

# Population synthesis of exoplanets

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SERGEI POPOV

# Population synthesis in astrophysics

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A population synthesis is a method of a direct modeling of relatively large populations of weakly interacting objects with non-trivial evolution.

As a rule, the evolution of the objects is followed from their birth up to the present moment.

## Evolutionary and Empirical

1. Evolutionary PS.

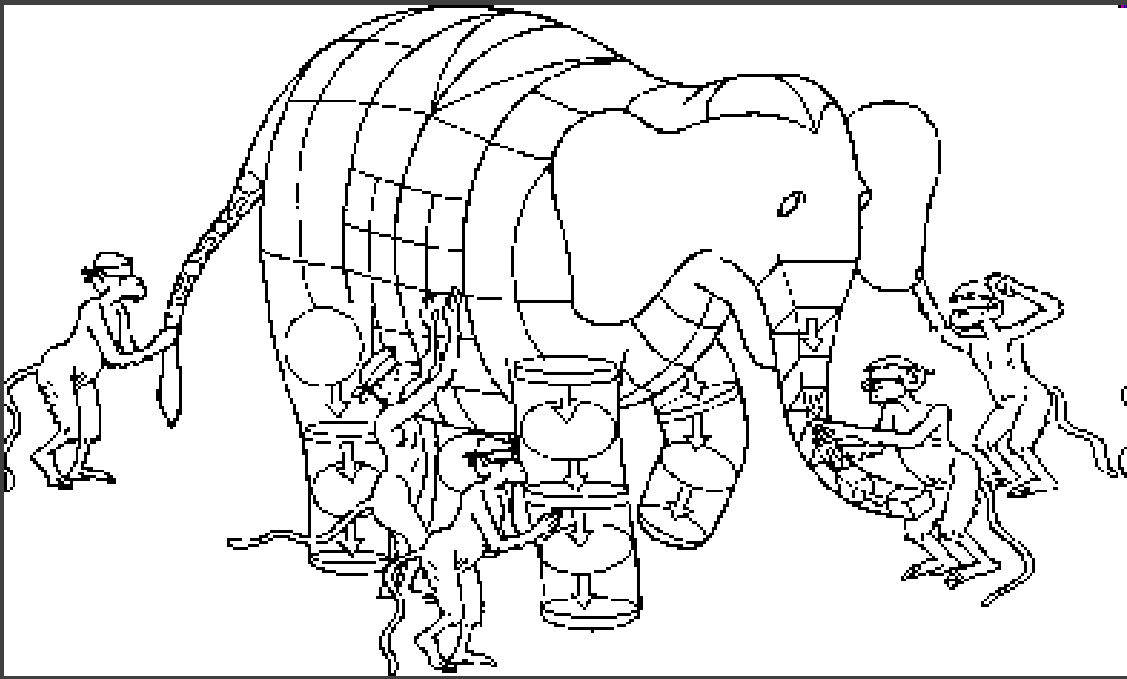
The evolution is followed from some early stage.

Typically, an artificial population is formed (especially, in Monte Carlo simulations)

2. Empirical PS.

It is used, for example, to study integral properties (spectra) of unresolved populations.

A library of spectra is used to predict integral properties.



Ingredients:

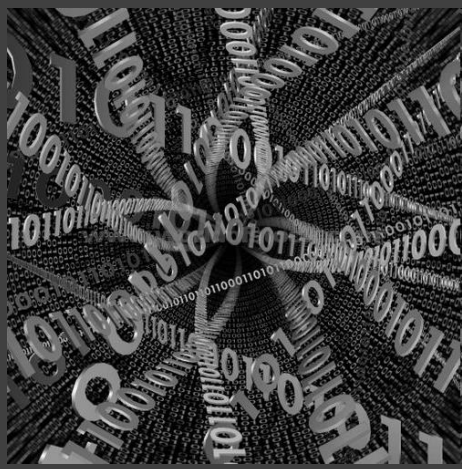
- initial condition
- evolutionary laws



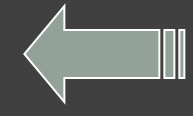
«Artificial observed universe»



