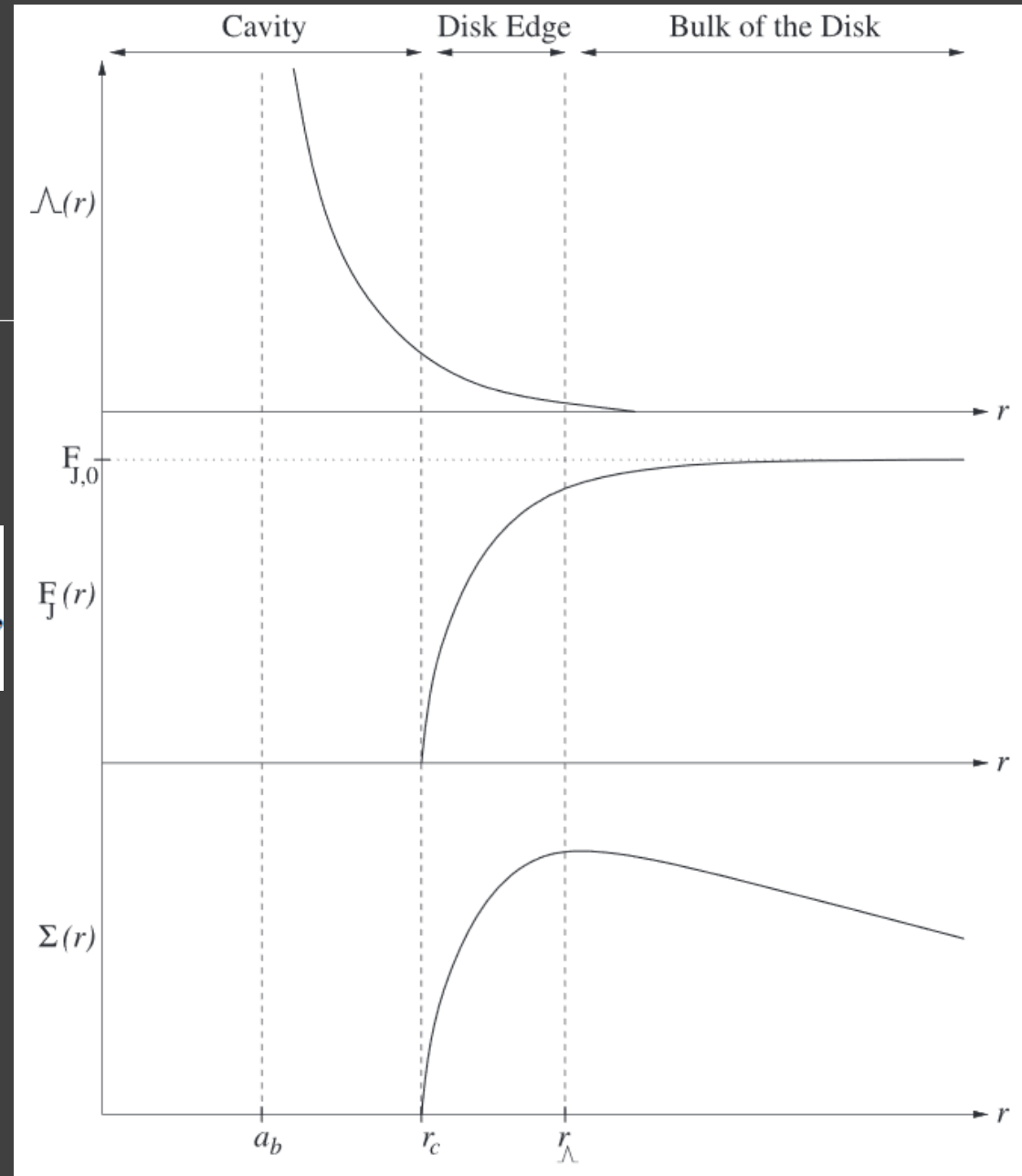
$$F_J \equiv -2\pi\nu\Sigma r^3 \frac{d\Omega}{dr} = 3\pi\nu\Sigma l,$$

Angular momentum injected to the disc from the binary

$$\frac{\partial}{\partial t} \left(\frac{F_J}{D_J} \right) = \frac{\partial}{\partial l} \left[\frac{\partial F_J}{\partial l} - \frac{2F_J}{D_J} \frac{d \ln l}{d \ln r} \Lambda(l) \right],$$

$$\dot{M}(l, t) = \frac{\partial F_J}{\partial l} - \frac{2F_J}{D_J} \frac{d \ln l}{d \ln r} \Lambda(l).$$

Accretion to the binary from the disc can be small or even zero.



Energy in the disc

Disc has three sources of energy:

- Viscosity;
- Illumination;
- Dissipation of shock generated by the binary.

$$\mathcal{F}_v = \frac{1}{4\pi r} \frac{d\dot{E}_v}{dr} = \frac{3}{8\pi} \frac{F_j \Omega}{r^2},$$

$$\mathcal{F}_{\text{irr}} = \frac{1}{2} \frac{L_c}{4\pi r^2} \zeta,$$

$$\mathcal{F}_{\text{tid}} = \frac{1}{2} (\Omega_b - \Omega) \Lambda \Sigma,$$

$$\tau \ll 1$$

$$\tau \sigma T^4 \approx \mathcal{F}_v + \mathcal{F}_{\text{tid}} + \tau \mathcal{F}_{\text{irr}}.$$

$$\tau \gg 1$$

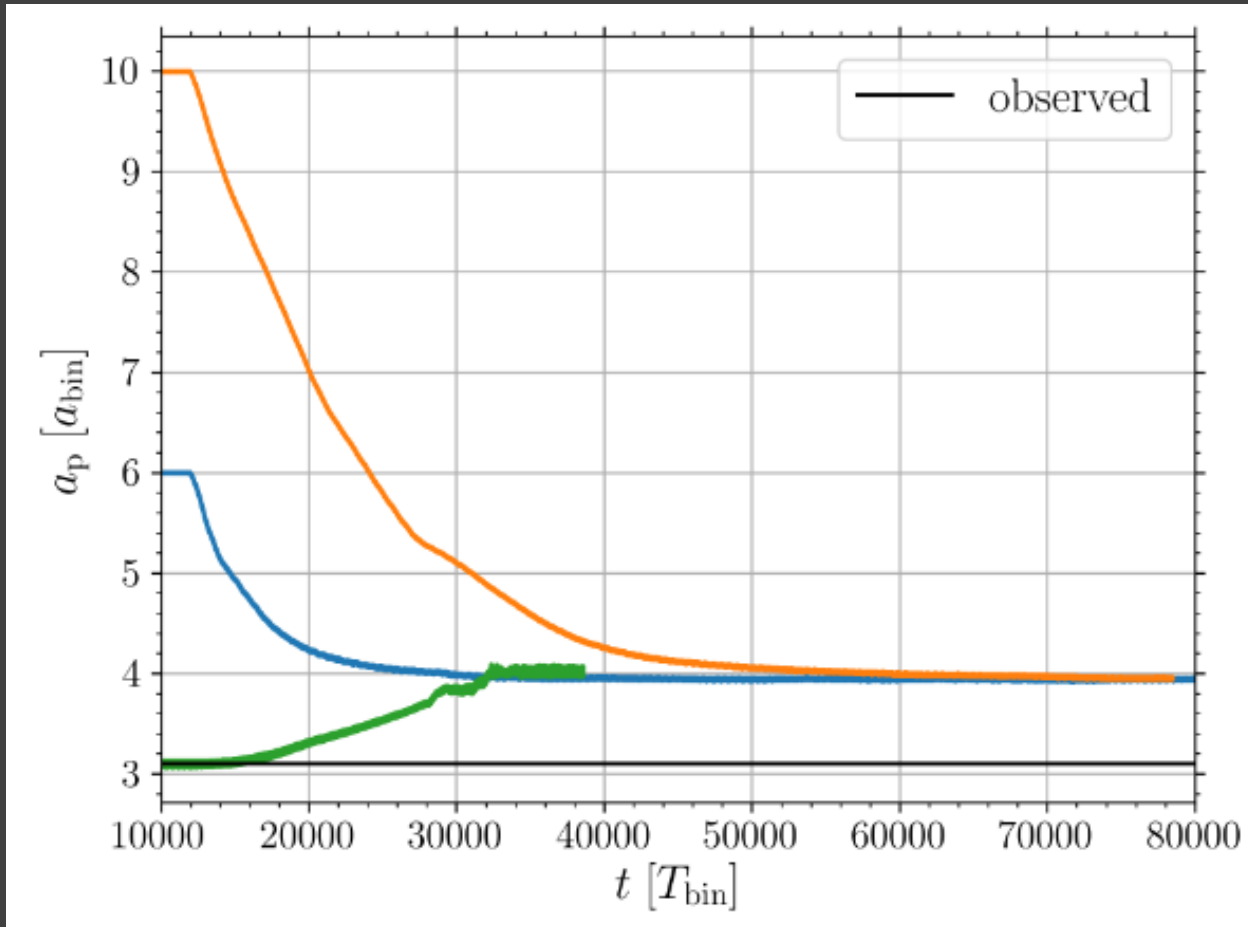
$$\sigma T^4 \approx \frac{3}{8} \tau (\mathcal{F}_v + \mathcal{F}_{\text{tid}}) + \mathcal{F}_{\text{irr}}.$$



$$\sigma T^4 = f(\tau) (\mathcal{F}_v + \mathcal{F}_{\text{tid}}) + \mathcal{F}_{\text{irr}},$$

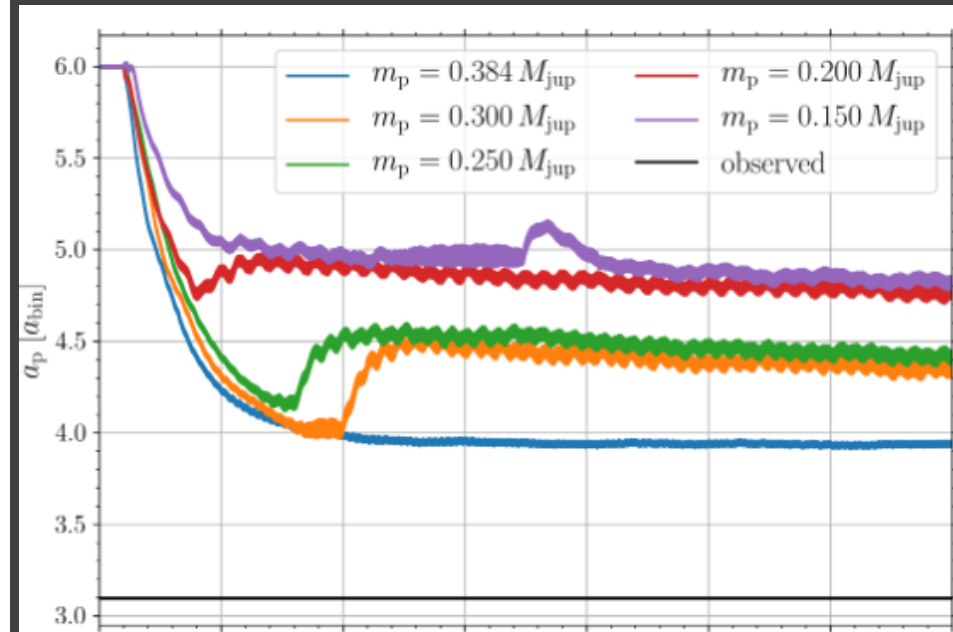
$$f(\tau) \approx \frac{3}{8} \tau + \tau^{-1}.$$

Migration of circumbinary planets



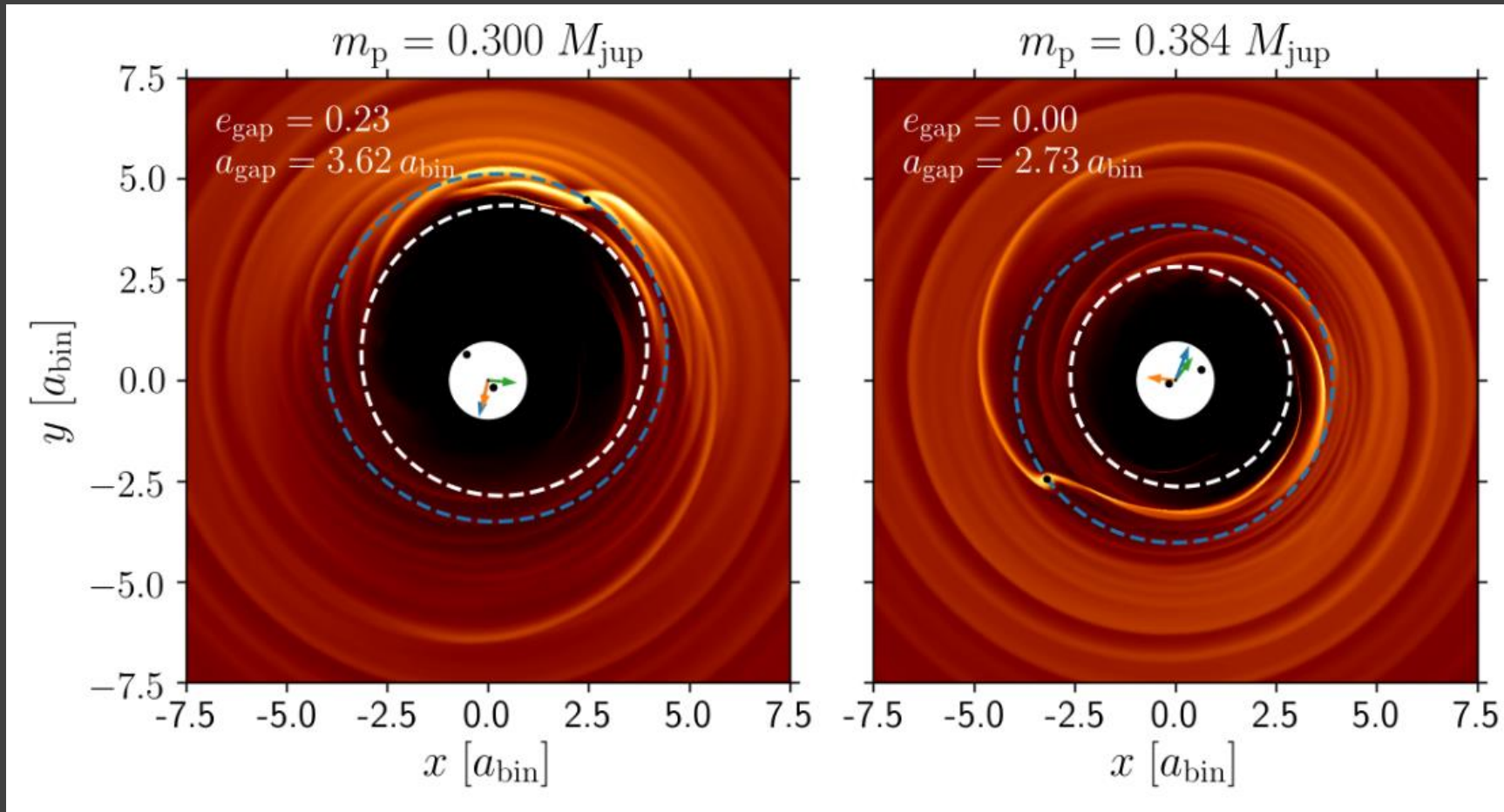
Example shows migration in a system similar to Kepler-38.

