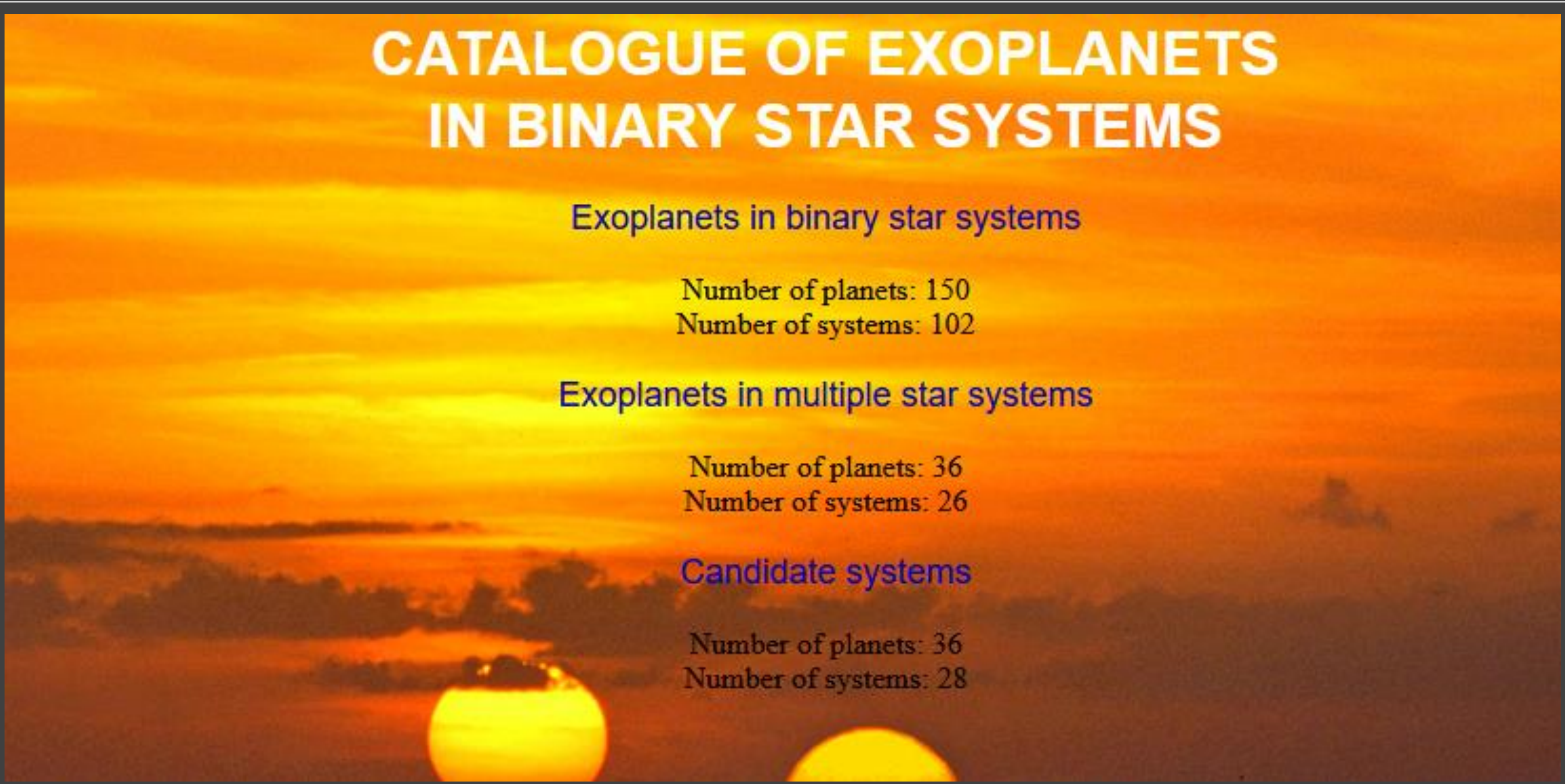
# Planets in binaries

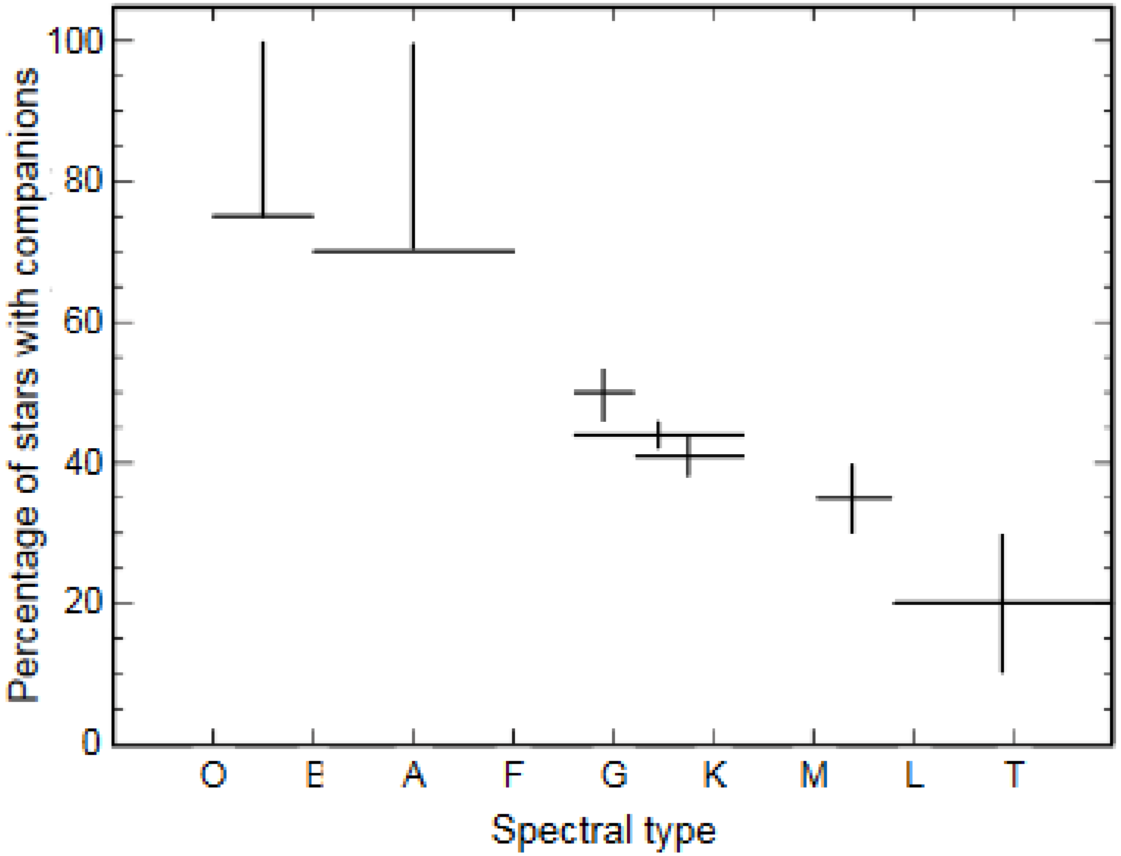
SERGEI POPOV

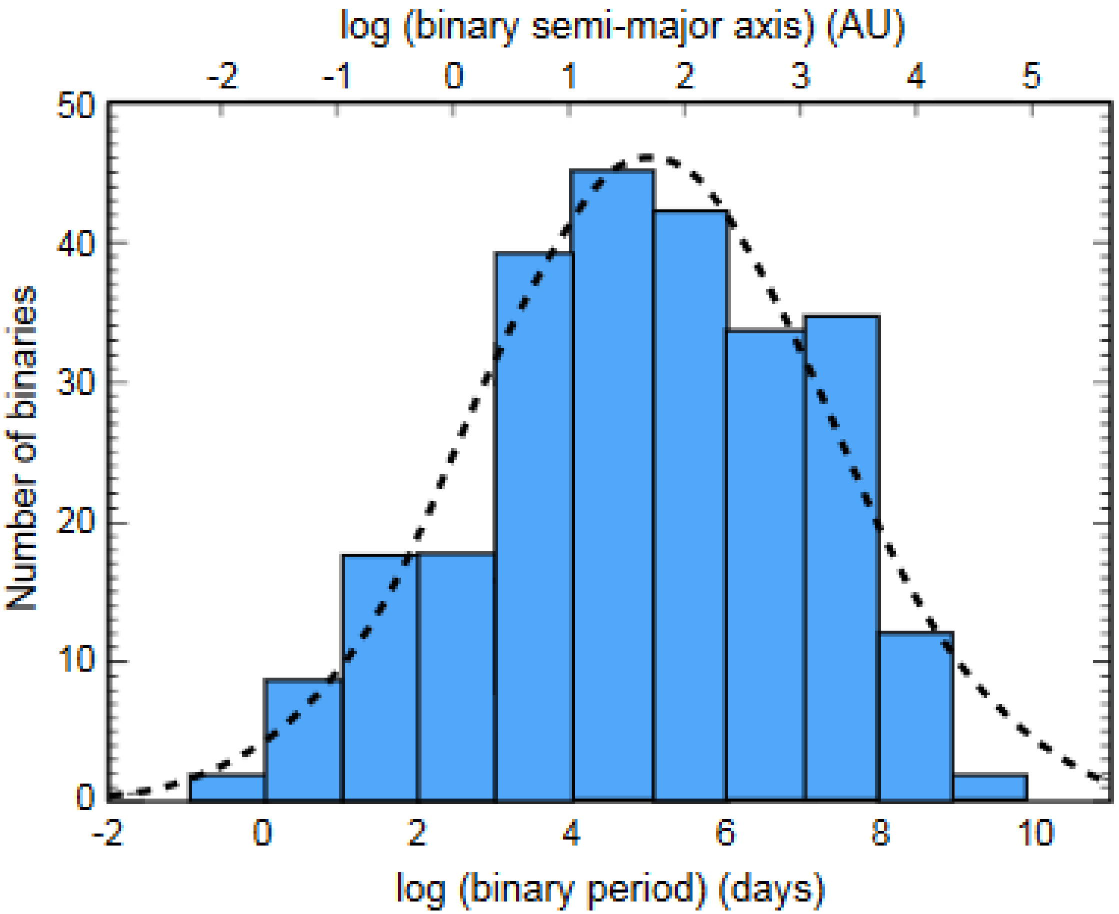
# Catalogue of planets in binaries




http://www.univie.ac.at/adg/schwarz/multiple.html

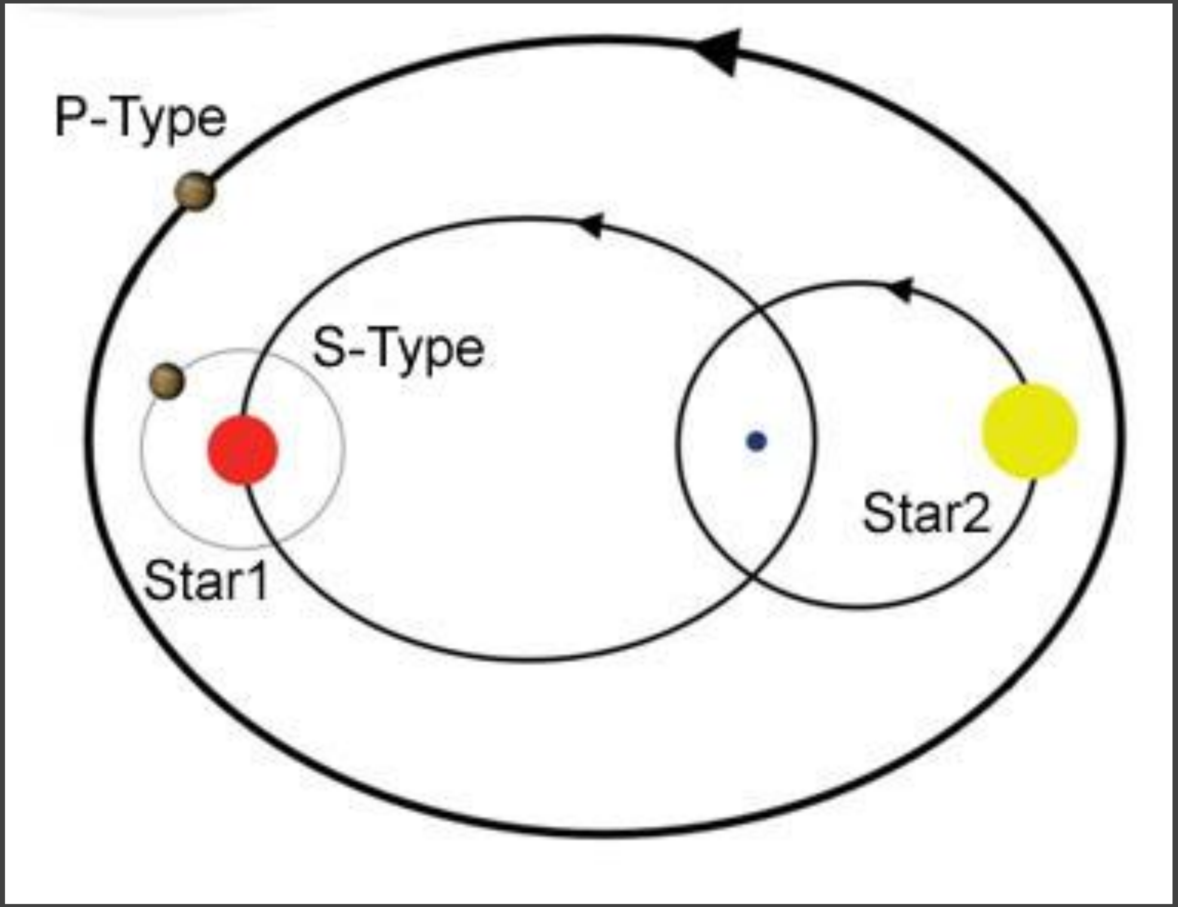
# Binary systems statistics



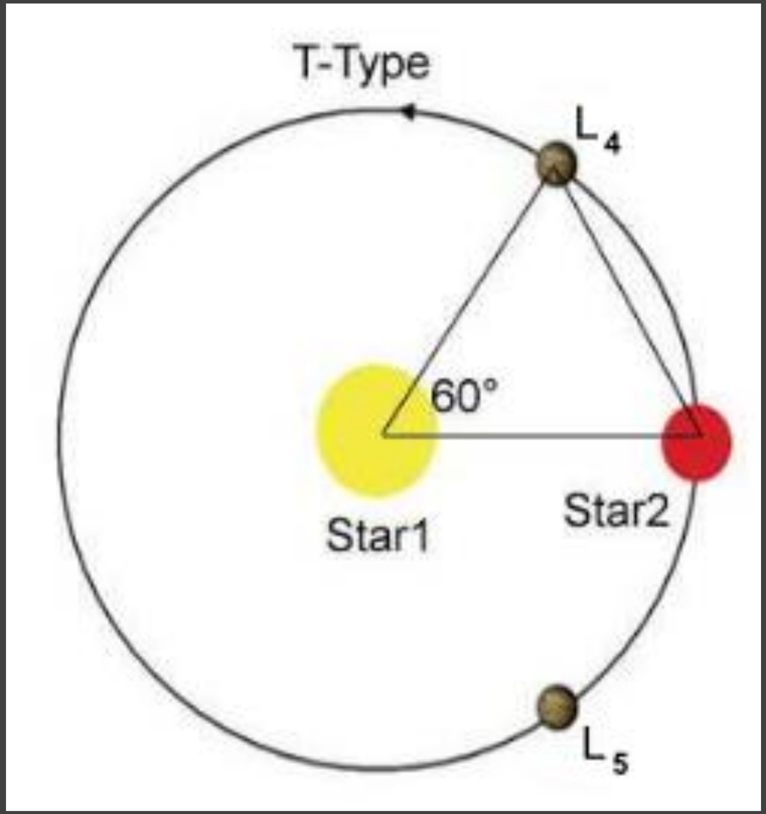

From Raghavan et al. 2010

# S-type and P-type. And T-type!




The central stars strongly perturb the region around them, clearing out orbits to distances of 2–5 times the binary separation.

In addition to known S-type and P-type planetary orbits, also so-call T-type planets, similar to Troyans, can exist.



# Orbit stability

1204.2014

$$R_p = \left(0.464 - 0.38\frac{m_2}{m_1+m_2}\right)a_*$$

S-type
Valid for the primary (more massive star)
$a_*$ - binary semimajor axis

$$a_c = (1.60 + 4.12\mu - 5.09\mu^2)a_b$$

P-type

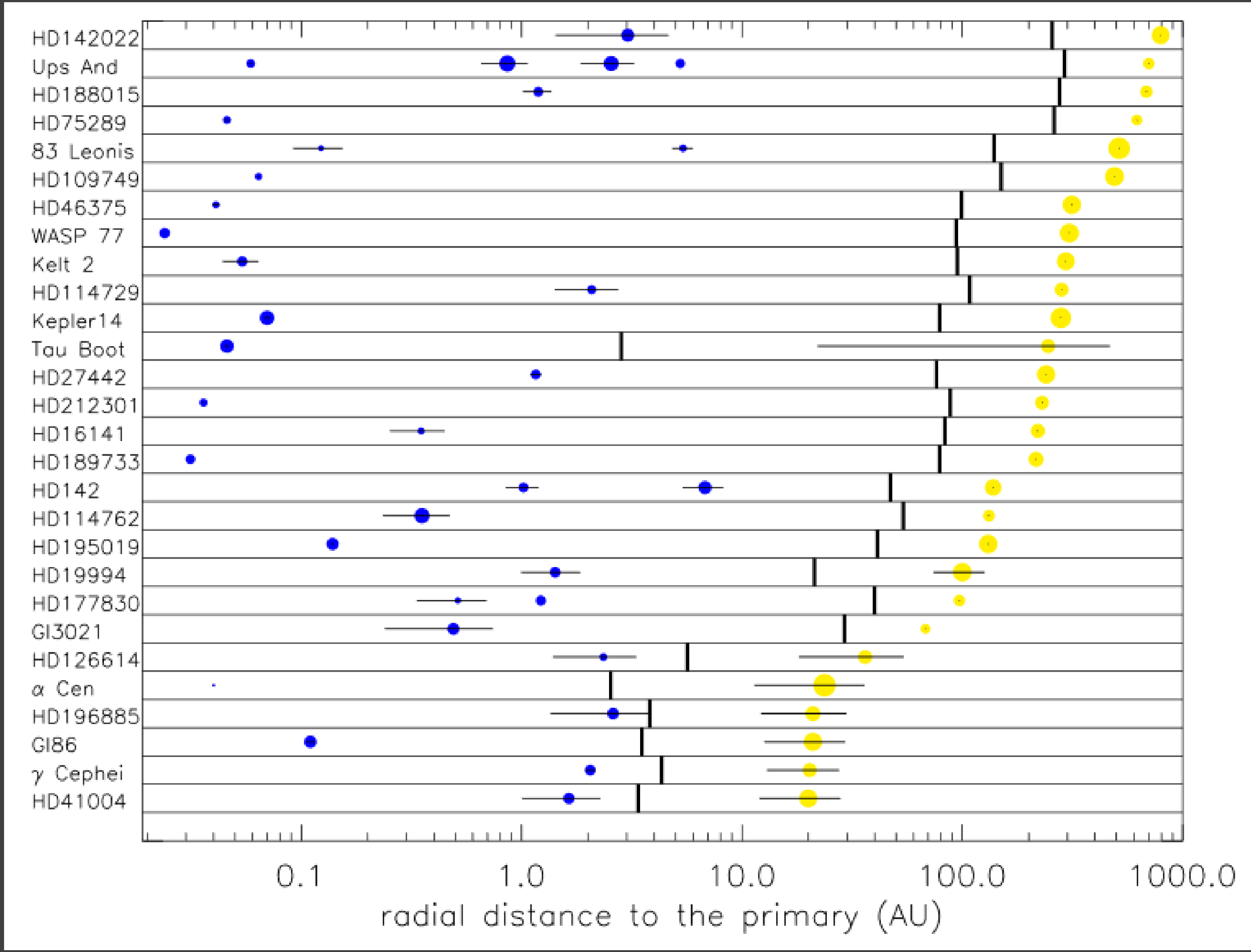
$$\mu = m_2/(m_1+m_2)$$

Both estimates are given for e=0.

Plus:
islands of stability
and instability
due to resonances, etc.

$$a_{\rm crit} \approx 1.60 + 5.10\,e_{\rm bin} - 2.22\,e_{\rm bin}^2 + 4.12\frac{M_s}{M_p+M_s} - 4.27\,e_{\rm bin}\frac{M_s}{M_p+M_s} - 5.09\frac{M_s^2}{(M_p+M_s)^2} + 4.61\,e_{\rm bin}^2\frac{M_s^2}{(M_p+M_s)^2},$$

1503.03876



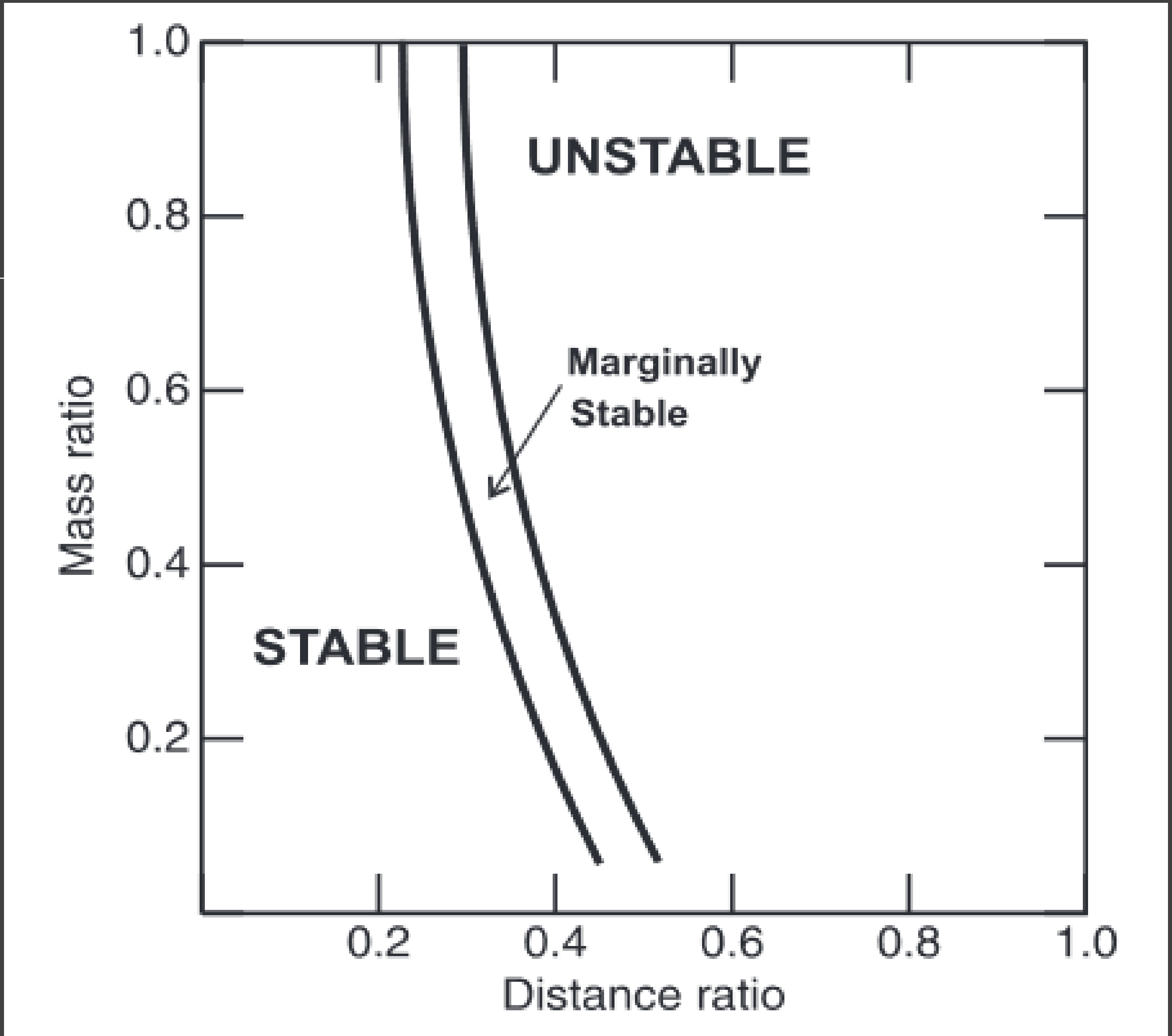

$$R_p = \left(0.464 - 0.38\frac{m_2}{m_1 + m_2}\right) a_*$$

1406.1357

# S-type

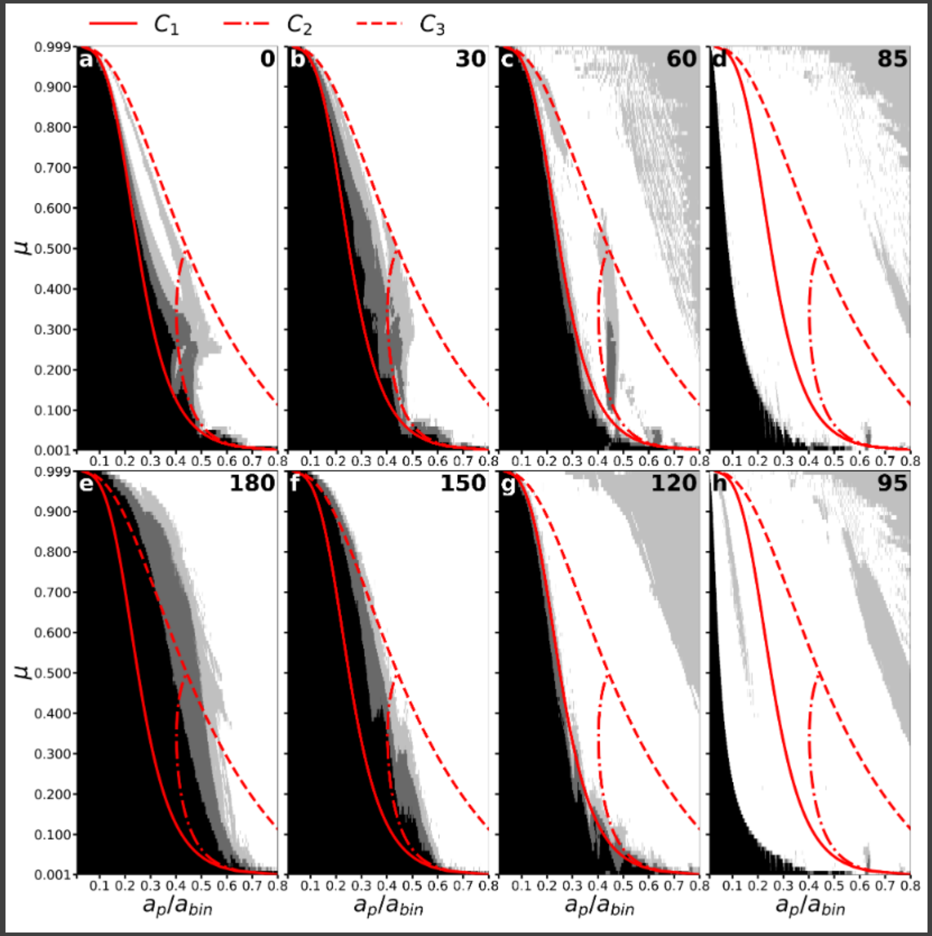
Distance ratio: $R_{planet\text{-}A}/R_{AB}$

Mass ratio: $M_B/M_A$




Musielak et al. (2005)

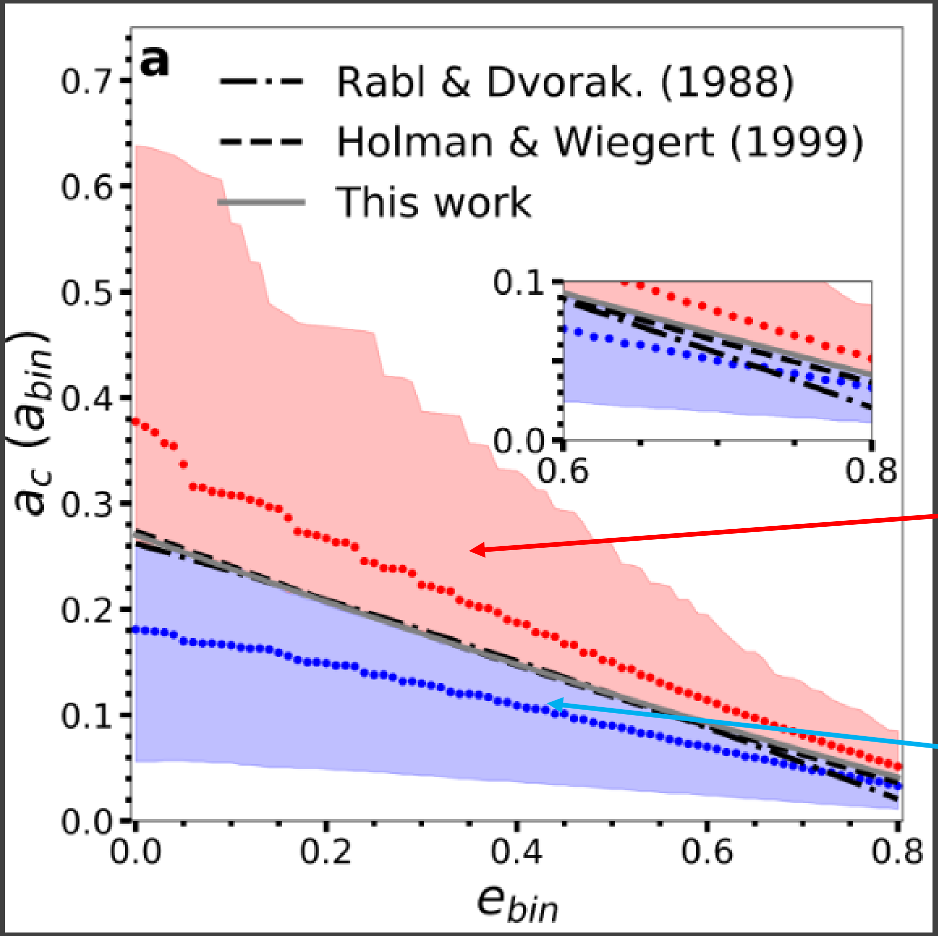
# Detailed studies of S-planets stability




$\mu = M_B/(M_A+M_B)$

Circular inclined orbits.

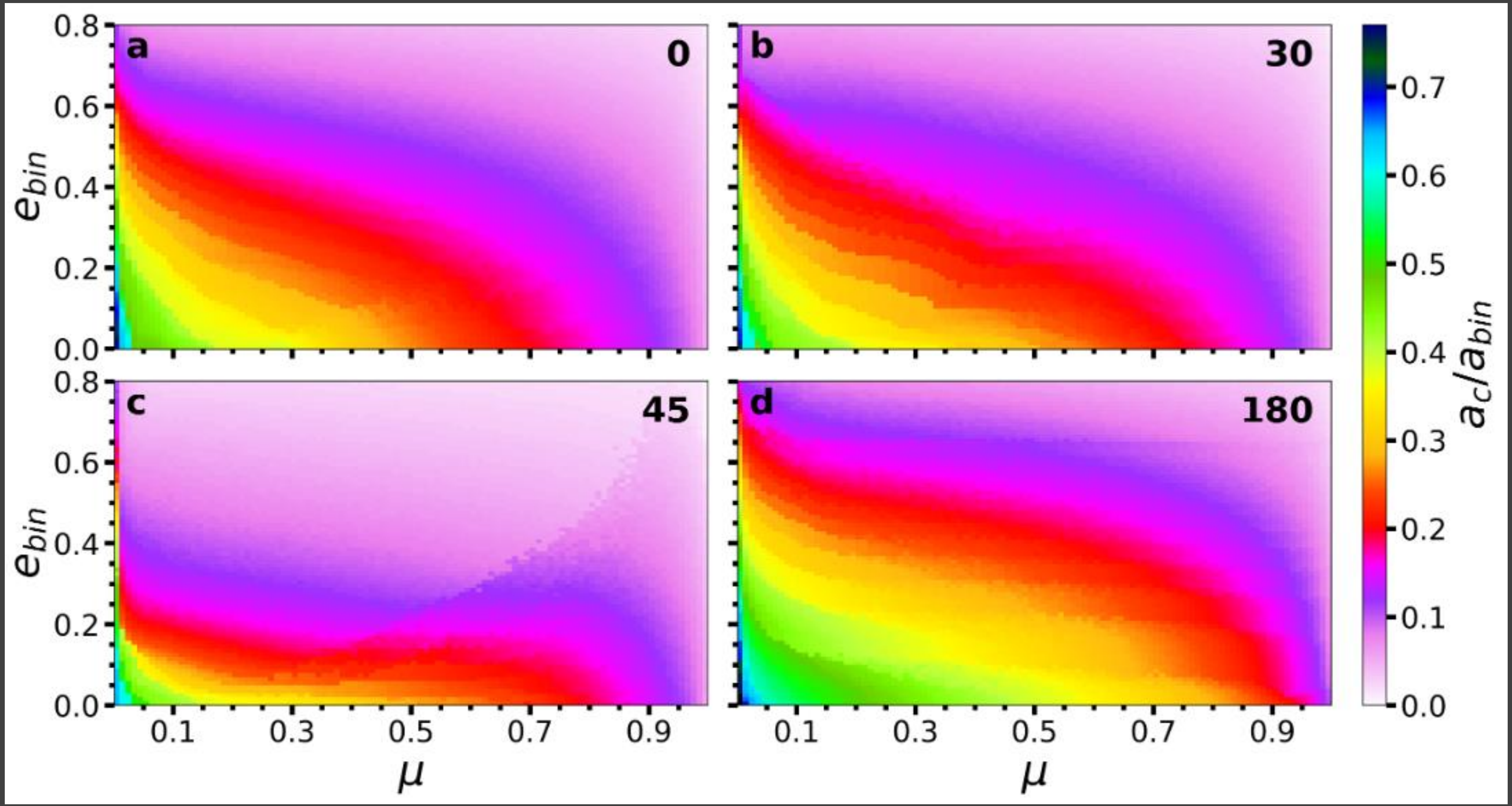
Black – stable.