*Modeling  
observations*

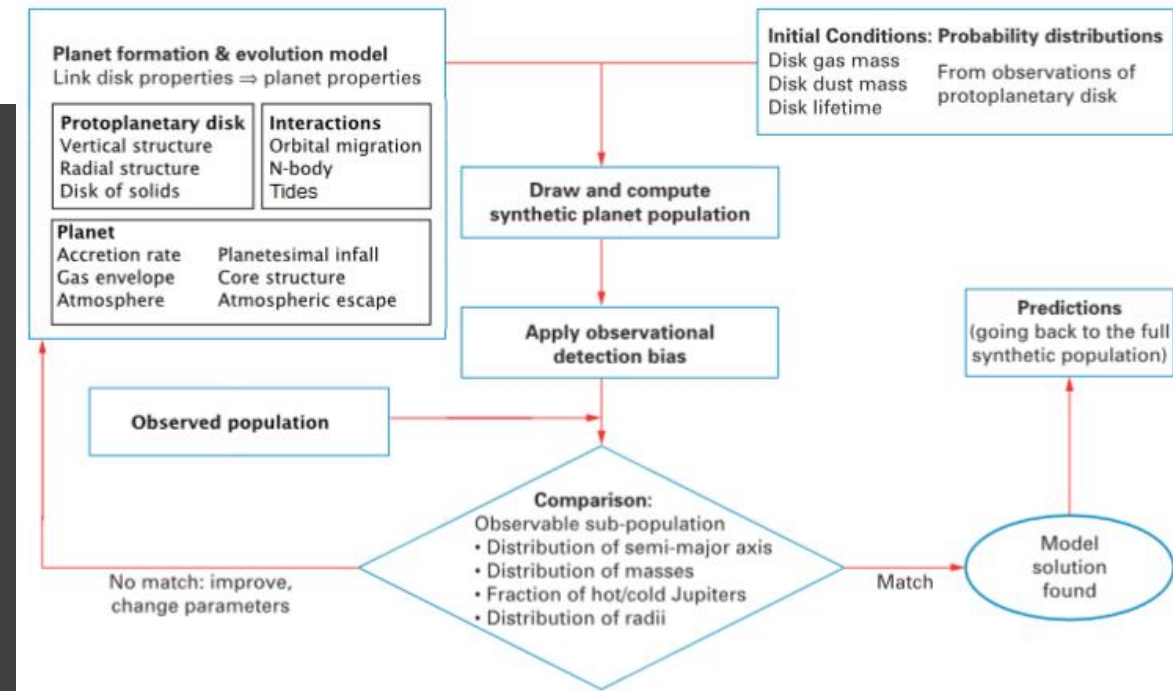


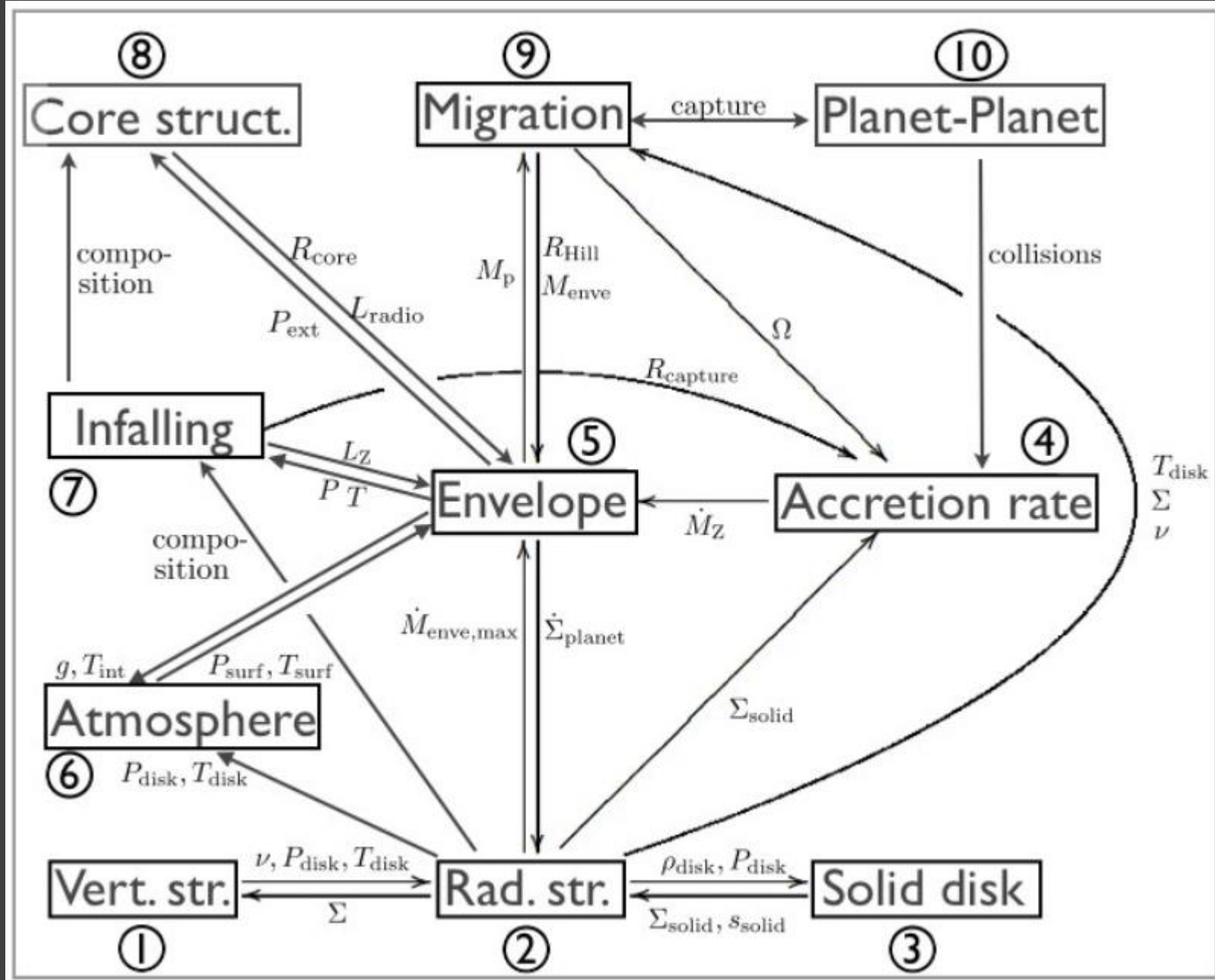
«Artificial universe»