Independently on the initial position
a planet migrates towards the inner edge of the gap
formed by disc-binary interaction.



Result
slightly
depends
on the
planet
and disc
masses.

Planets in a disc around a binary



Important issues for planet formation

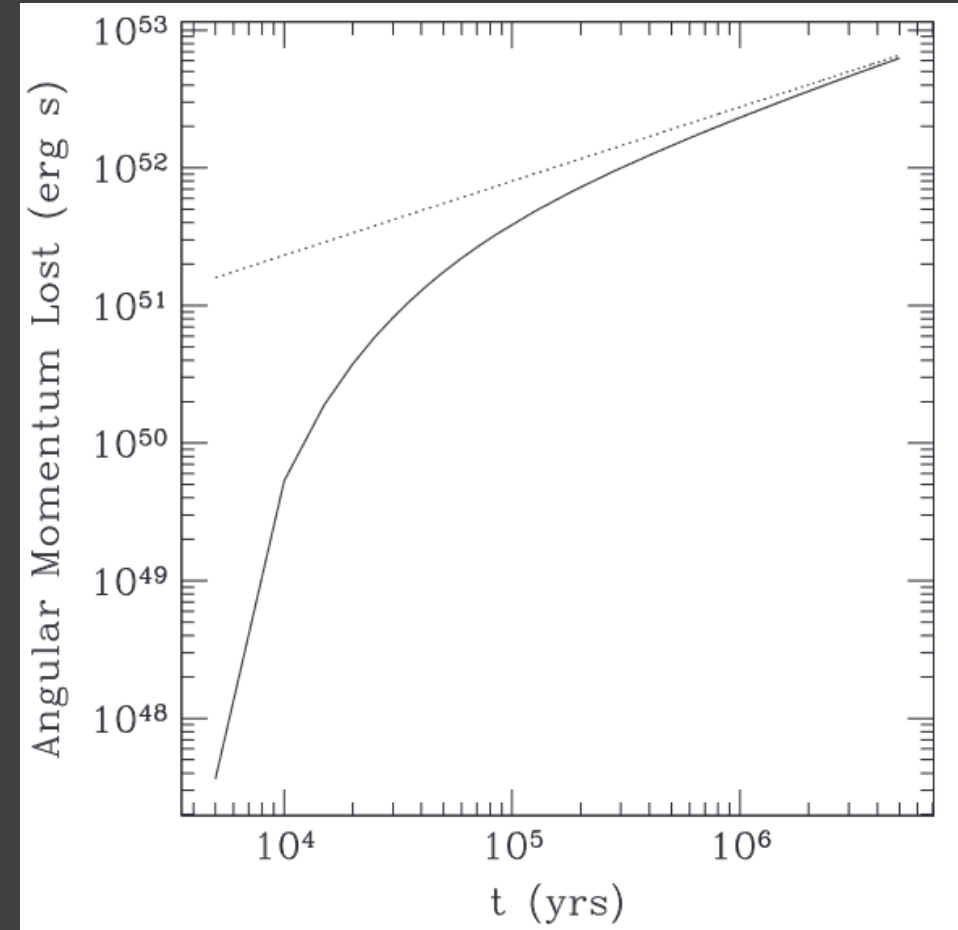
- Disc is more massive than around a single star
- Relative speeds at collisions are smaller
- Isolation masses are larger
- Ice line is shifted outwards
- Dissipation of the binary-driven density waves dominates heating of the inner disk, within 1–2AU

Circumbinary disks are in many ways more favorable sites of planet formation than their analogs around single stars

Binary evolution due to the disc

$$L_b = \frac{q}{(1+q)^2} (GM_c^3 a_b)^{1/2} \approx 4 \times 10^{52} \text{ g cm}^2 \text{ s}^{-1} \\ \times \frac{q}{(1+q)^2} M_{c,1}^{3/2} \left(\frac{a_b}{0.2 \text{ AU}} \right)^{1/2}.$$

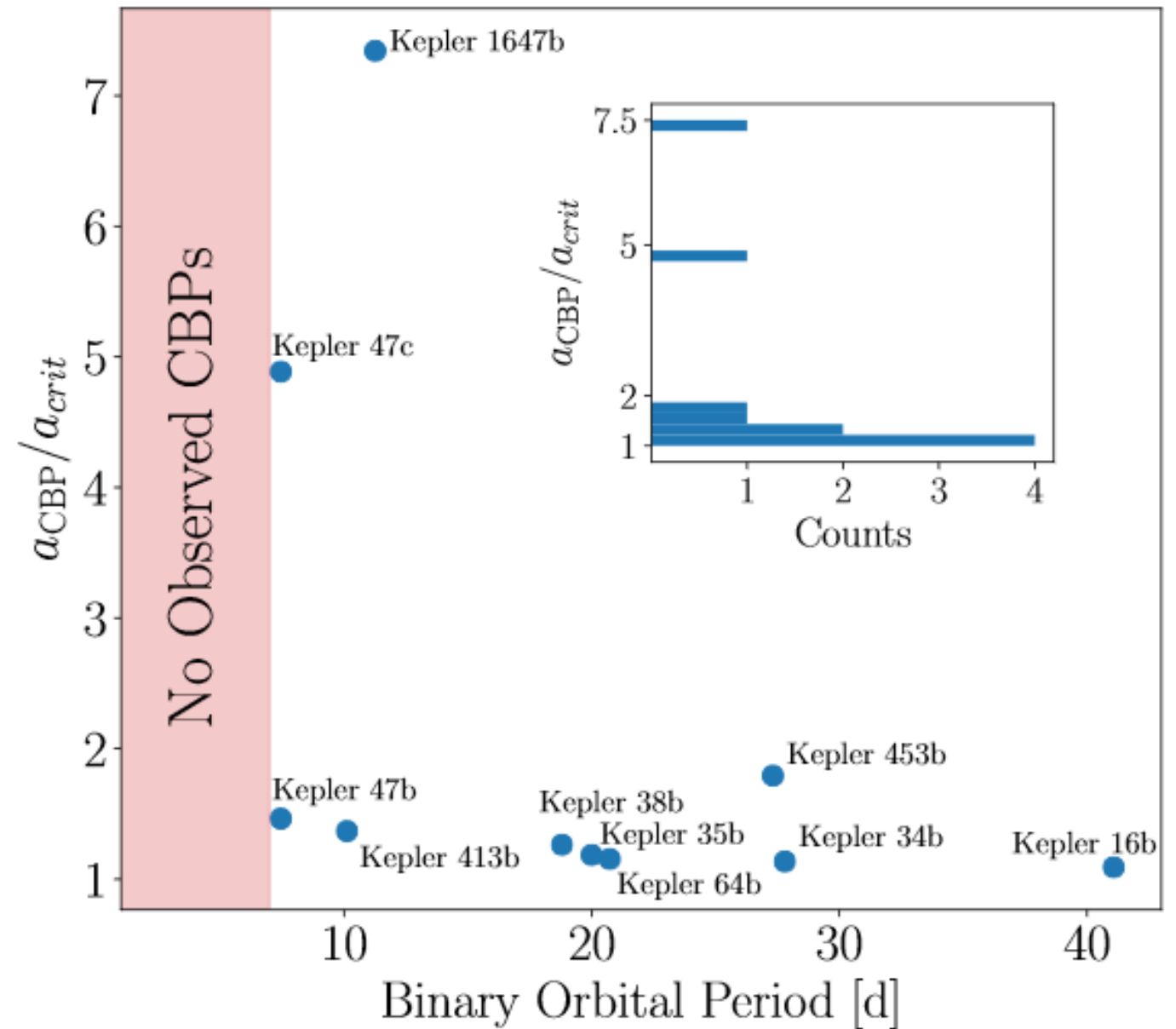
The binary can coalesce due to tidal interaction with the disc.



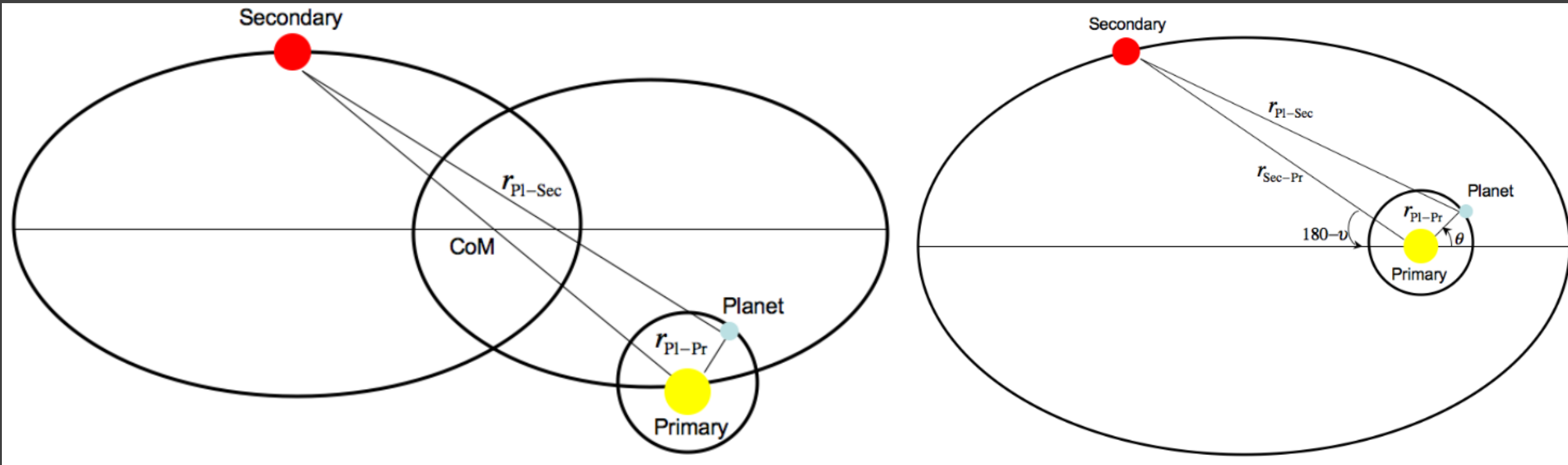
P-type systems

Absence of planets around binaries with $P < \text{few days}$ can be due to evolution of binary separation.

Mostly planets are observed close to the stability limit.