$\mu<0.5$

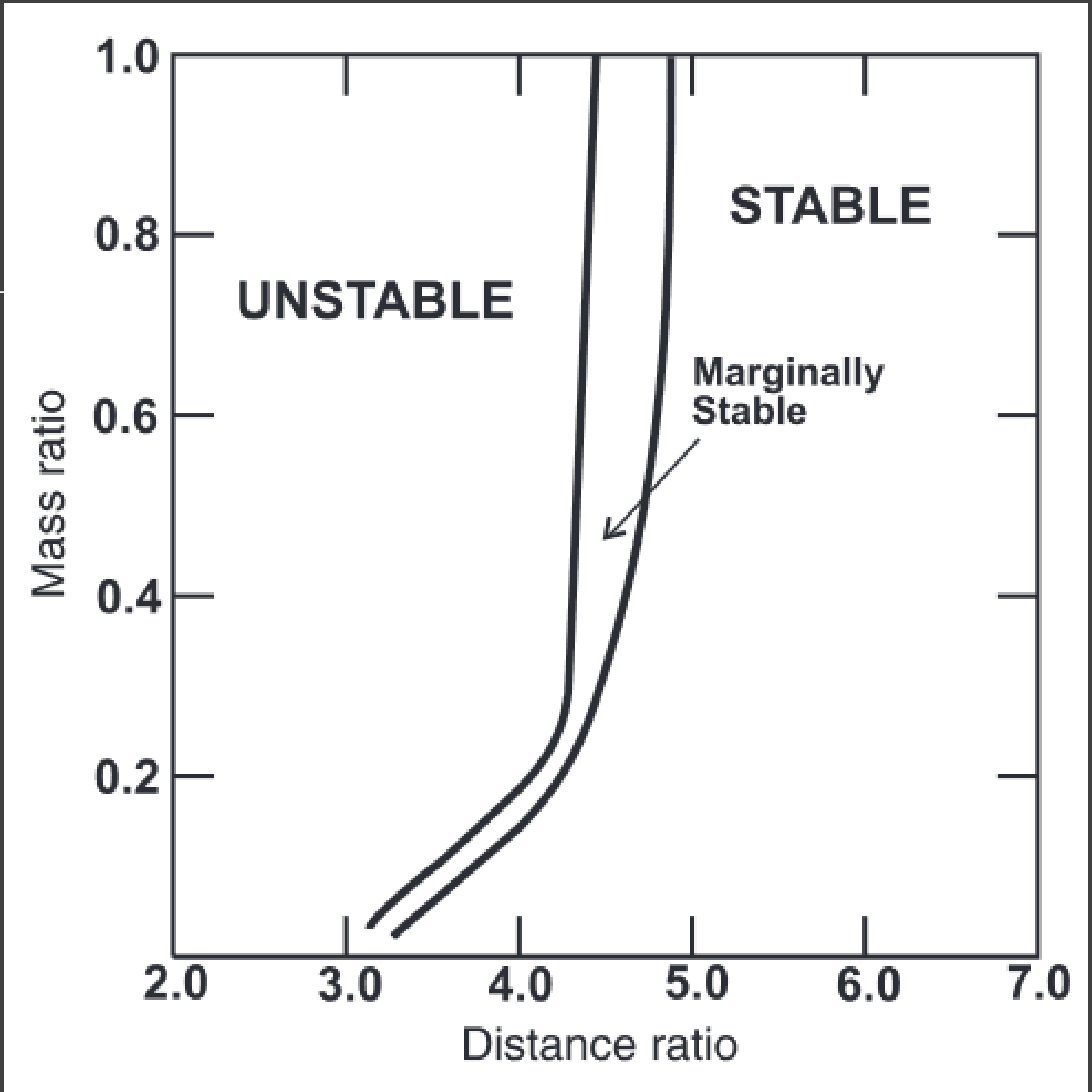
$\mu>0.5$

# Stability in inclined eccentric cases




1912.11019

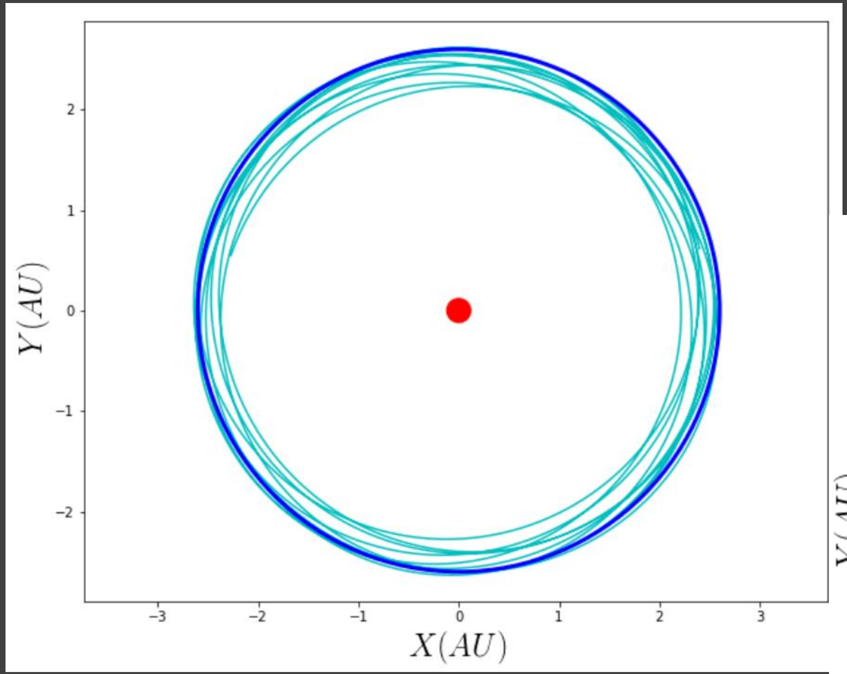
# P-type




See a more detailed study for non-zero eccentricities in 1802.08868

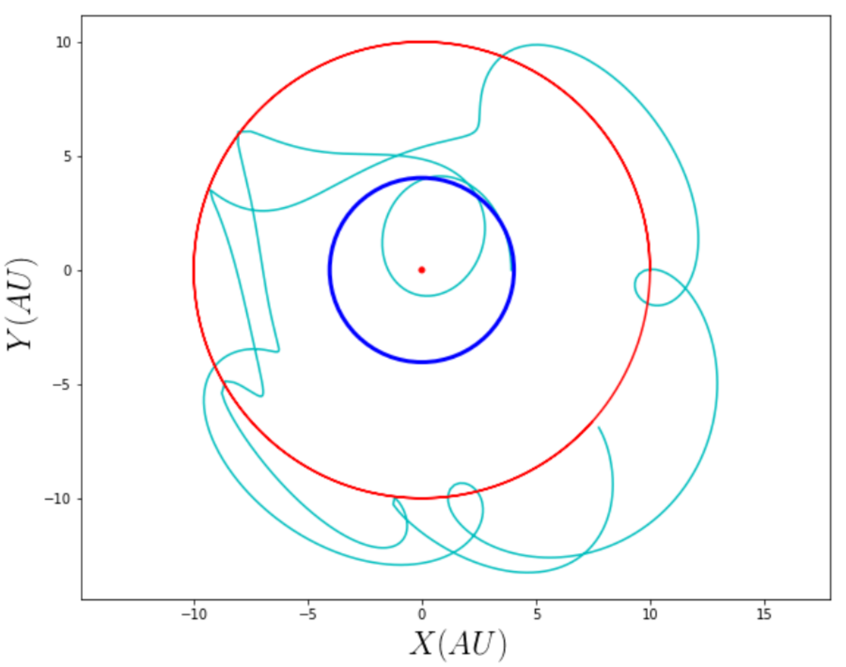
Musielak et al. (2008)

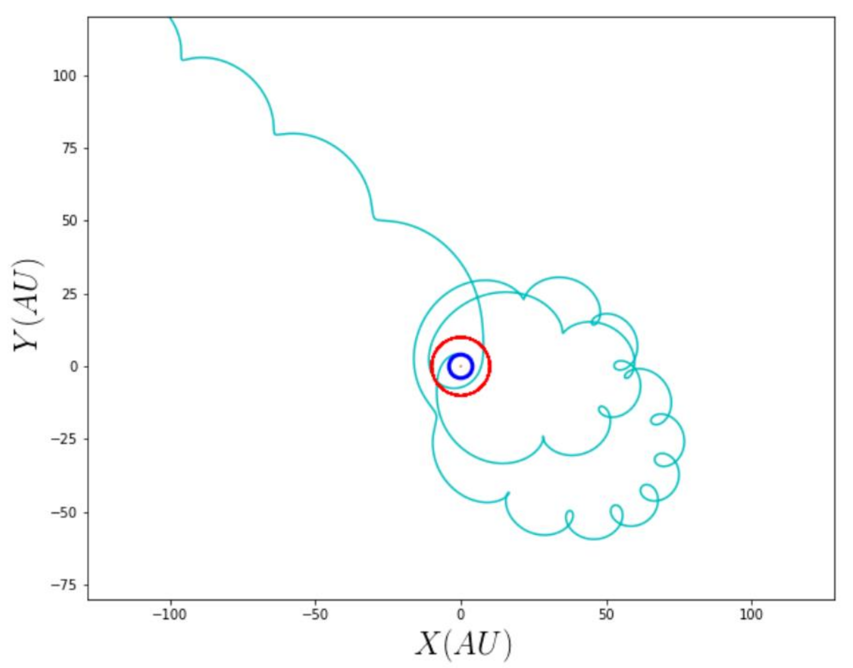
# Examples of trajectories




Stable orbit

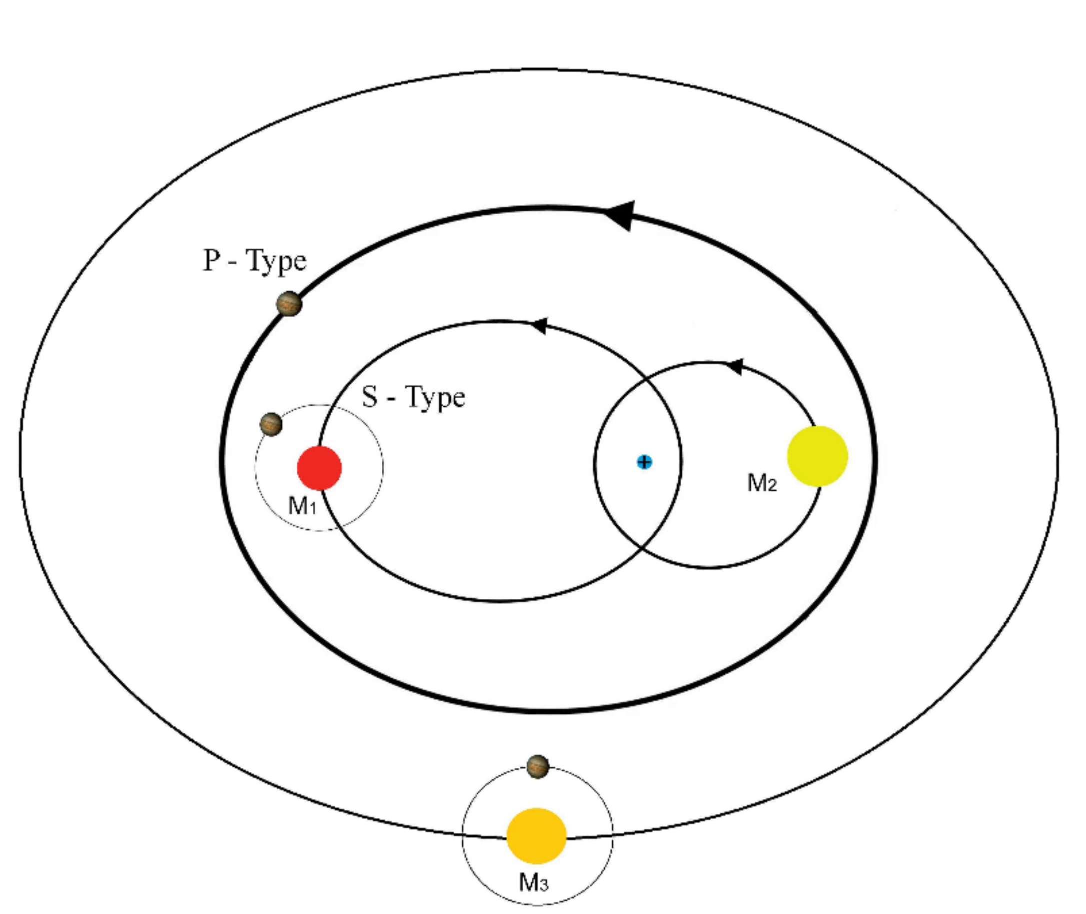
Collision with the secondary



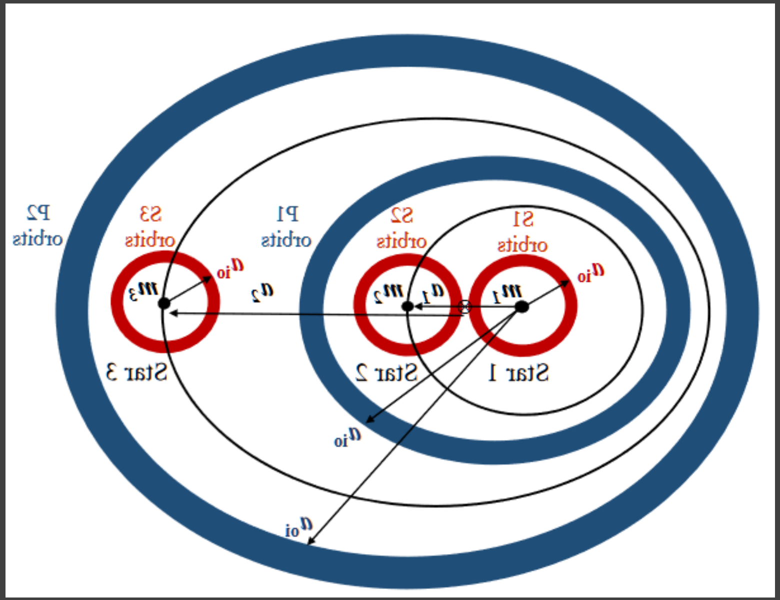

Ejection

# Planets in triple-star systems




Stable orbits of S-, P-, and T-types
are possible in different kinds of multiple systems
(see arXiv:1811.08221 ).

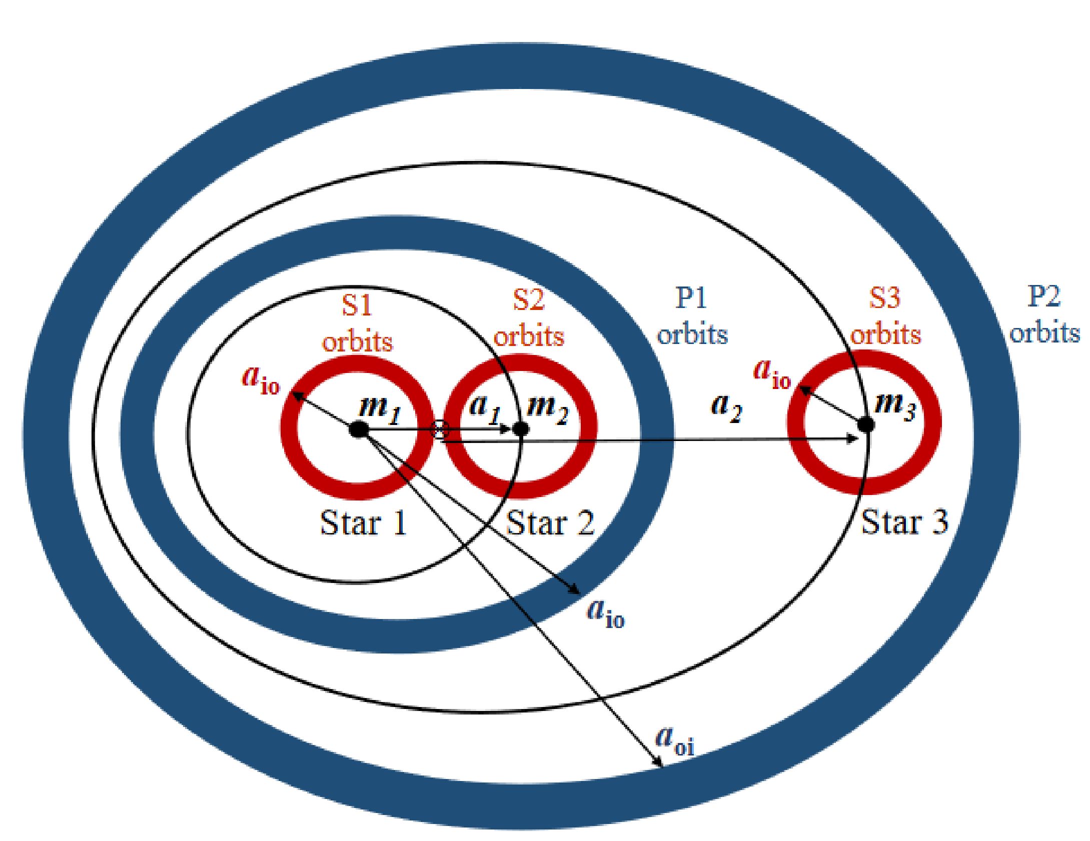
24 triple systems with planets (Dec. 2020):

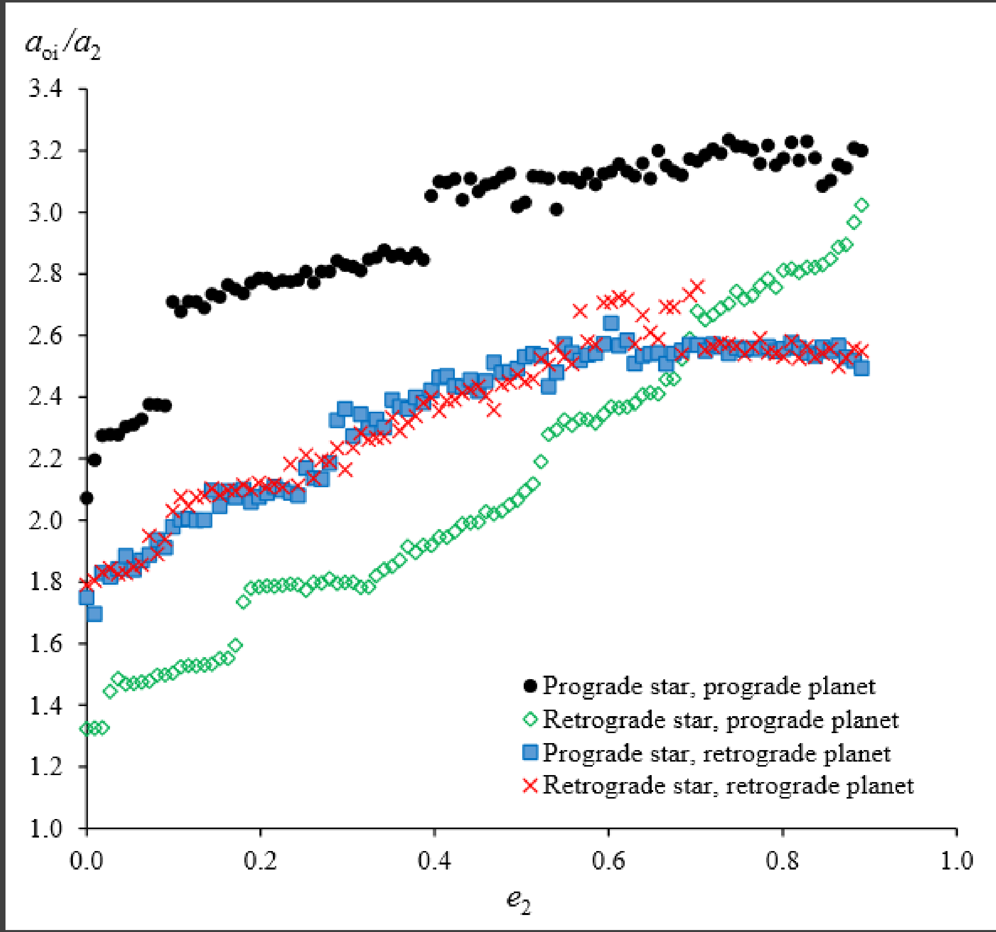


1608.00764

# Stability of planets in triples

Eccentricity of the outer stellar orbit is the most crucial parameter for stability of P-type planetary orbits.

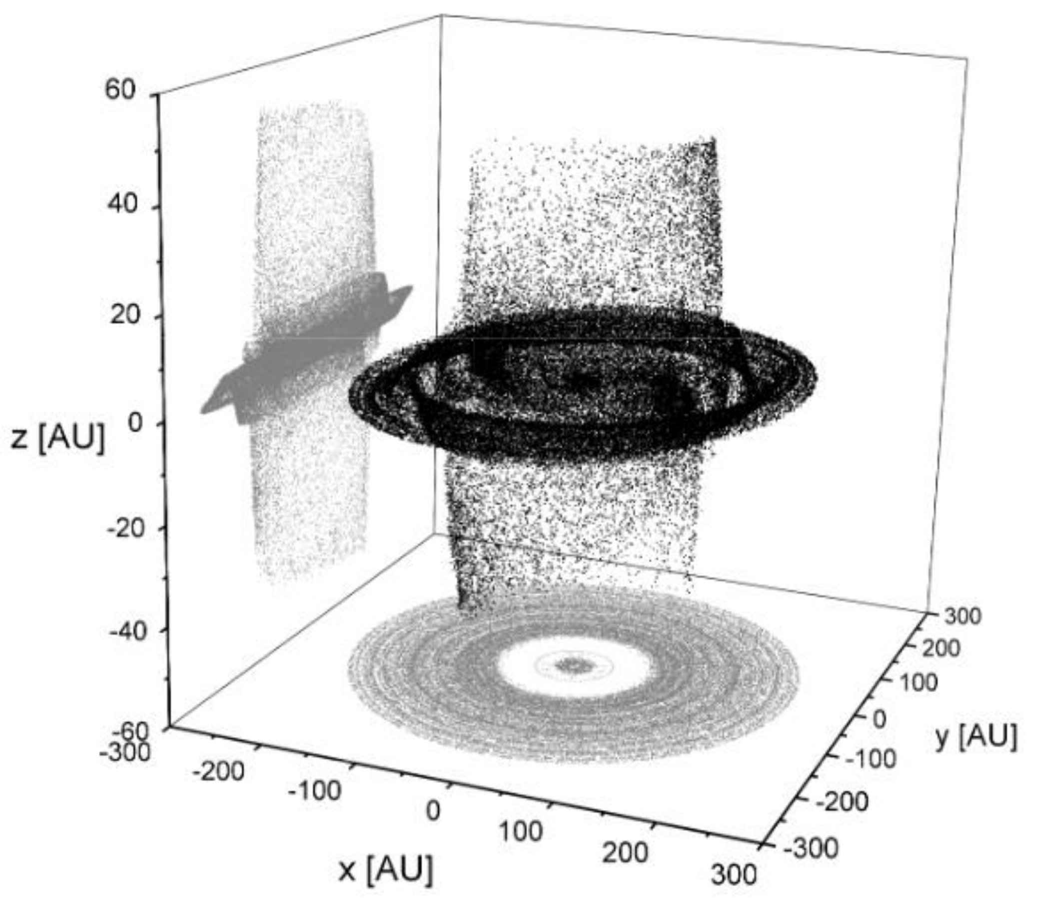



P2-orbit

$e_2$ – outer eccentricity

# Orbits in non-coplanar triples

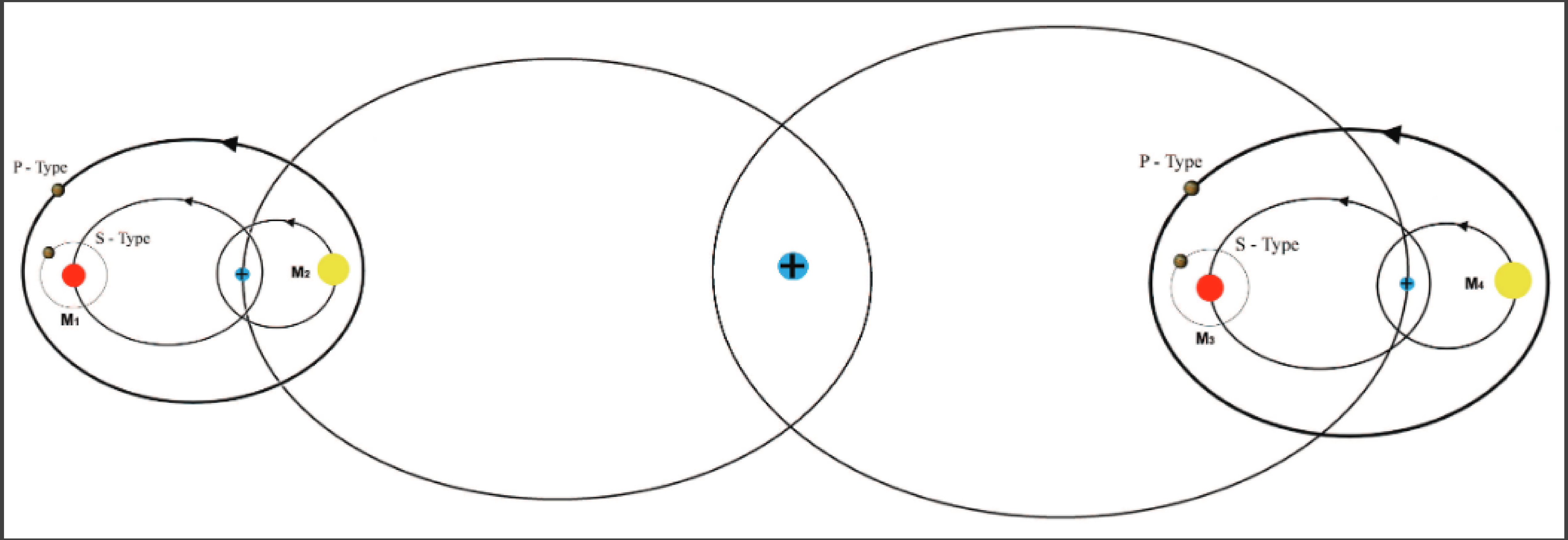



The typical annular vertical "chimney" consists of test particles in circular orbits of constant semi-major axis but varying inclination.

Some test particles develop Kozai resonance and some of these eventually undergo orbit flipping in to retrograde motion.

**Fig. 4.** Position in space of surviving test particles after 100 kyr. P1 and P2 prograde orbits, with $a = 40$ AU, $e_1 = e_2 = 0$, $i_1 = 0°$, $i_2 = 65°$, $\mu_1 = 0.5$, $\mu_2 = 2.0$.

# Quadruple star systems

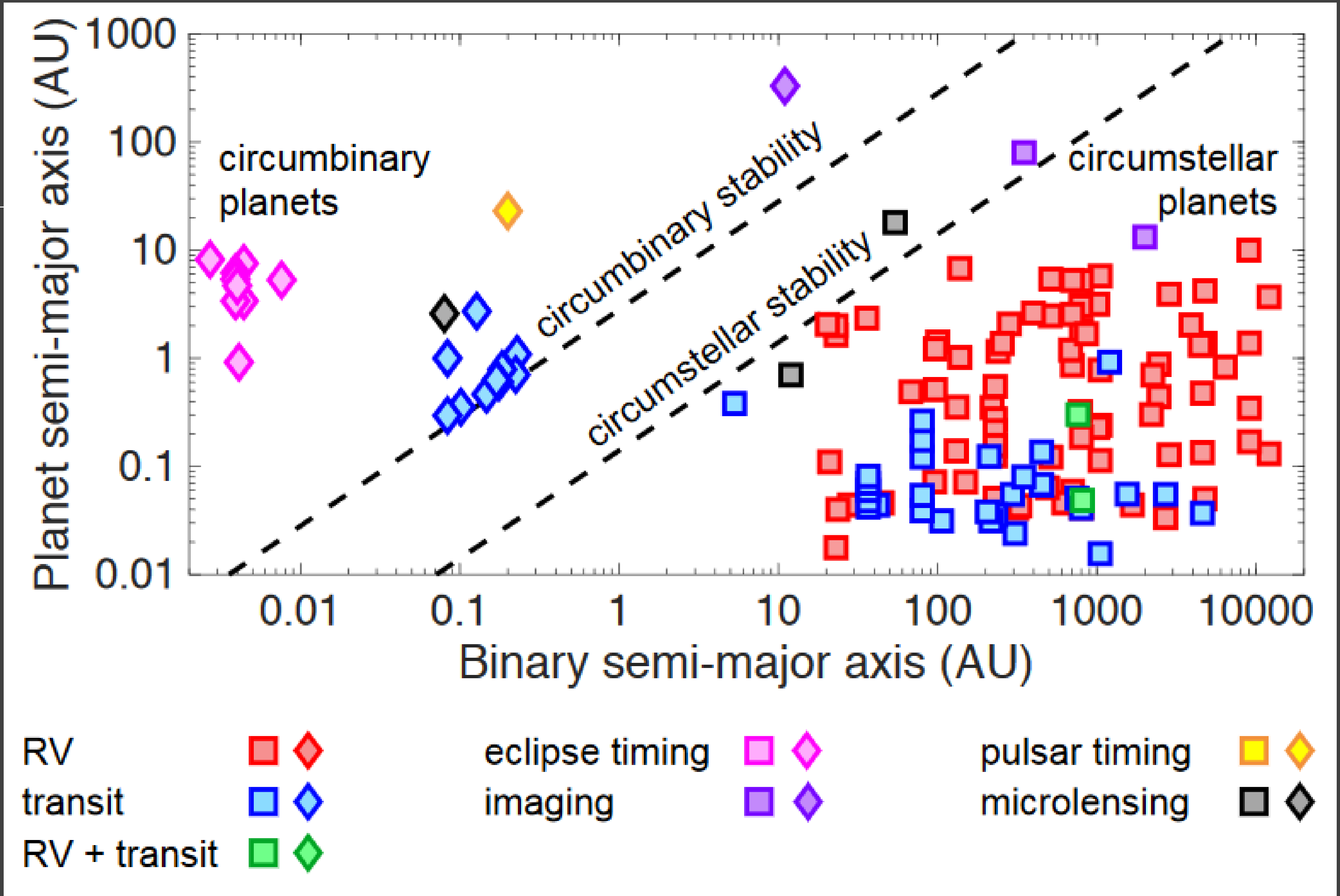



2 planetary systems systems in quadruple stellar systems (Dec. 2020)

# Statistics

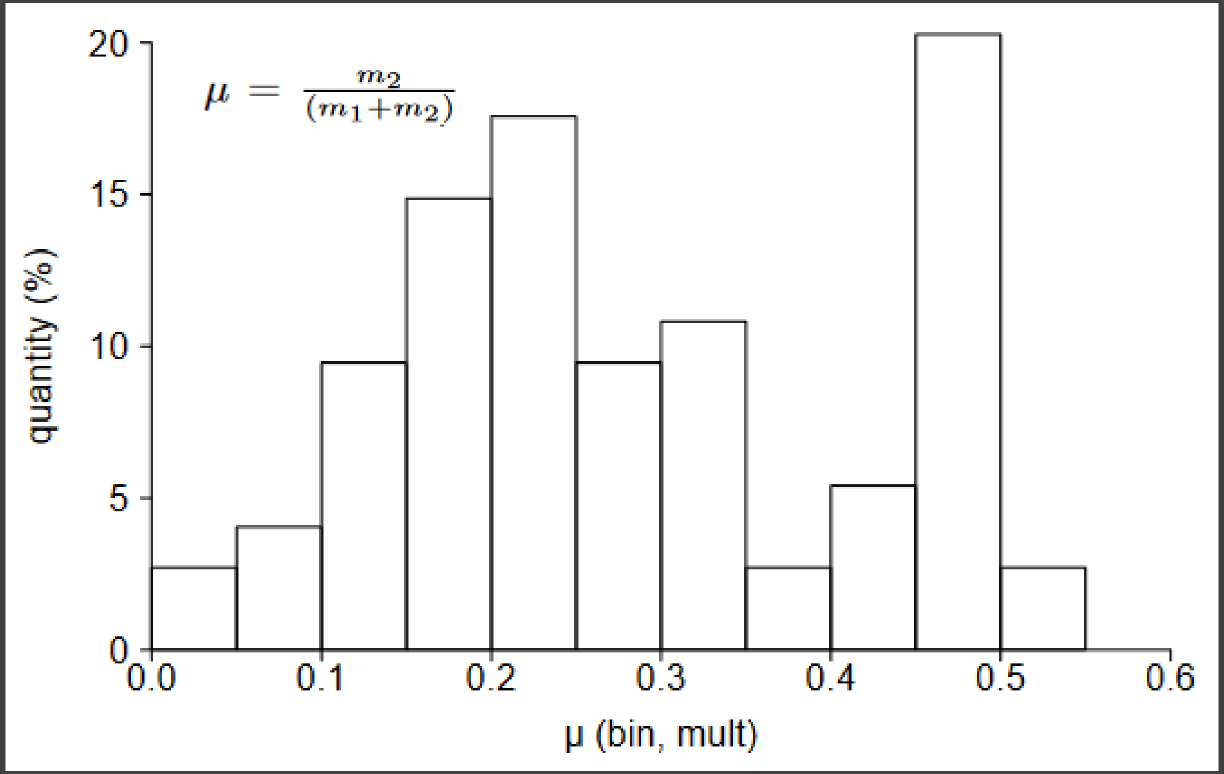
Diamonds – curcumbinary,
Squares - circumstellar

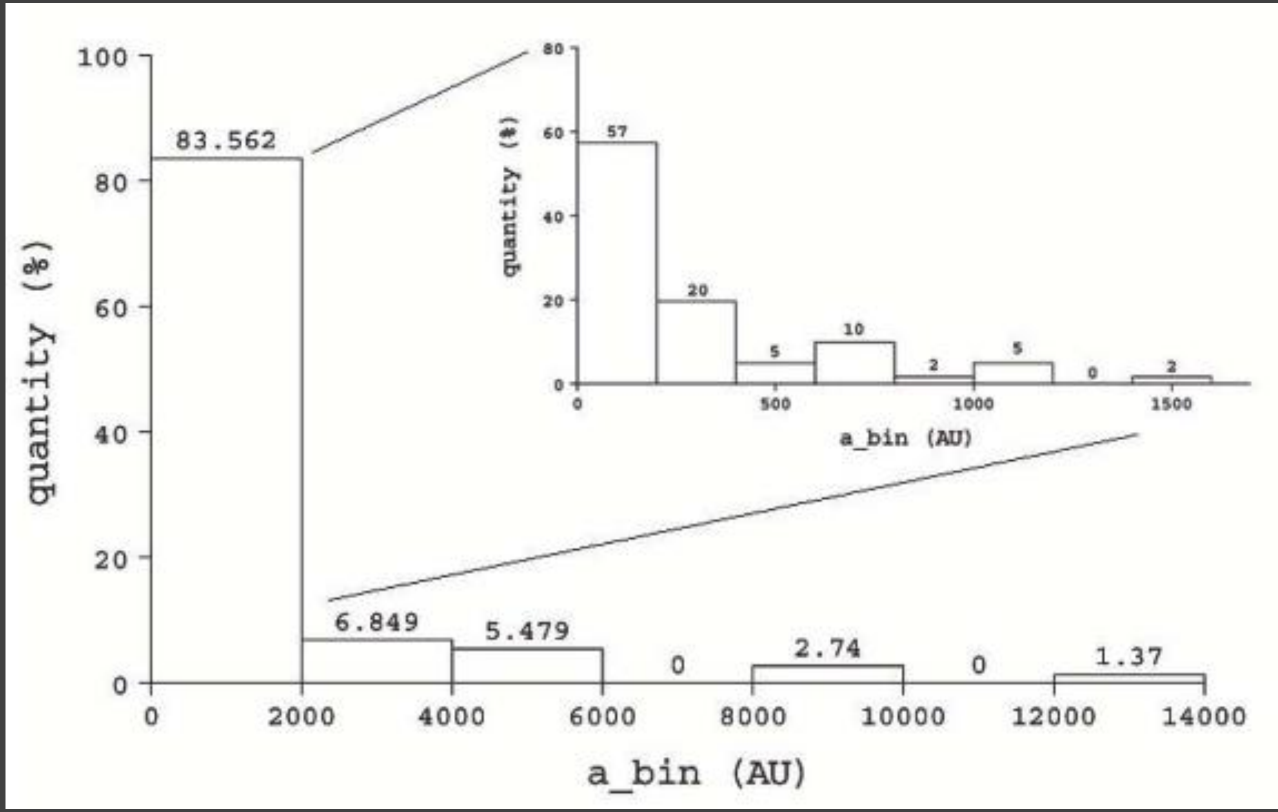
Analysis of selection effects and biases for transits can be found in 1904.04832.