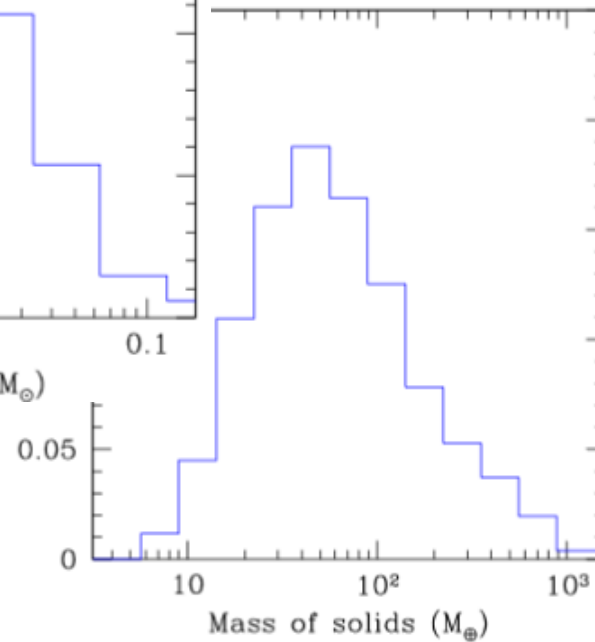
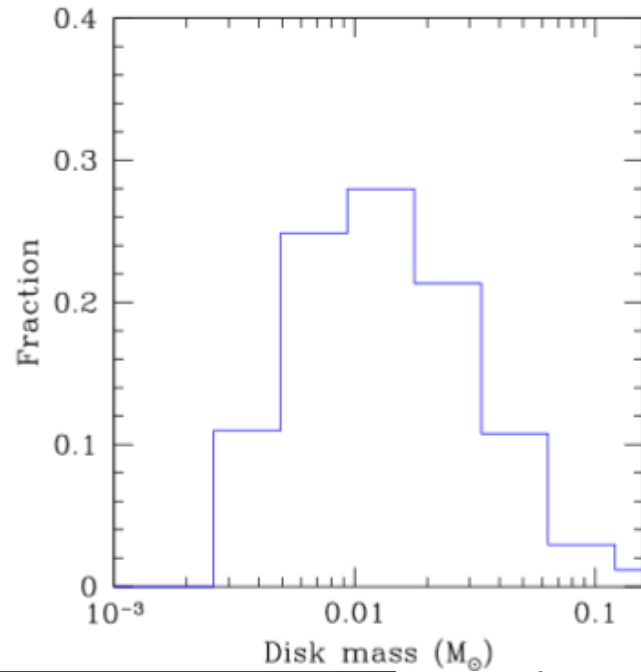
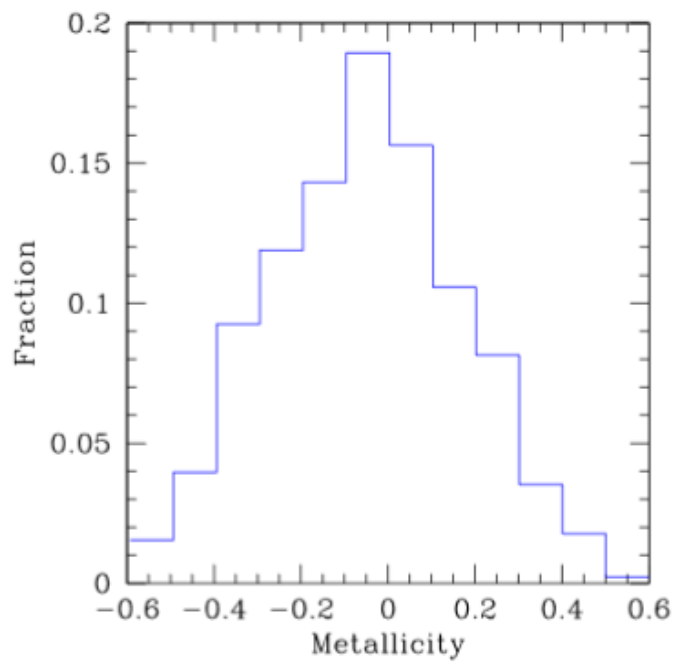
# Ingredients for planetary PS

1. The structure and evolution of the protoplanetary gas disk
2. The structure and evolution of the disk of solids (dust, pebbles, planetesimals)
3. The accretion of solids leading to the growth of the planetary solid core
4. The accretion of H/He leading to the growth of the planetary gaseous envelope
5. Orbital migration resulting from the exchange of angular momentum
6. N-body interaction among (proto)planets