Habitable zone calculations



Calculations

$$F_{\text{Pl}}(f, T_{\text{Pr}}, T_{\text{Sec}}) = W_{\text{Pr}}(f, T_{\text{Pr}}) \frac{L_{\text{Pr}}(T_{\text{Pr}})}{r_{\text{Pl-Pr}}^2} + W_{\text{Sec}}(f, T_{\text{Sec}}) \frac{L_{\text{Sec}}(T_{\text{Sec}})}{r_{\text{Pl-Sec}}^2}.$$

$$W_{\text{Pr}}(f, T_{\text{Pr}}) \frac{L_{\text{Pr}}(T_{\text{Pr}})}{l_{\text{x-Bin}}^2} + W_{\text{Sec}}(f, T_{\text{Sec}}) \frac{L_{\text{Sec}}(T_{\text{Sec}})}{r_{\text{Pl-Sec}}^2} = \frac{L_{\text{Sun}}}{l_{\text{x-Sun}}^2}.$$

$$l_{\text{x-Star}} = l_{\text{x-Sun}} \left[\frac{L/L_{\text{Sun}}}{1 + \alpha_{\text{x}}(T_i) l_{\text{x-Sun}}^2} \right]^{1/2}, \quad l_{\text{x}} = (l_{\text{in}}, l_{\text{out}}) \text{ is in AU, } T_i(\text{K}) = T_{\text{Star}}(\text{K}) - 5780.$$

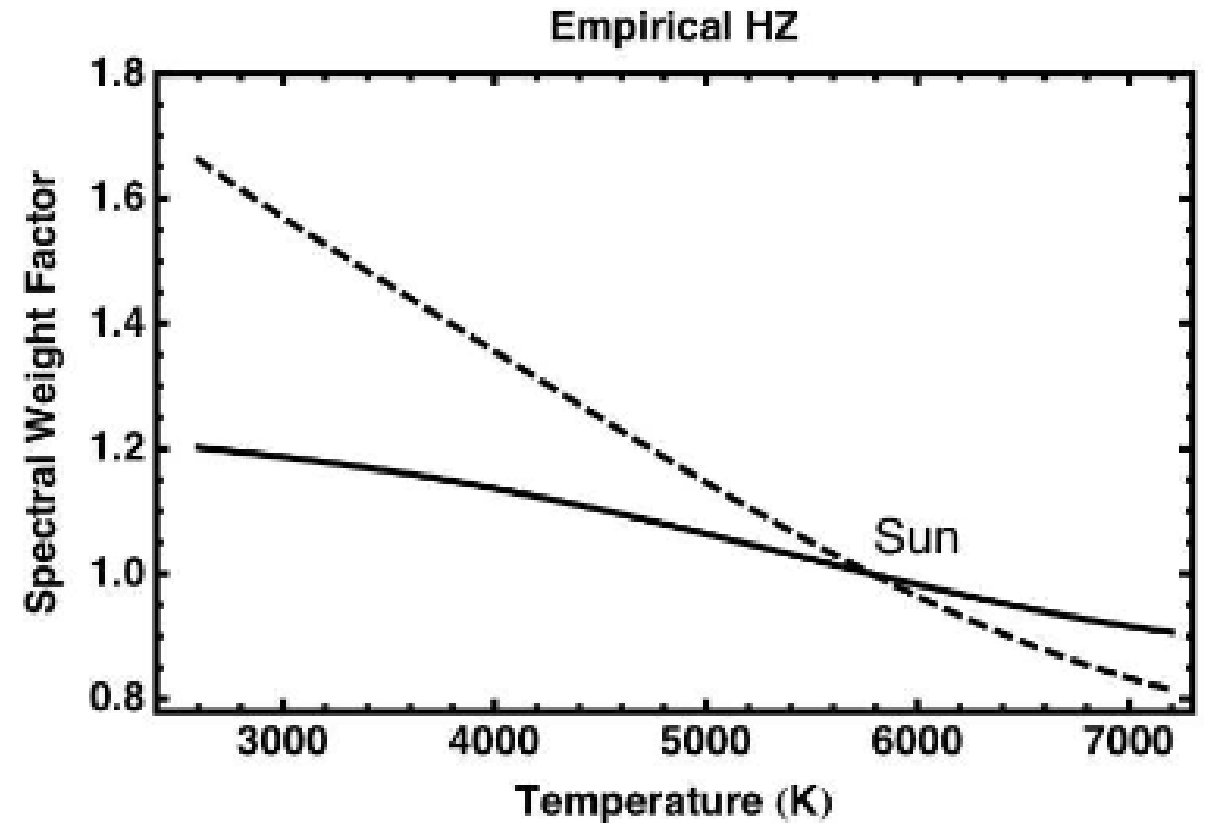
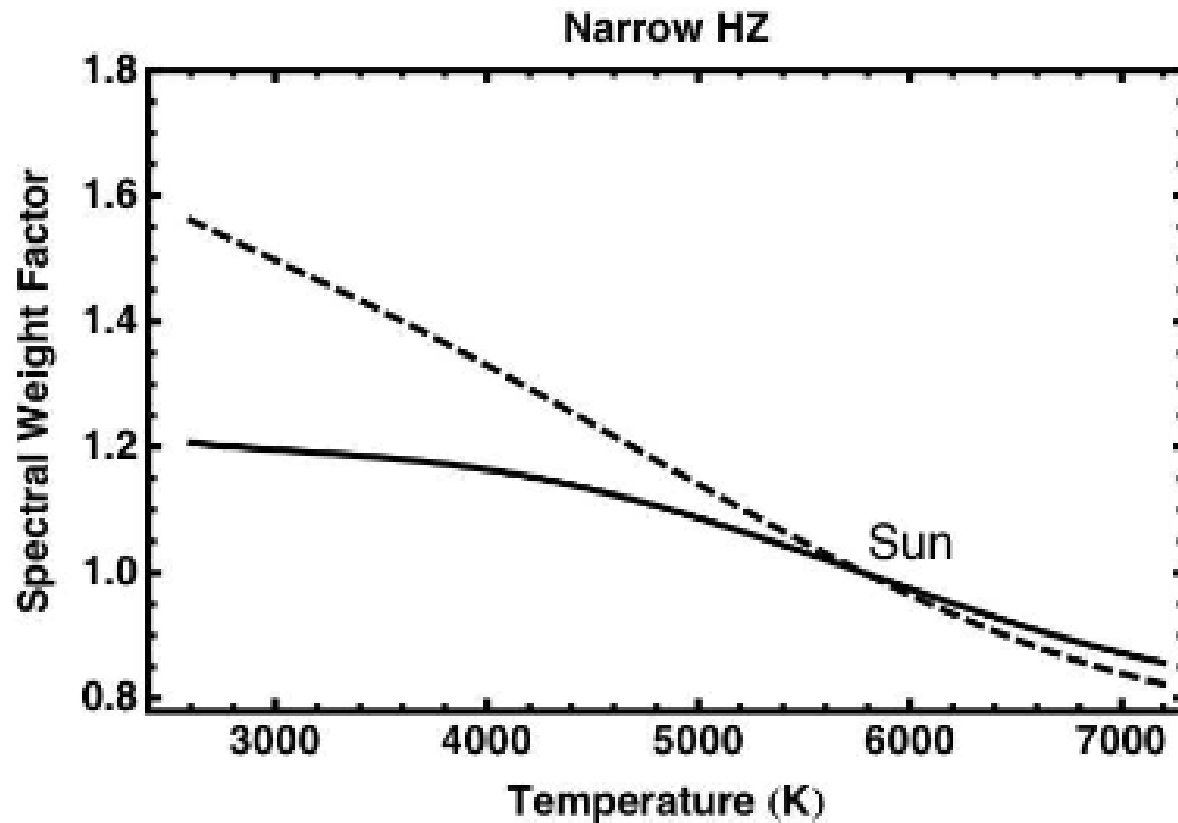
$$\alpha_{\text{x}}(T_i) = a_{\text{x}} T_i + b_{\text{x}} T_i^2 + c_{\text{x}} T_i^3 + d_{\text{x}} T_i^4,$$

	Narrow HZ		Empirical HZ	
	Runaway Greenhouse	Maximum Greenhouse	Recent Venus	Early Mars
$l_{\text{x-Sun}}$ (AU)	0.97	1.67	0.75	1.77
Flux (Solar Flux @ Earth)	1.06	0.36	1.78	0.32
a	1.2456×10^{-4}	5.9578×10^{-5}	1.4335×10^{-4}	5.4471×10^{-5}
b	1.4612×10^{-8}	1.6707×10^{-9}	3.3954×10^{-9}	1.5275×10^{-9}
c	-7.6345×10^{-12}	-3.0058×10^{-12}	-7.6364×10^{-12}	-2.1709×10^{-12}
d	-1.7511×10^{-15}	-5.1925×10^{-16}	-1.1950×10^{-15}	-3.8282×10^{-16}

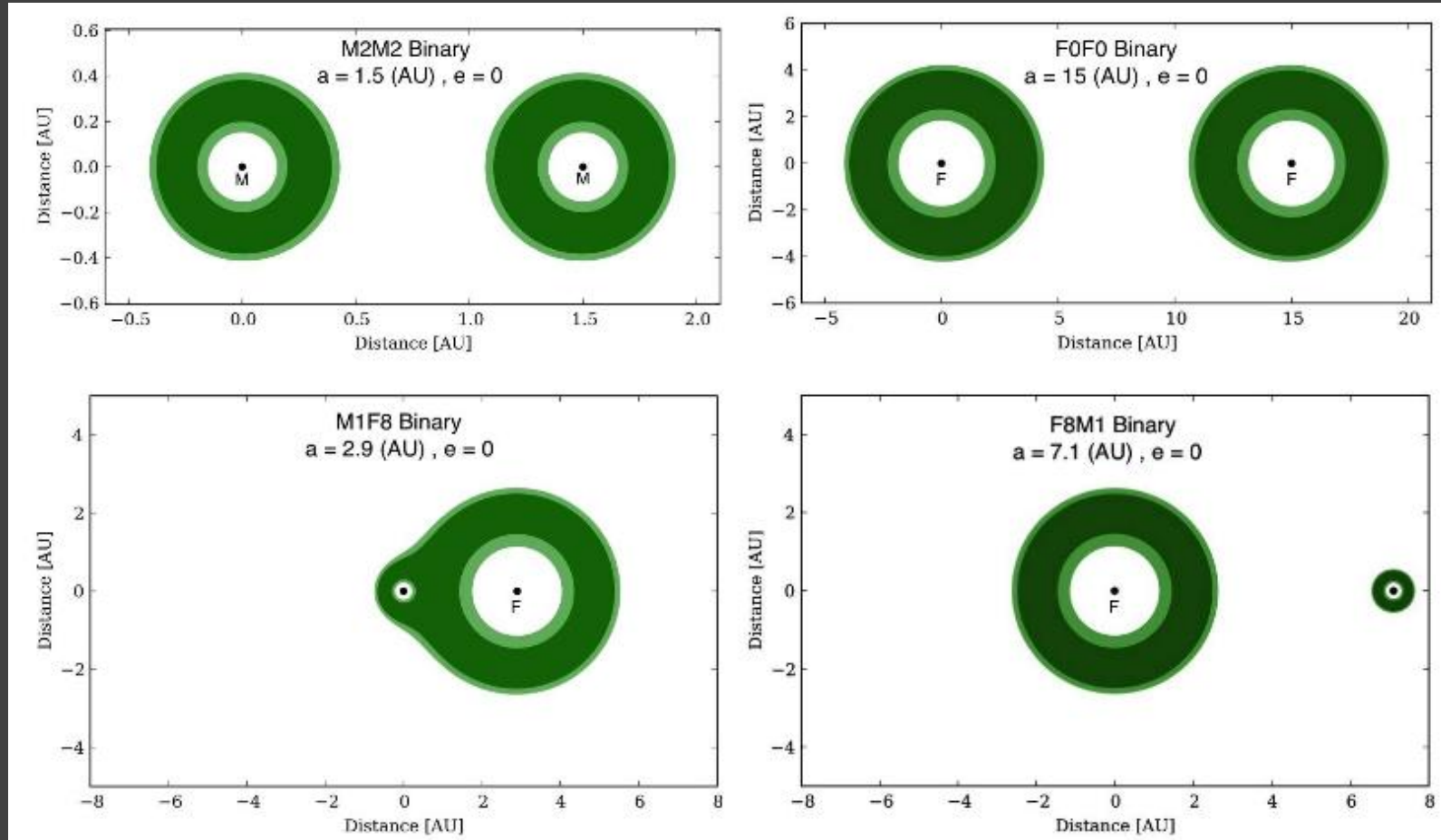
$$F_{\text{x-Star}}(f, T_{\text{Star}}) = F_{\text{x-Sun}}(f, T_{\text{Star}}) \left[1 + \alpha_{\text{x}}(T_i) l_{\text{x-Sun}}^2 \right]$$

$$W_i(f, T_i) = \left[1 + \alpha_{\text{x}}(T_i) l_{\text{x-Sun}}^2 \right]^{-1}$$

Weight coefficients



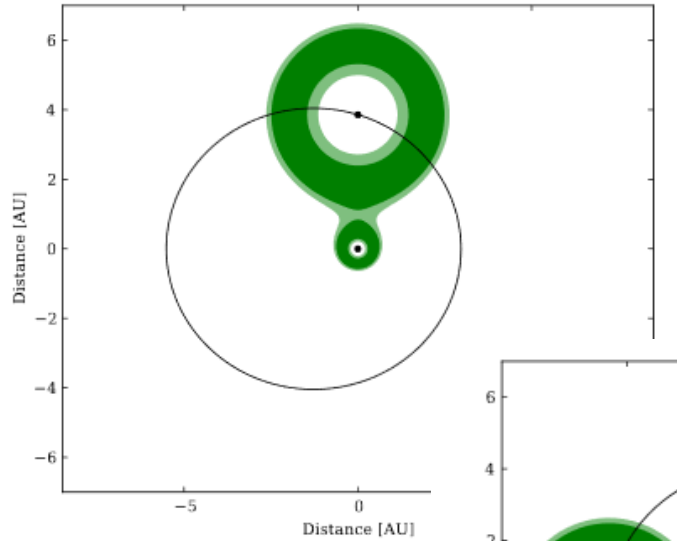
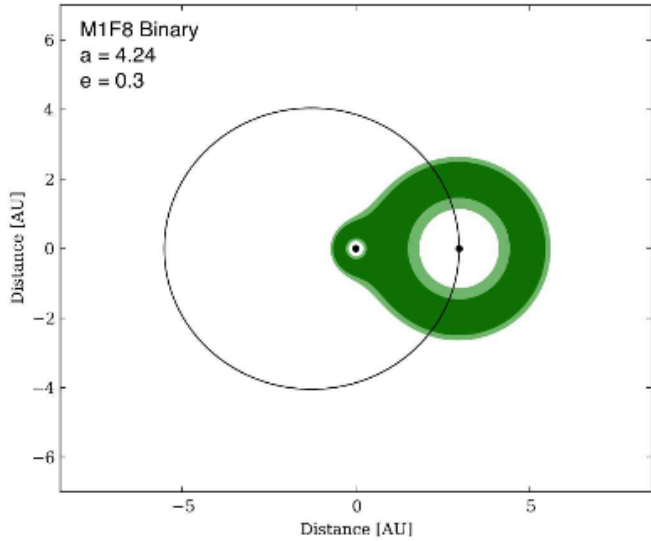
Examples