1802.08693

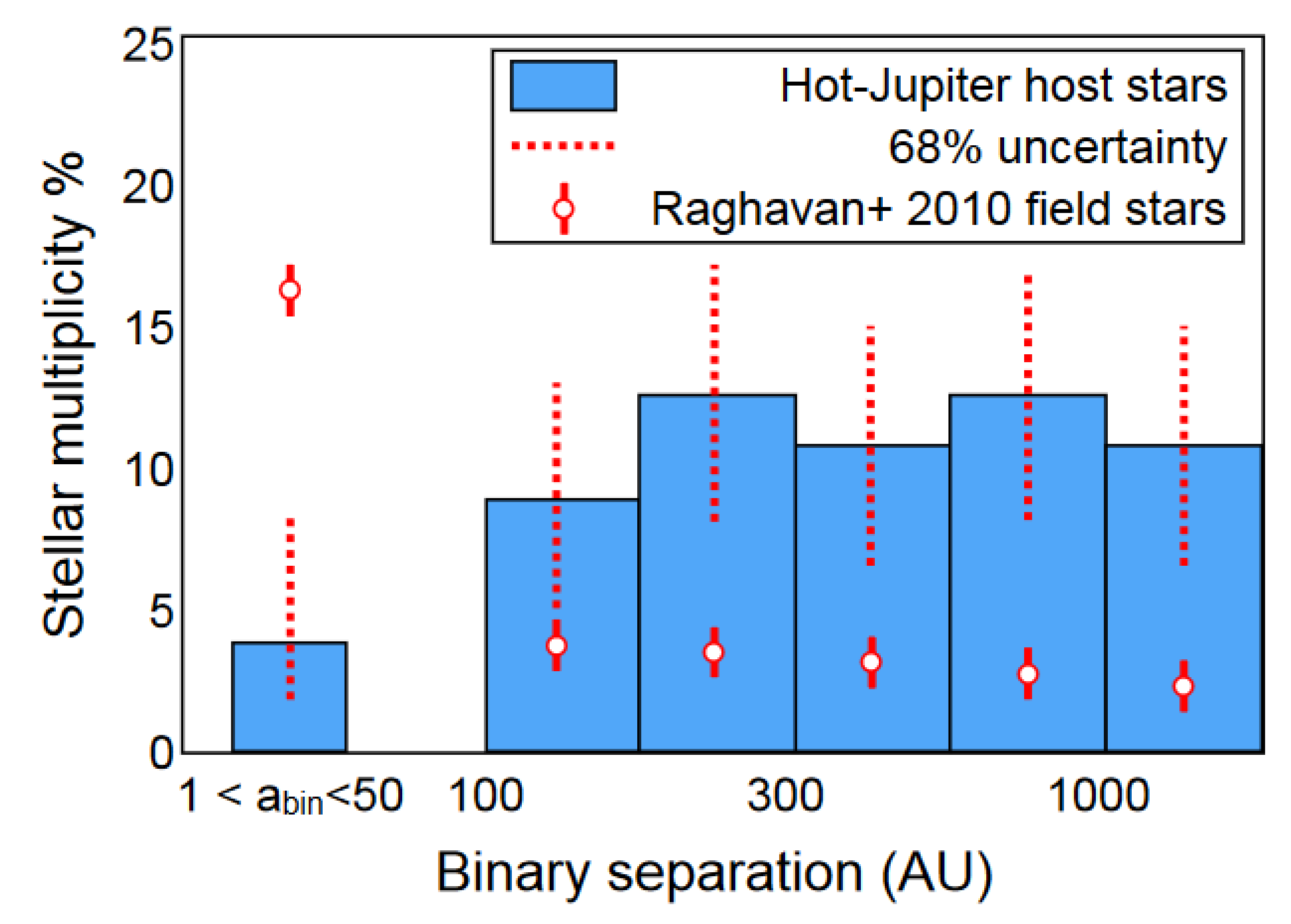
# Statistics




1608.00764

# Statistics for hot jupiters hosts




Less close companions,
but more farther away.

Recent studies (2002.11734) confirm that large planet (S-type) occurrence rate in not-too-narrow binaries is similar to the rate for single stars.
In very close binaries there are less planets.

# First circumbinary planet found by microlensing

Only triple lensing model (star+2 planets or planet+ 2stars) can fit the light curve.

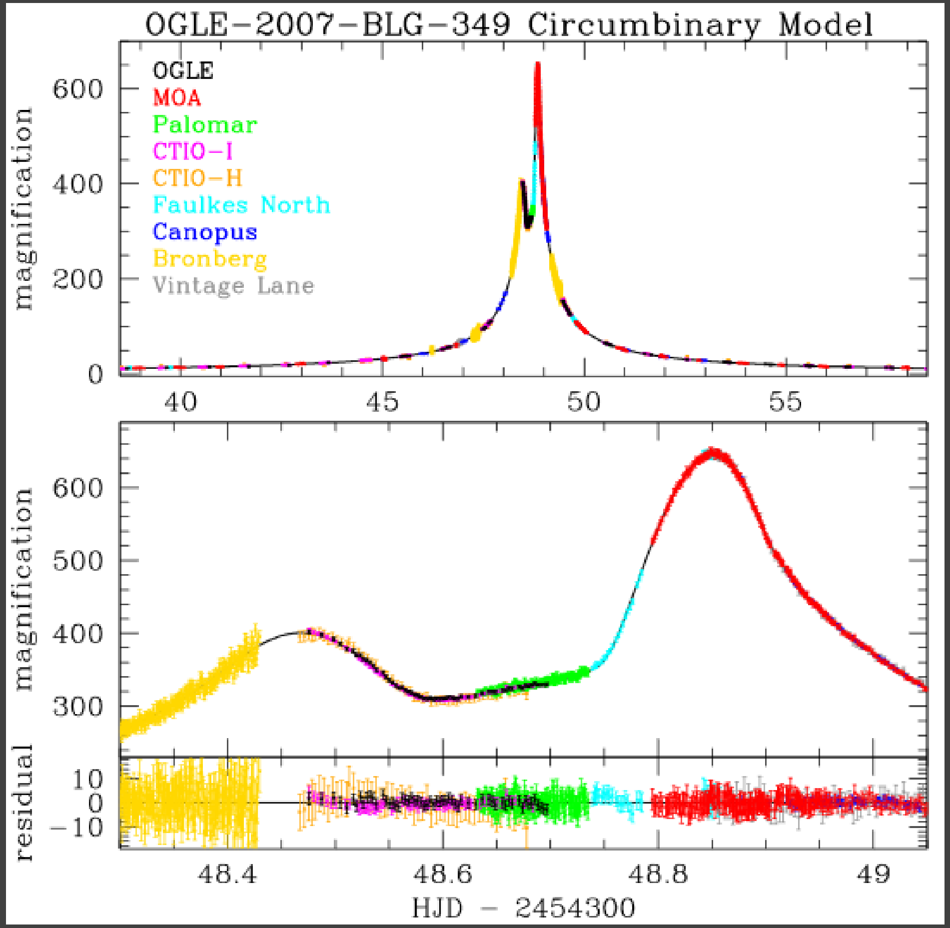
Subsequent HST observations favour the circumbinary model.

| Parameter | units | average value | 2-$\sigma$ range |
|---|---|---|---|
| $D_L$ | kpc | $2.76 \pm 0.38$ | 2.06-3.56 |
| $M_{\rm A+B}$ | $M_\odot$ | $0.71 \pm 0.12$ | 0.50-0.95 |
| $M_{\rm A}$ | $M_\odot$ | $0.41 \pm 0.07$ | 0.28-0.54 |
| $M_{\rm B}$ | $M_\odot$ | $0.30 \pm 0.07$ | 0.15-0.45 |
| $m_c$ | $M_\oplus$ | $80 \pm 13$ | 56-107 |
| $a_{\perp \rm AB}$ | AU | $0.061 \pm 0.007$ | 0.048-0.074 |
| $a_{\rm AB}$ | AU | $0.080^{+0.027}_{-0.015}$ | 0.054-0.162 |
| $P_{\rm AB}$ | days | $9.7^{+5.4}_{-2.5}$ | 5.7-28.1 |
| $a_{\perp \rm CMc}$ | AU | $2.59^{+0.43}_{-0.34}$ | 1.97-3.89 |

Planetary orbit ~3.2AU
Orbital period ~7 years

$80 \pm 13 M_\oplus$

Planetary orbit




1609.06720

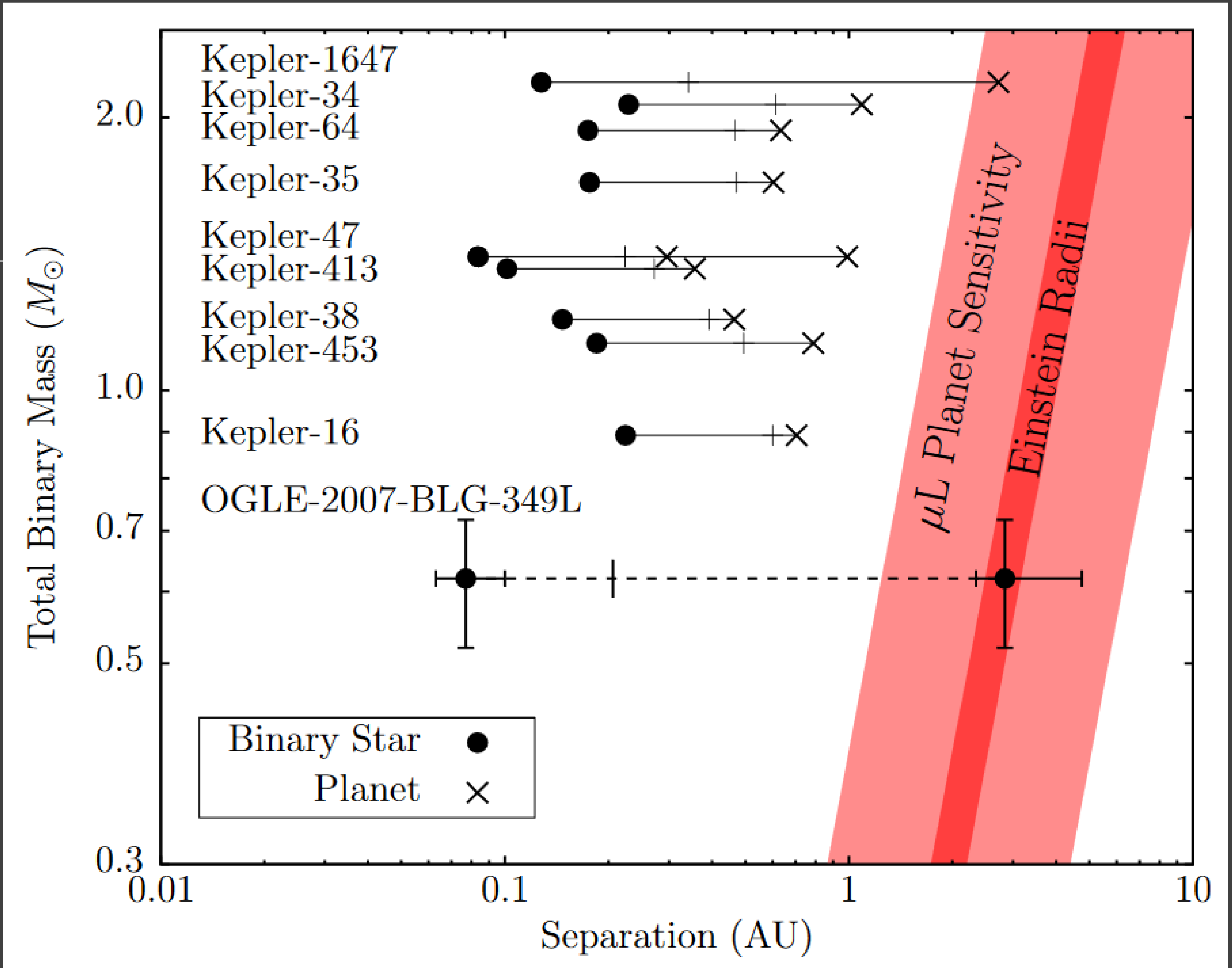
# Comparison

Kepler planets have tight orbits, as if they moved close to their stars after formation.
They are close to the stability limit.

$$a_c \simeq (2.28 \pm 0.01) + (3.8 \pm 0.3)e + (1.7 \pm 0.1)e^2,$$

$a_c$ is measured in binary semi-major axis

Circles show binary separation,
And crosses – planetary.
Vertical ticks mark the stability limit.



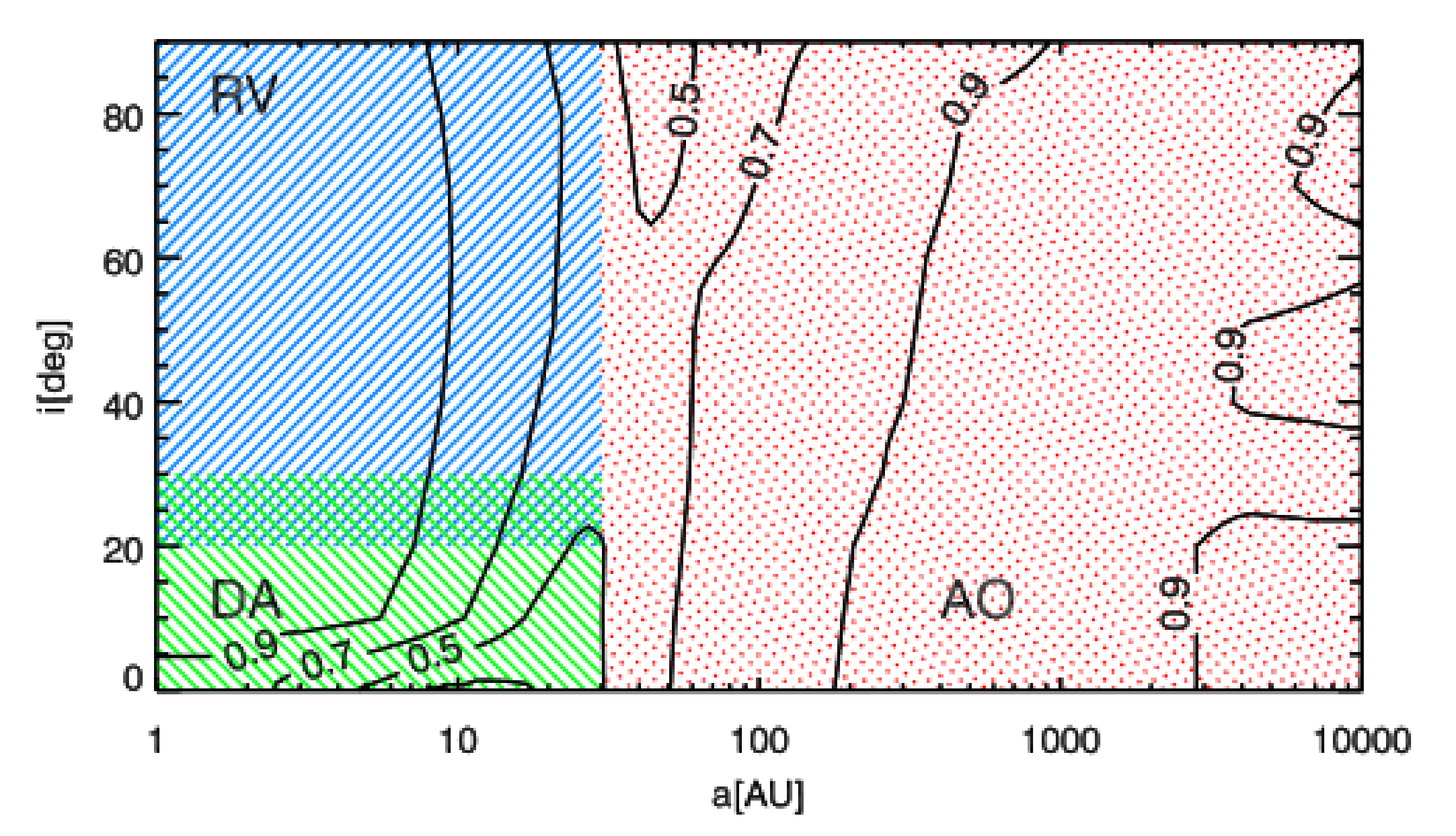
# Search for binary component around planets hosts

Three techniques:

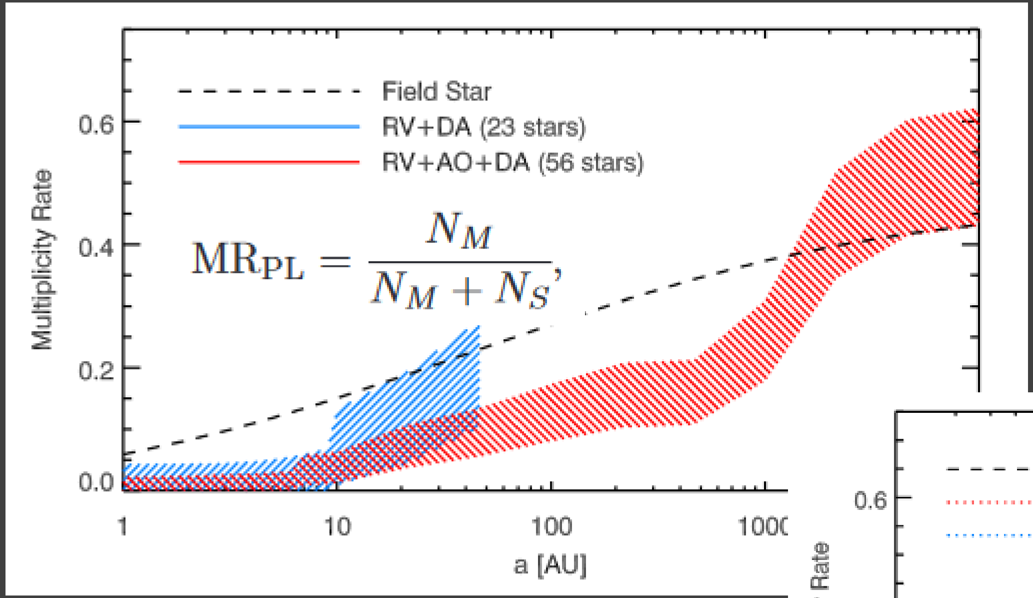
- RV
- AO
- DA

DA (dynamical analysis) works for multiplanet systems.

Mutial inclination of a companion and planetary orbit – i.
90 degrees corresponds to coplanar orbits.



# Statistics of planets in close binaries

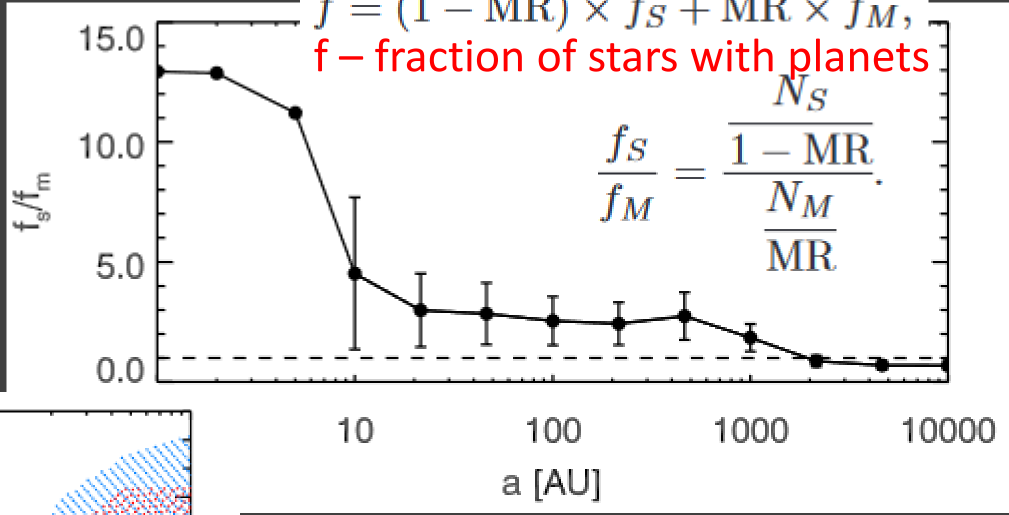



$$\mathrm{MR_{PL}} = \frac{N_M}{N_M + N_S},$$

MR – global stellar multiplicity rate

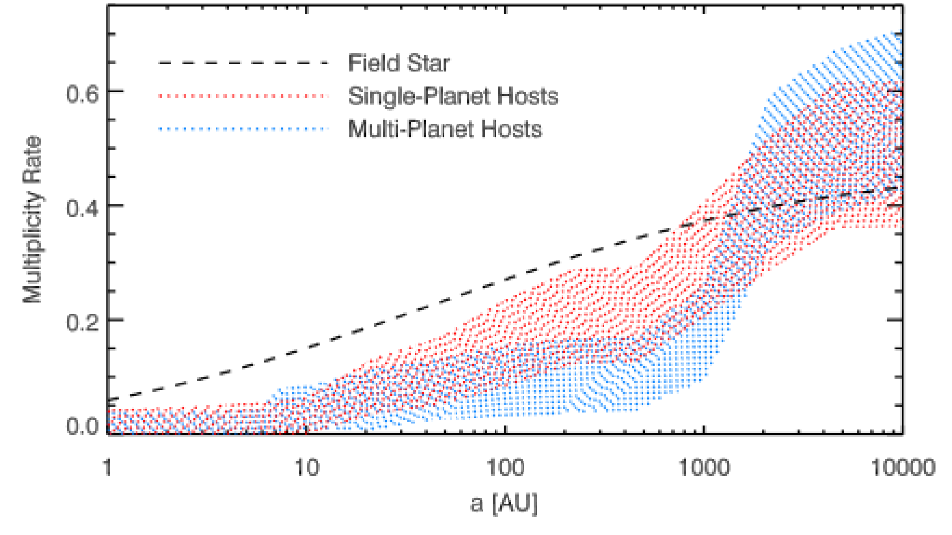
$$f = (1 - \mathrm{MR}) \times f_S + \mathrm{MR} \times f_M,$$

f – fraction of stars with planets




$$\frac{f_S}{f_M} = \frac{\frac{N_S}{1 - \mathrm{MR}}}{\frac{N_M}{\mathrm{MR}}}.$$

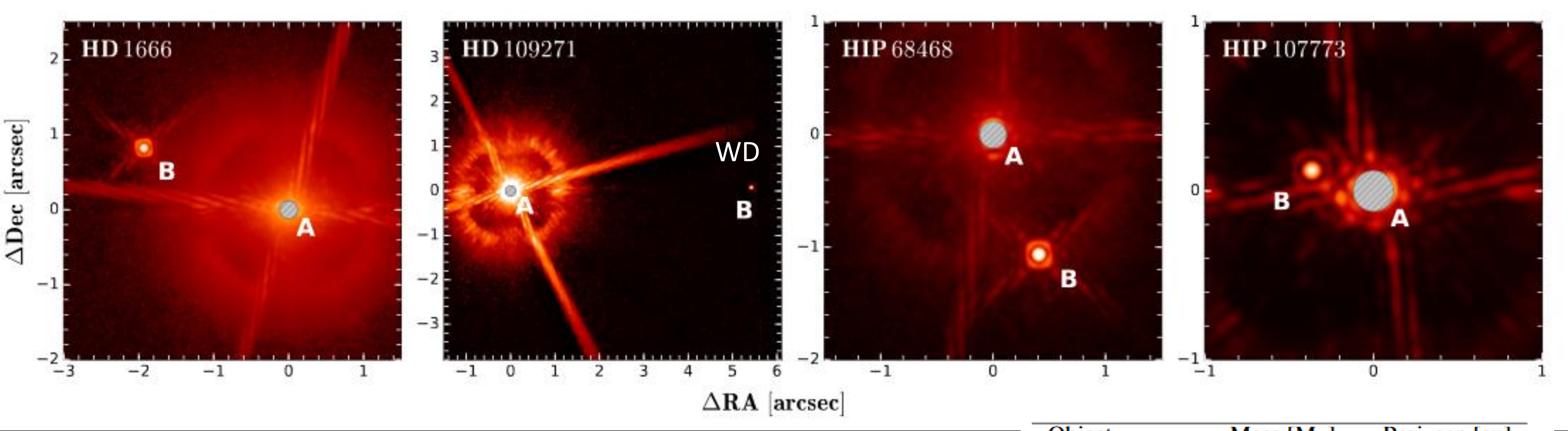
Multiplicity rate of planet hosting stars is smaller than in the field for binary separations <~1000-1500 AU.




At binary separations >~1000-1500 AU S-type planets are as frequent as around single stars.
For smaller separations S-type planets are less numerous.

1407.3344

# VLT/SPHERE search for companions



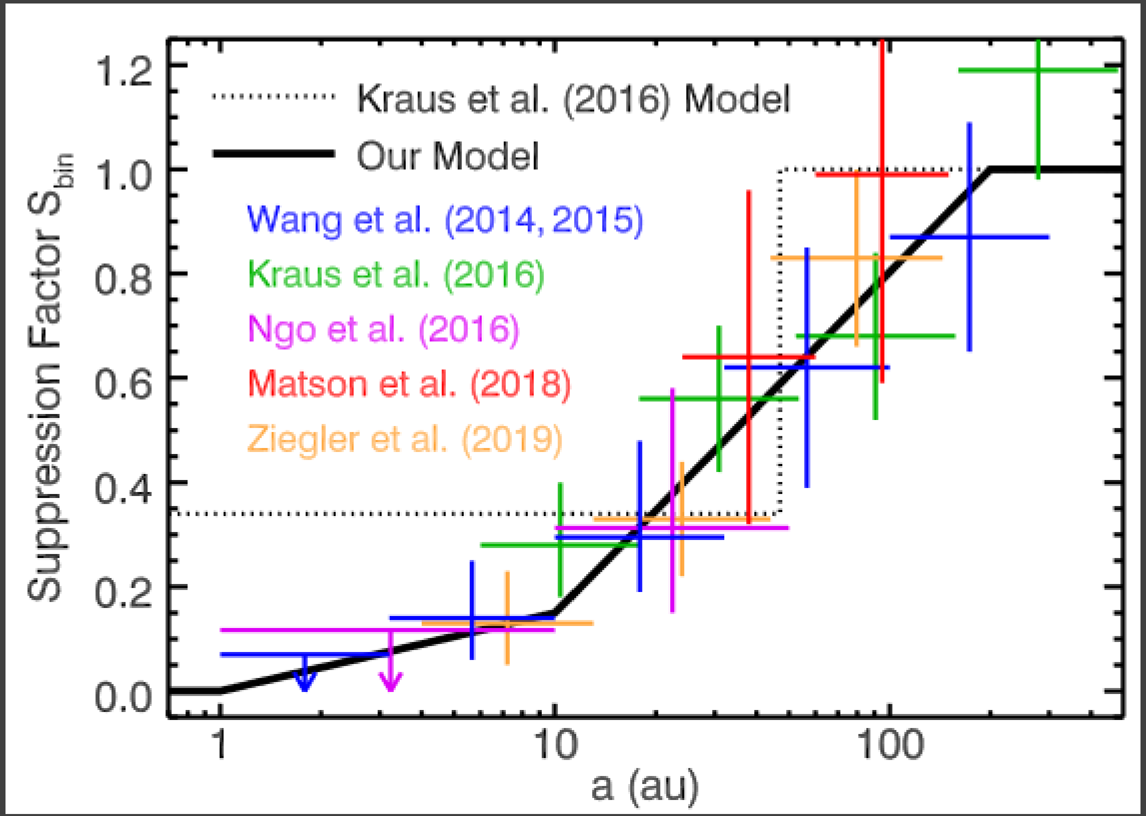

| Object | Mass [$M_\odot$] | Proj. sep [au] |
|---|---|---|
| HD 1666 B | 0.39±0.01 $M_\odot$ | 248 |
| HD 109271 B | ~0.6 $M_\odot$ | 304 |
| HIP 68468 B | 0.36±0.01 $M_\odot$ | 114 |
| HIP 107773 B | 0.63±0.04 $M_\odot$ | 40 |

# Close binaries suppress S-type planets

Binaries with a~1 au fully suppress S-type planets.

Binaries with a≈10 au host close planets
at ≈15% the occurrence rate of single stars.

Wide binaries with a~200 au have
a negligible effect on planet formation.




Ratio of the stellar companion fraction in planet hosts versus field stars, as a function of binary separation.

# ETV: Eclipse timing variations

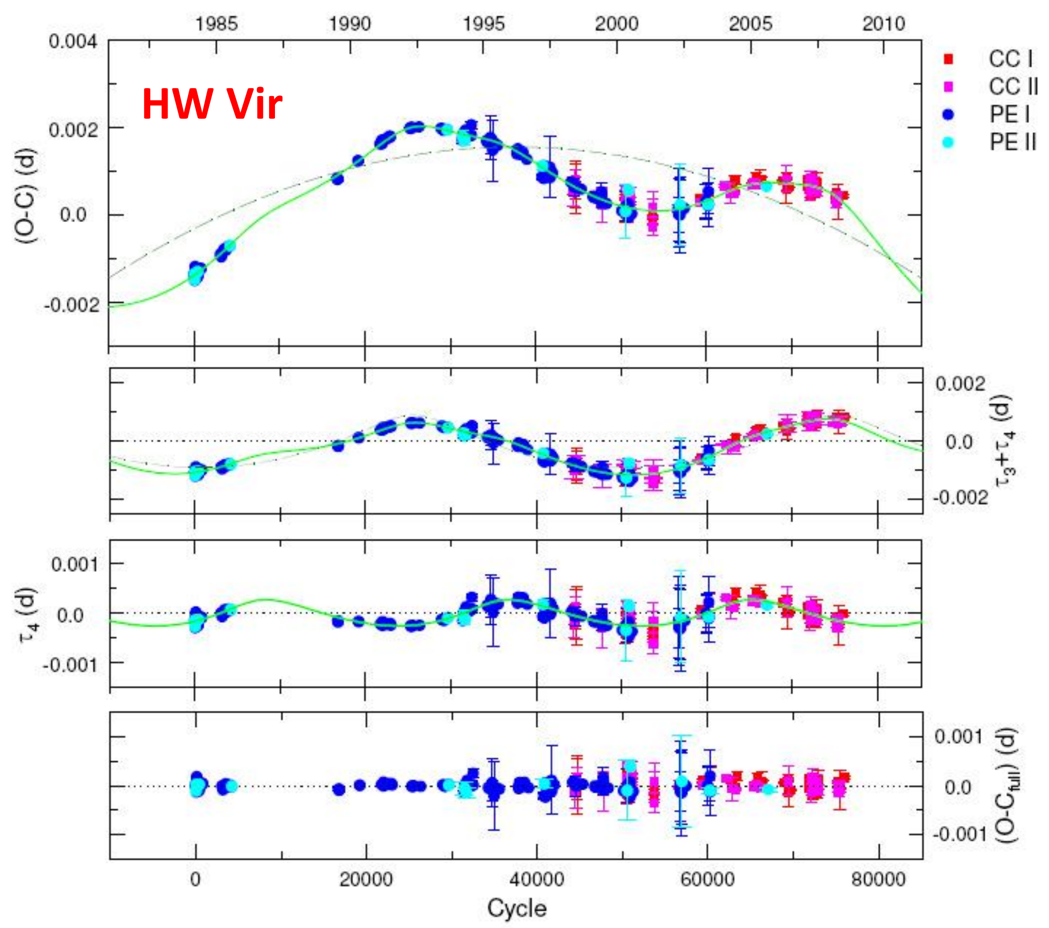
CoRoT: 4 sec – for bright stars (12 mag),
and 16 sec – for dim stars (15.2 mag).
Kepler: 0.5 sec – for bright stars (9 mag),
and 4 sec – for dim stars (14.5 mag).