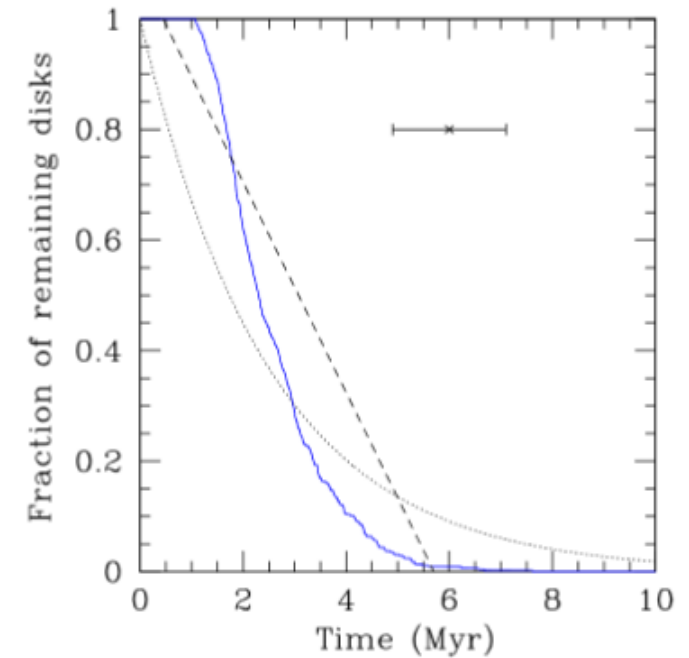




# Initial distributions



Initial conditions for discs around 1 solar mass stars.



In addition, initial positions of embryos is also a random parameter.



# The first PS model for exoplanets

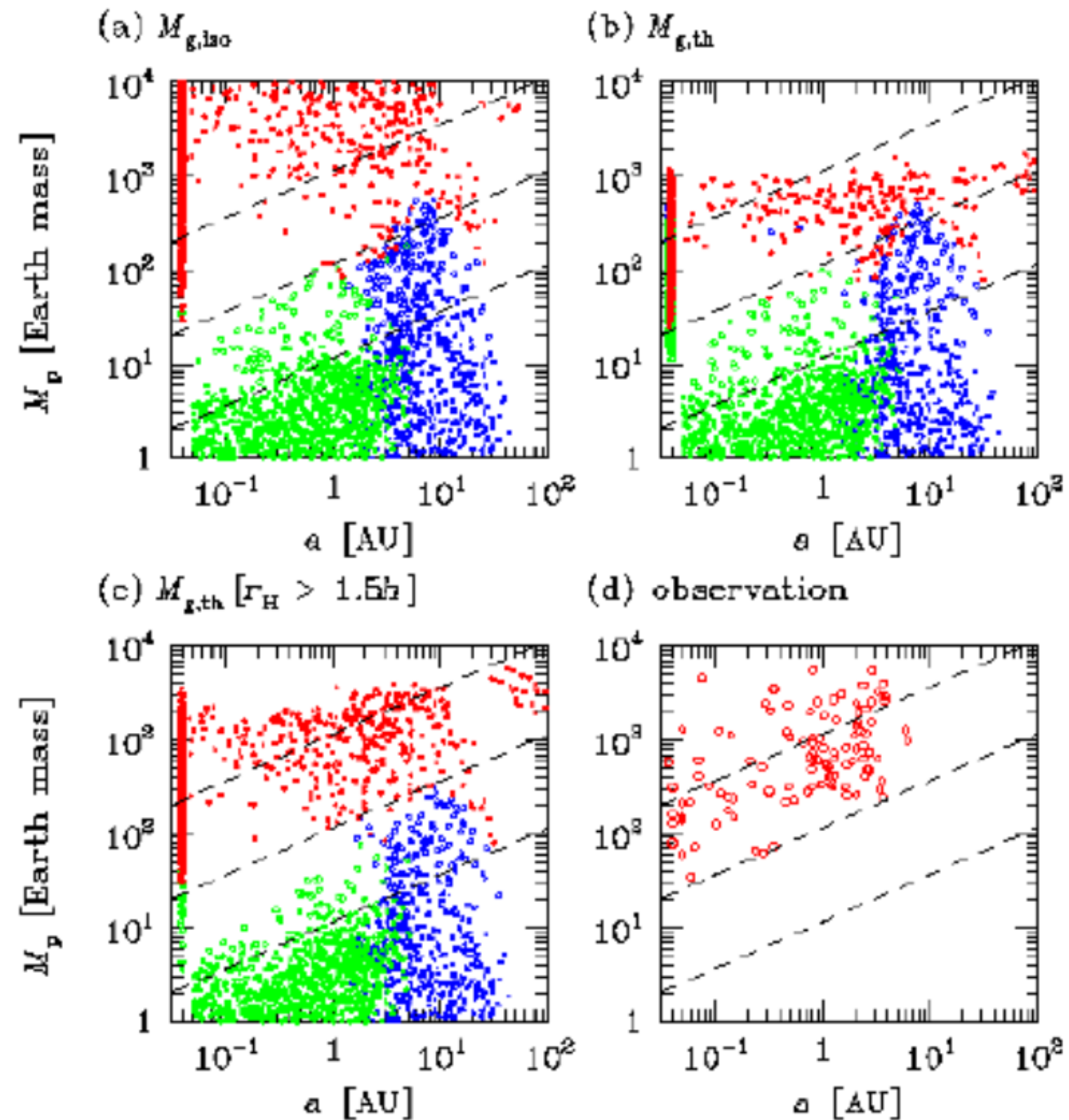
Authors modeled formation and migration (I&II) of exoplanets in order to reproduce so-called “desert” in mass-semi-major axis distribution (masses 10-100 Earth mass, and  $a < 3$  AU).