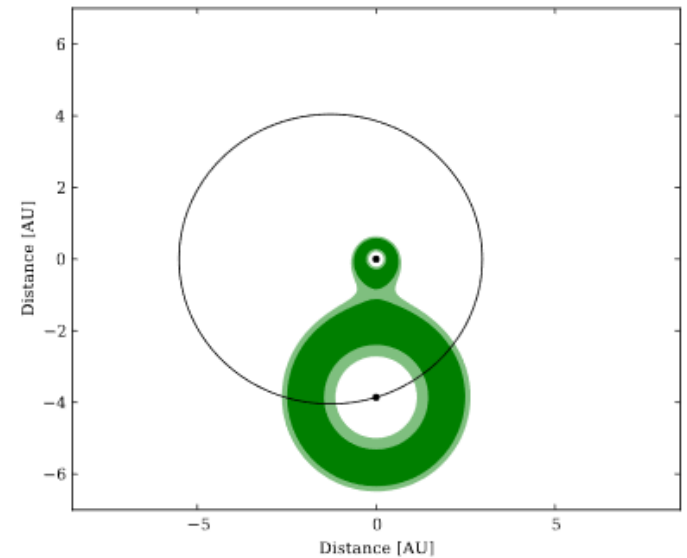
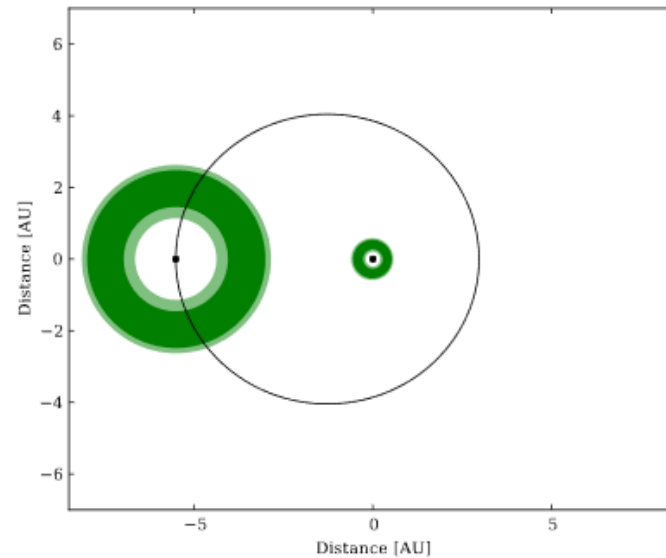
Dark green – narrow HZ.

Light green- empirical HZ.

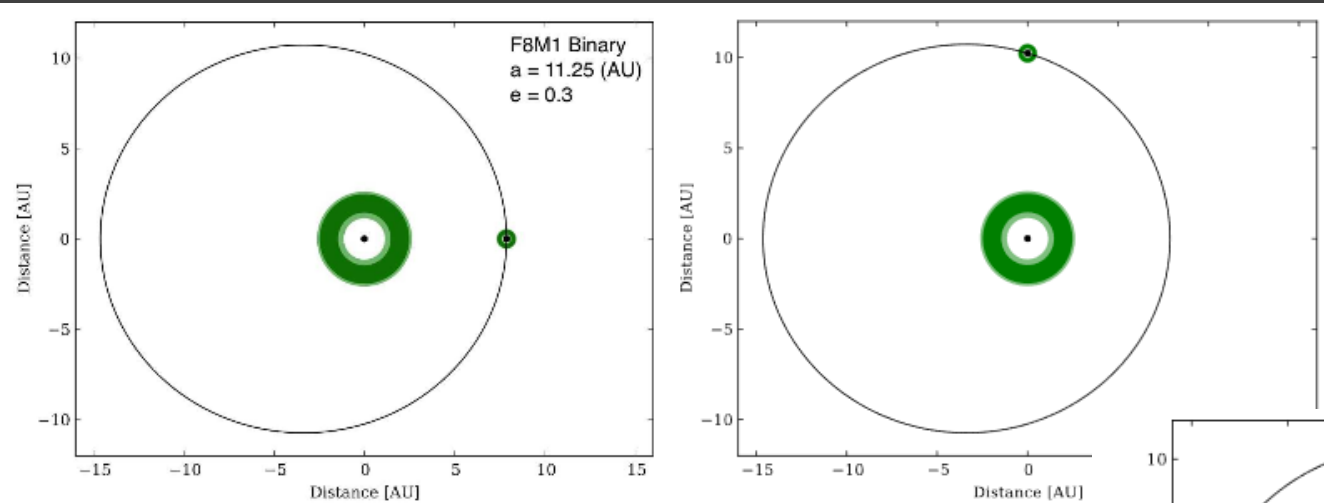
Examples: eccentricity=0.3



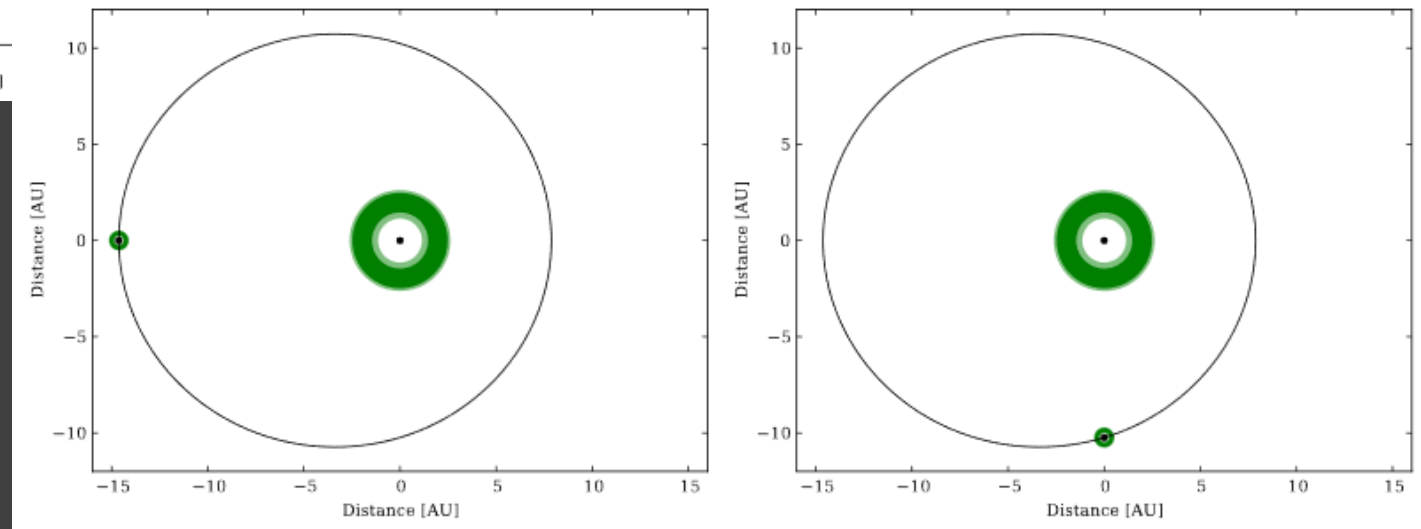
M-dwarf is the primary



Examples: eccentricity=0.3



F-star is the primary



Habitable zone calculation. II. Circumbinary

Let us start with single stars

$$L_{\star} = 4\pi R_{\star}^2 \sigma T_{\text{eff}}^4$$

$$S_{\text{inner}} = 4.190 \times 10^{-8} T_{\text{eff}}^2 - 2.139 \times 10^{-4} T_{\text{eff}} + 1.268$$

$$S_{\text{outer}} = 6.190 \times 10^{-9} T_{\text{eff}}^2 - 1.319 \times 10^{-5} T_{\text{eff}} + 0.2341$$

$$r_{\text{inner}} = \sqrt{L_{\star} / S_{\text{inner}}}$$
$$r_{\text{outer}} = \sqrt{L_{\star} / S_{\text{outer}}}$$

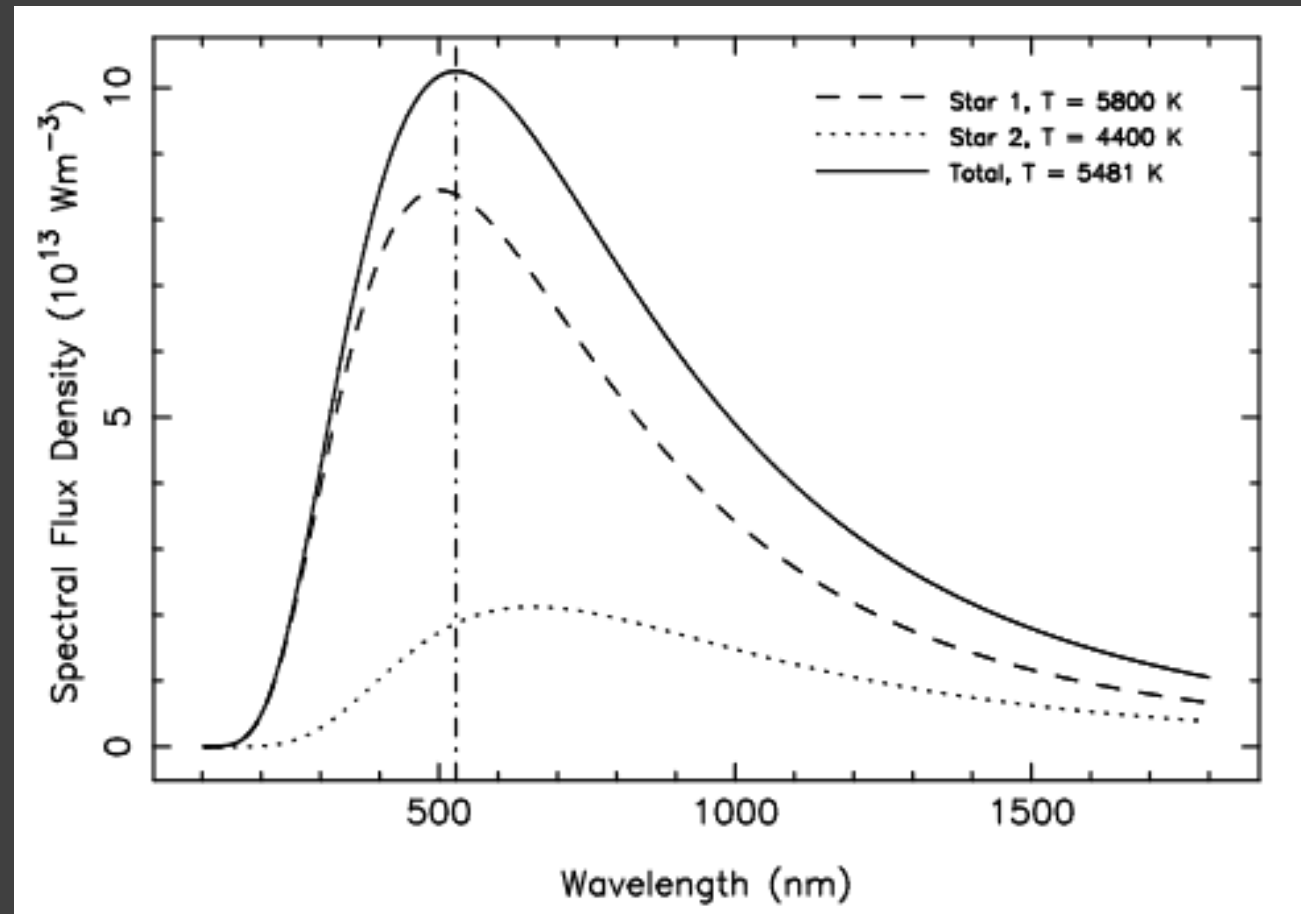
Here luminosity and flux are in solar units, and distance – in AU.