HW Vir – short period (2.8 h) binary
P-type planets: 15.8 and 9.1 yrs

Residuals

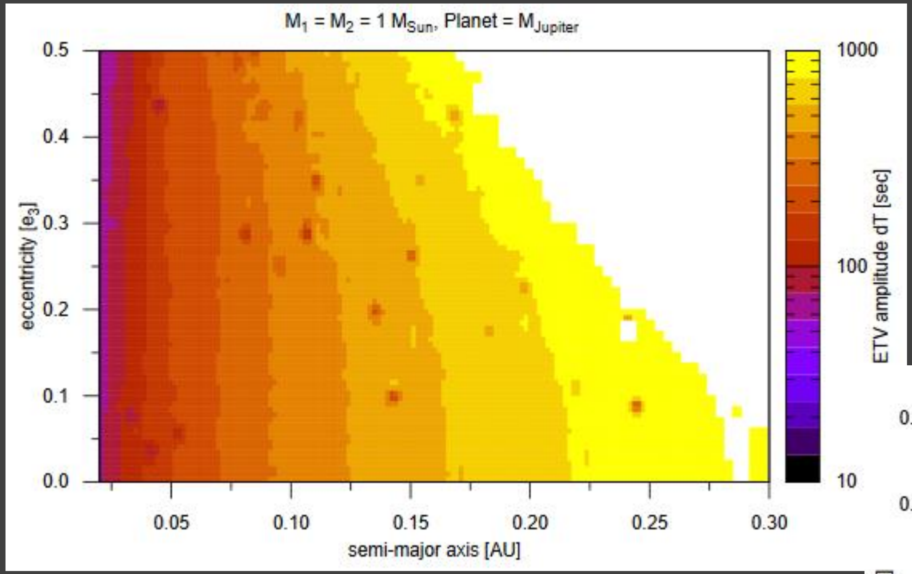
One planet model

Two planets model

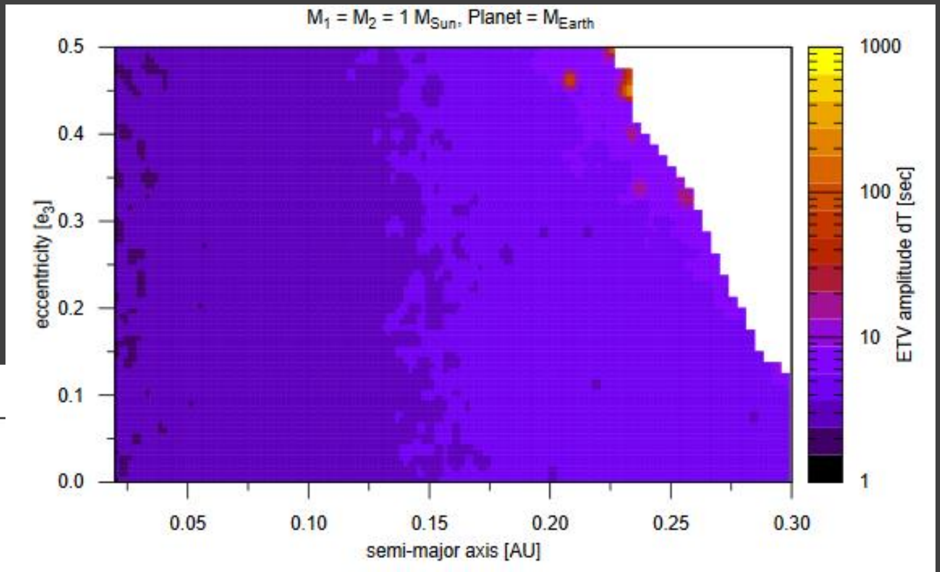



http://astronomy.science.upjs.sk/projectdwarf/scientific-background/

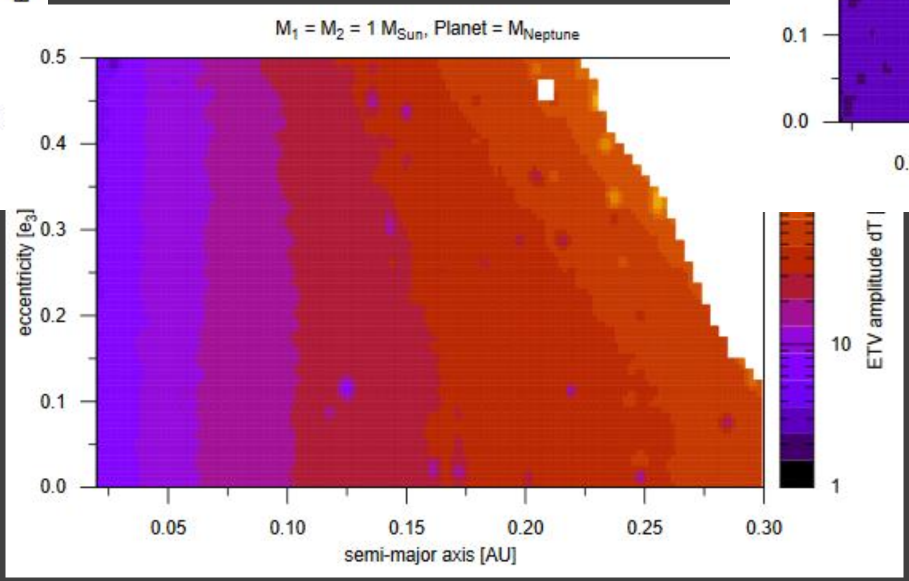
# Modeling ETV. S-type systems




$a_{bin}$=1AU
i=0 (planar orbit)
$e_3$ – planetary eccentricity

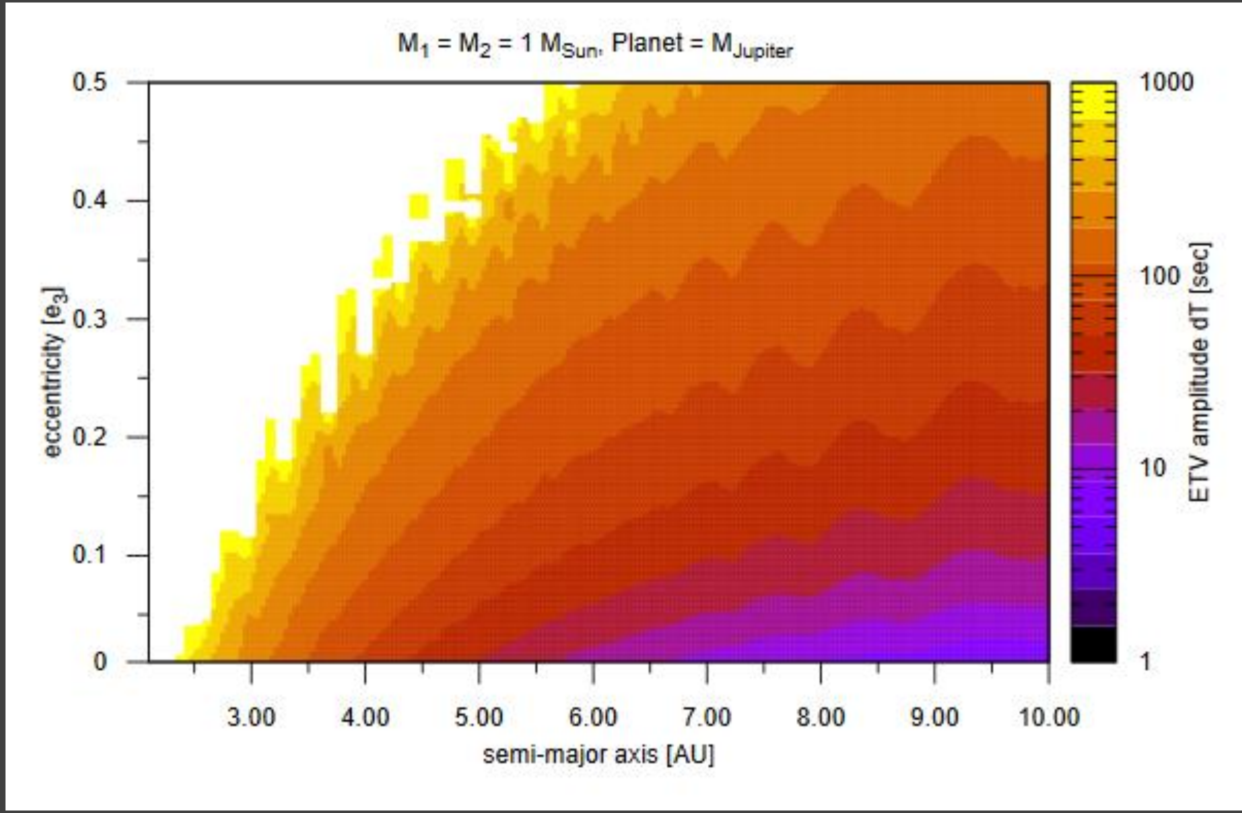


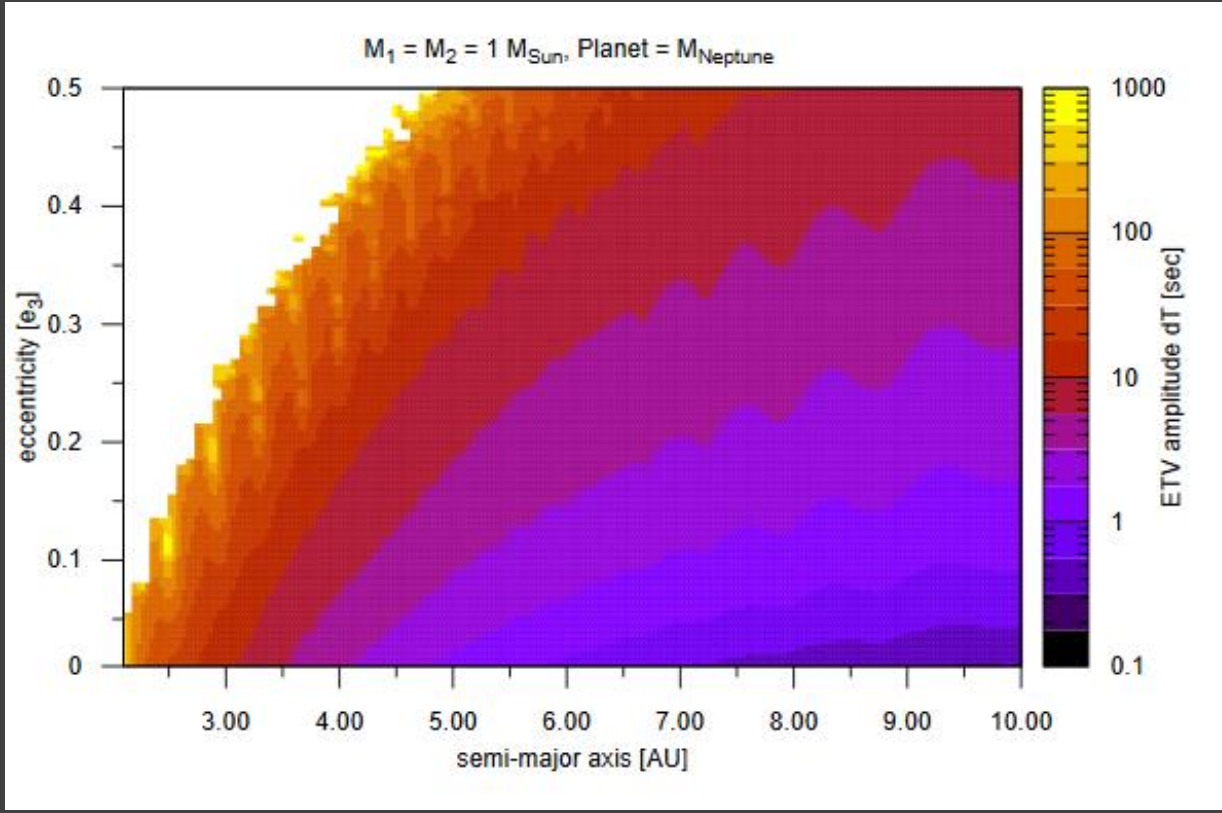

Jupiters are 100% detectable




Earth-like planets are undetectable

# Modeling ETV. P-type systems

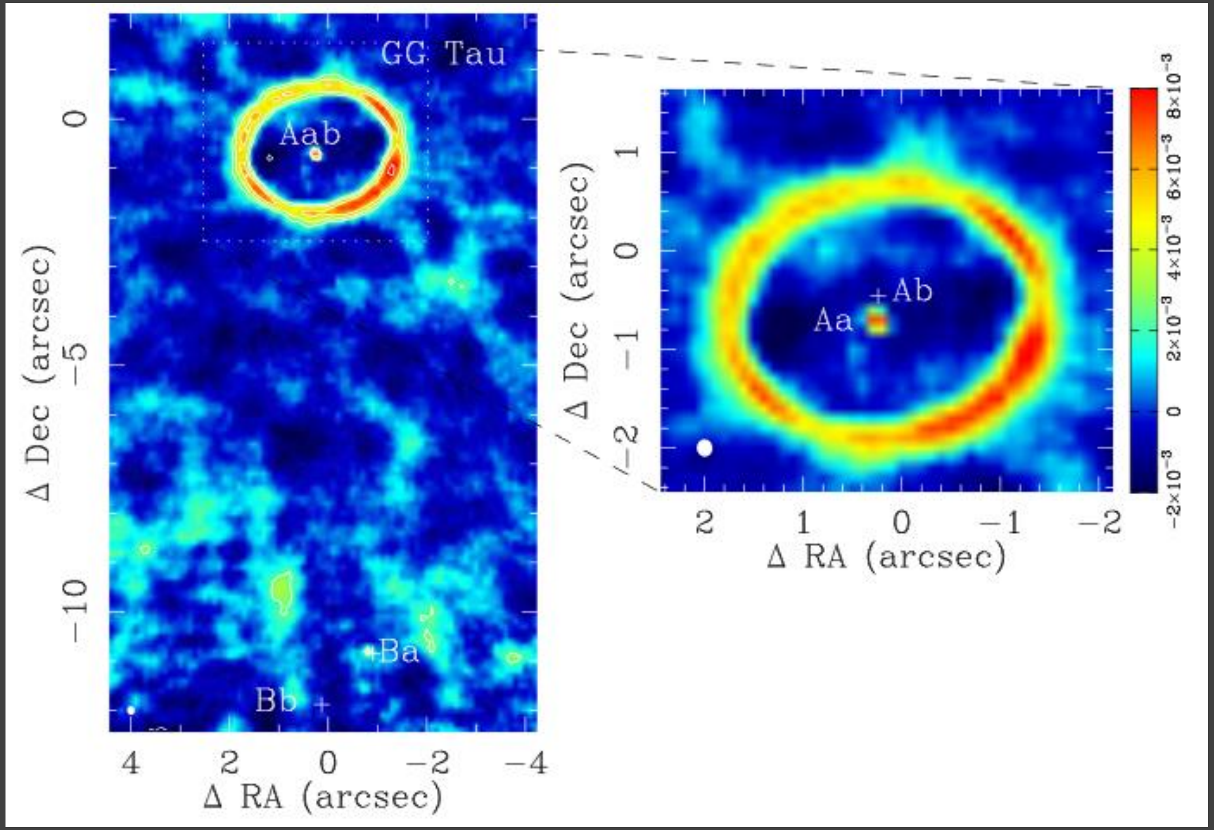


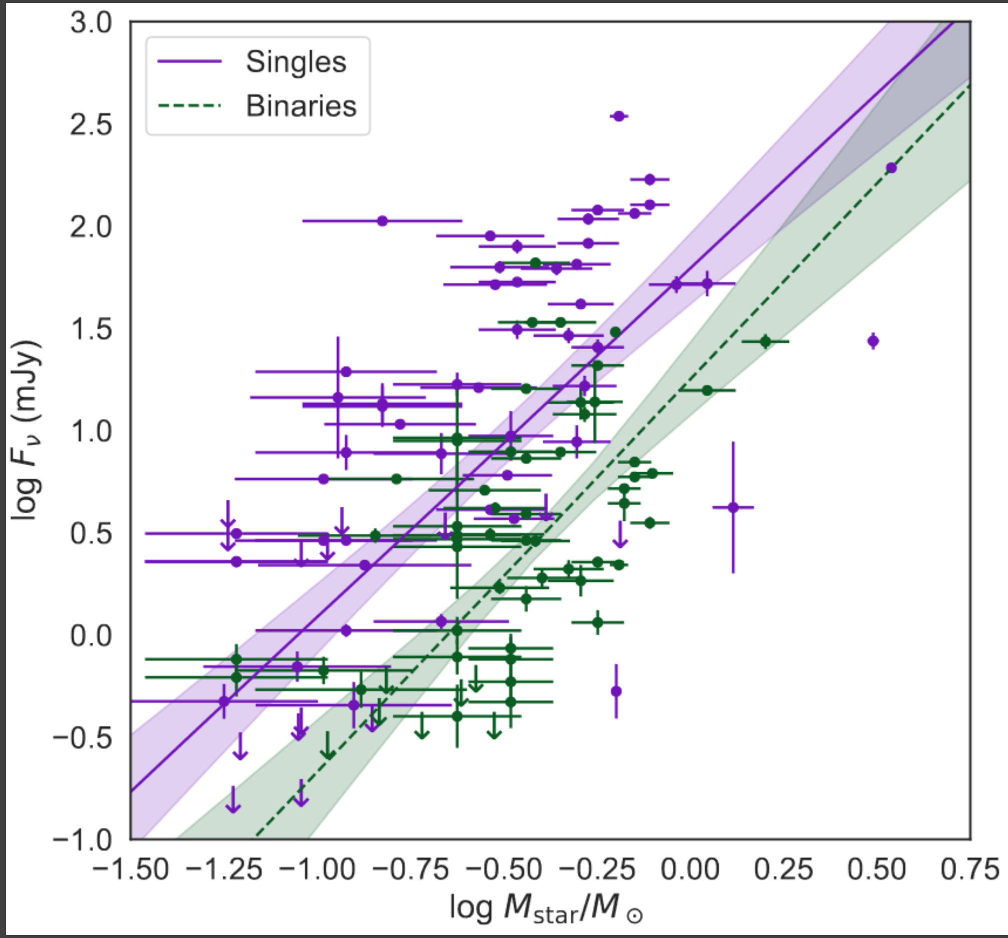




$a_{bin}$=1 AU; $e_{bin}$=0

# Protoplanetary discs in binaries

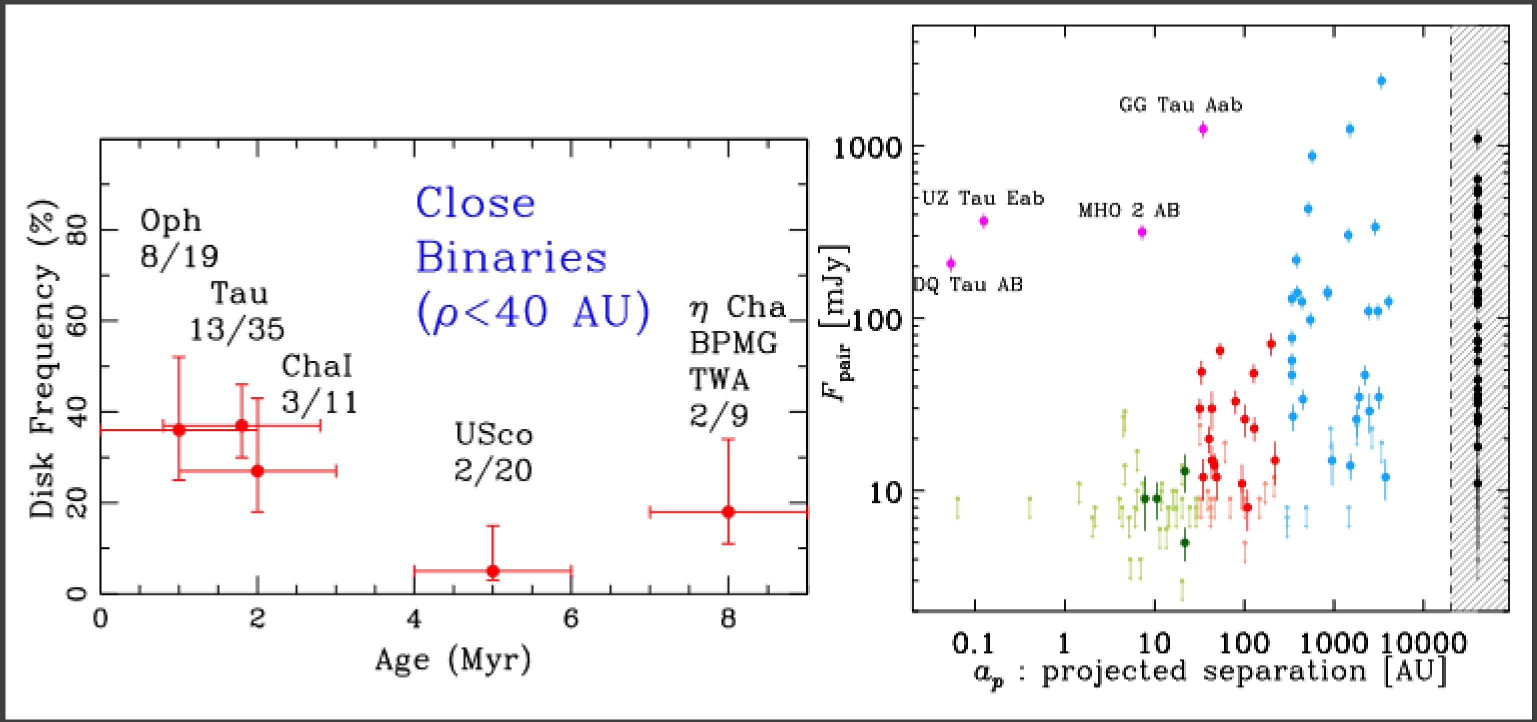
Many discs are resolved by ALMA in binary systems.
Discs in binaries are lighter (in close binaries lighter than in wide).



# Protoplanetary discs in binaries. S-type

Discs in binaries might be truncated (at 1/3 - 1/4 of the orbital separation).




Perturbations in the disc also modifies planet growth.

Disc frequency for wide binaries is similar to that in single stars, but for close binaries it is lower.

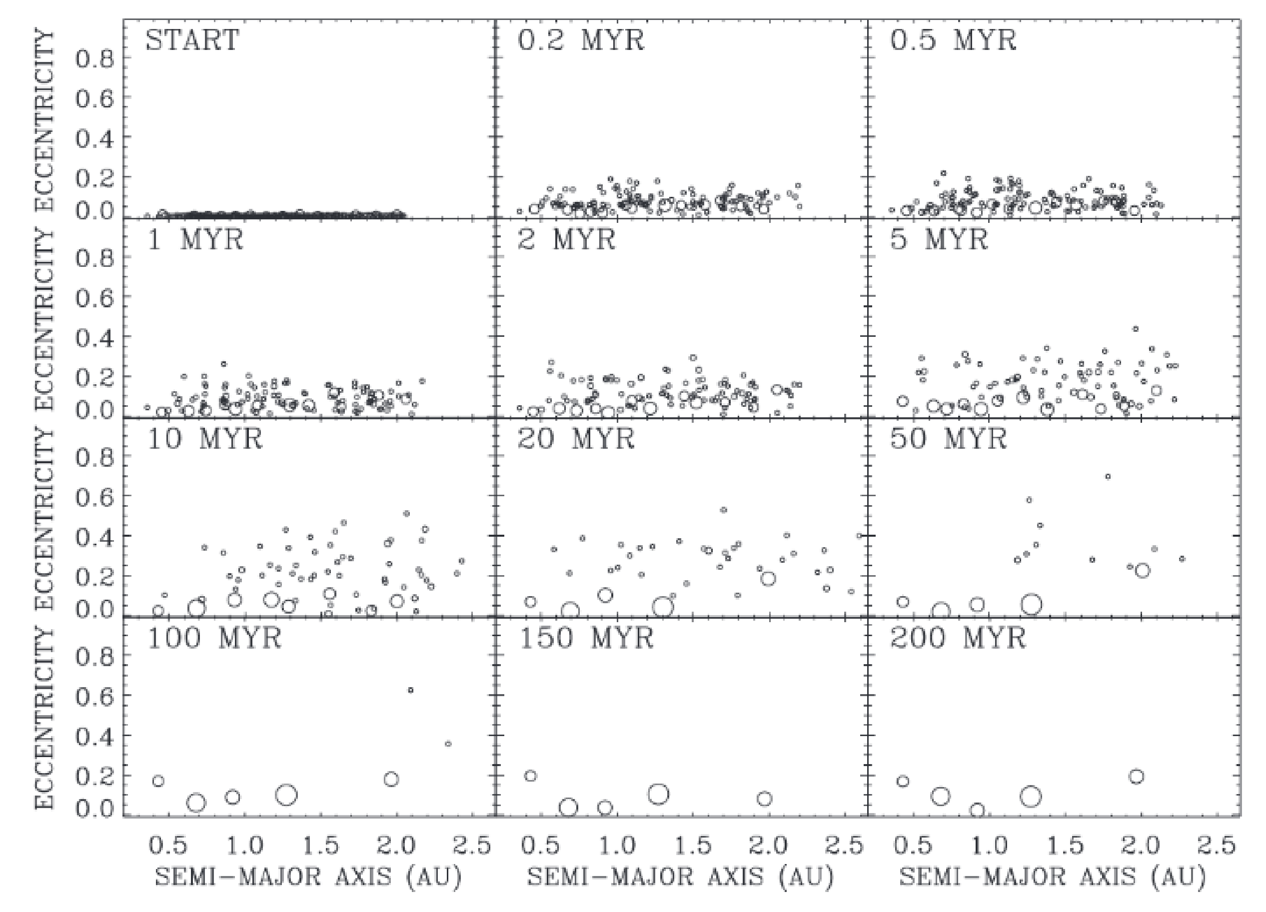
Dust mass is smaller for smaller binary separation.

Truncated discs with lower dust mass can be a bad place to form planets, especially massive.

Discs in close binaries are also short lived. Also bad for planet formation, especially for giants. In 2/3 of close binaries discs live for <1 Myr.

Temperature is higher in such discs, so dust growth can be less efficient.

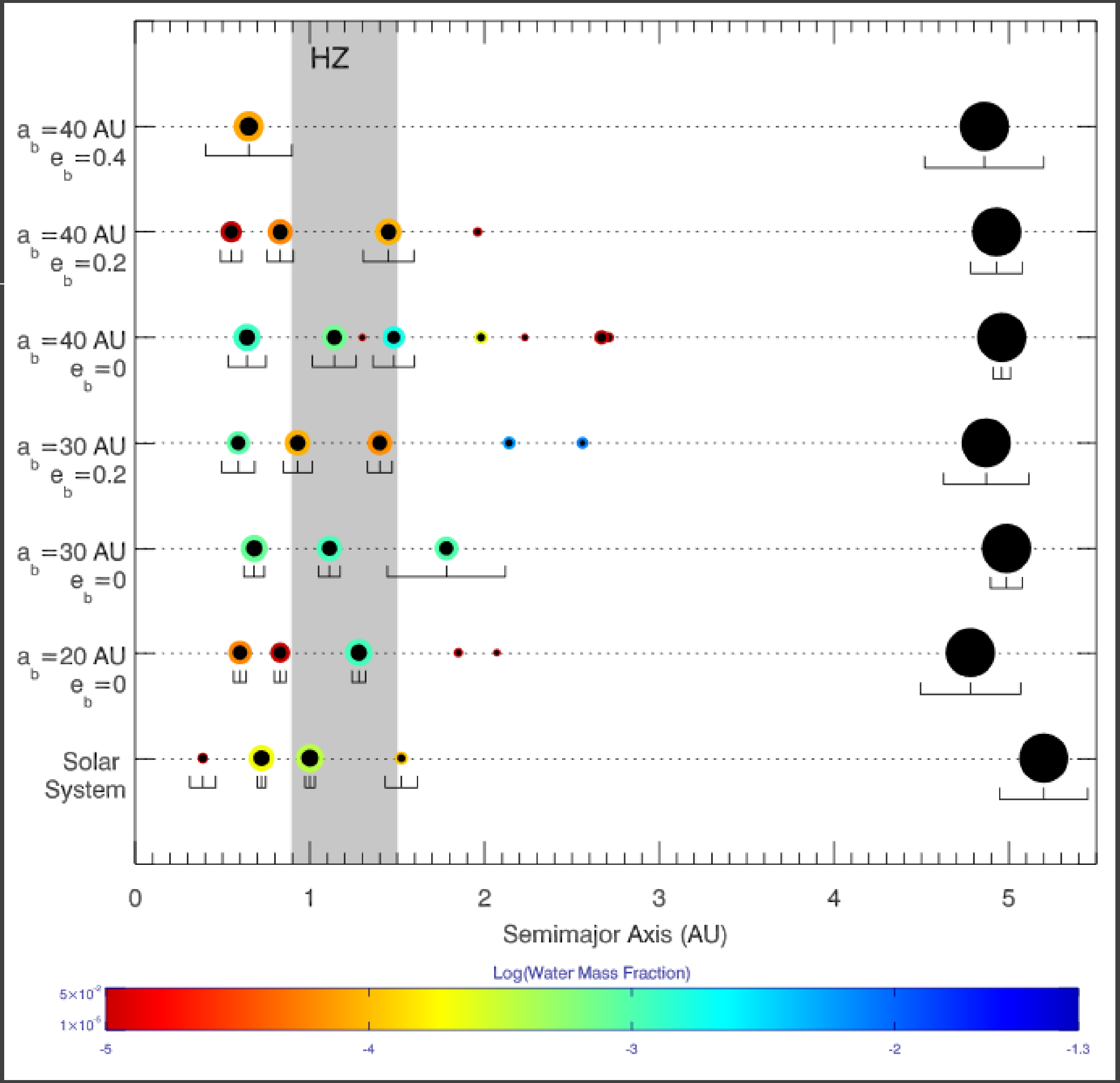
# Alpha Centauri-like binary




In a compact binary system planet formation is possible only at small distances from the host star.

# Water delivery

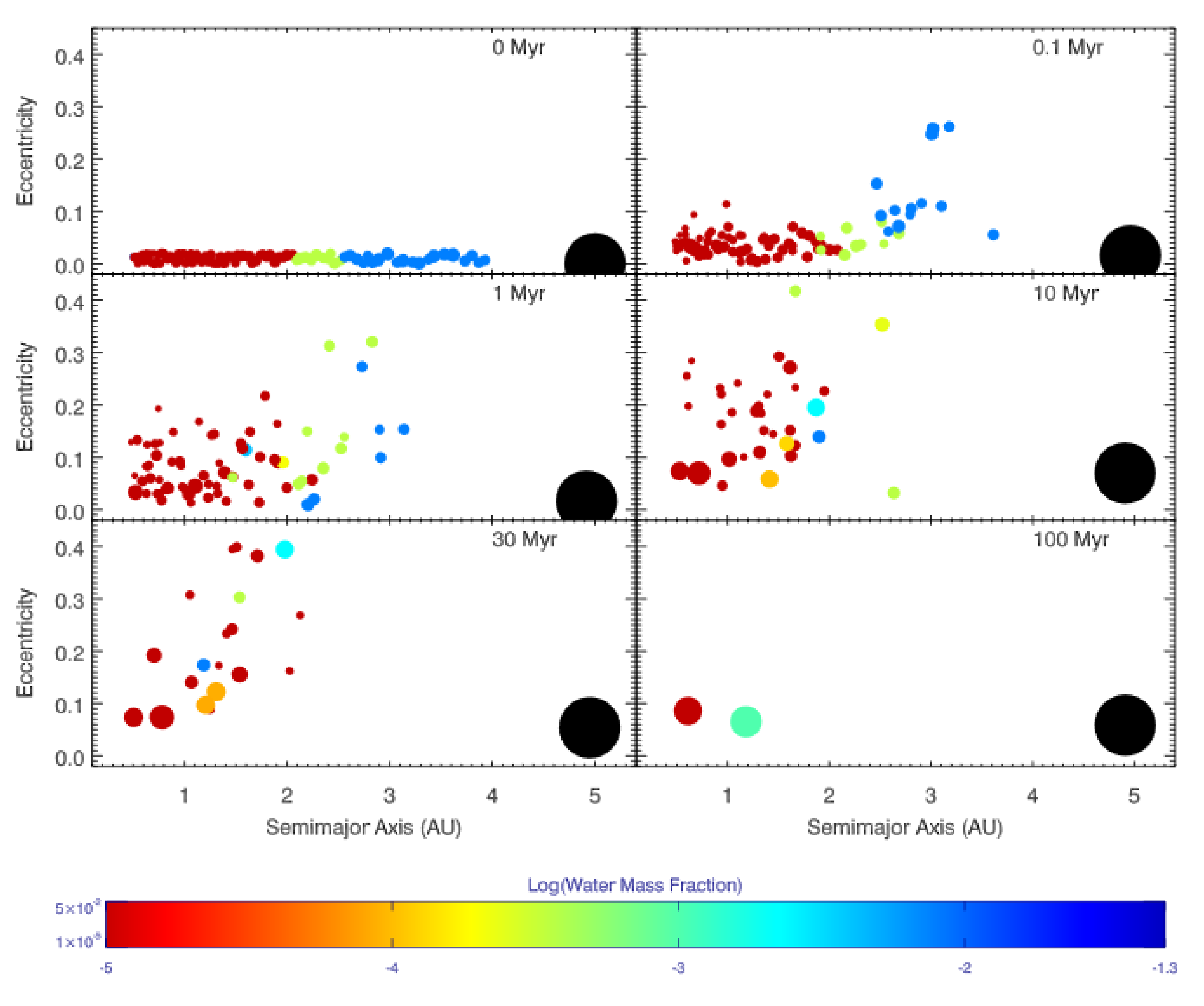
Earth-like planets can be formed
in a habitable zone in a compact binary.



# Earth-like

Snap shots of formation of Earth-like planets in a HZ.