Main ingredients:

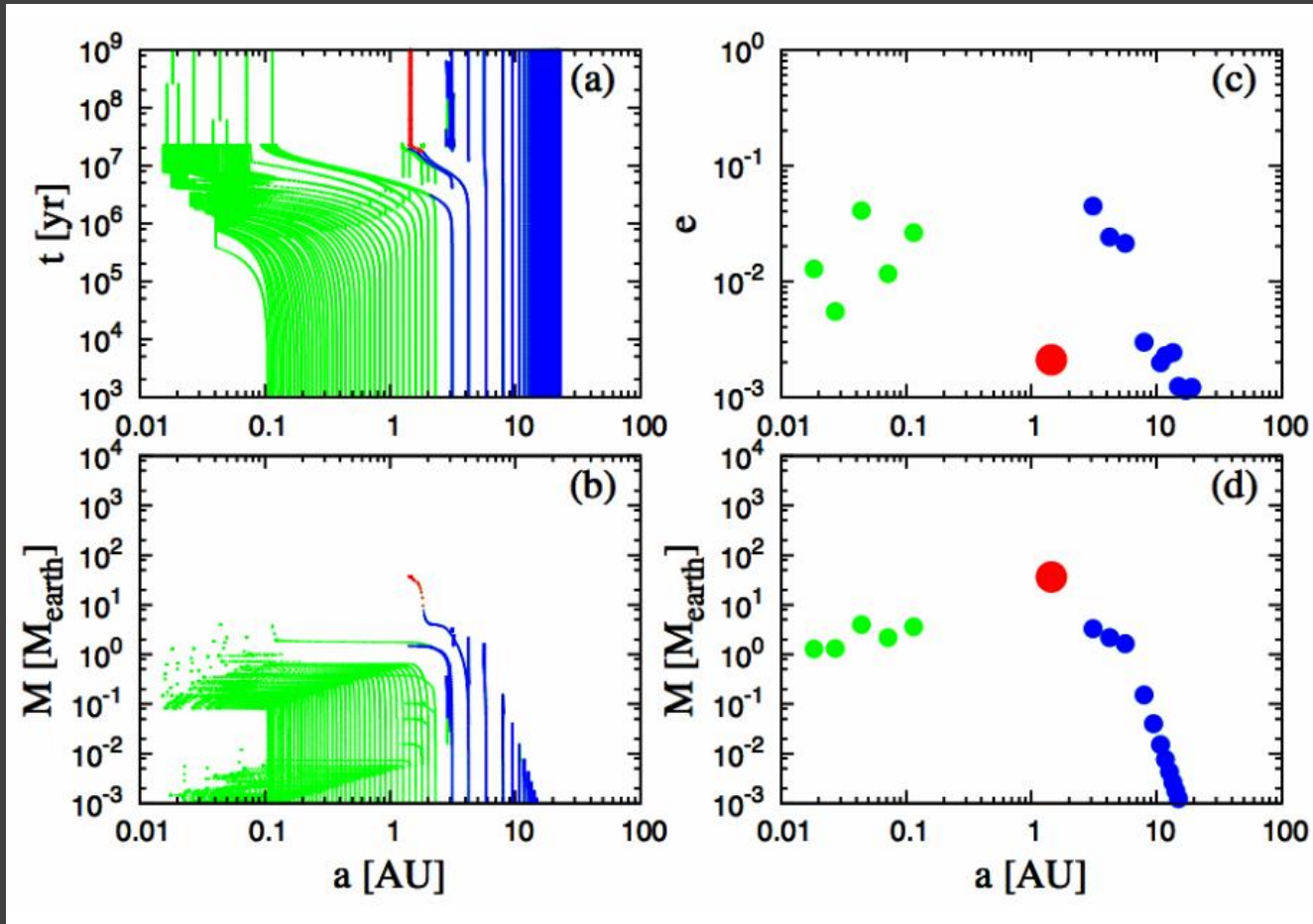
- Disk model;
- Accretion model;
- Migration model.

The rate of type I migration was significantly reduced to avoid rapid planet displacement.

Red- giants;  
green – rocky;  
blue – ice.