Binary stars

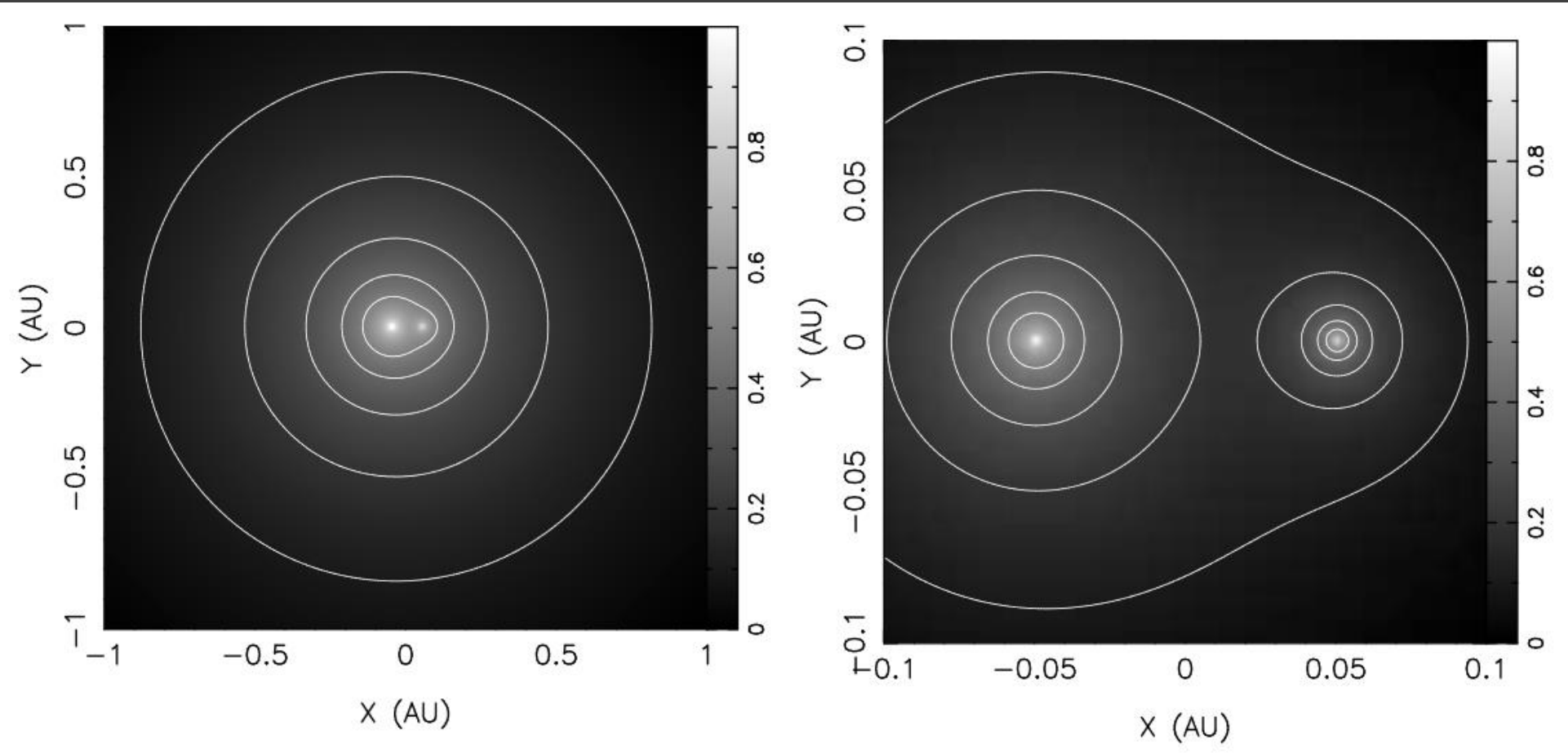
$$S_1 = \frac{2\pi hc^2}{\lambda^5} \frac{1}{e^{hc/\lambda k T_{\text{eff},1}} - 1} \left(\frac{R_{*,1}}{r_1} \right)^2$$
$$S_2 = \frac{2\pi hc^2}{\lambda^5} \frac{1}{e^{hc/\lambda k T_{\text{eff},2}} - 1} \left(\frac{R_{*,2}}{r_2} \right)^2$$

$$S = S_1 + S_2$$

G2V+K5V
0.1 AU



Stellar flux map

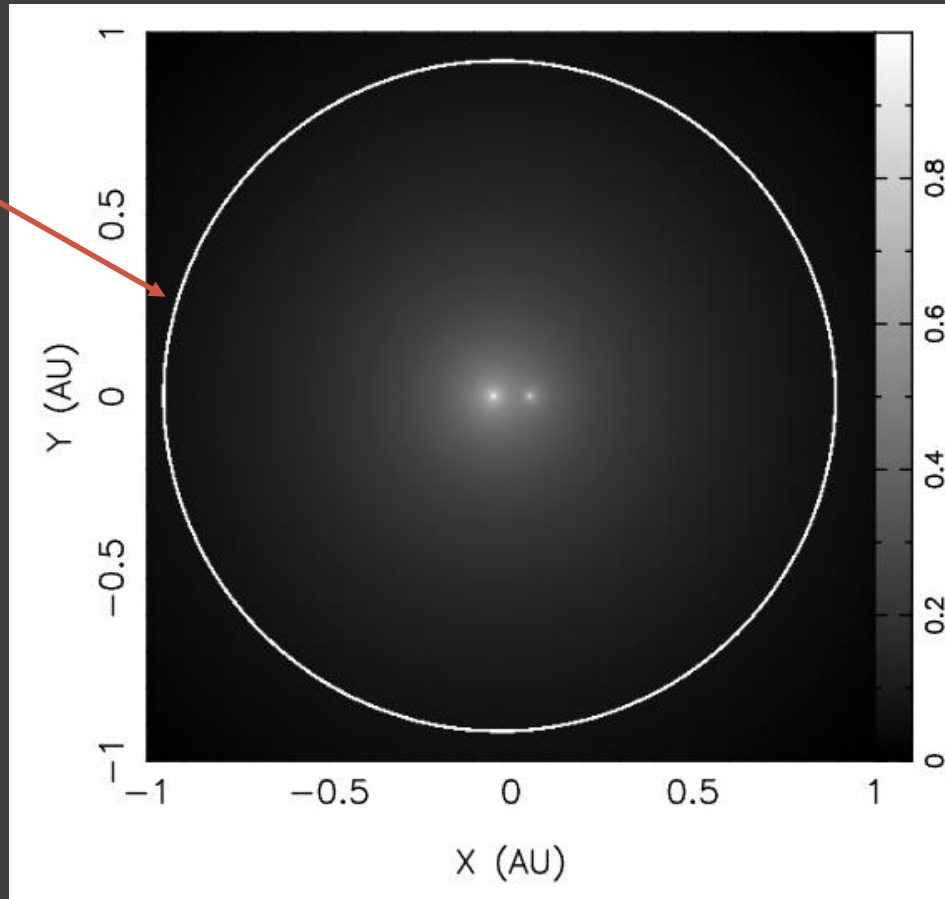


G2V+K5V
0.1 AU

HZ edges

Inner edge

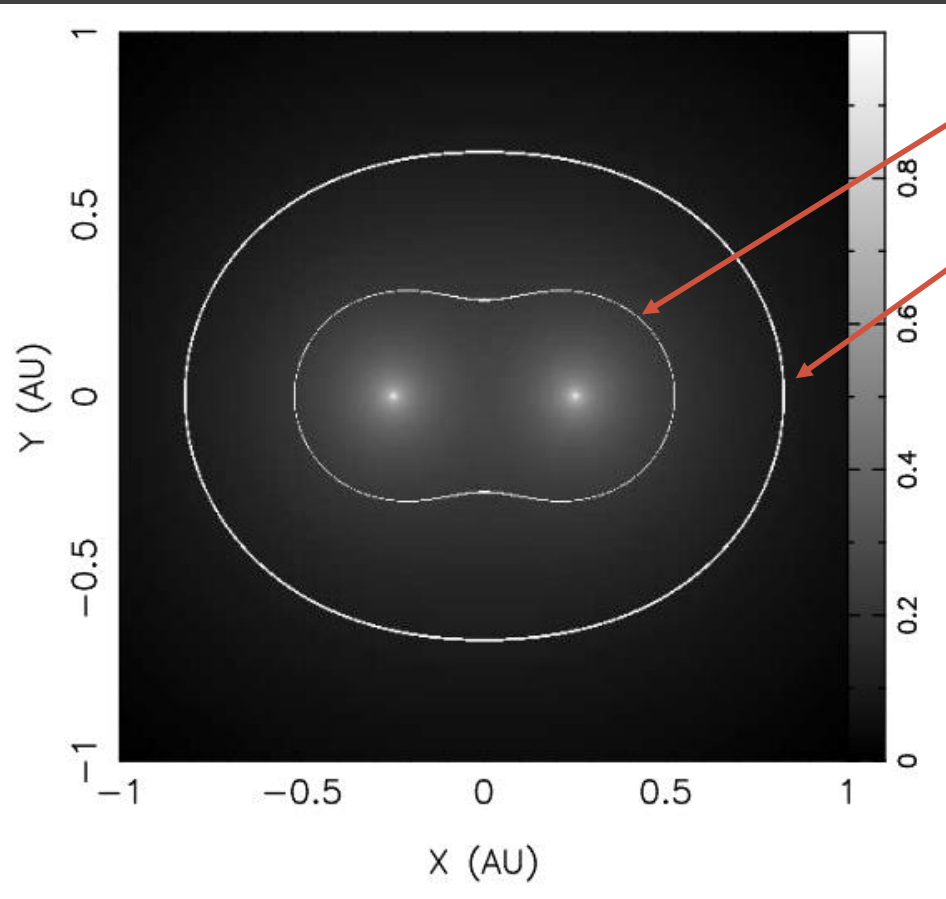
G2V+K5V
0.1 AU



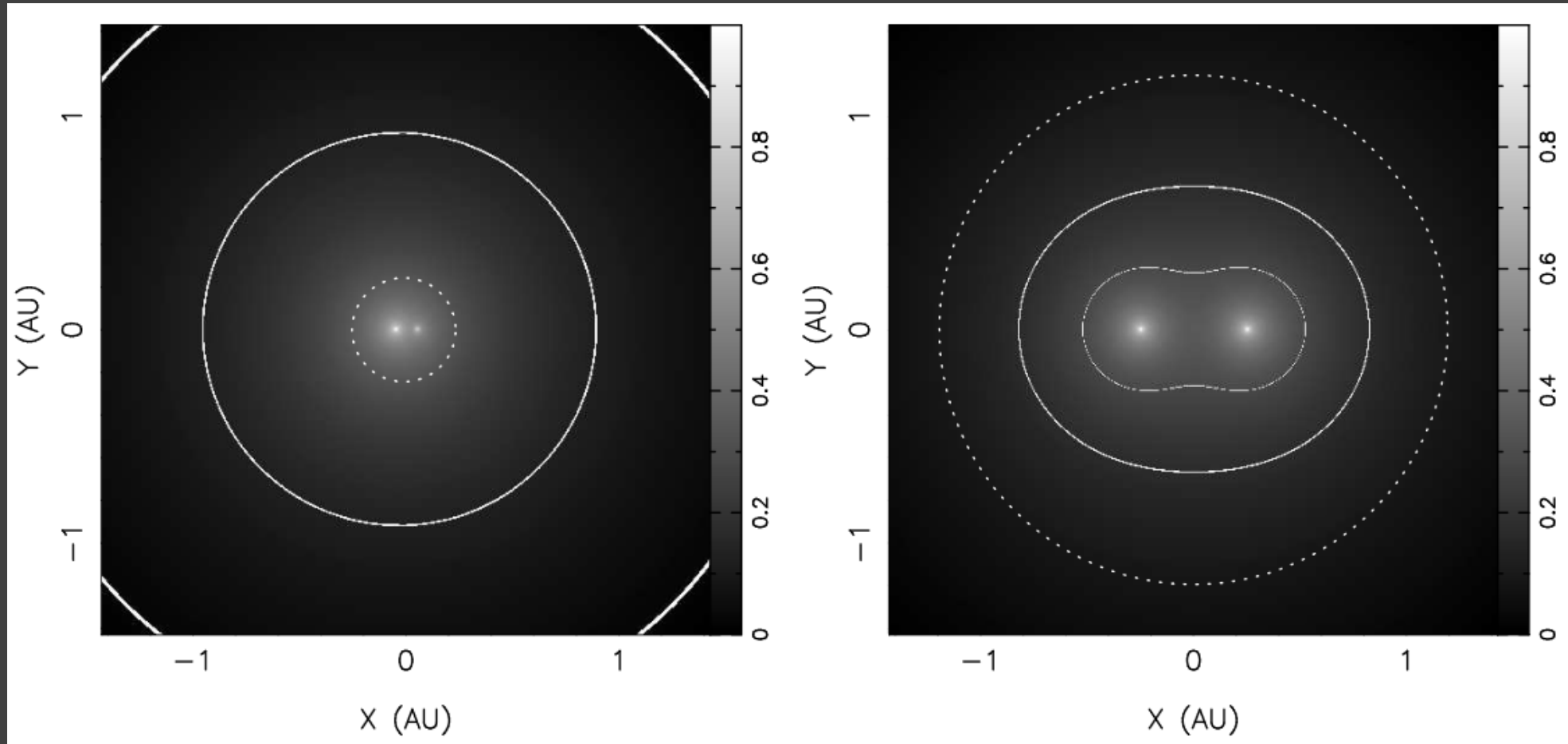
Inner edge

Outer edge

MOV+MOV
0.5 AU



Stability of the planetary orbit



On-line calculator

Described in 1401.0601

T, L, and M can be changed independently
(i.e., there is not fit for the MS, etc.)