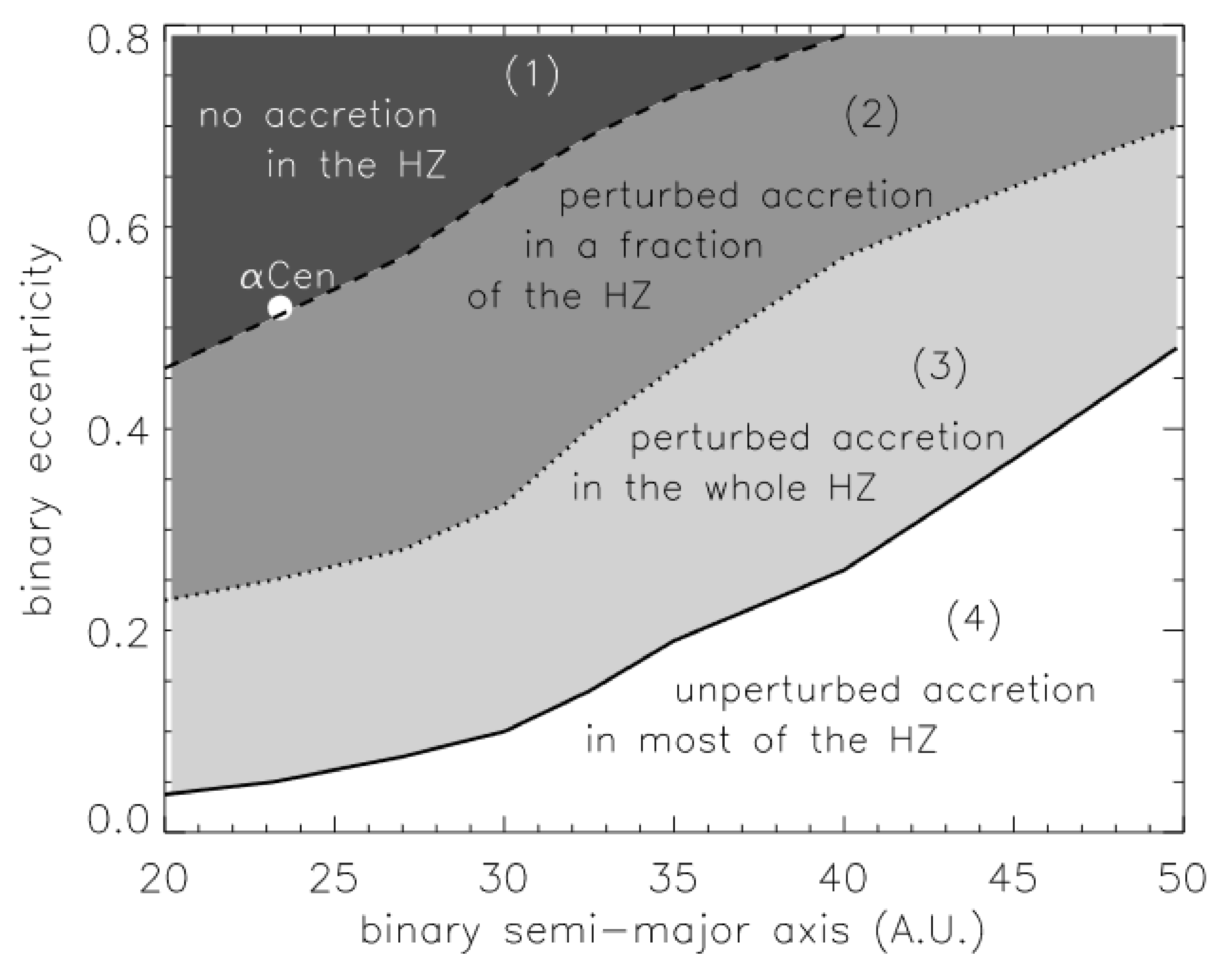
In this study it was assumed that the initial phases of planet formation have been successful.



# Accretion

Accretion onto forming planets in a binary system is influenced dynamically by the companion.

High velocity collisions results in erosion, not in growth.



# Other ideas for S-type

Several other ideas are discussed

- Planet migration
- Changing of the binary separation
- Gravitational instability

1. Planets could be formed at different orbits, and then migrate outwards. Seems problematic to increase the planetary orbit significantly.
2. It is possible that some compact binaries have been wider, but then their orbits shrink due to interaction with stars in a cluster.
3. Gravitational instability in a protoplanetary disc can help to form planets at larger distances avoiding problems with dust growth, accretion, etc.

Also, in multi-planets systems interaction between disk, planets and stars can result in retrograde orbits and other features (1911.00520).

1406.1357

# Stability of orbits

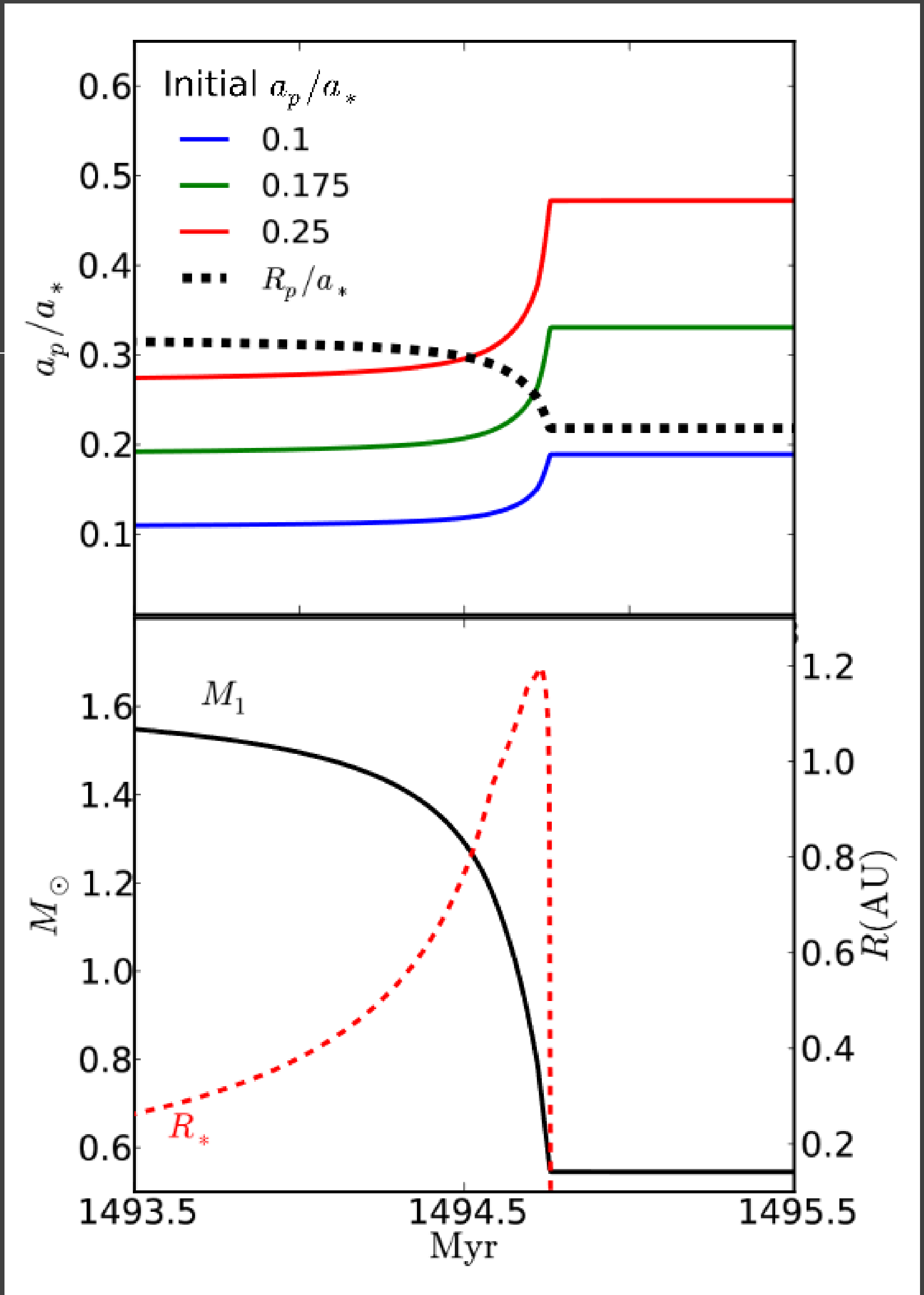
$$R_p = \left(0.464 - 0.38\frac{m_2}{m_1+m_2}\right) a_*$$

For slow (adiabatic) mass loss the orbit expands as:

$$a_f = \frac{M_i}{M_f} a_i,$$

If we consider the planetary mass as a very small value:

$$\left(\frac{a_{p,f}}{a_{*,f}}\right) / \left(\frac{a_{p,i}}{a_{*,i}}\right) = \left(\frac{m_{1,i}}{m_{1,f}}\right)\left(\frac{m_{1,f}+m_2}{m_{1,i}+m_2}\right)$$

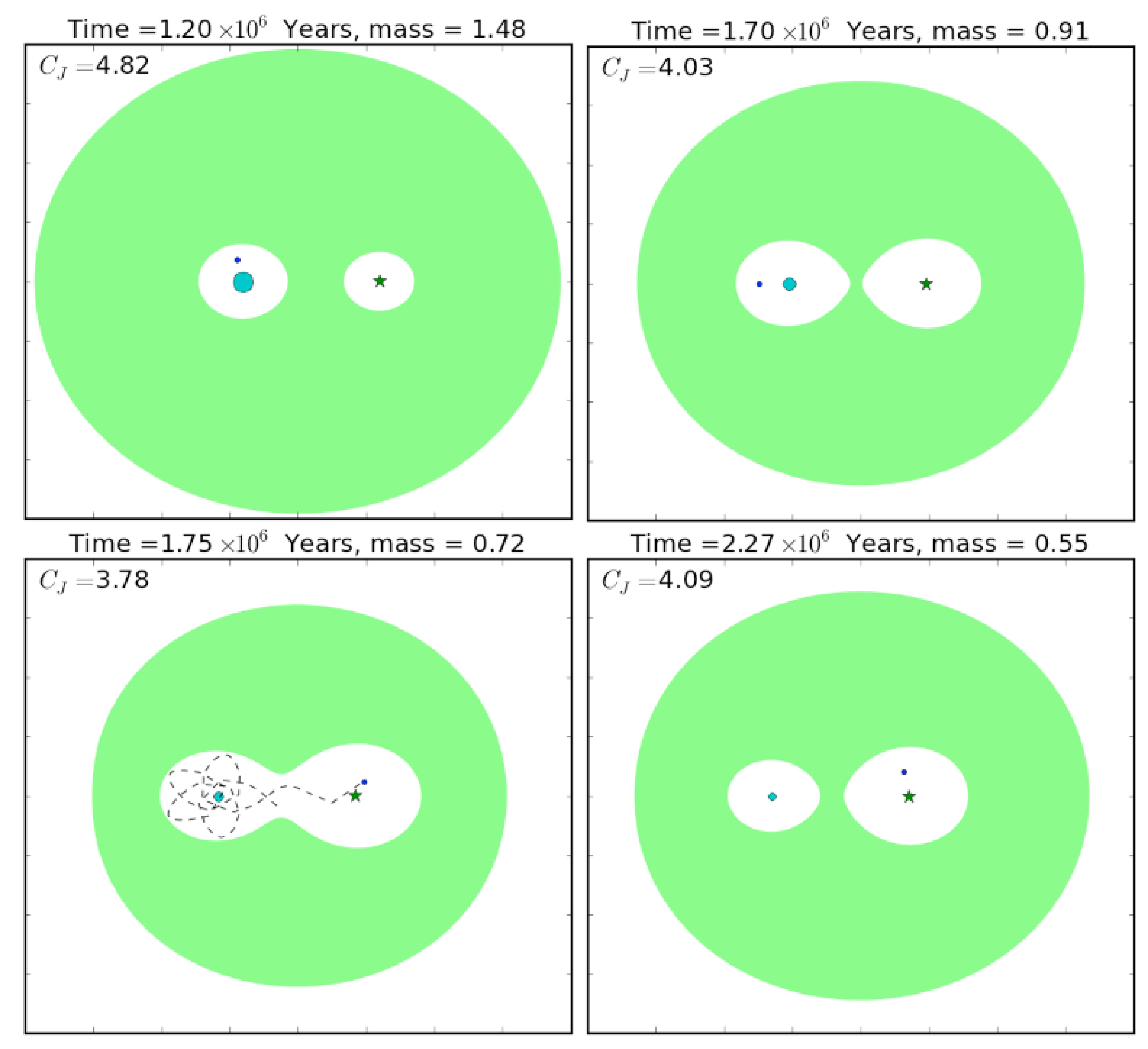


# Star-hoppers

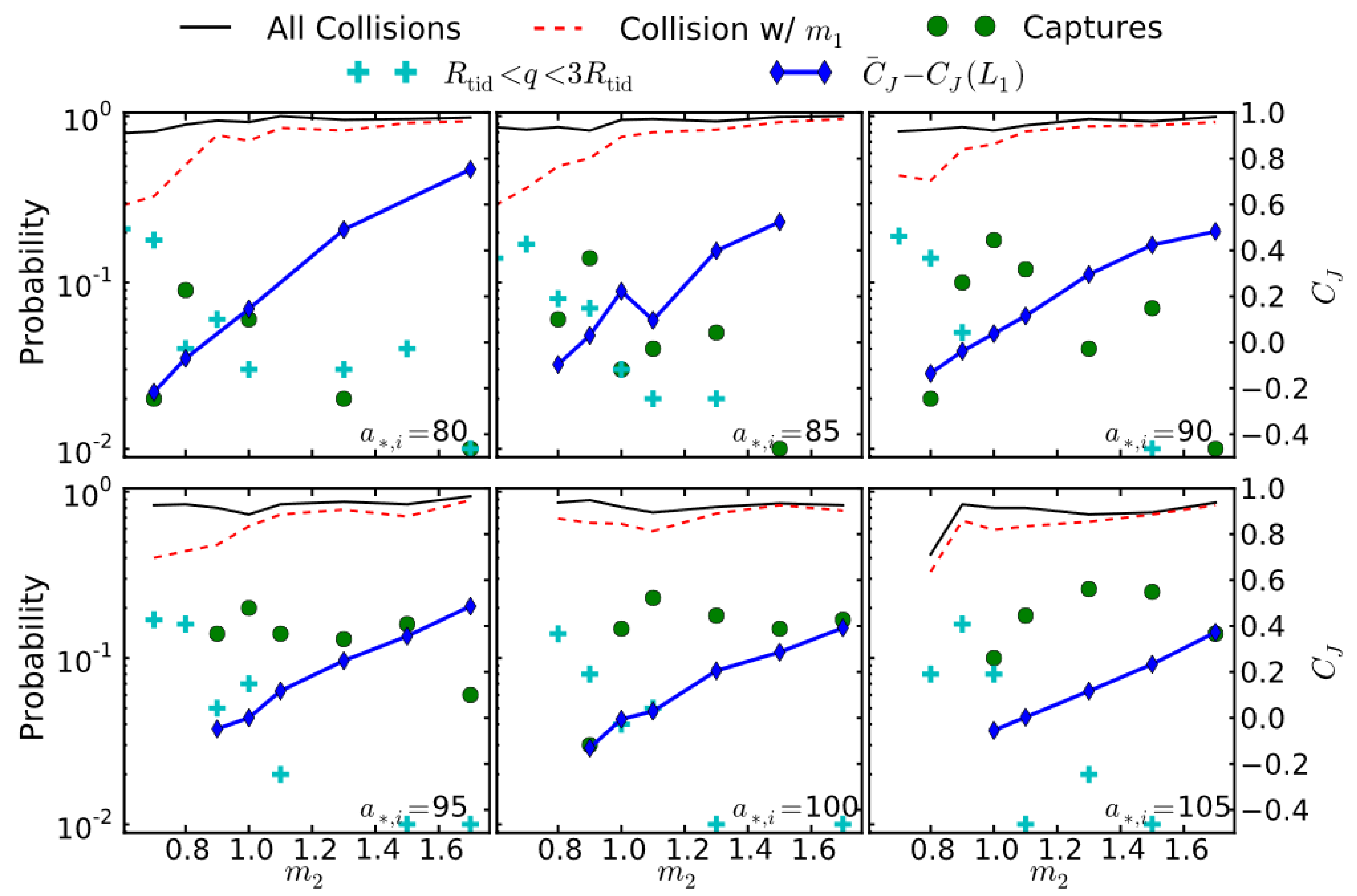
In a binary if the primary is loosing mass planets can change the host.

This is important to form some peculiar types of planets.

$M_1$=2 -> 0.55 $M_{solar}$
$M_2$=1 $M_{solar}$
$a_{ini}$=90 AU



Primary mass
2 solar masses.
It evolves
down to M=0.55.



# Circumbinary (P-type) planets

$$
\begin{aligned}
a_{\mathrm{crit}} \quad \approx \quad & 1.60 + 5.10\, e_{\mathrm{bin}} - 2.22\, e_{\mathrm{bin}}^2 + 4.12 \frac{M_s}{M_p + M_s} - 4.27\, e_{\mathrm{bin}} \frac{M_s}{M_p + M_s} \\
& -5.09 \frac{M_s^2}{(M_p + M_s)^2} + 4.61\, e_{\mathrm{bin}}^2 \frac{M_s^2}{(M_p + M_s)^2},
\end{aligned}
$$

Inside the critical radius orbits of light satellites are unstable.

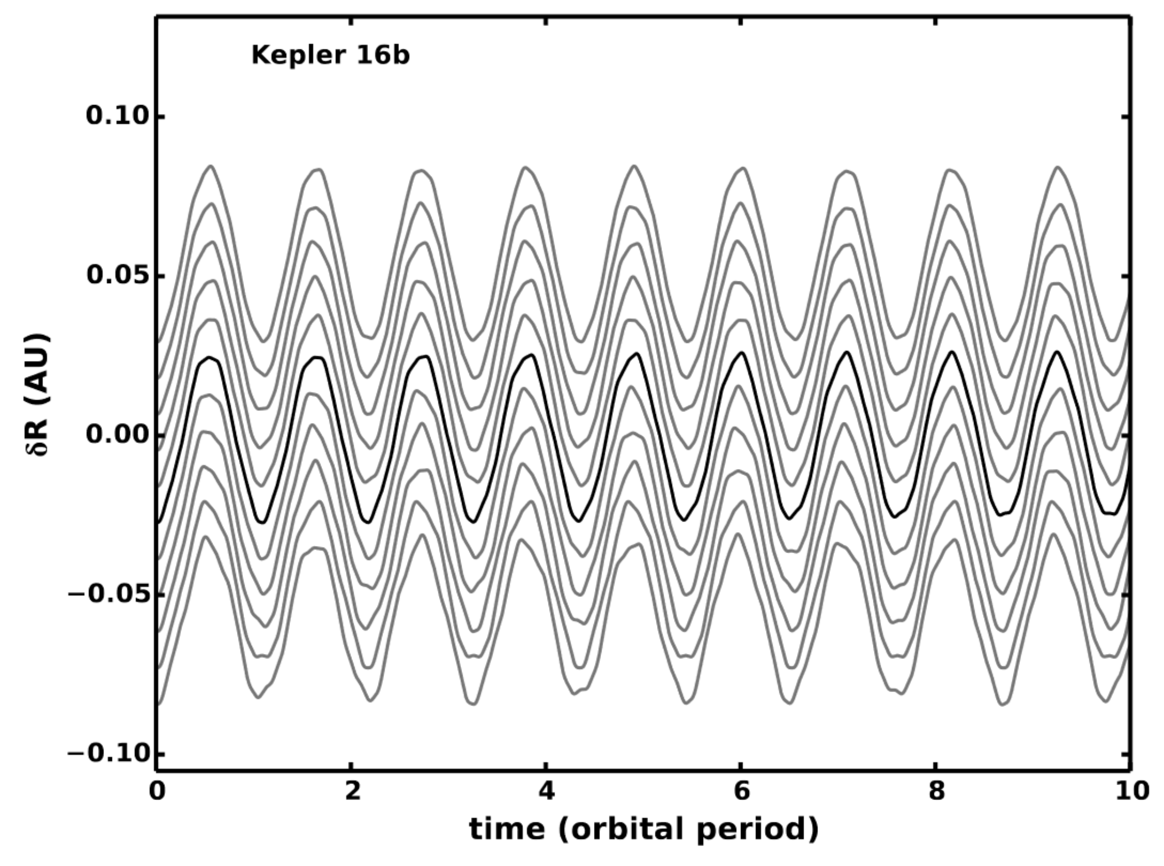
A binary cleans out orbits around it up to 2-5 binary separations.

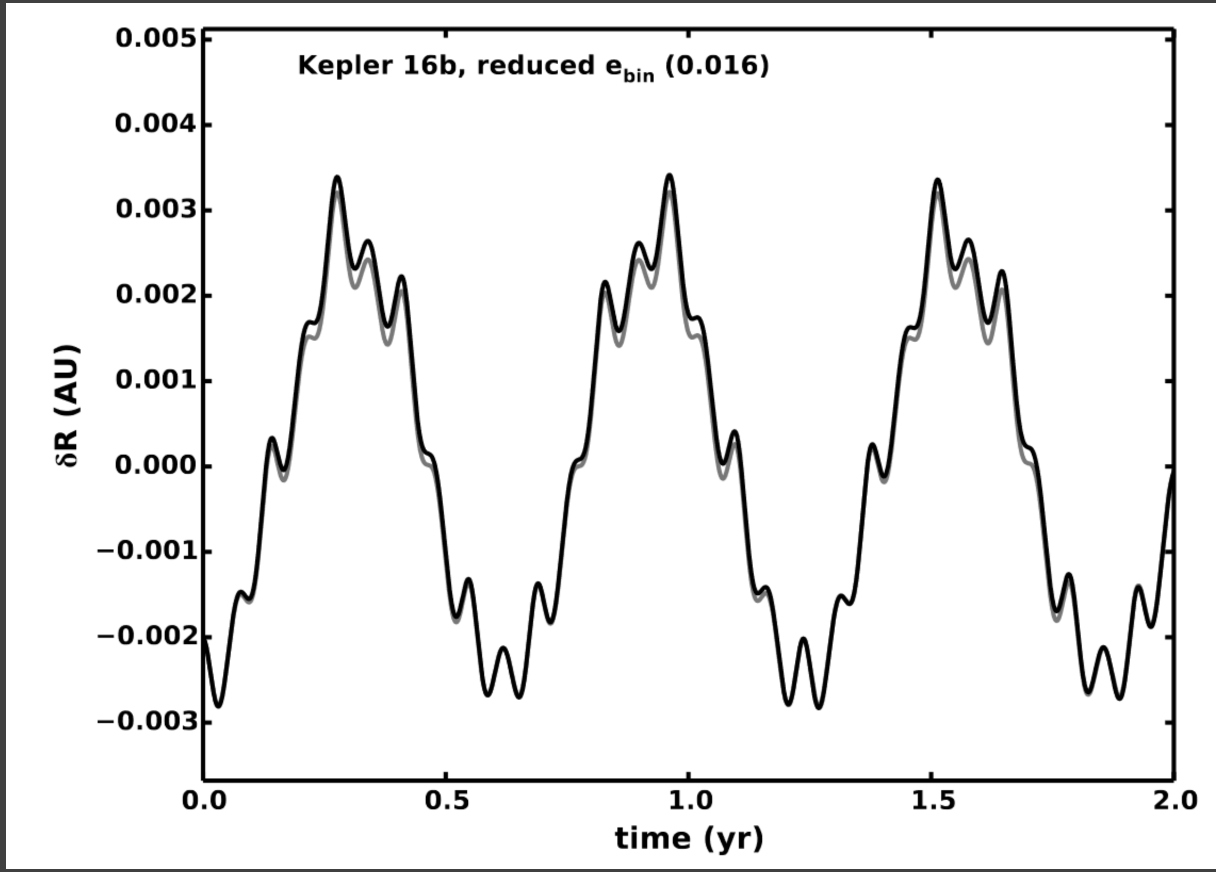
Outside critical distance (except some resonances) there is a family of nested quasi circular (most circular) orbits, which behave quite similar to orbits around single stars.

Beyond 6:1 resonance orbits are stable for small binary eccentricity.
This allows to form planets around binary stars (in the circumbinary regime) in a usual way.

# Most circular orbits

Particles having these orbits make minimal radial excursions and never collide.




1503.03876

# Planetary formation in P-type binaries

Gas and small particles quickly settle to most circular orbits.

$$v_{\rm dest} \gtrsim 0.1 \text{ km/s} \qquad [\text{destructive collisions, } r = 1\,\text{km}]$$

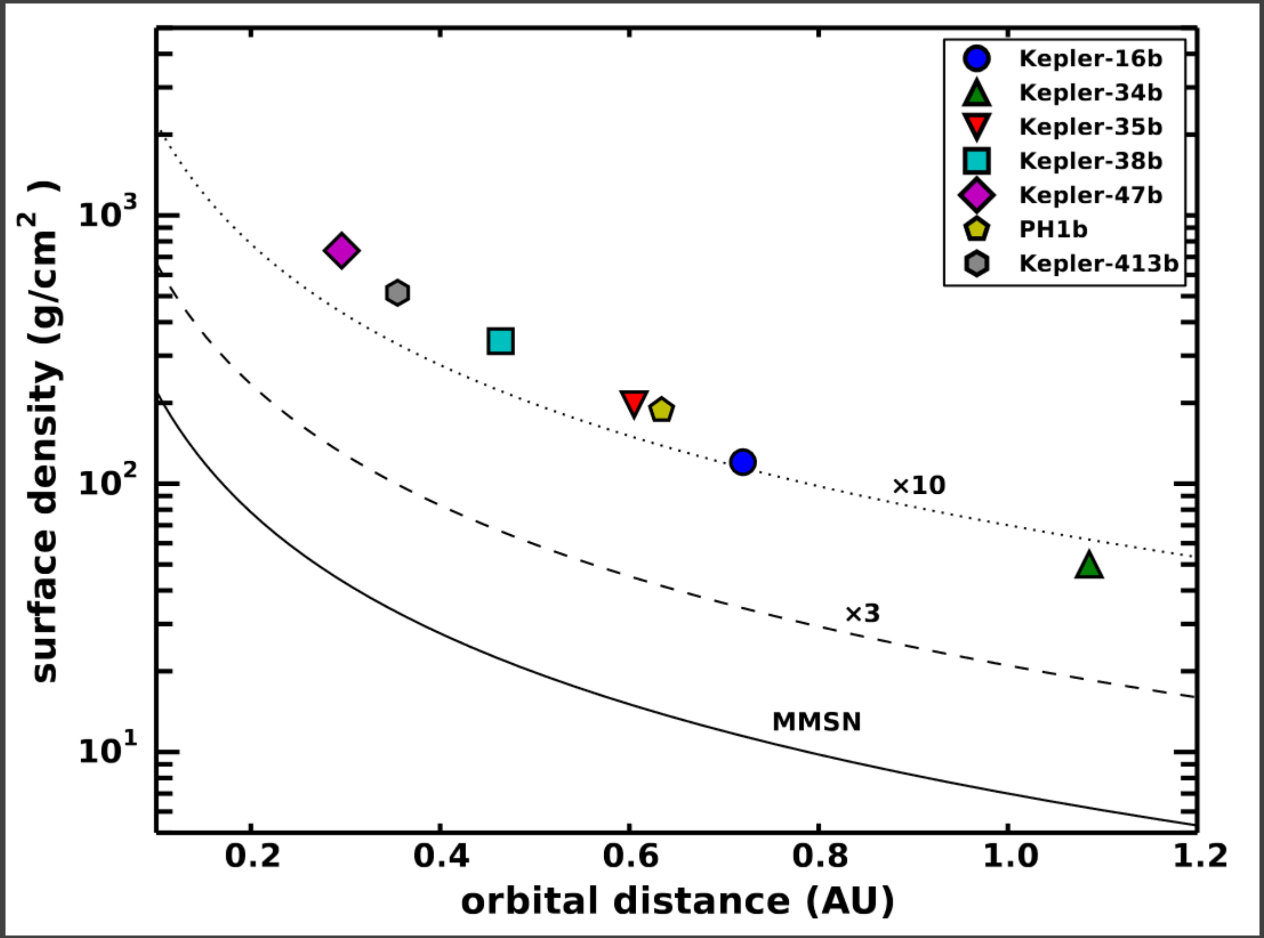
In most circular orbits particles have low relative velocity.
So, collisions are not destructive.
And a set of lunar-size objects form planets as around single stars.

Still, there are problems with some known planets, as they are situated close to their hosts.
It requires too massive discs (>10 times more massive than in the classical MMSN scenario).
Four scenarios are discussed:

- *In situ* formation
- Migration – then assemble
- Migration through a gas disk
- Planet scattering

Analysis of six known planets favours "migration –then assemble" or "disc migration", and in few cases – scattering, but not *in situ* formation.

# *In situ* formation in massive discs




Minimal surface density necessary to build known planets.

Lower curve – MMSN (Hayashi 1981)

Upper – multiplied.

Typical disc masses in the MMSN 0.01 Msolar.

It is difficult to explain massive planets by *in situ* formation.

Light planets can form at small distances without migration.

1503.03876

# Another circumbinary disc model

$$\frac{\partial \Sigma}{\partial t} = -\frac{1}{r}\frac{\partial}{\partial r}\left[\left(\frac{dl}{dr}\right)^{-1}\frac{\partial}{\partial r}\left(r^3 \nu \Sigma \frac{d\Omega}{dr}\right) + 2\frac{\Sigma \Lambda}{\Omega}\right].$$

$$\Lambda(r) = \mathrm{sgn}(r - a_b) f \frac{q^2 G M_c}{a_b}\left(\frac{a_b}{r - a_b}\right)^4,$$

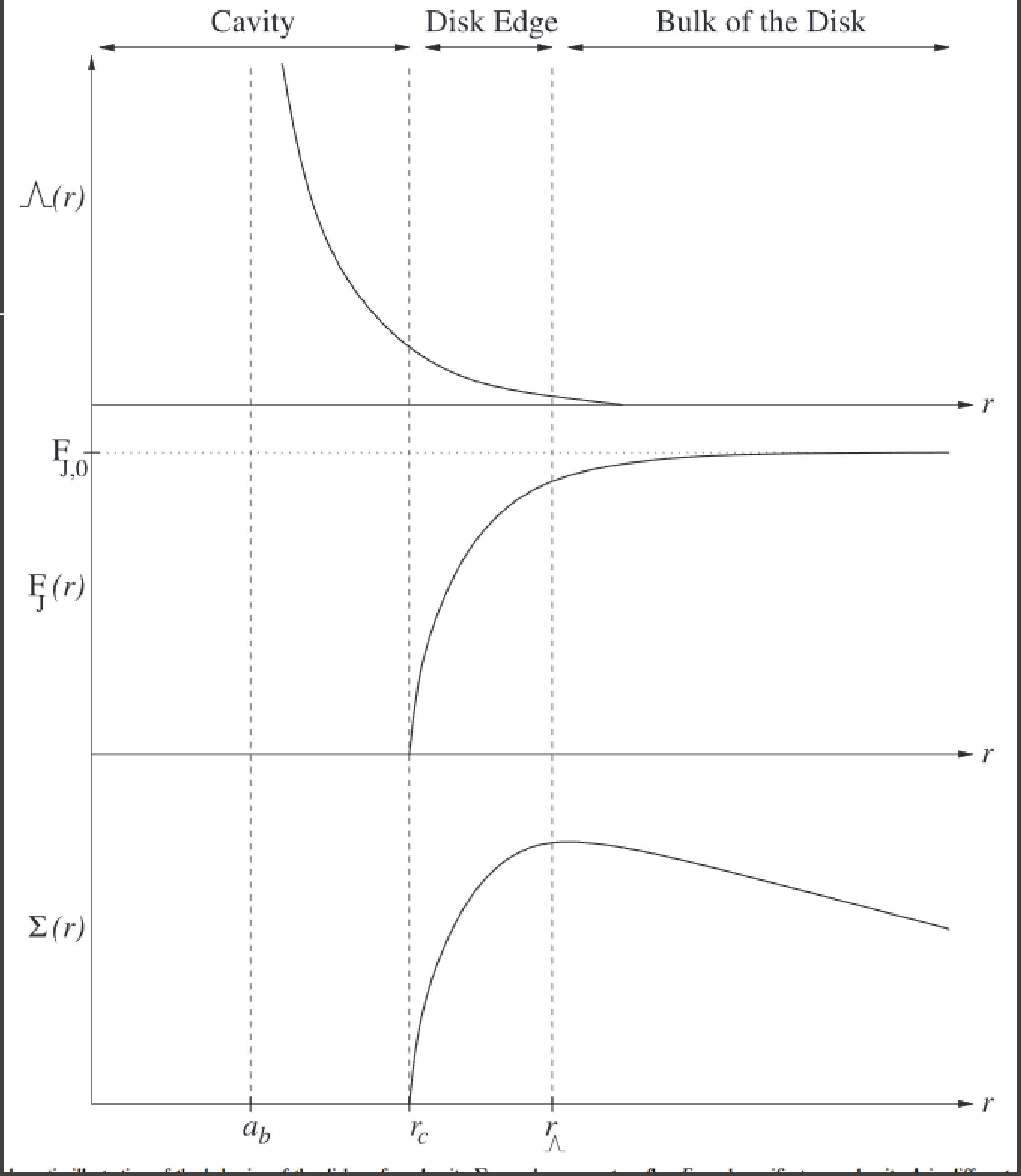
$$F_J \equiv -2\pi \nu \Sigma r^3 \frac{d\Omega}{dr} = 3\pi \nu \Sigma l,$$

Angular momentum in injected to the disc from the binary

$$\frac{\partial}{\partial t}\left(\frac{F_J}{D_J}\right) = \frac{\partial}{\partial l}\left[\frac{\partial F_J}{\partial l} - \frac{2F_J}{D_J}\frac{d\ln l}{d\ln r}\Lambda(l)\right],$$

$$\dot{M}(l, t) = \frac{\partial F_J}{\partial l} - \frac{2F_J}{D_J}\frac{d\ln l}{d\ln r}\Lambda(l).$$

Accretion to the binary from the disc can be small or even zero.