# Individual tracks



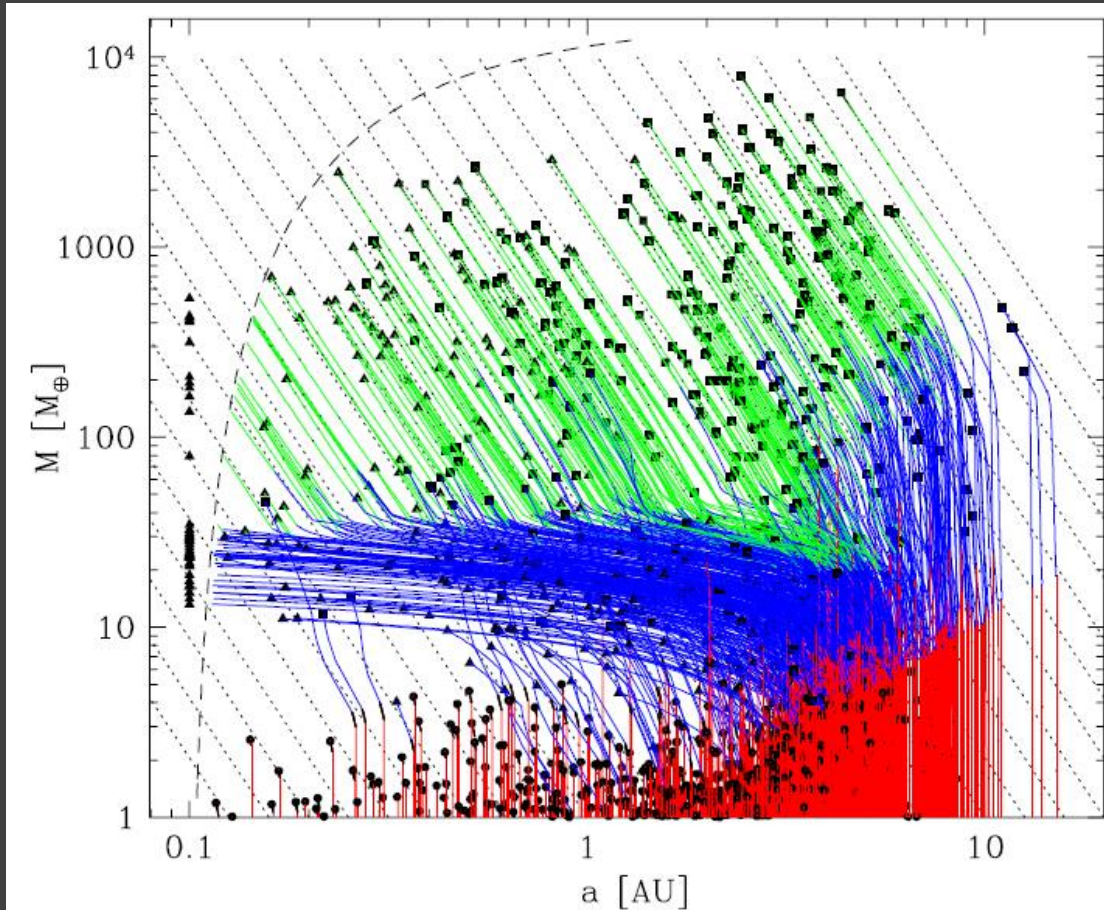
Green - rock  
Red - gas  
Blue - ice

$$\begin{aligned}
 \tau_{\text{mig1}} &= \frac{a}{\dot{a}} \\
 &= \frac{1}{C_1} \frac{1}{3.81} \left( \frac{c_s}{a\Omega_K} \right)^2 \frac{M_*}{M_{\text{planet}}} \frac{M_*}{a^2 \Sigma_g} \Omega_K^{-1} \\
 &\simeq 1.5 \times 10^5 \frac{1}{C_1 f_g} \left( \frac{M_c}{M_{\oplus}} \right)^{-1} \left( \frac{a}{1\text{AU}} \right) \\
 &\quad \times \left( \frac{M_*}{M_{\odot}} \right)^{3/2} \text{ yrs.}
 \end{aligned}$$