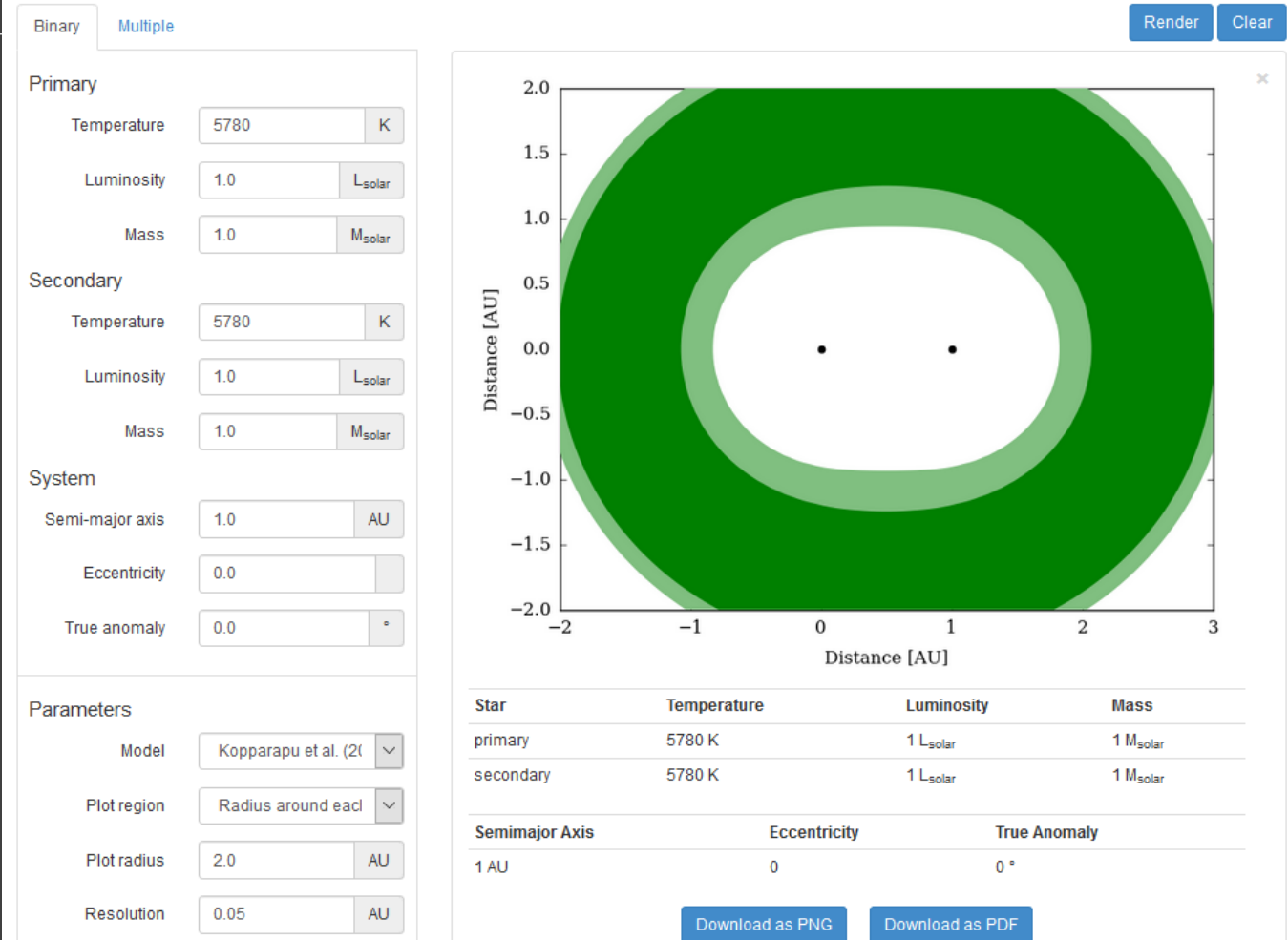
Habitable Zones in Multiple Star Systems

Using this website, you can calculate the habitable zones of single, binary and multiple star systems (for single stars use the multiple star option with only one star. You can then compare the results with the [HZ Gallery](#) and the [HZ Calculator](#)). The methodology for calculating the HZ is described in [Müller & Haghighipour \(2014\)](#). The HZ can be calculated using the models by [Kopparapu et al \(2014\)](#) (assuming $M_{\text{planet}} = 1 M_{\text{Earth}}$), [Kopparapu et al \(2013\)](#), [Selsis et al \(2007\)](#), or [Kasting et al \(1993\)](#). The stability radii in the binary cases are calculated using the formulae given by [Holman & Wiegert \(1999\)](#).

Movies of time-dependent habitable zones can be found at <http://astro.twam.info/hz-ptype> and <http://astro.twam.info/hz-multi>.

If you encounter any problems while using this website please contact Tobias_Mueller@twam.info.

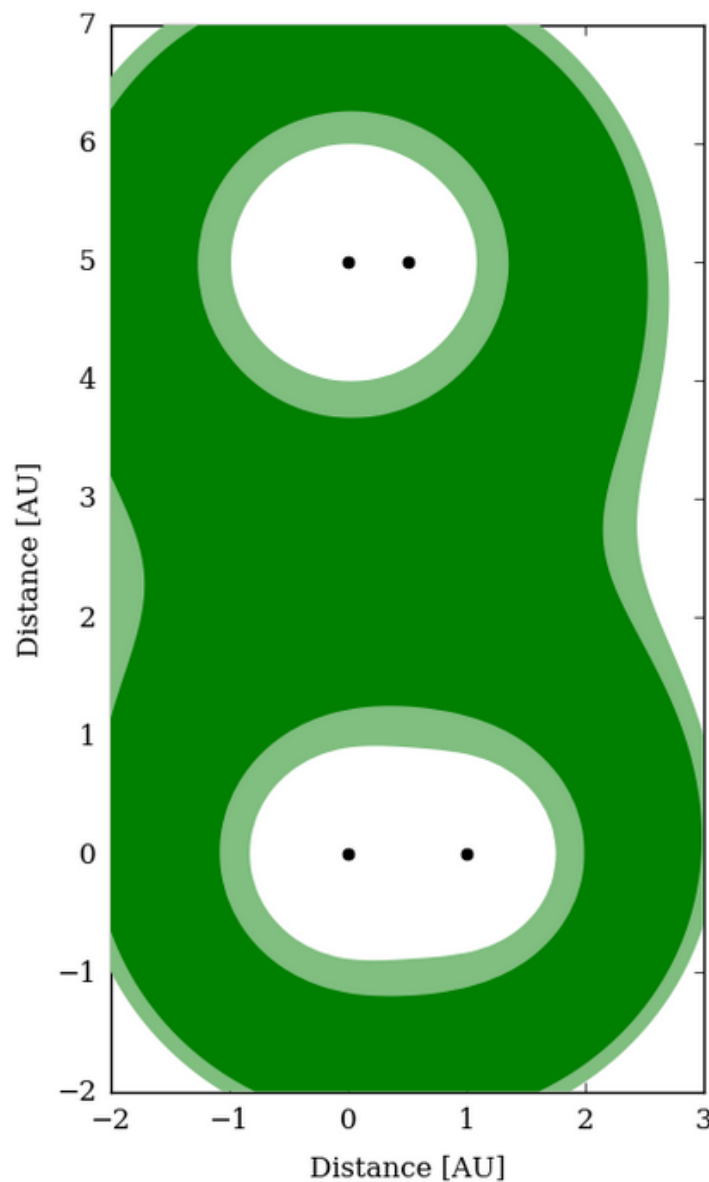
You are welcome to use any of the figures created with this website in your papers, presentations and for teaching. In that case, we ask you that you kindly cite the paper [Müller & Haghighipour \(2014\)](#), and mention the URL address to the website.



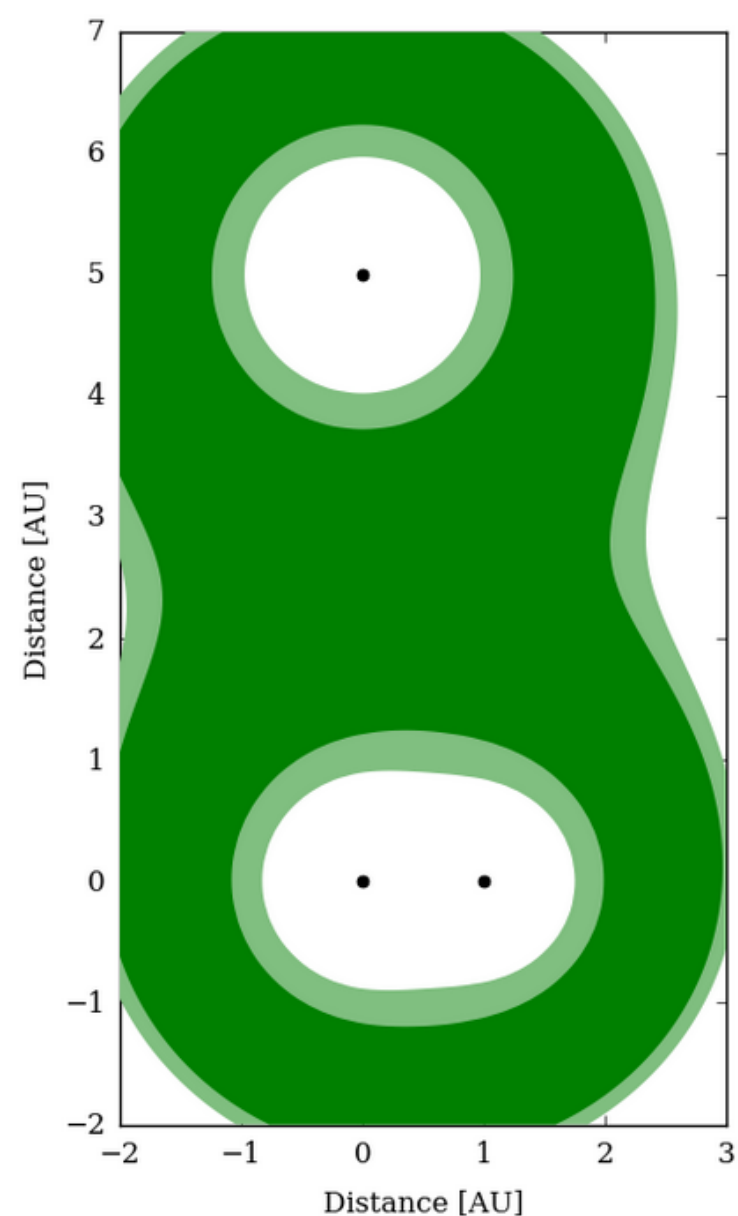
Multiple systems

The method allows to make plots for any number of stars.

However, consistency of all conditions (orbital stability, etc.) is not automatically controlled.

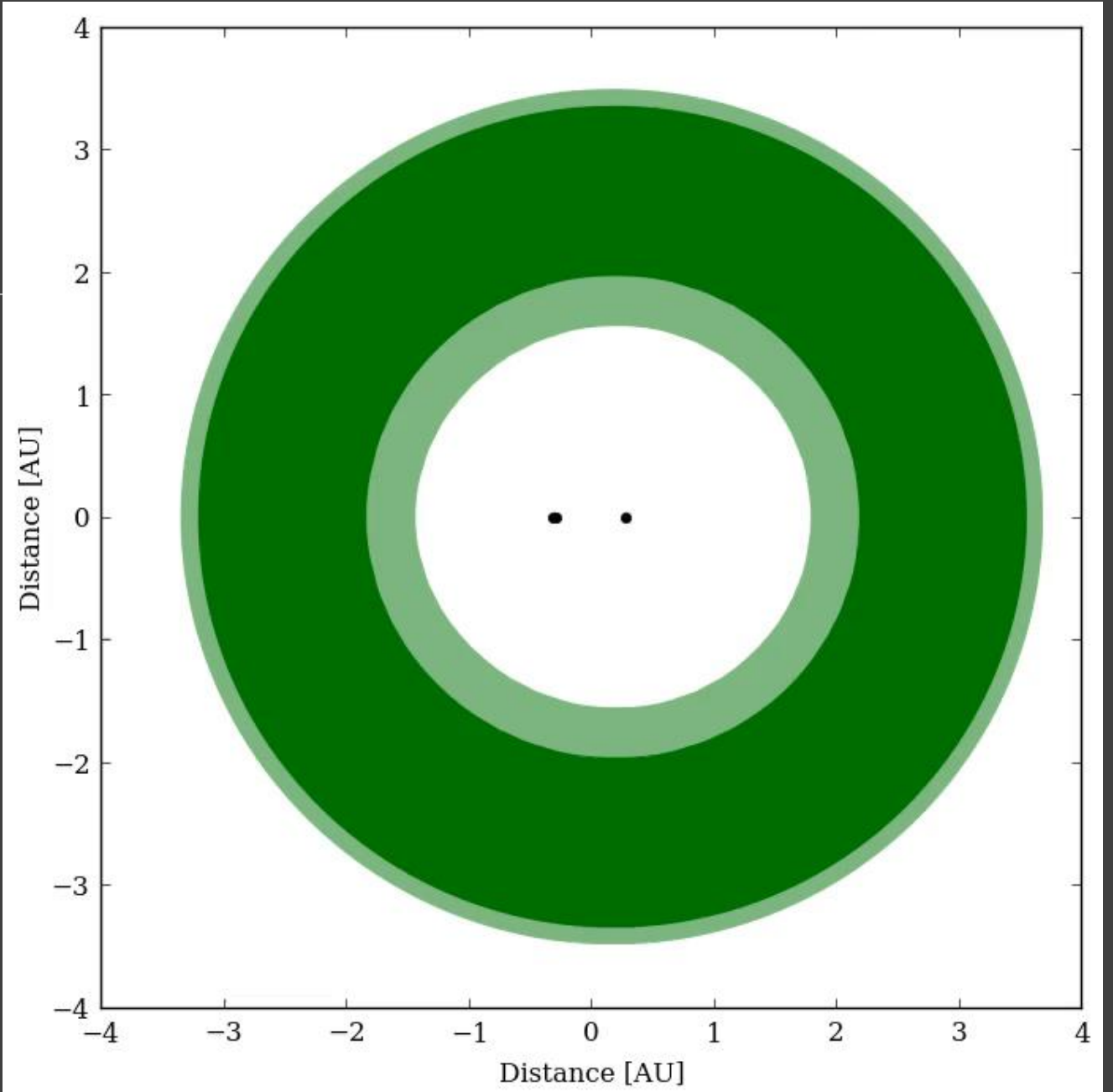


Star	Temperature	Luminosity	Mass	X	Y
A	5780 K	1 L_{solar}	1 M_{solar}	0 AU	0 AU
B	4780 K	0.7 L_{solar}	0.7 M_{solar}	1 AU	0 AU
C	6780 K	1.8 L_{solar}	1.4 M_{solar}	0 AU	5 AU
D	3780 K	0.1 L_{solar}	0.2 M_{solar}	0.5 AU	5 AU