# Energy in the disc

Disc has three sources of energy:

- Viscosity;
- Illumination;
- Dissipation of shock generated by the binary.

$$\mathcal{F}_\nu = \frac{1}{4\pi r}\frac{d\dot{E}_\nu}{dr} = \frac{3}{8\pi}\frac{F_J \Omega}{r^2},$$

$$\mathcal{F}_{\rm irr} = \frac{1}{2}\frac{L_c}{4\pi r^2}\zeta,$$

$$\mathcal{F}_{\rm tid} = \frac{1}{2}(\Omega_b - \Omega)\Lambda\Sigma,$$

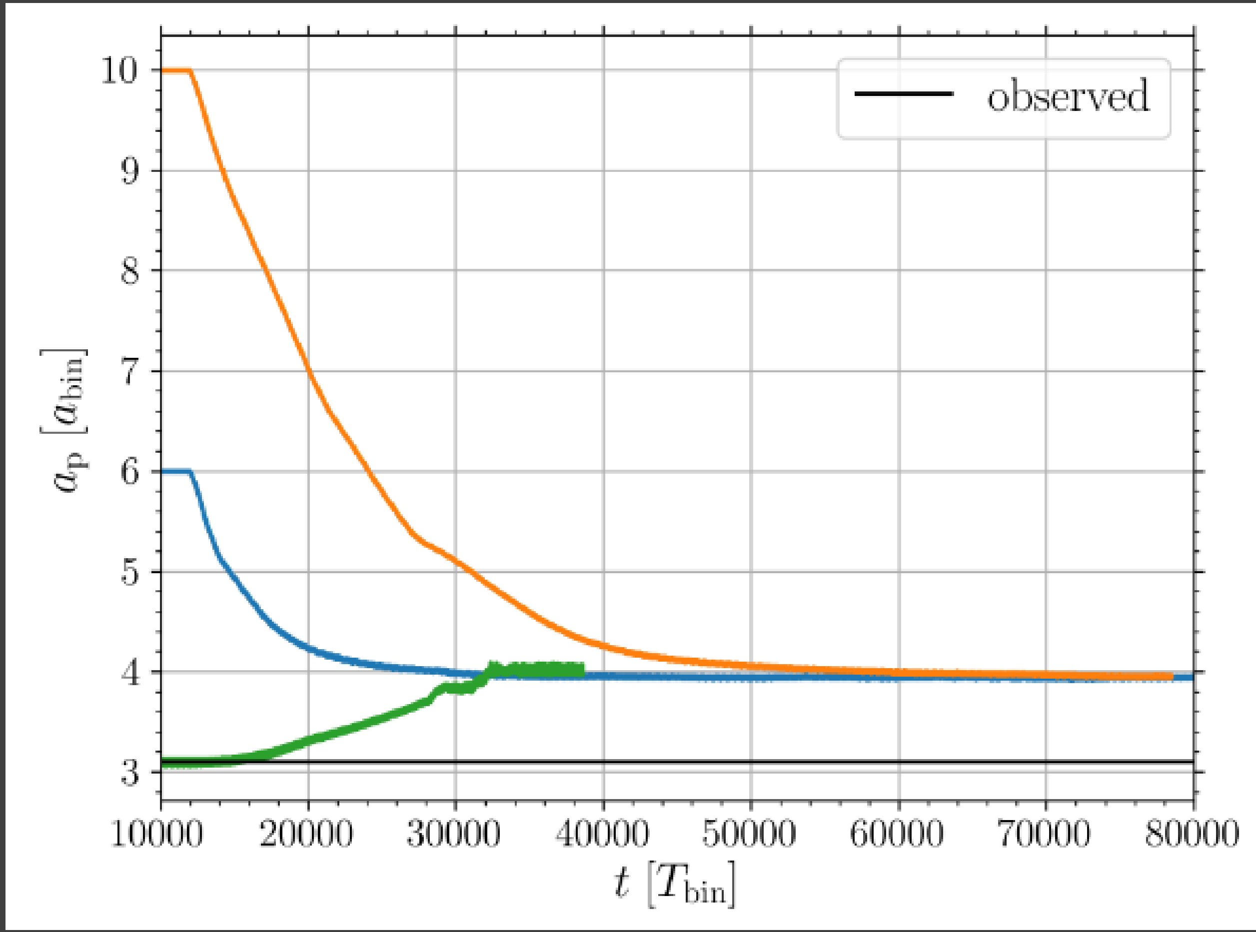
$\tau \ll 1$ $\quad \tau\sigma T^4 \approx \mathcal{F}_\nu + \mathcal{F}_{\rm tid} + \tau\mathcal{F}_{\rm irr}.$

$\tau \gg 1$ $\quad \sigma T^4 \approx \frac{3}{8}\tau\left(\mathcal{F}_\nu + \mathcal{F}_{\rm tid}\right) + \mathcal{F}_{\rm irr}.$

$$\sigma T^4 = f(\tau)\left(\mathcal{F}_\nu + \mathcal{F}_{\rm tid}\right) + \mathcal{F}_{\rm irr},$$

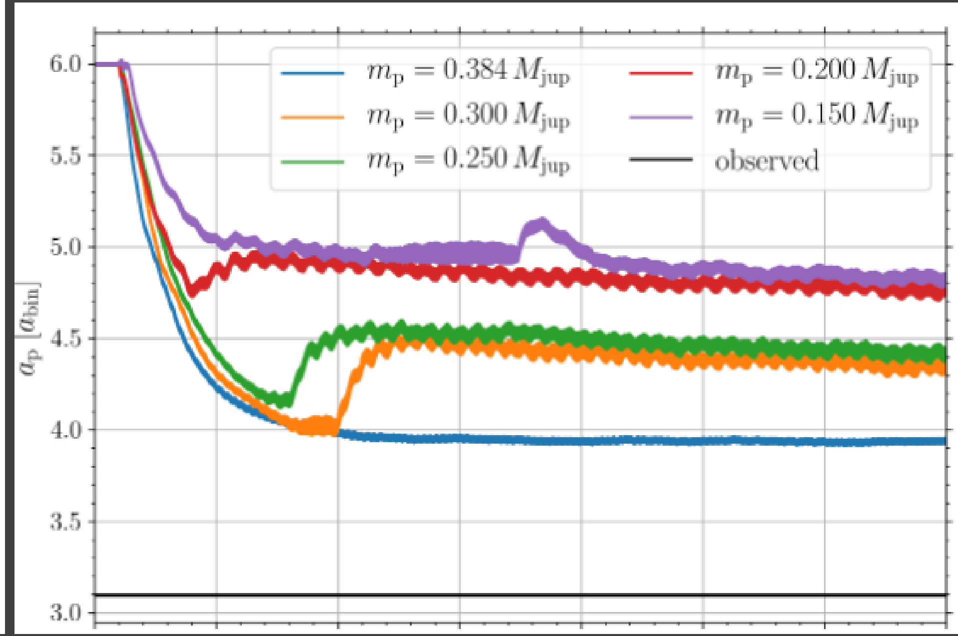
$$f(\tau) \approx \frac{3}{8}\tau + \tau^{-1}.$$

# Migration of circumbinary planets



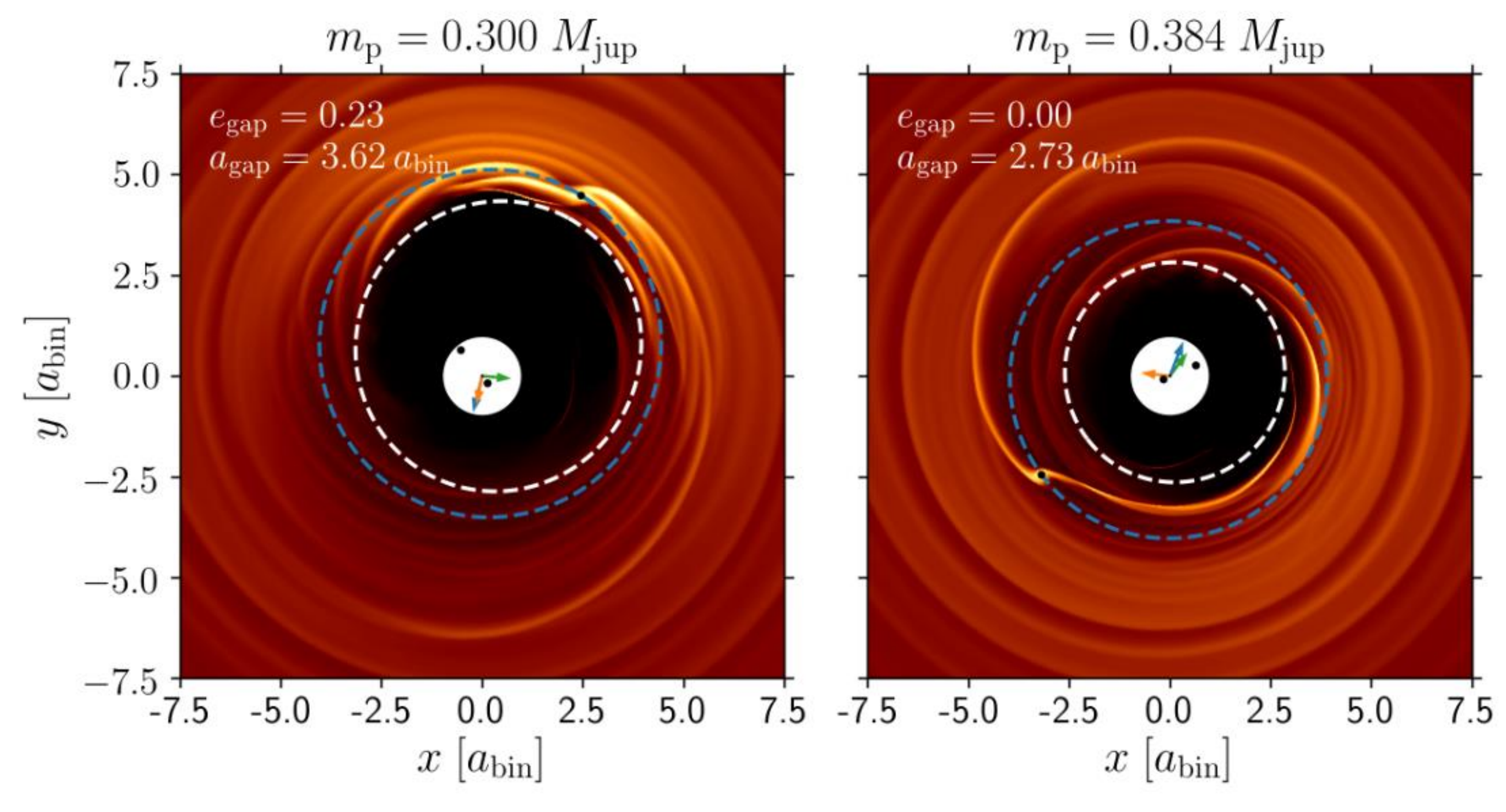

Example shows migration in a system similar to Kepler-38.

Independently on the initial position
a planet migrates towards the inner edge of the gap
formed by disc-binary interaction.



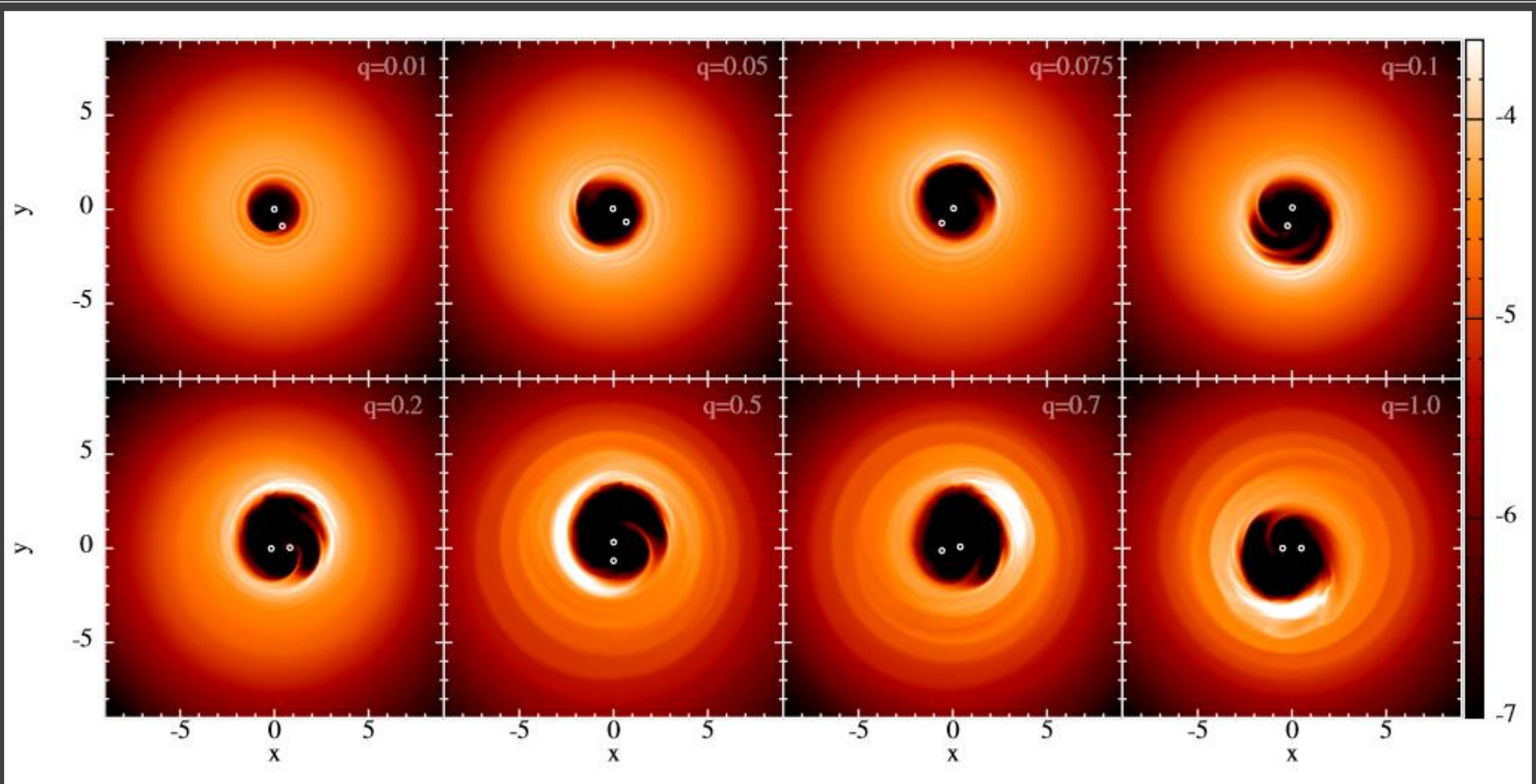

Result slightly depends on the planet and disc masses.

# Planets in a disc around a binary




1806.00314, see new calculations for P-type planet migration in 2012.03651

# Dependence on the mass ratio



# Important issues for planet formation

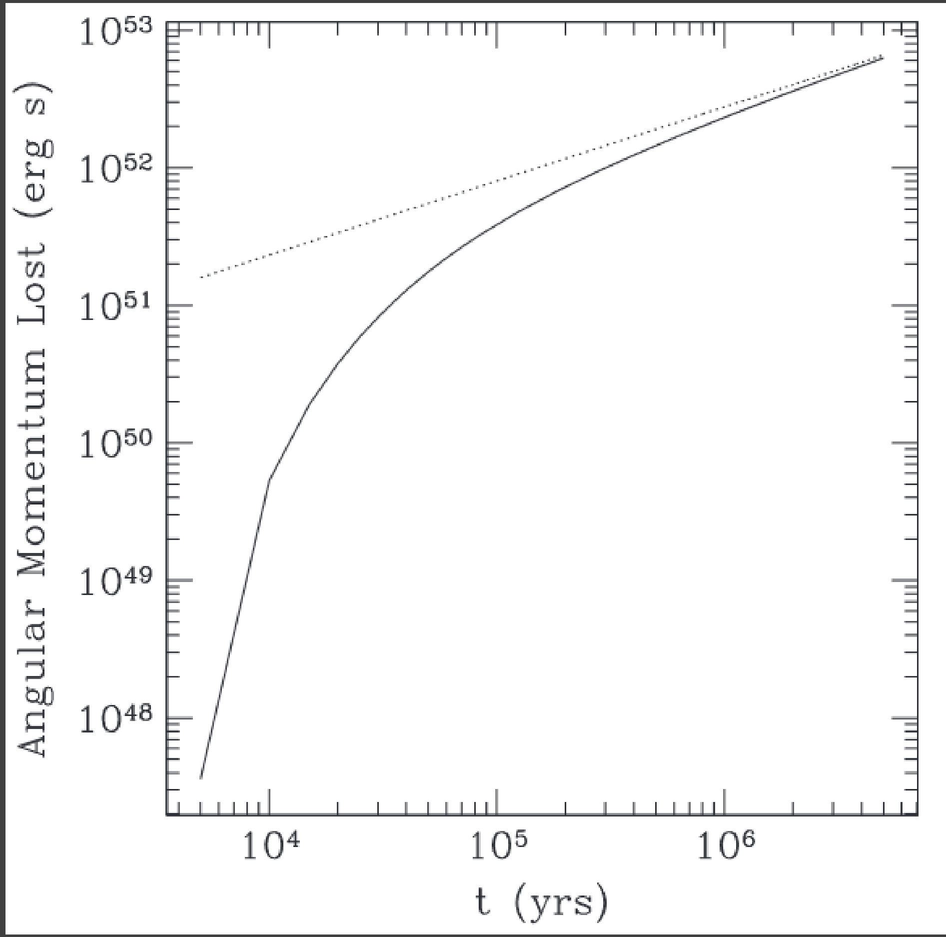
- Disc is more massive than around a single star
- Relative speeds at collisions are smaller
- Isolation masses are larger
- Ice line is shifted outwards
- Dissipation of the binary-driven density waves dominates heating of the inner disk, within 1–2AU

Circumbinary disks are in many ways more favorable sites of planet formation than their analogs around single stars

# Binary evolution due to the disc

$$L_b = \frac{q}{(1+q)^2}\left(GM_c^3 a_b\right)^{1/2} \approx 4 \times 10^{52}\ \mathrm{g\,cm^2\,s^{-1}} \times \frac{q}{(1+q)^2} M_{c,1}^{3/2}\left(\frac{a_b}{0.2\ \mathrm{AU}}\right)^{1/2}.$$

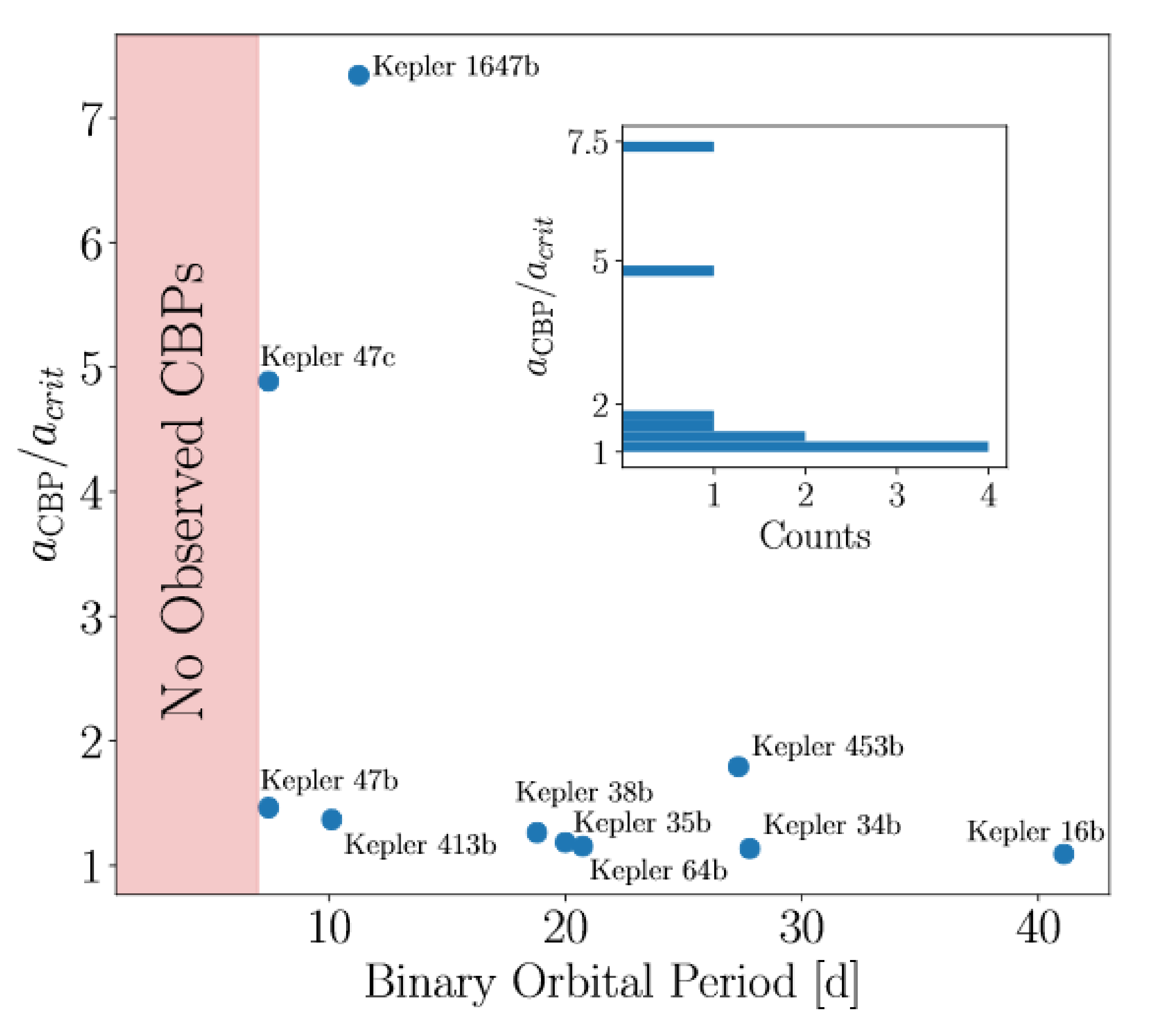
The binary can coalesce due to tidal interaction with the disc.



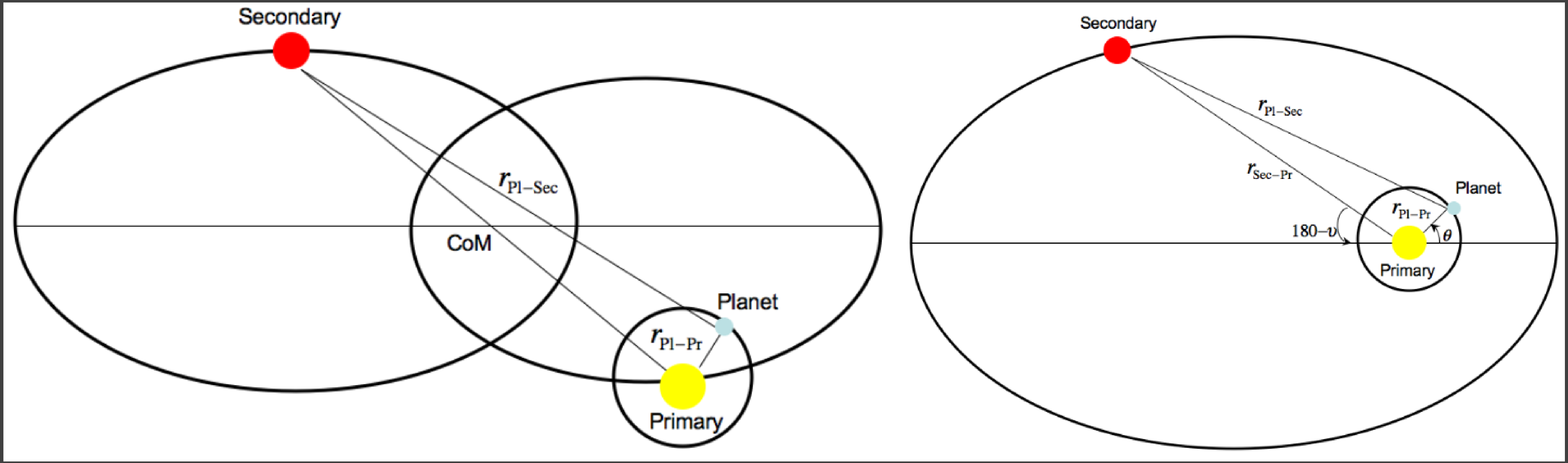
# P-type systems

Absence of planets around binaries with P<few days can be due to evolution of binary separation.

Mostly planets are observed close to the stability limit.



# Habitable zone calculations




1406.1357, see also 1901.11171 for a slightly different approach and 1911.02983, 2009.00144 for new calculations

# Calculations

$$F_{\mathrm{Pl}}(f, T_{\mathrm{Pr}}, T_{\mathrm{Sec}}) = W_{\mathrm{Pr}}(f, T_{\mathrm{Pr}}) \frac{L_{\mathrm{Pr}}(T_{\mathrm{Pr}})}{r^2_{\mathrm{Pl-Pr}}} + W_{\mathrm{Sec}}(f, T_{\mathrm{Sec}}) \frac{L_{\mathrm{Sec}}(T_{\mathrm{Sec}})}{r^2_{\mathrm{Pl-Sec}}} .$$

$$W_{\mathrm{Pr}}(f, T_{\mathrm{Pr}}) \frac{L_{\mathrm{Pr}}(T_{\mathrm{Pr}})}{l^2_{\mathrm{x-Bin}}} + W_{\mathrm{Sec}}(f, T_{\mathrm{Sec}}) \frac{L_{\mathrm{Sec}}(T_{\mathrm{Sec}})}{r^2_{\mathrm{Pl-Sec}}} = \frac{L_{\mathrm{Sun}}}{l^2_{\mathrm{x-Sun}}} .$$

$$l_{\mathrm{x-Star}} = l_{\mathrm{x-Sun}} \left[ \frac{L/L_{\mathrm{Sun}}}{1 + \alpha_{\mathrm{x}}(T_i)\, l^2_{\mathrm{x-Sun}}} \right]^{1/2}$$

, $l_{\mathrm{x}} = (l_{\mathrm{in}}, l_{\mathrm{out}})$ is in AU, $T_i(\mathrm{K}) = T_{\mathrm{Star}}(\mathrm{K}) - 5780$,

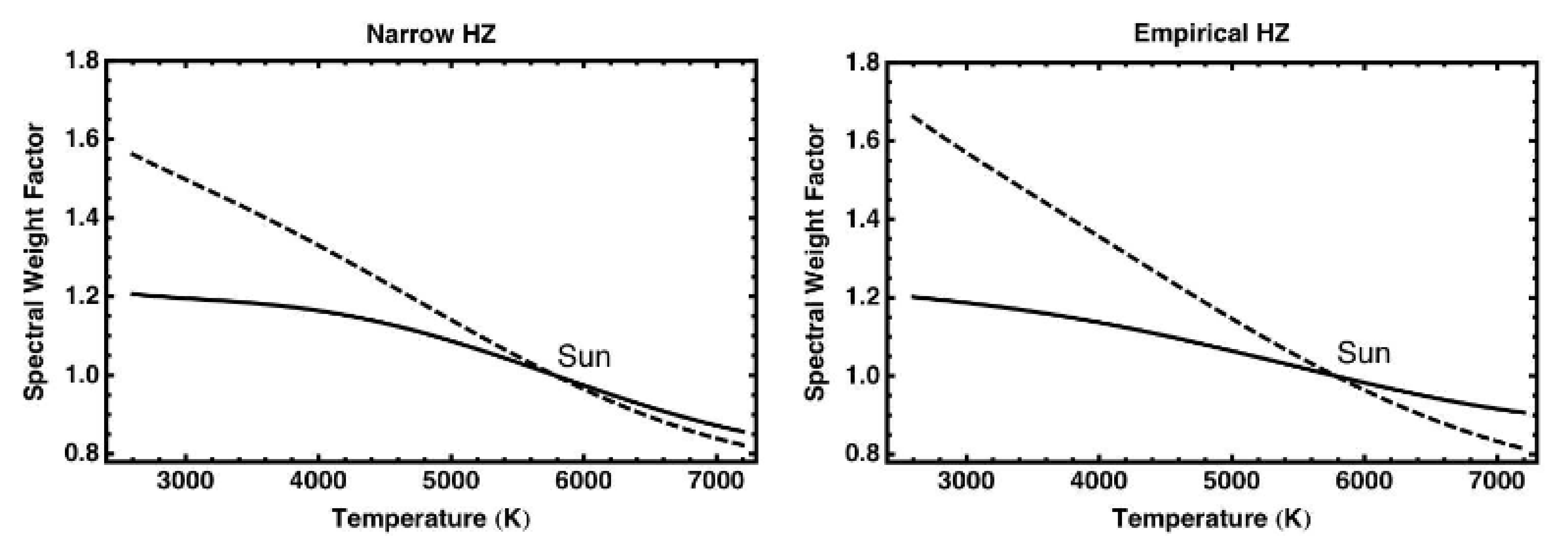
$$\alpha_{\mathrm{x}}(T_i) = a_{\mathrm{x}} T_i + b_{\mathrm{x}} T_i^2 + c_{\mathrm{x}} T_i^3 + d_{\mathrm{x}} T_i^4 ,$$

| | Narrow HZ | | Empirical HZ | |
|---|---|---|---|---|
| | Runaway Greenhouse | Maximum Greenhouse | Recent Venus | Early Mars |
| $l_{\mathrm{x-Sun}}$ (AU) | 0.97 | 1.67 | 0.75 | 1.77 |
| Flux (Solar Flux @ Earth) | 1.06 | 0.36 | 1.78 | 0.32 |
| a | $1.2456 \times 10^{-4}$ | $5.9578 \times 10^{-5}$ | $1.4335 \times 10^{-4}$ | $5.4471 \times 10^{-5}$ |
| b | $1.4612 \times 10^{-8}$ | $1.6707 \times 10^{-9}$ | $3.3954 \times 10^{-9}$ | $1.5275 \times 10^{-9}$ |
| c | $-7.6345 \times 10^{-12}$ | $-3.0058 \times 10^{-12}$ | $-7.6364 \times 10^{-12}$ | $-2.1709 \times 10^{-12}$ |
| d | $-1.7511 \times 10^{-15}$ | $-5.1925 \times 10^{-16}$ | $-1.1950 \times 10^{-15}$ | $-3.8282 \times 10^{-16}$ |

$$F_{\mathrm{x-Star}}(f, T_{\mathrm{Star}}) = F_{\mathrm{x-Sun}}(f, T_{\mathrm{Star}}) \left[ 1 + \alpha_{\mathrm{x}}(T_i)\, l^2_{\mathrm{x-Sun}} \right]$$

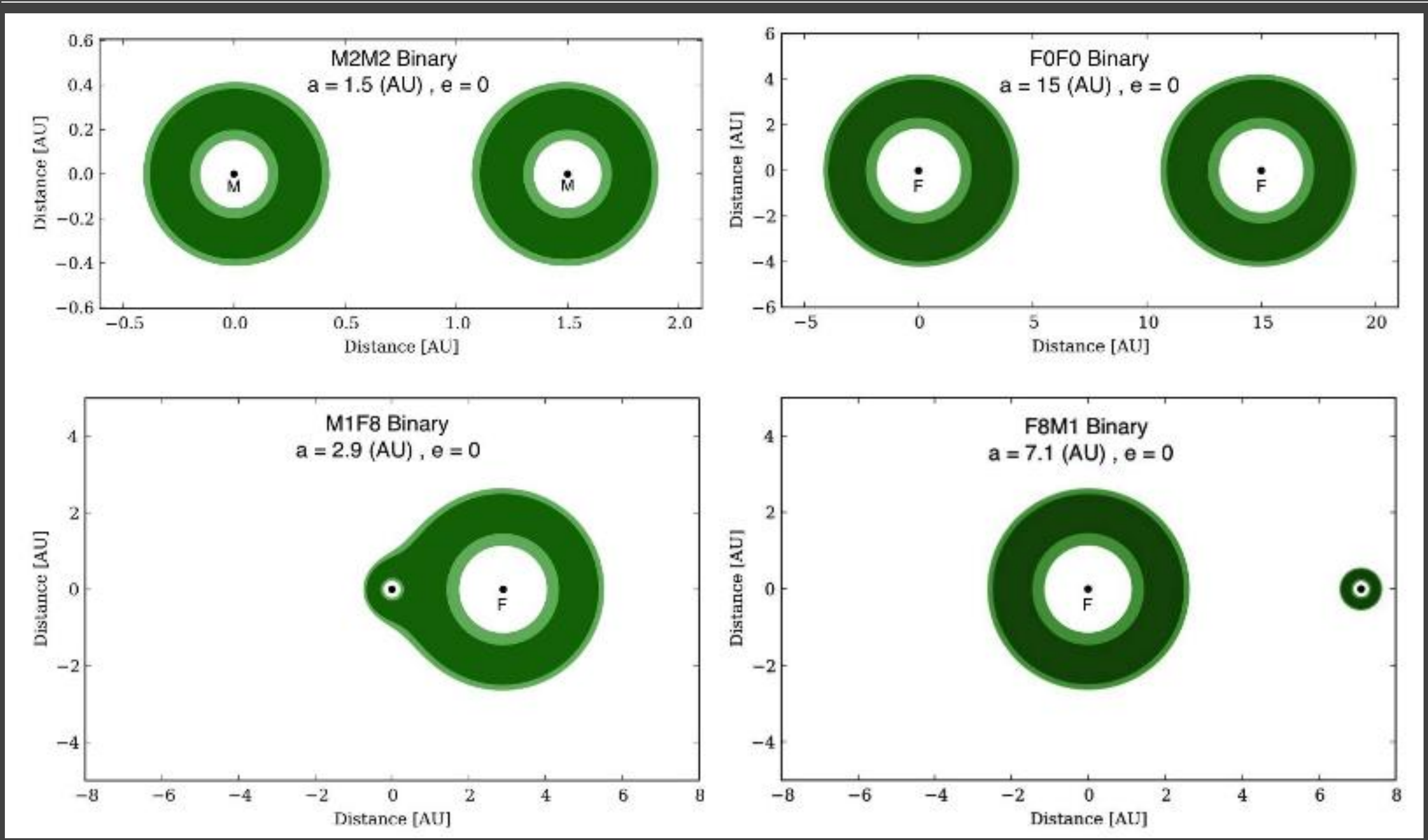
$$W_i(f, T_i) = \left[ 1 + \alpha_{\mathrm{x}}(T_i)\, l^2_{\mathrm{x-Sun}} \right]^{-1}$$

# Weight coefficients




1406.1357

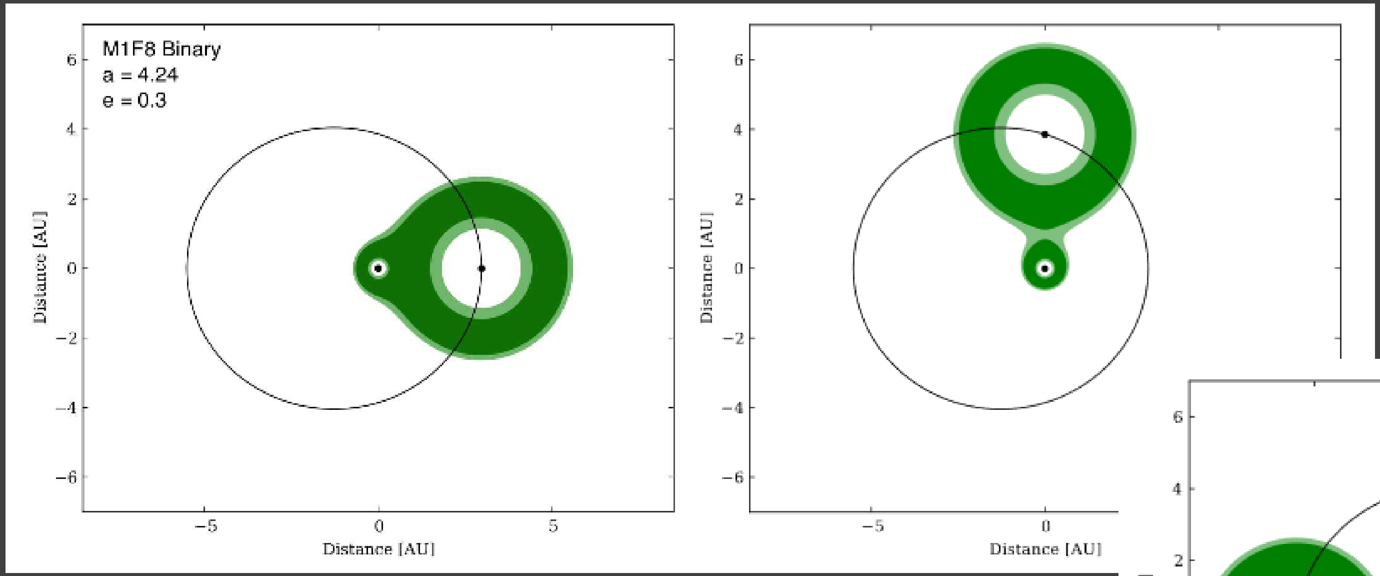
# Examples



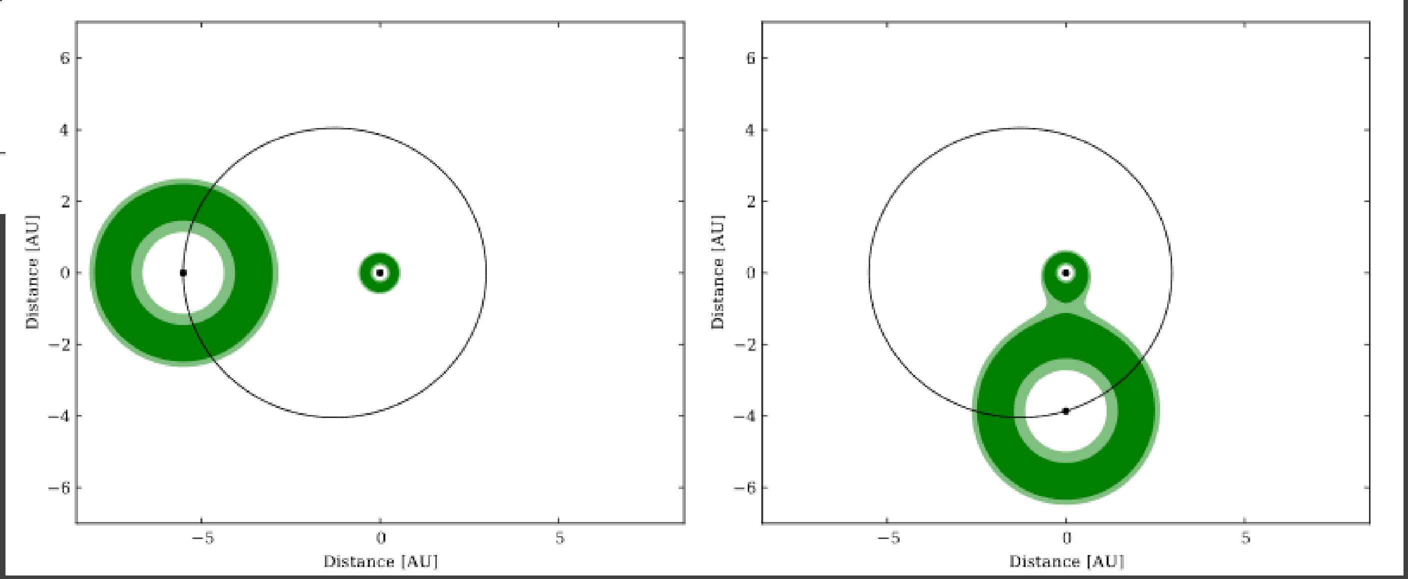

Dark green – narrow HZ.