$$C_1 = 0.1; f_{d,0} = 2$$



# Mordasini et al. models



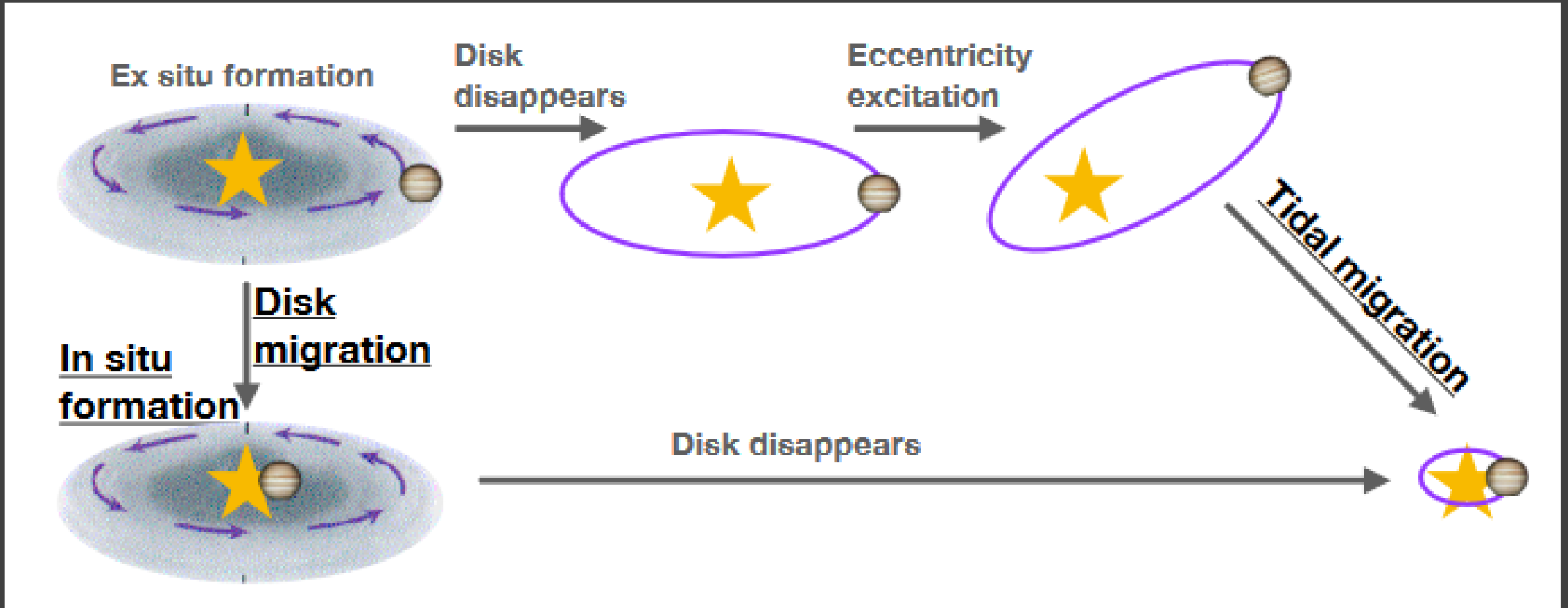
Mordasini et al. published a series of papers (0904.2524, 0904.2542, 1101.0513, 1201.1036) on population synthesis of exoplanets, using an approach generally similar to the one by Ida, Lin.

Then this studies were continued in 1206.6103, 1206.3303, 1708.00868. A review is given in 1402.7086.

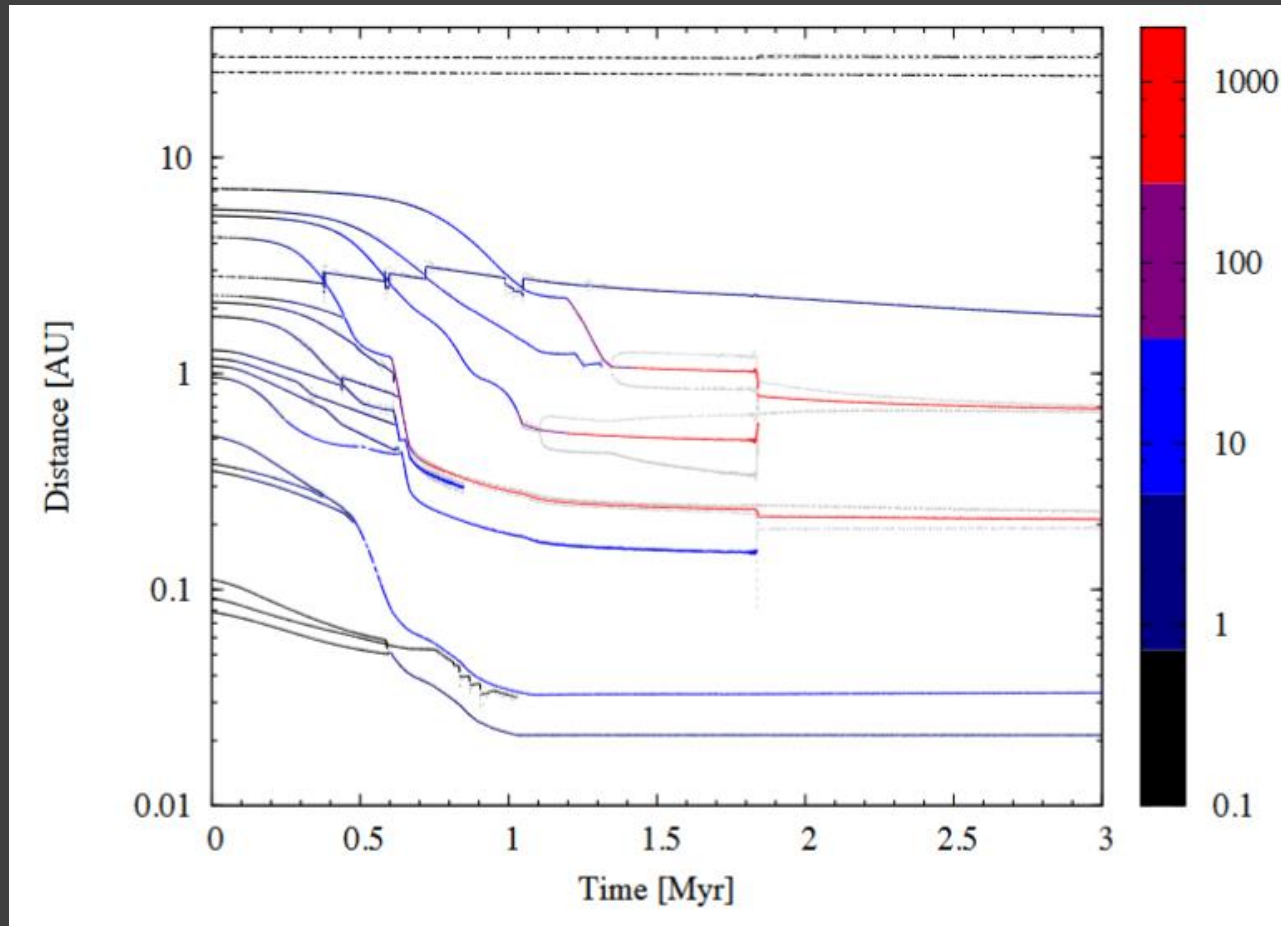
An important step is too include planet-planet interactions.