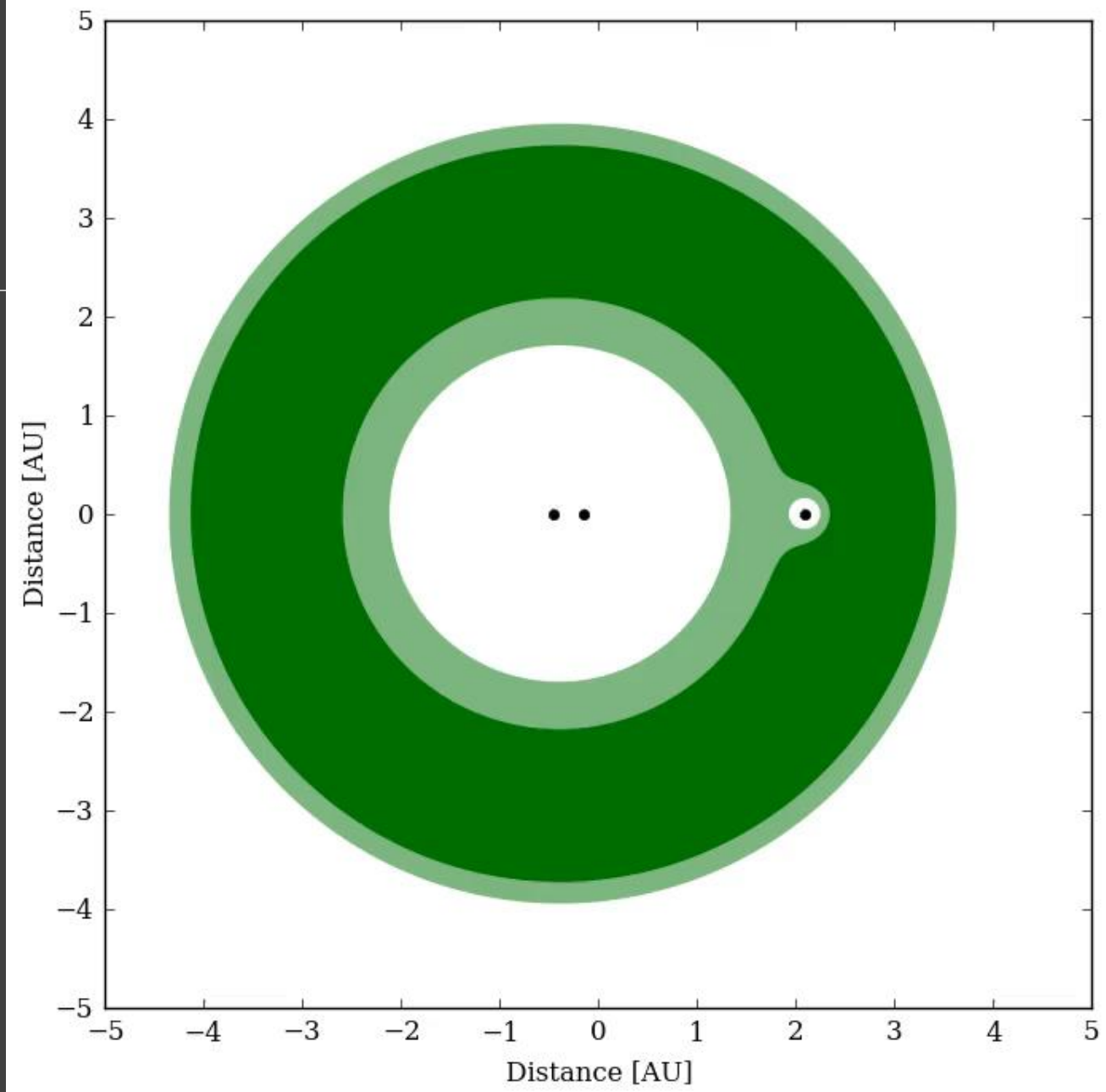


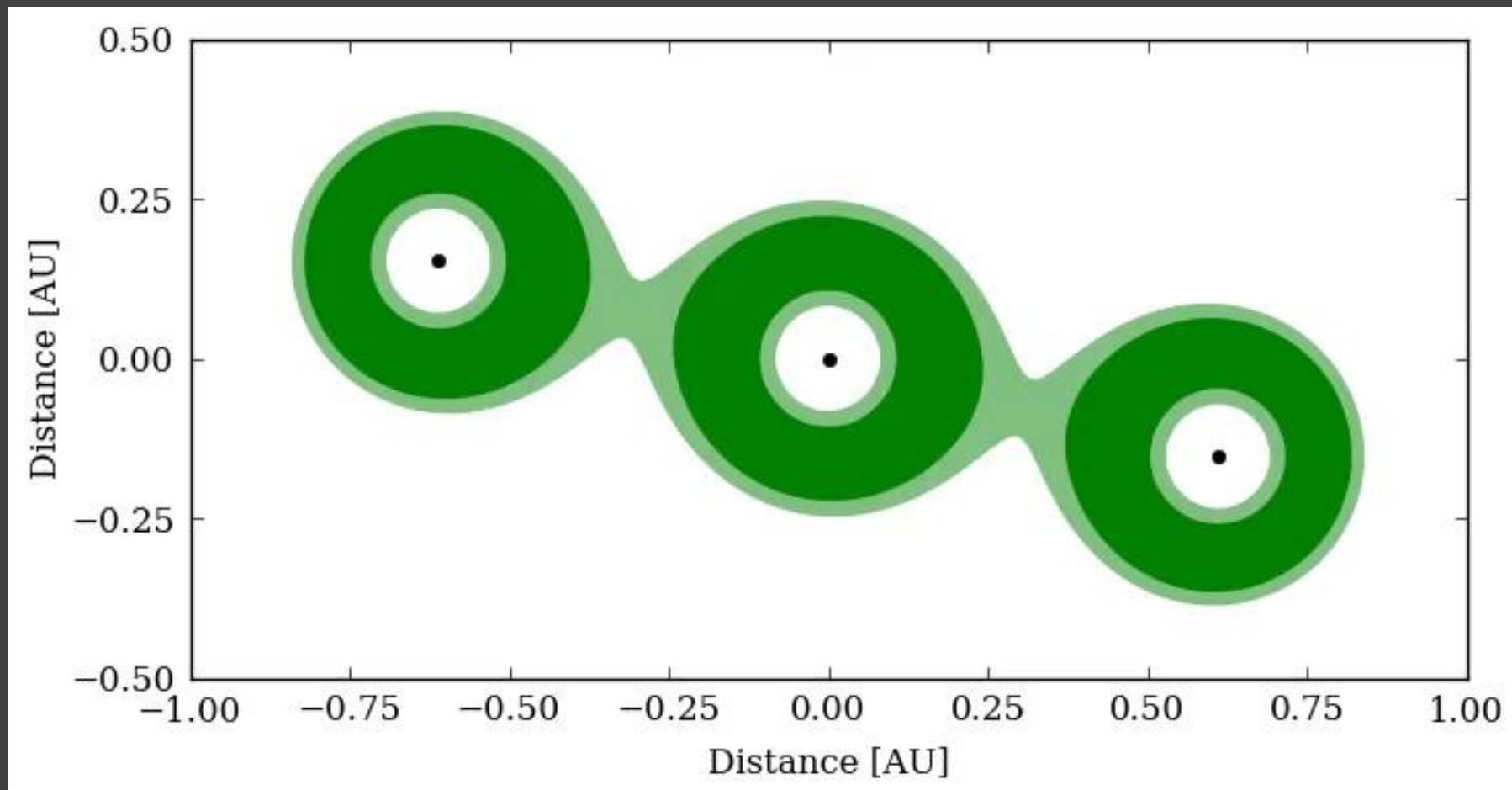
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KIC 4150611



KID 5653126



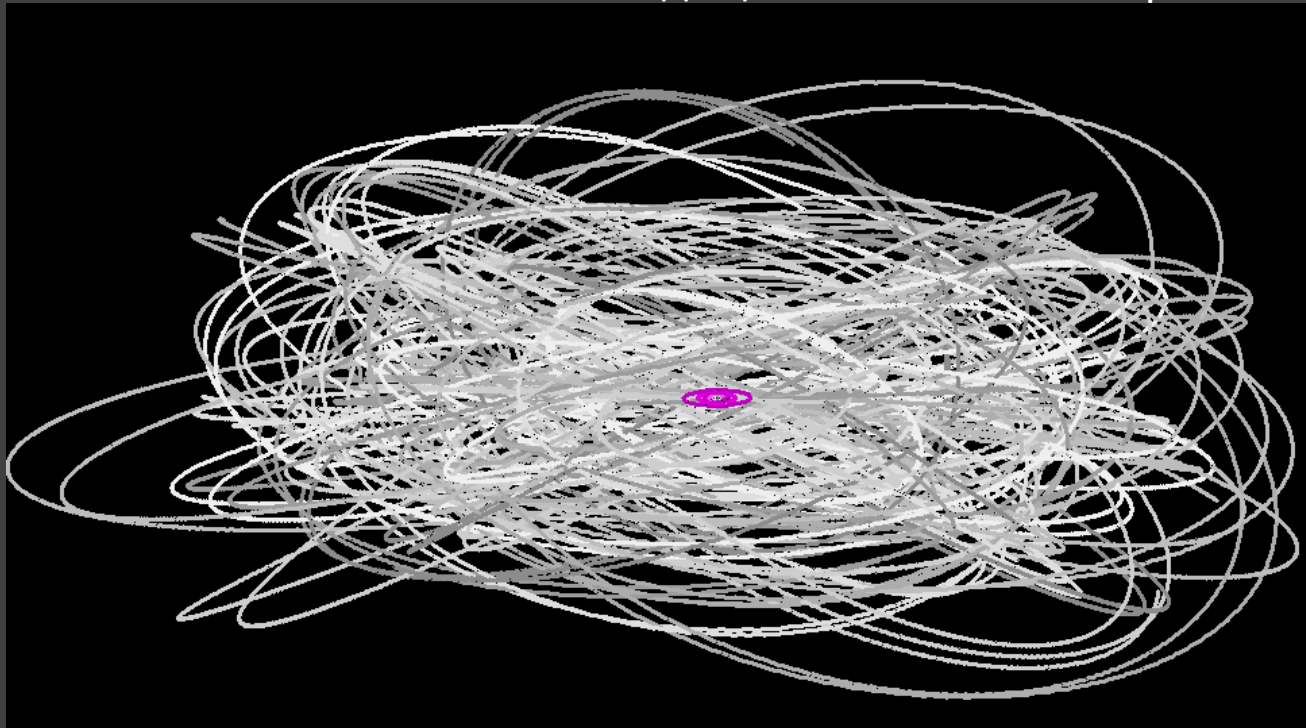


Эффект Лидова-Козаи

У орбиты могут одновременно меняться наклонение эксцентриситет.

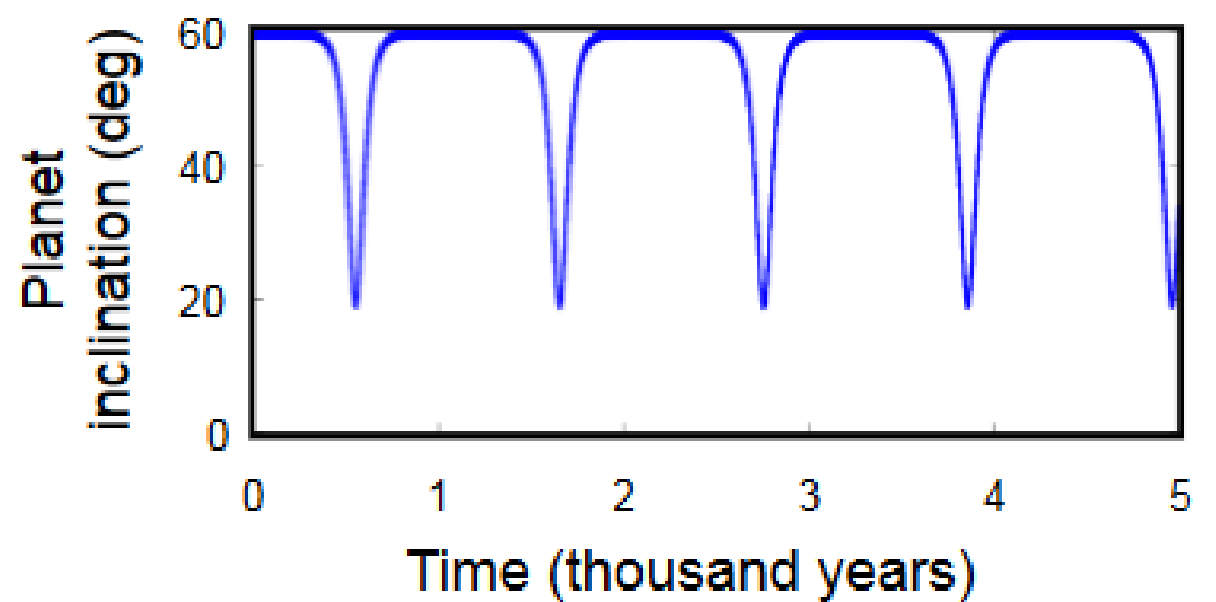
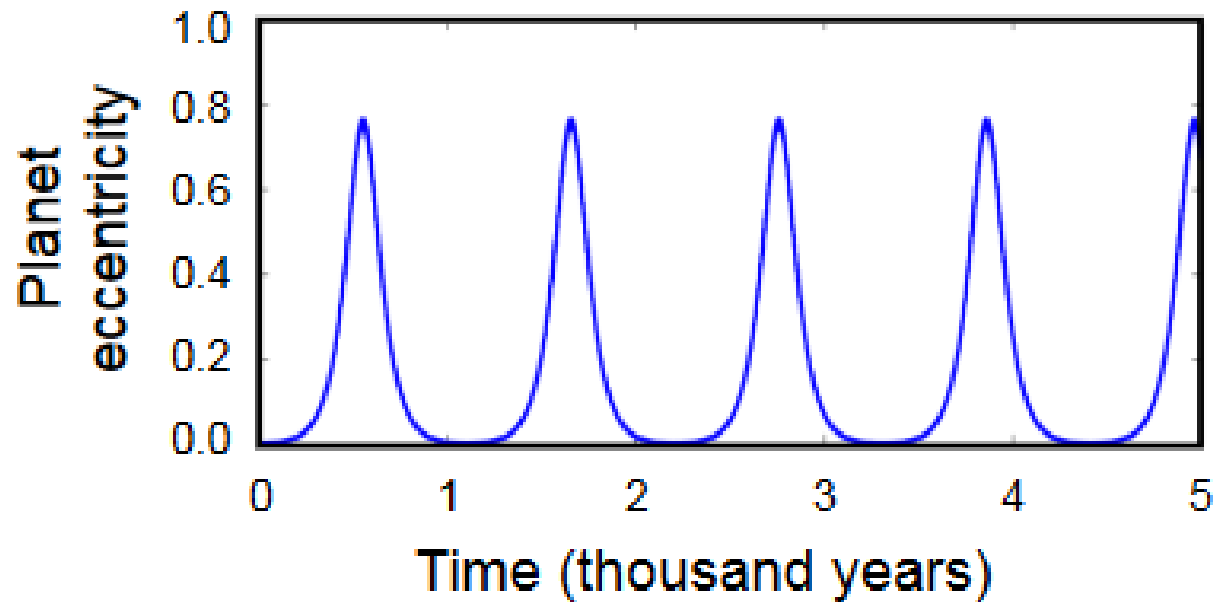
Эффект связан с воздействием тела, находящегося на внешней орбите.