Light green- empirical HZ.

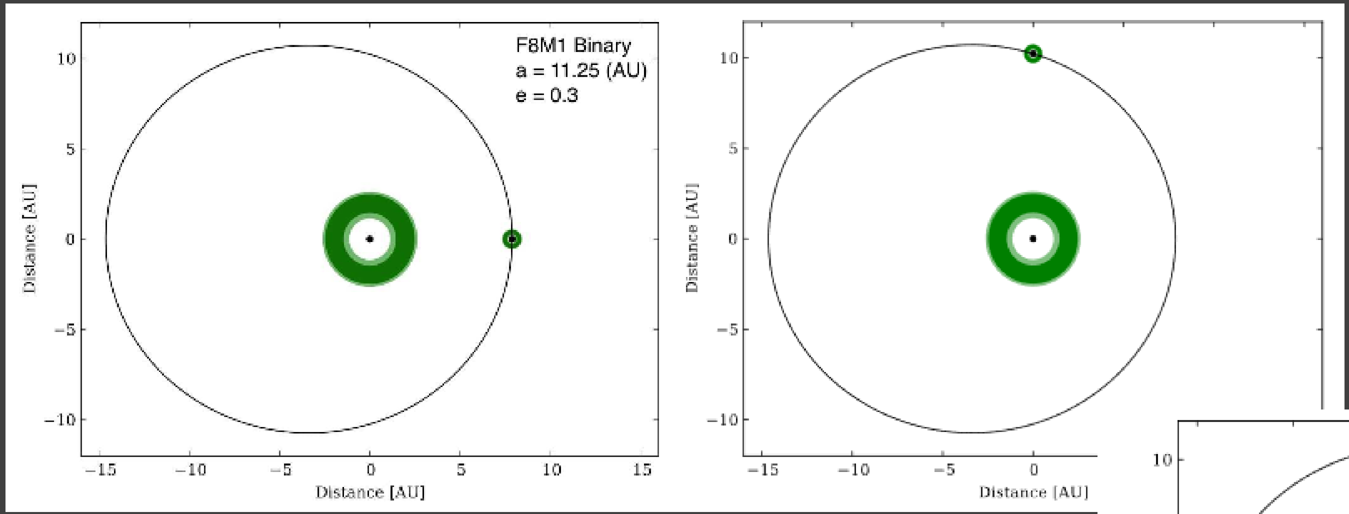
# Examples: eccentricity=0.3



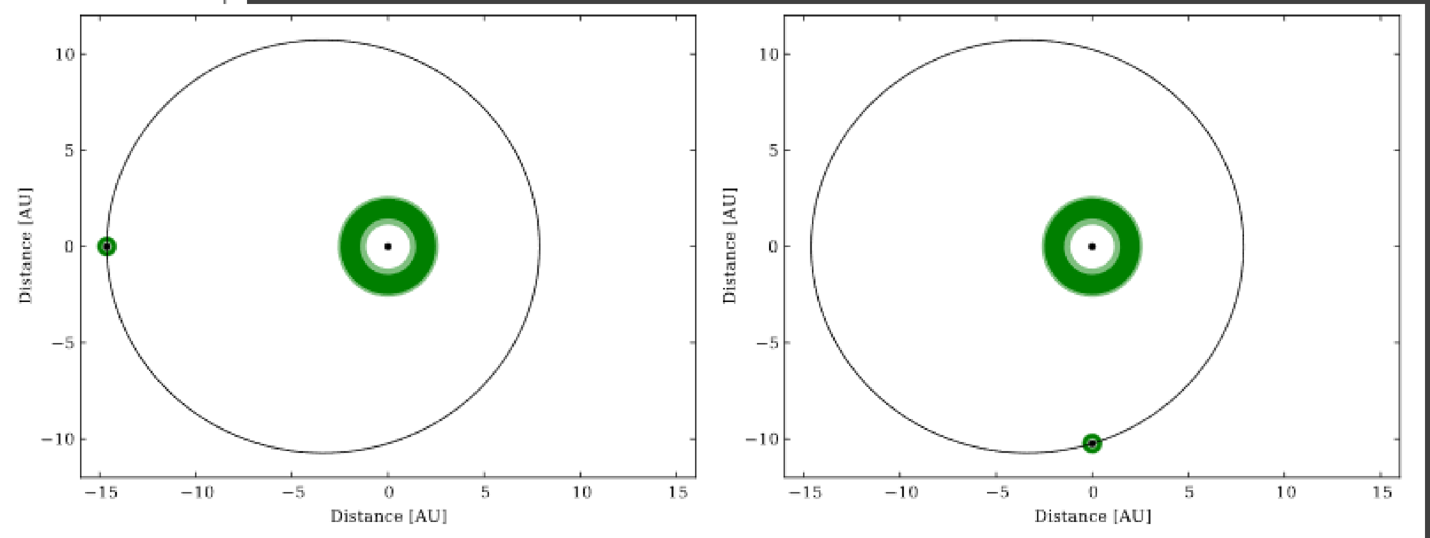

M-dwarf is the primary



# Examples: eccentricity=0.3




F-star is the primary




1406.1357

# Habitable zone calculation. II. Circumbinary

Let us start with single stars

$$L_\star = 4\pi R_\star^2 \sigma T_{\rm eff}^4$$

$$S_{\rm inner} = 4.190 \times 10^{-8} T_{\rm eff}^2 - 2.139 \times 10^{-4} T_{\rm eff} + 1.268$$

$$S_{\rm outer} = 6.190 \times 10^{-9} T_{\rm eff}^2 - 1.319 \times 10^{-5} T_{\rm eff} + 0.2341 .$$

$$r_{\rm inner} = \sqrt{L_\star / S_{\rm inner}}$$

$$r_{\rm outer} = \sqrt{L_\star / S_{\rm outer}}$$

Here luminosity and flux are in solar units, and distance – in AU.

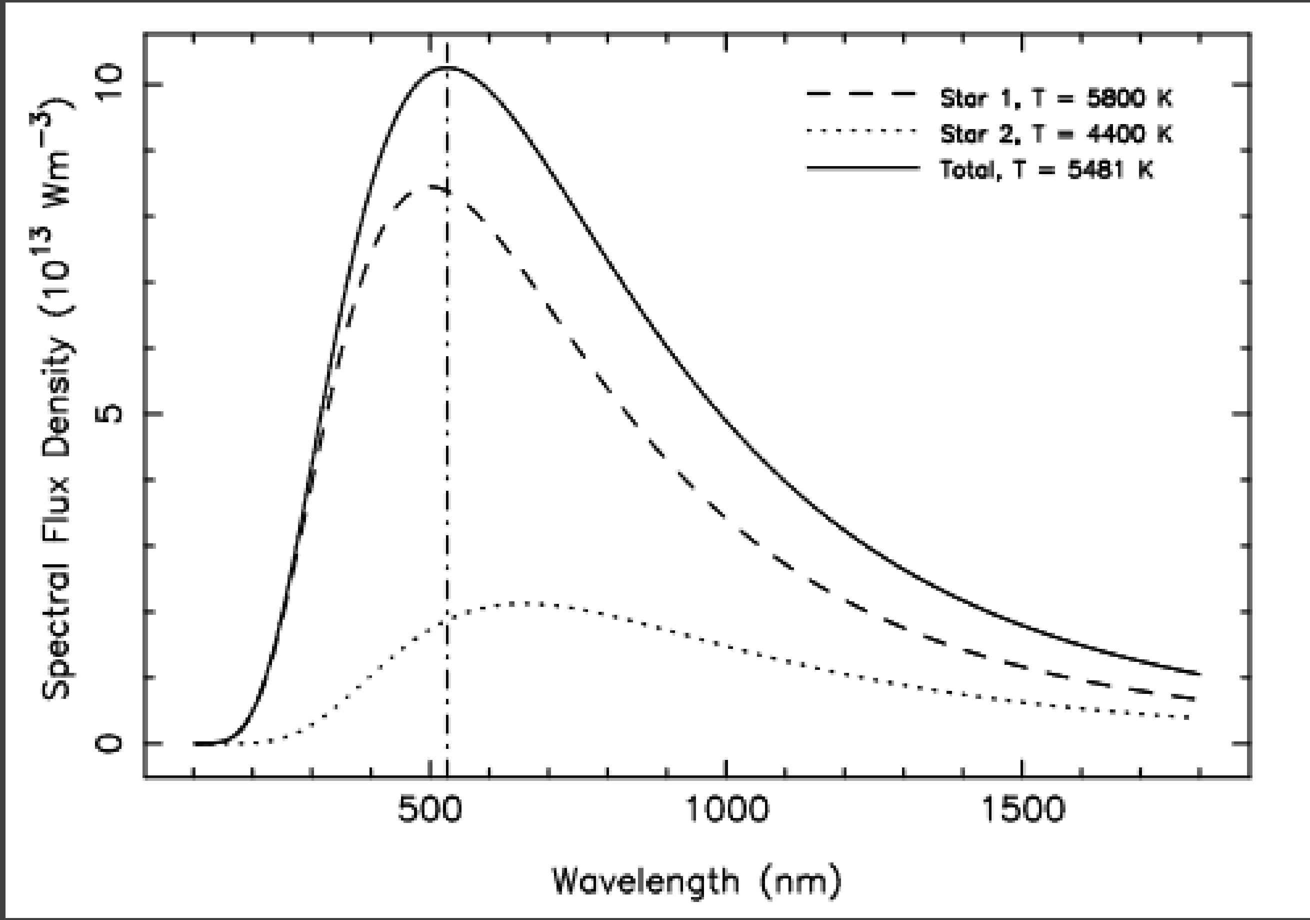
# Binary stars

$$S_1 = \frac{2\pi hc^2}{\lambda^5}\frac{1}{e^{hc/\lambda kT_{\mathrm{eff},1}}-1}\left(\frac{R_{\star,1}}{r_1}\right)^2$$

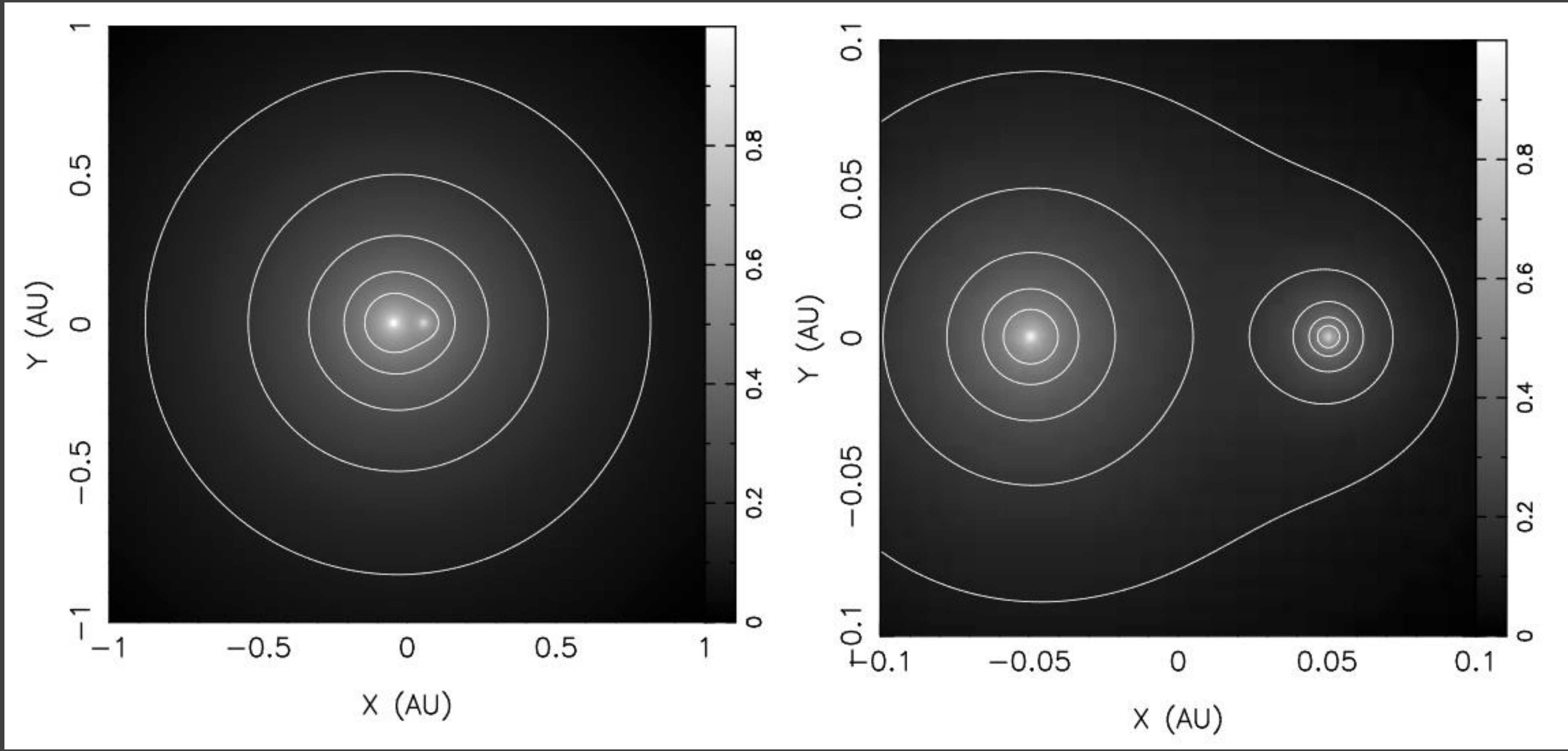
$$S_2 = \frac{2\pi hc^2}{\lambda^5}\frac{1}{e^{hc/\lambda kT_{\mathrm{eff},2}}-1}\left(\frac{R_{\star,2}}{r_2}\right)^2$$

$$S = S_1 + S_2.$$

G2V+K5V
0.1 AU



# Stellar flux map

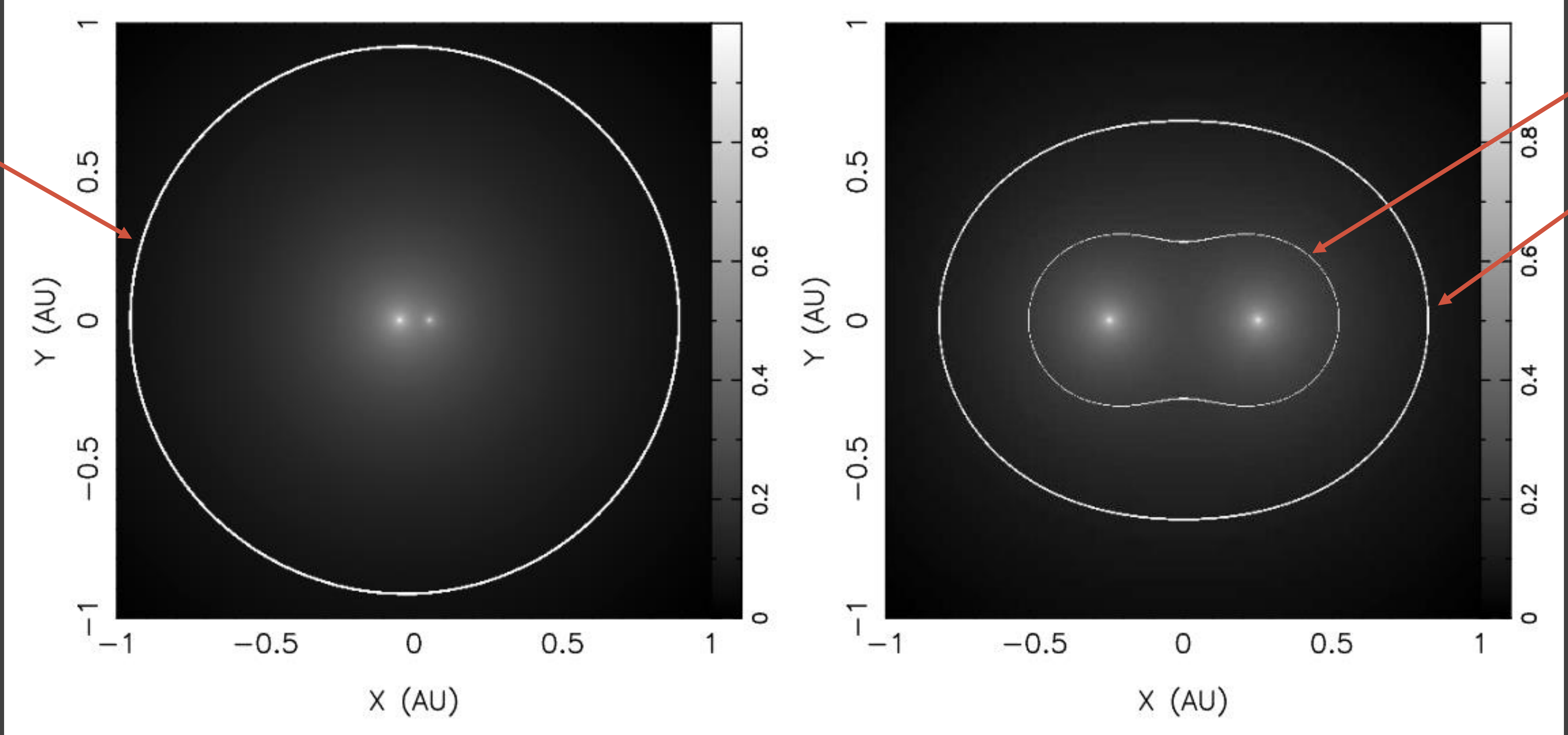



G2V+K5V
0.1 AU

# HZ edges

Inner edge

G2V+K5V
0.1 AU

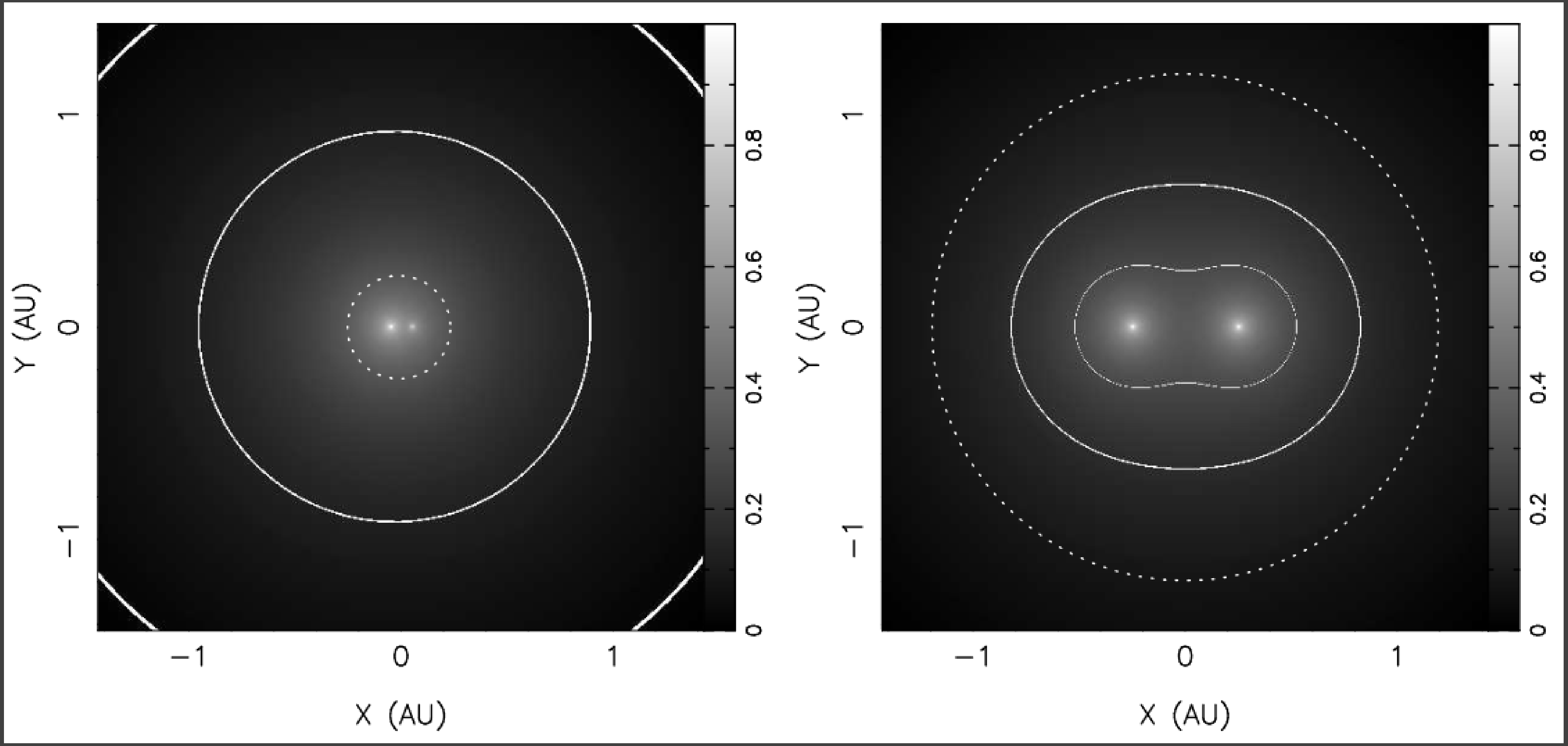



Inner edge

Outer edge

M0V+M0V
0.5 AU

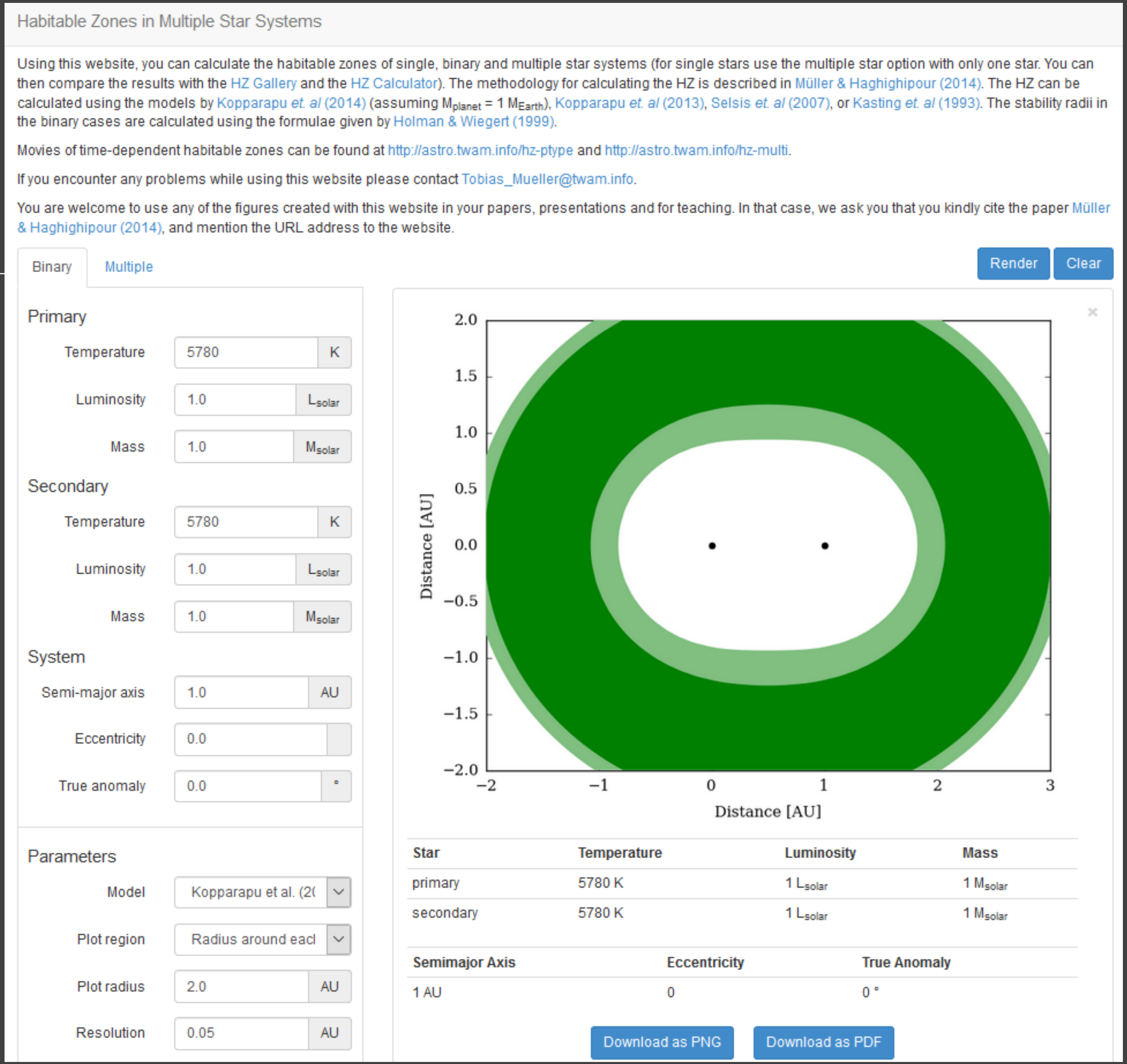
# Stability of the planetary orbit



# On-line calculator

Described in 1401.0601

T, L, and M can be changed independently (i.e., there is not fit for the MS, etc.)



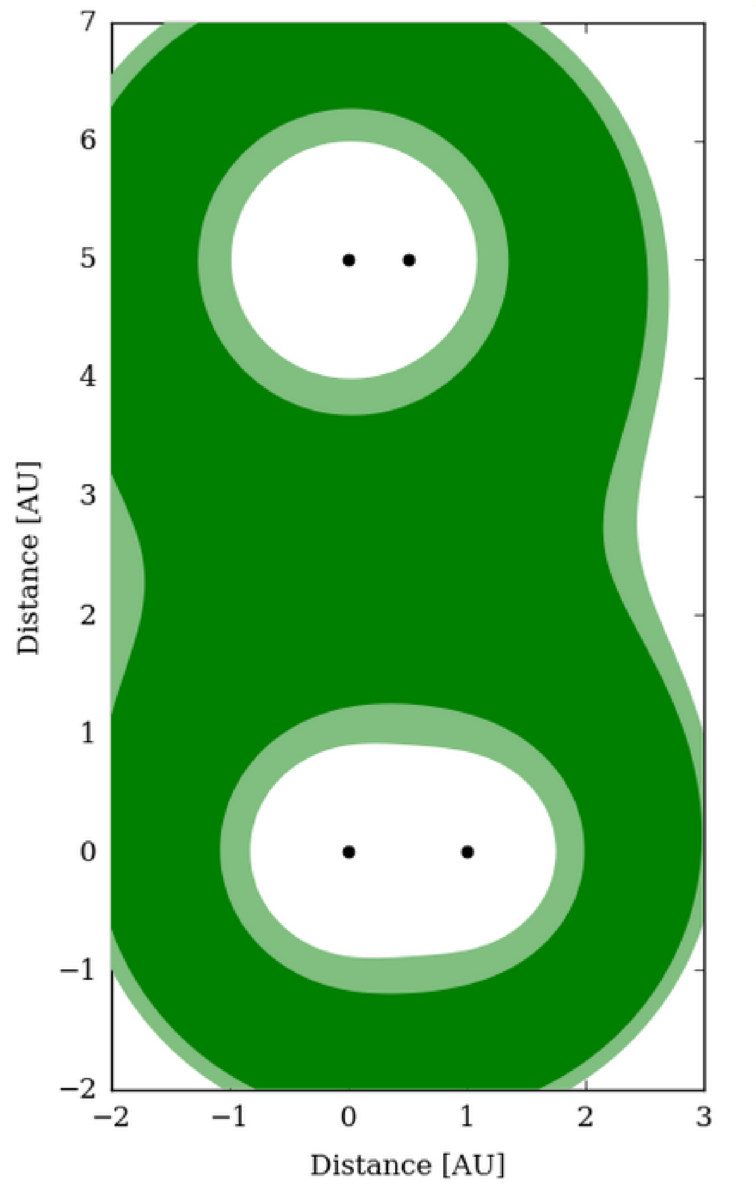

http://astro.twam.info/hz/

# Multiple systems

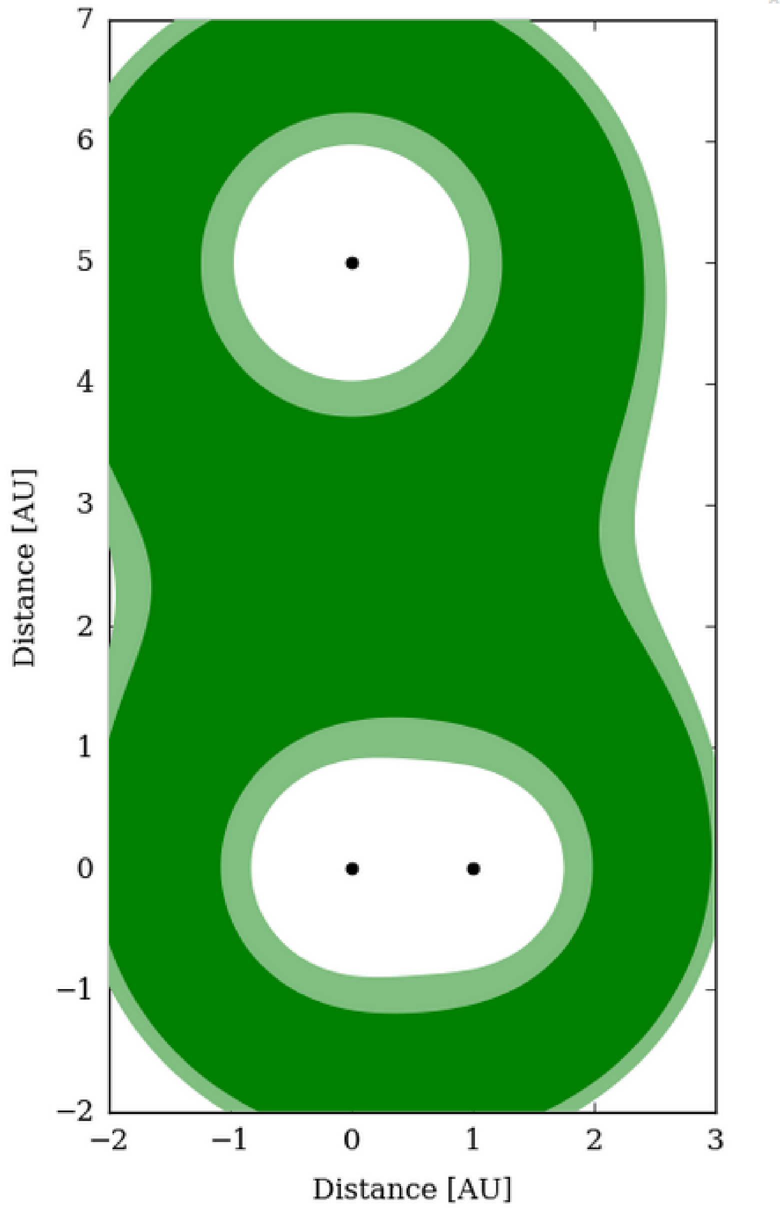
The method allows to make plots for any number of stars.

However, consistency of all conditions (orbital stability, etc.) is not automatically controlled.

http://astro.twam.info/hz/

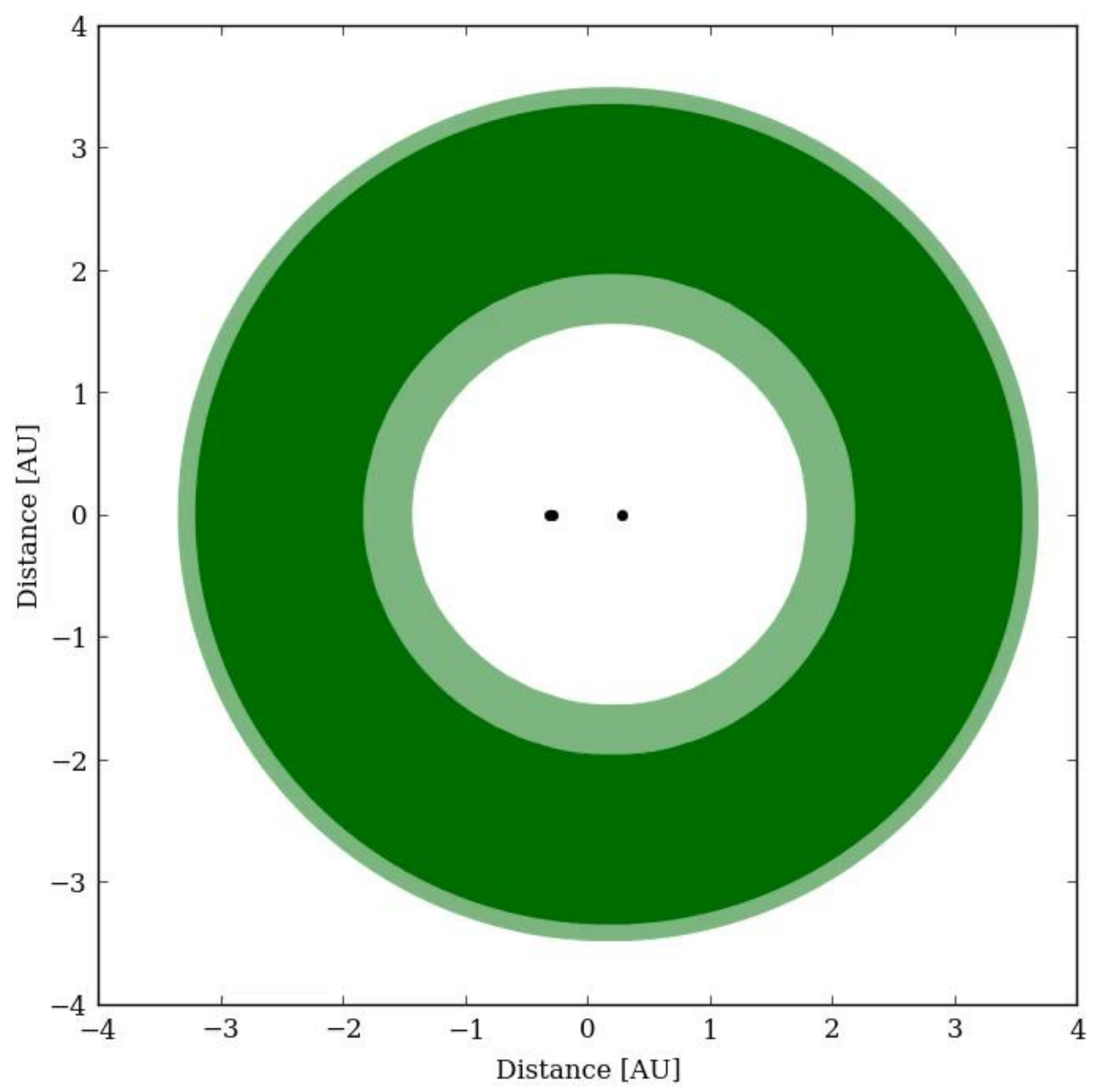



| Star | Temperature | Luminosity | Mass | X | Y |
|---|---|---|---|---|---|
| A | 5780 K | 1 $L_{solar}$ | 1 $M_{solar}$ | 0 AU | 0 AU |
| B | 4780 K | 0.7 $L_{solar}$ | 0.7 $M_{solar}$ | 1 AU | 0 AU |
| C | 6780 K | 1.8 $L_{solar}$ | 1.4 $M_{solar}$ | 0 AU | 5 AU |
| D | 3780 K | 0.1 $L_{solar}$ | 0.2 $M_{solar}$ | 0.5 AU | 5 AU |



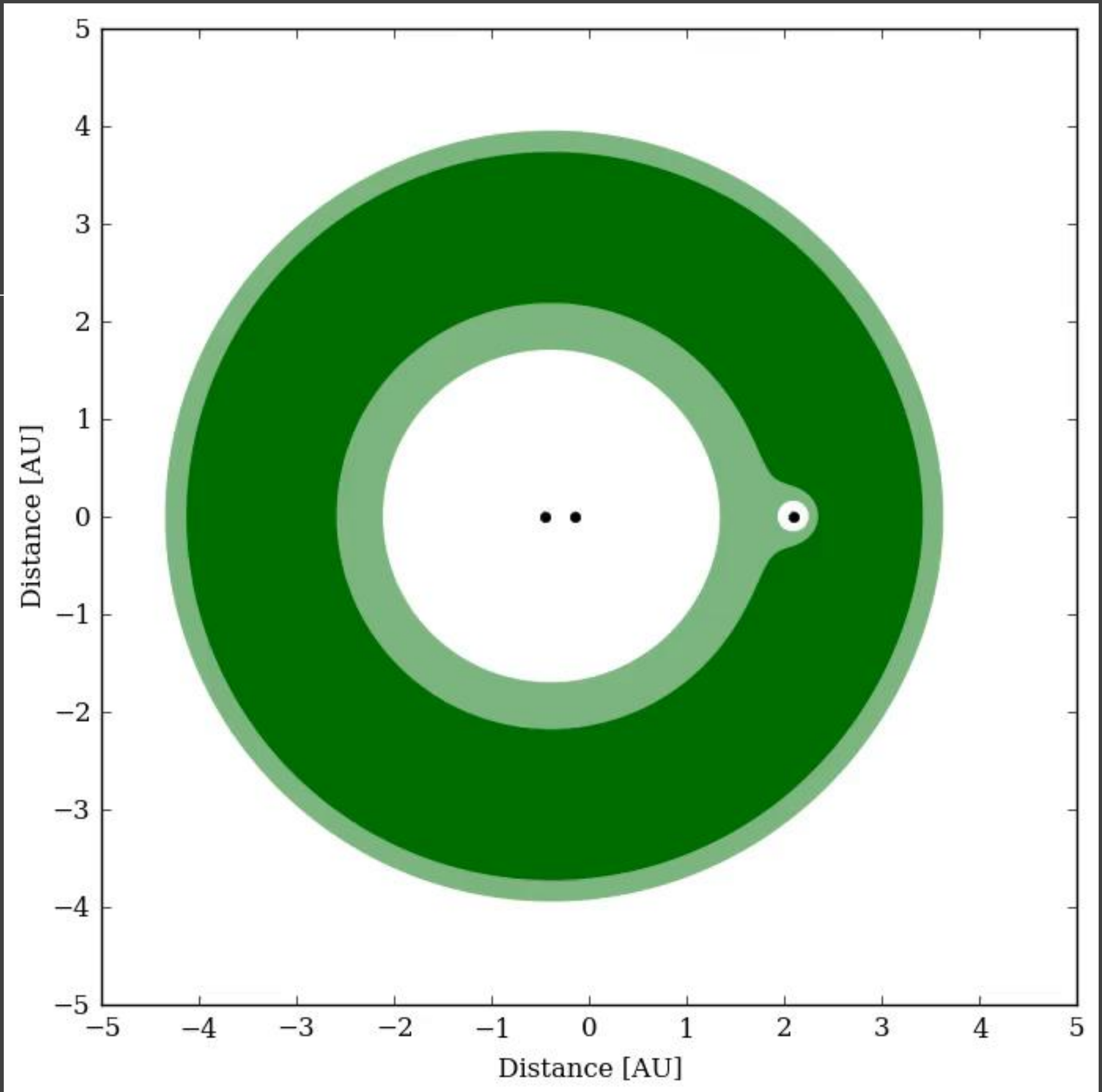

| Star | Temperature | Luminosity | Mass | X | Y |
|---|---|---|---|---|---|
| A | 5780 K | 1 $L_{solar}$ | 1 $M_{solar}$ | 0 AU | 0 AU |
| B | 4780 K | 0.7 $L_{solar}$ | 0.7 $M_{solar}$ | 1 AU | 0 AU |
| C | 6780 K | 1.8 $L_{solar}$ | 1.4 $M_{solar}$ | 0 AU | 5 AU |

# KIC 4150611

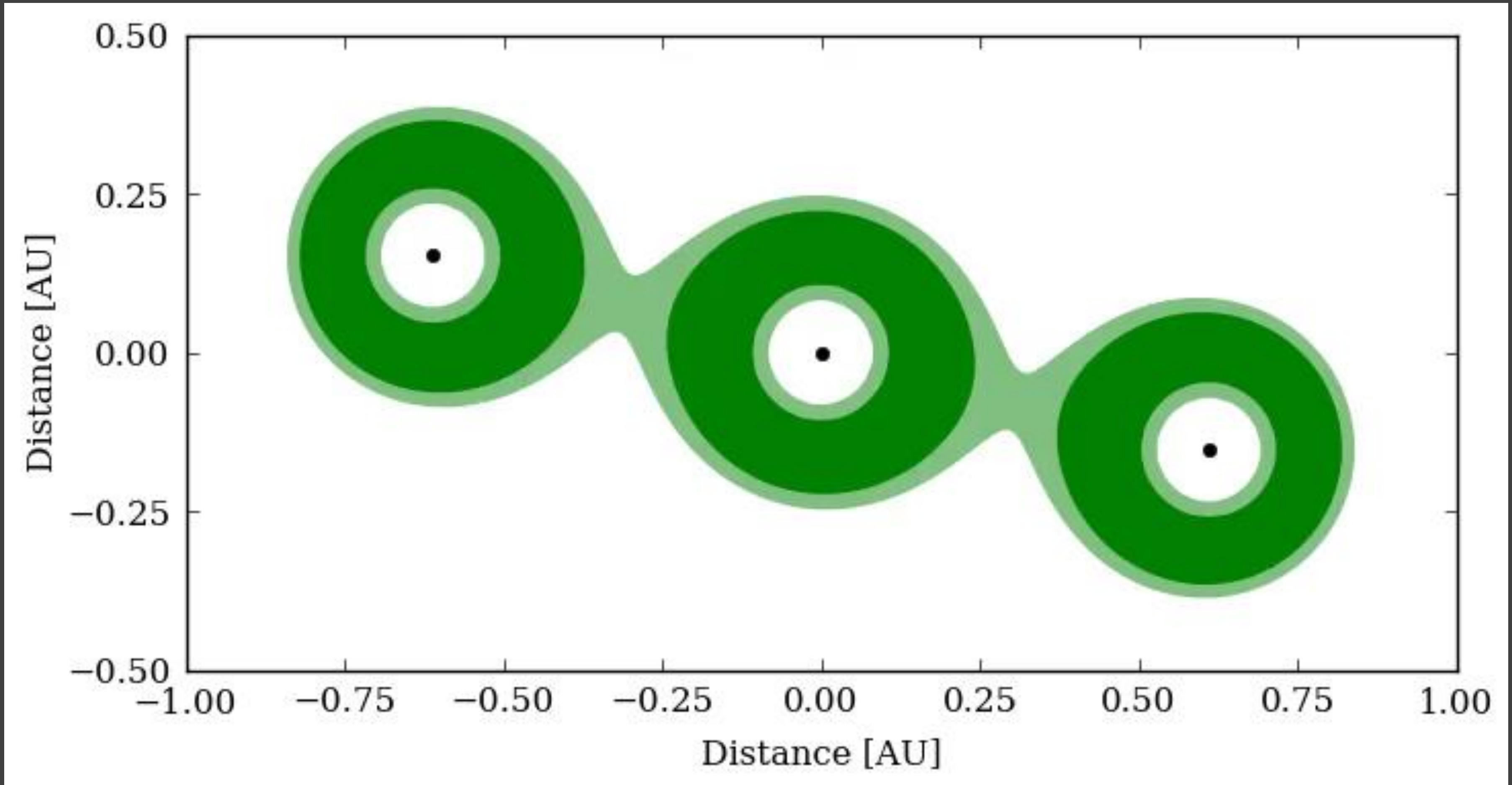



http://astro.twam.info/hz/

# KID 5653126

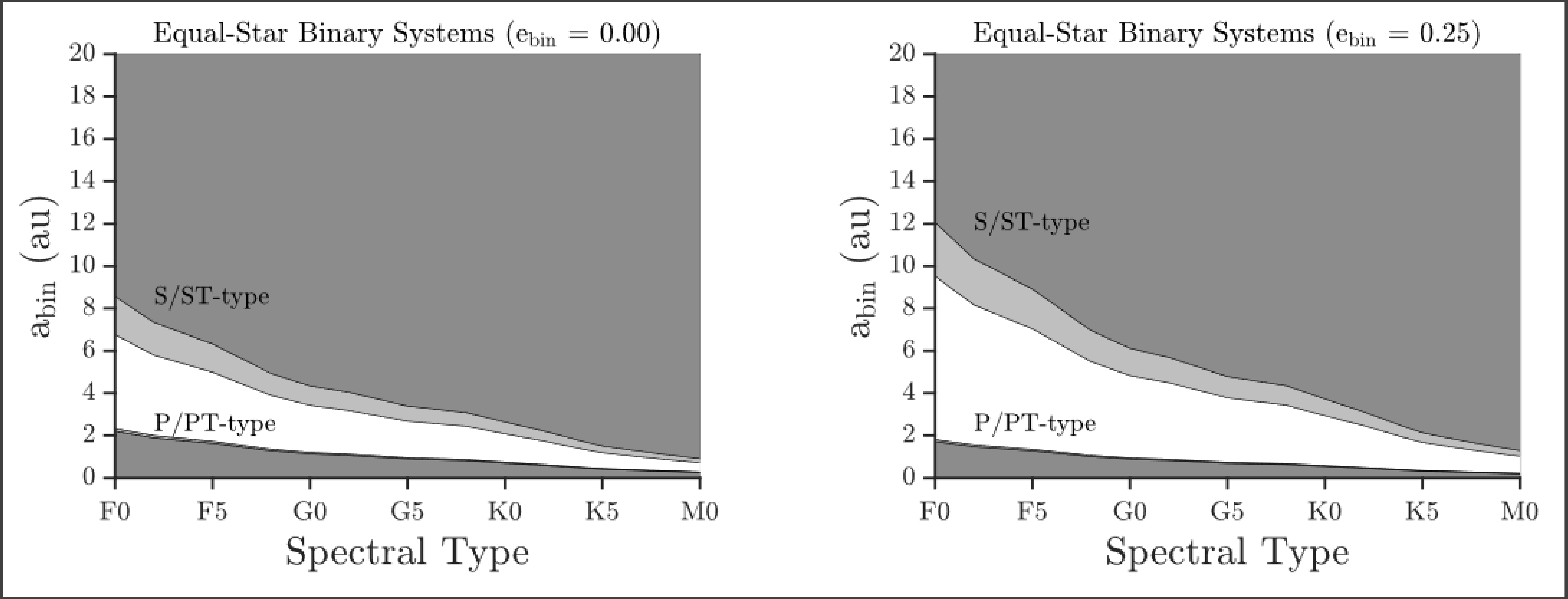



http://astro.twam.info/hz/




http://astro.twam.info/hz/

# Joint S- and P-type description



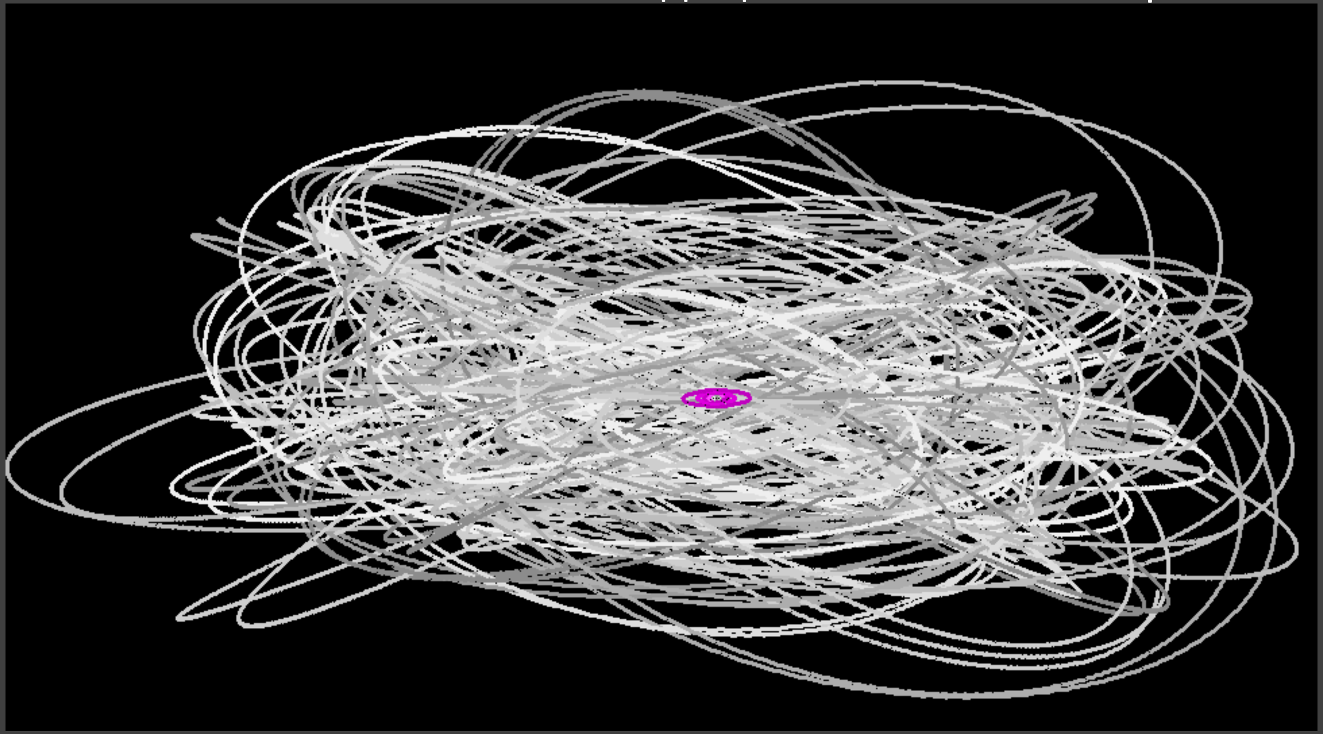

2001.01754

# Эффект Лидова-Козаи

У орбиты могут одновременно меняться наклонение  эксцентриситет.

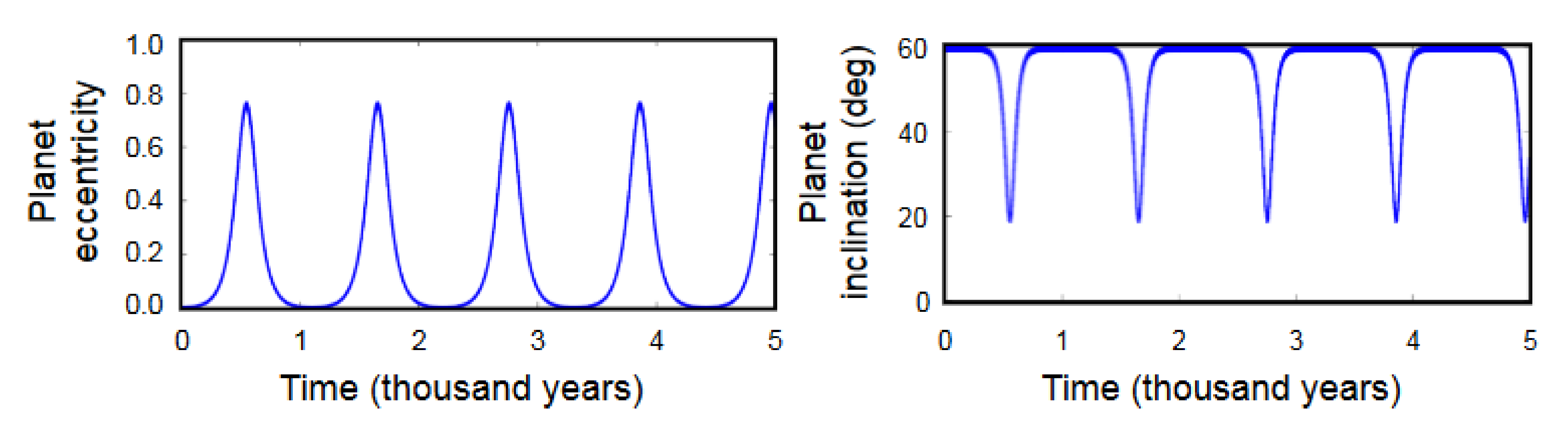
Эффект связан с воздействием тела, находящегося на внешней орбите.

$$e_{\max} \simeq \sqrt{1-(5/3)\cos^2 i_0}$$



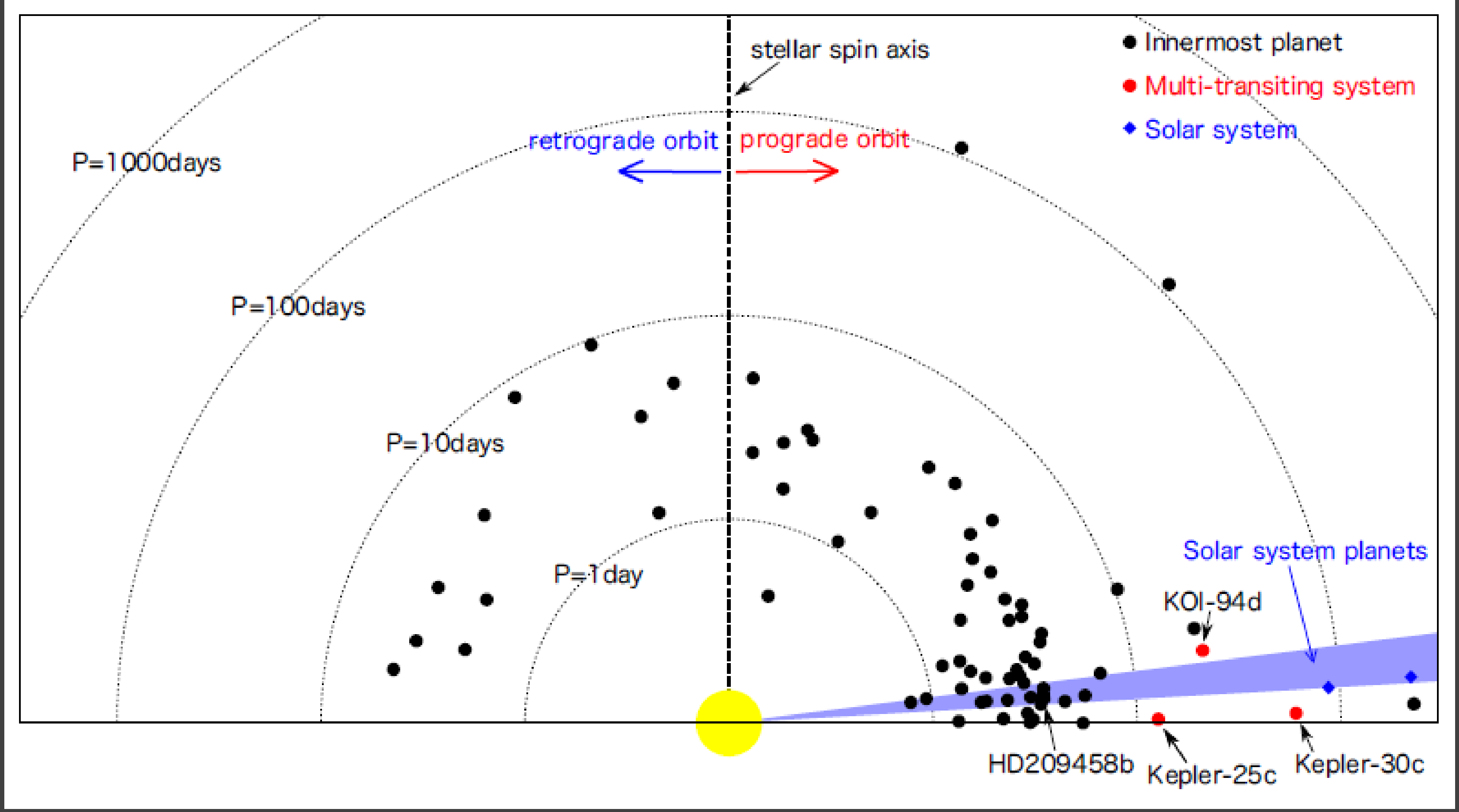
Эффект был впервые описан Михаилом Лидовым для спутников в 1961 г., а затем в 1962 г. был описан Козаи для астероидов.

# Example of orbital behaviour



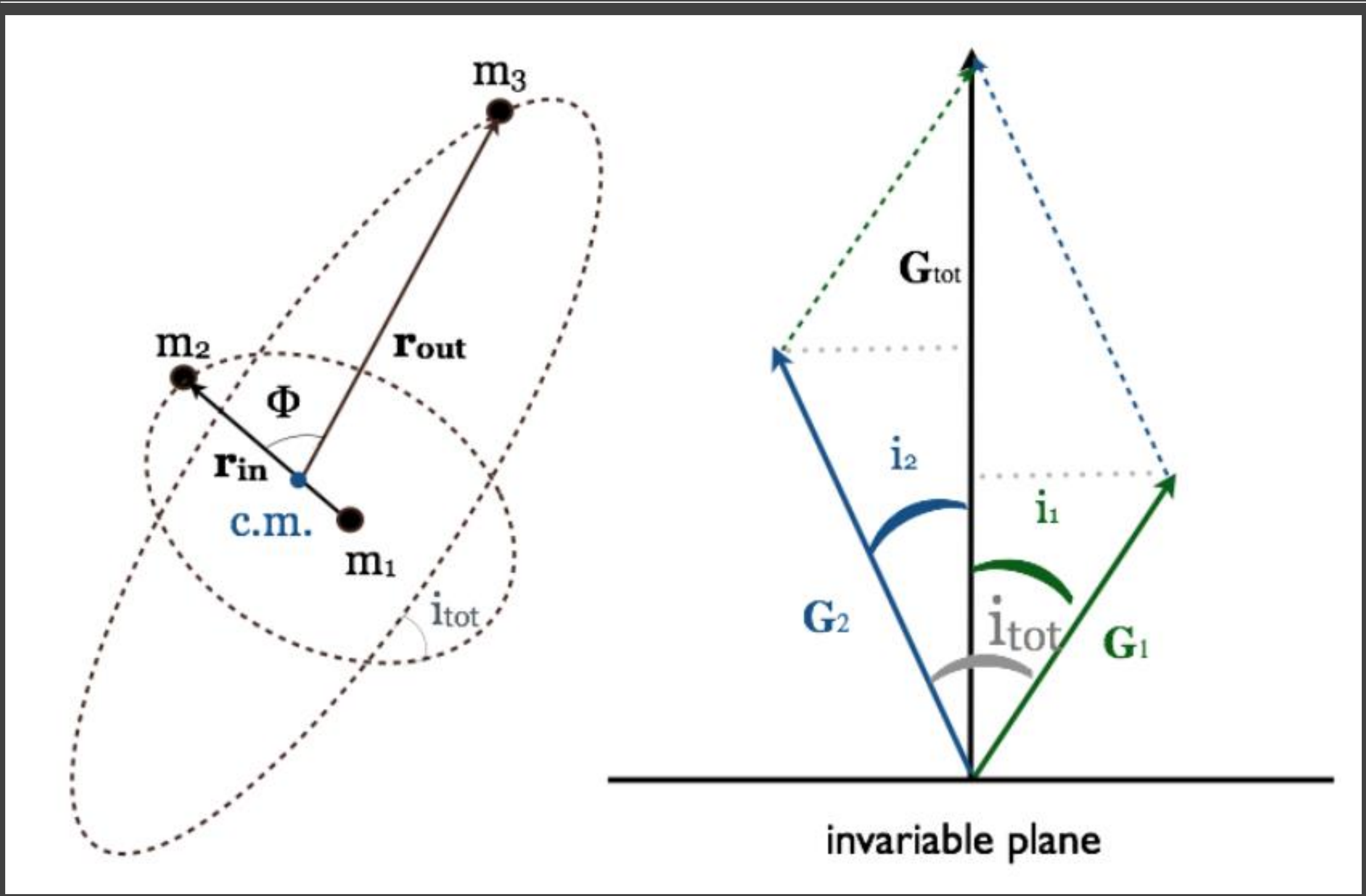

0.5 AU planet (around the primary) in a 5 AU binary.
Initial inclination 60 degrees.

# Распределение планет по ориентации орбиты




Есть планеты с полярными и даже обратными орбитами.

# Approximation




Wide outer body's orbit.
No resonances.

Two orbits exchange angular momentum, but not energy.
So, orbits can change shape and orientation, but not semi-major axes.

Conservation of projection of the angular momentum results in

$$j_z = \sqrt{1-e_1^2}\cos i_{tot} = \text{Const.}$$

# Circular outer orbit

$$j_{z,1} = \sqrt{1 - e^2_{1,max/min}} \cos i_{1,min/max} = \sqrt{1 - e^2_{1,0}} \cos i_{1,0}$$

$m_2 \longrightarrow 0$ (test particle approximation)

$e_{1,0} = 0$ and $\omega_{1,0} = 0$

$$E_0 = 2e^2_{1,min} - 2 + (1 - e^2_{1,min}) \cos i^2_{max}$$

$\Rightarrow$

$$e_{max} = \sqrt{1 - \frac{5}{3}\cos^2 i_0}$$

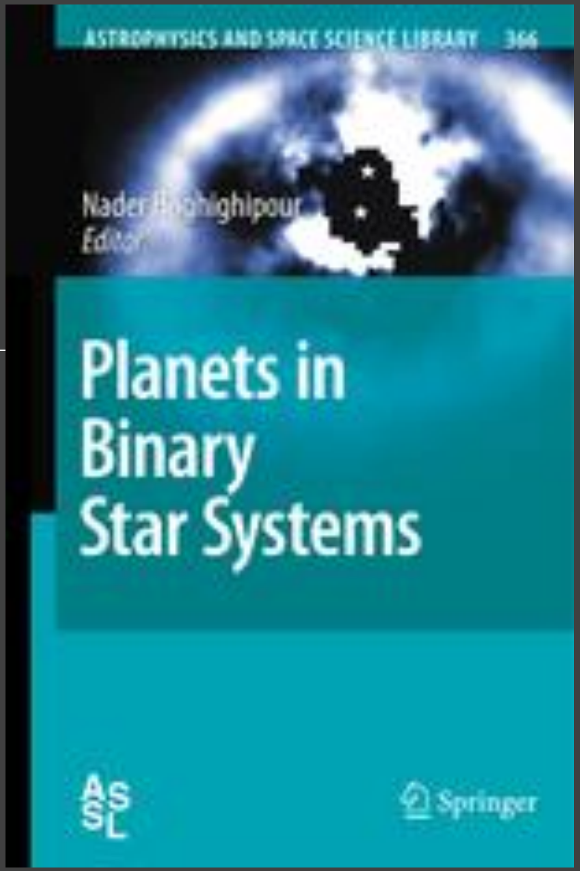
$$\cos i_{min} = \pm\sqrt{\frac{3}{5}}$$

~40 and 140 degrees
(Kozai angles)

The z-component of the angular momentum of the inner and outer orbits (i.e., the nominal $\sqrt{1 - e^2_{1,2}} \cos i_{1.2}$) are only conserved if one of the binary members is a test particle and the outer orbit is axisymmetric ($e_2 = 0$).

# Literature

- 1608.00764 New prospects for observing and cataloguing exoplanets in well detached binaries R. Schwarz et al.
- 1401.0601 Calculating the Habitable Zone of Multiple Star Systems Tobias Mueller, Nader Haghighipour
- 1601.07175 The Eccentric Kozai-Lidov Effect and Its Applications Smadar Naoz
- 1406.1357 Planet formation in Binaries P. Thebault, N. Haghighipour
- 1802.08693 Populations of planets in multiple star systems
- 2002.12006 Planets in Binaries: Formation and Dynamical Evolution




(2010)