$$e_{\max} \simeq \sqrt{1 - (5/3) \cos^2 i_0}$$



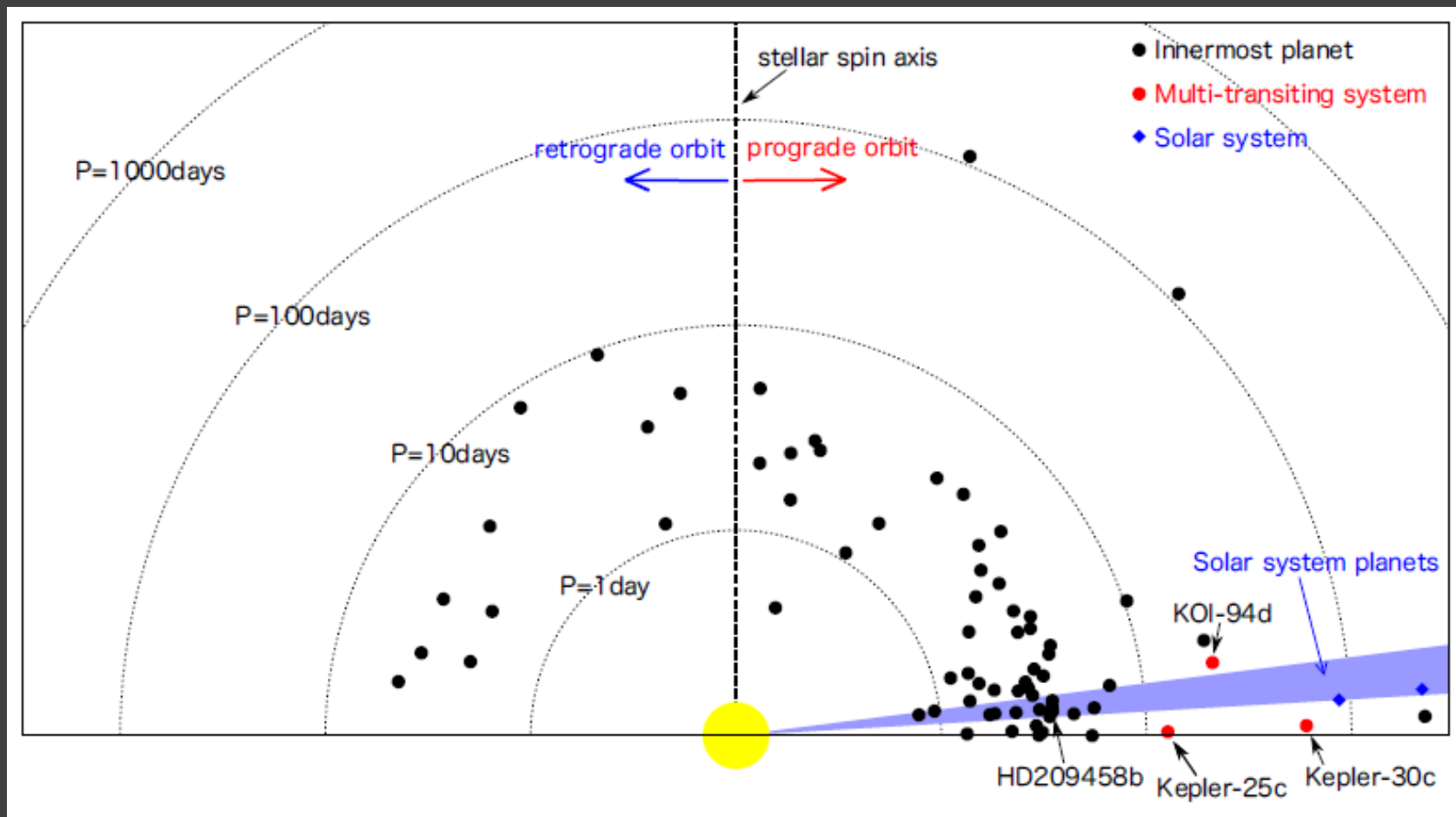
Эффект был впервые описан Михаилом Лидовым для спутников в 1961 г., а затем в 1962 г. был описан Козаи для астероидов.

Example of orbital behaviour



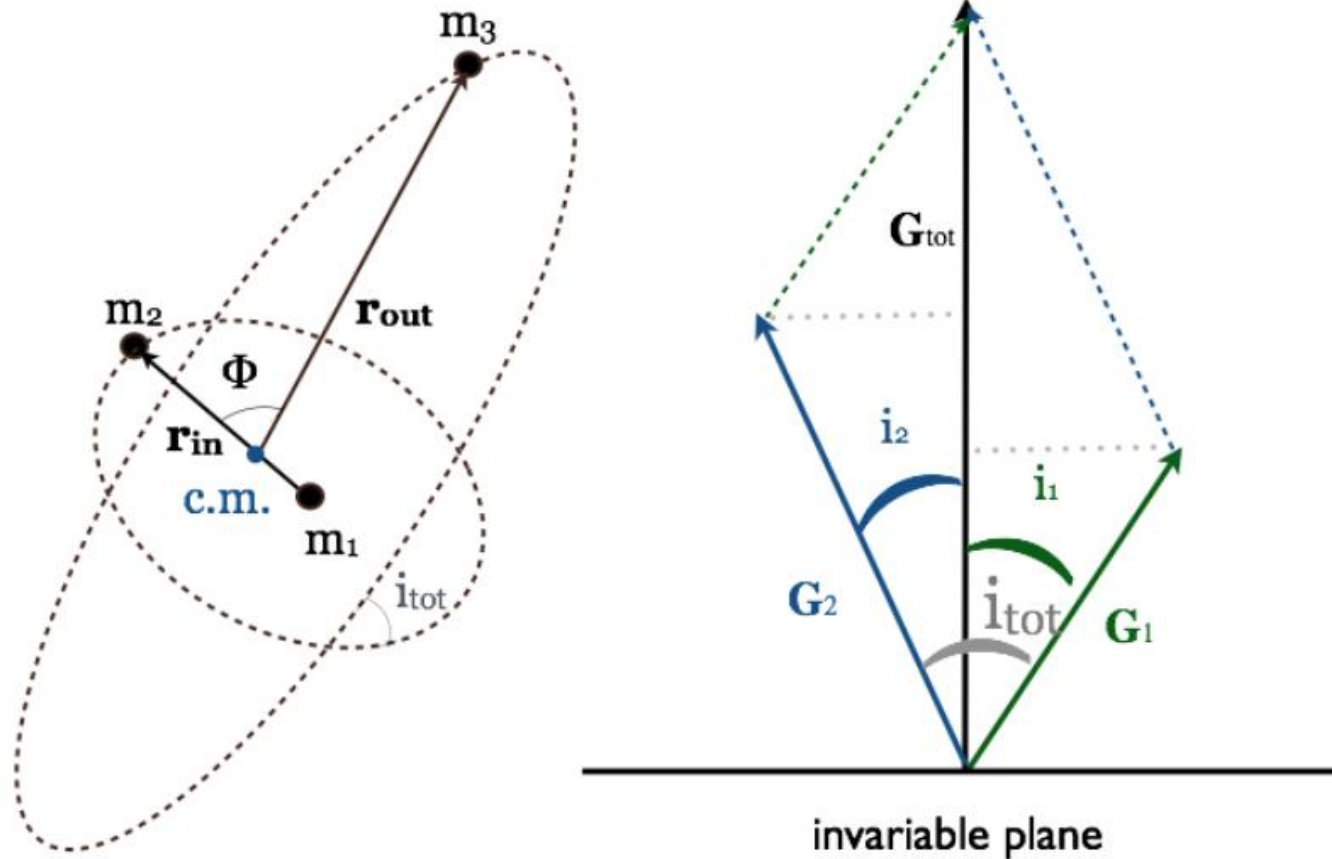
0.5 AU planet (around the primary) in a 5 AU binary.
Initial inclination 60 degrees.

Распределение планет по ориентации орбиты



Есть планеты с полярными и даже обратными орбитами.

Approximation



Wide outer body's orbit.
No resonances.

Two orbits exchange angular momentum,
but not energy.
So, orbits can change shape and
orientation, but not semi-major axes.

Conservation of projection of the
angular momentum results in

$$j_z = \sqrt{1 - e_1^2} \cos i_{tot} = \text{Const.}$$

Circular outer orbit

$$j_{z,1} = \sqrt{1 - e_{1,max/min}^2} \cos i_{1,min/max} = \sqrt{1 - e_{1,0}^2} \cos i_{1,0} \quad m_2 \longrightarrow 0 \text{ (test particle approximation)}$$

$$e_{1,0} = 0 \text{ and } \omega_{1,0} = 0.$$



$$e_{max} = \sqrt{1 - \frac{5}{3} \cos^2 i_0}$$

$$\cos i_{min} = \pm \sqrt{\frac{3}{5}}$$

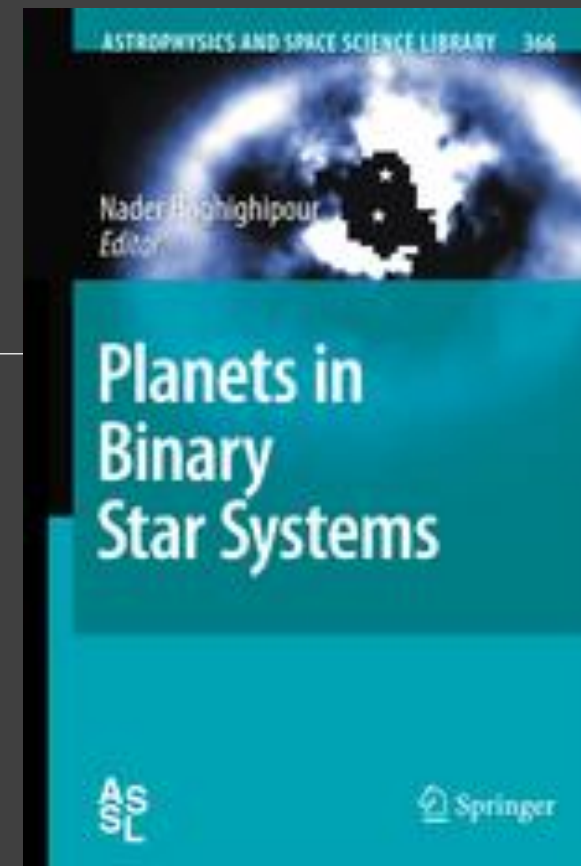
~40 and 140 degrees
(Kozai angles)

$$E_0 = 2e_{1,min}^2 - 2 + (1 - e_{1,min}^2) \cos^2 i_{max}$$

The z-component of the angular momentum of the inner and outer orbits (i.e., the nominal $\sqrt{1 - e_{1,2}^2} \cos i_{1,2}$) are only conserved if one of the binary members is a test particle and the outer orbit is axisymmetric ($e_2 = 0$).

Literature

- 1608.00764 New prospects for observing and cataloguing exoplanets in well detached binaries R. Schwarz et al.
- 1401.0601 Calculating the Habitable Zone of Multiple Star Systems Tobias Mueller, Nader Haghighipour
- 1601.07175 The Eccentric Kozai-Lidov Effect and Its Applications Smadar Naoz
- 1406.1357 Planet formation in Binaries P. Thebault, N. Haghighipour
- 1802.08693 Populations of planets in multiple star systems



(2010)