A separate subject is to follow long-term evolution.

# Alternative variants of hot planets formation

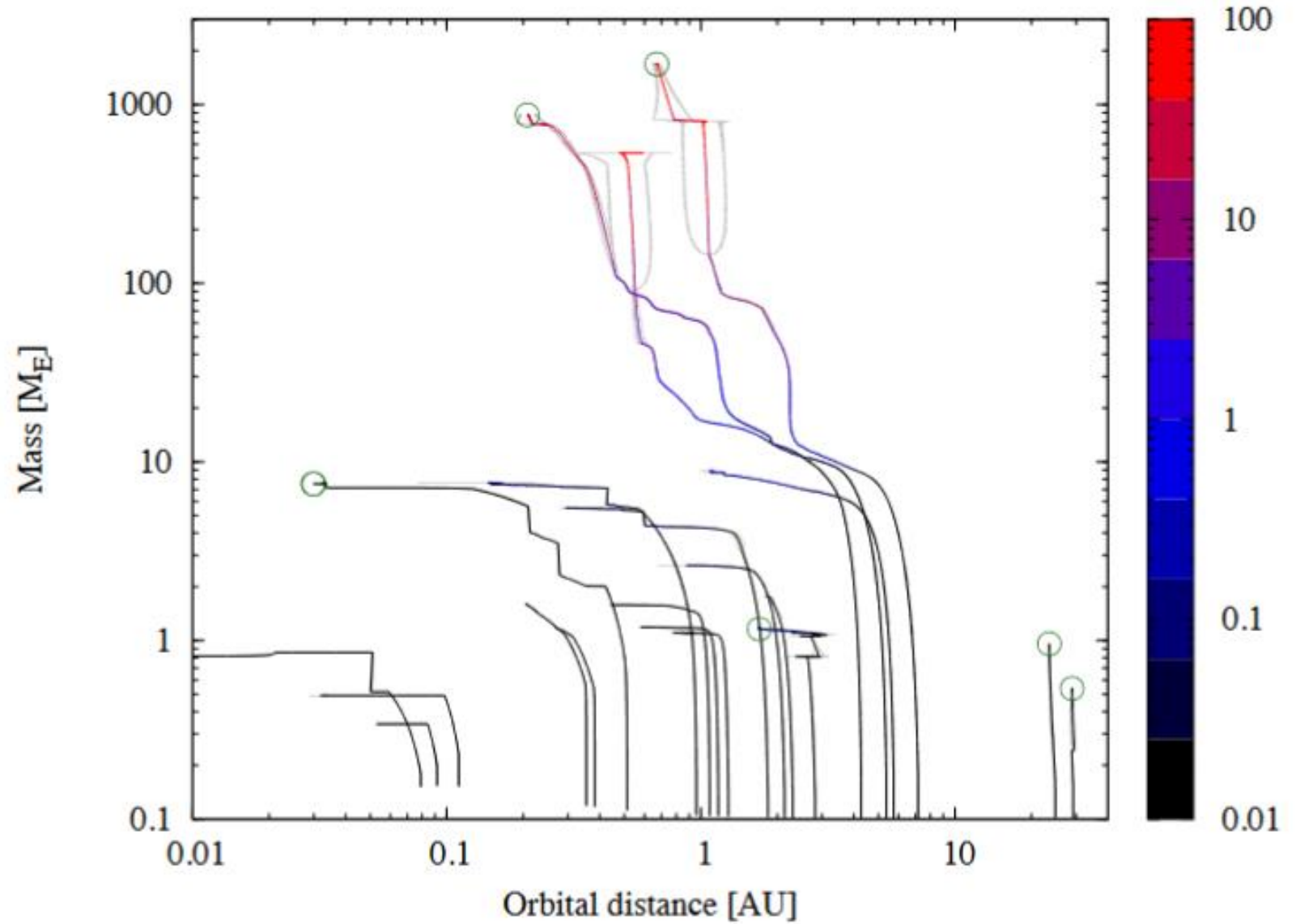


# Multi-embryo systems



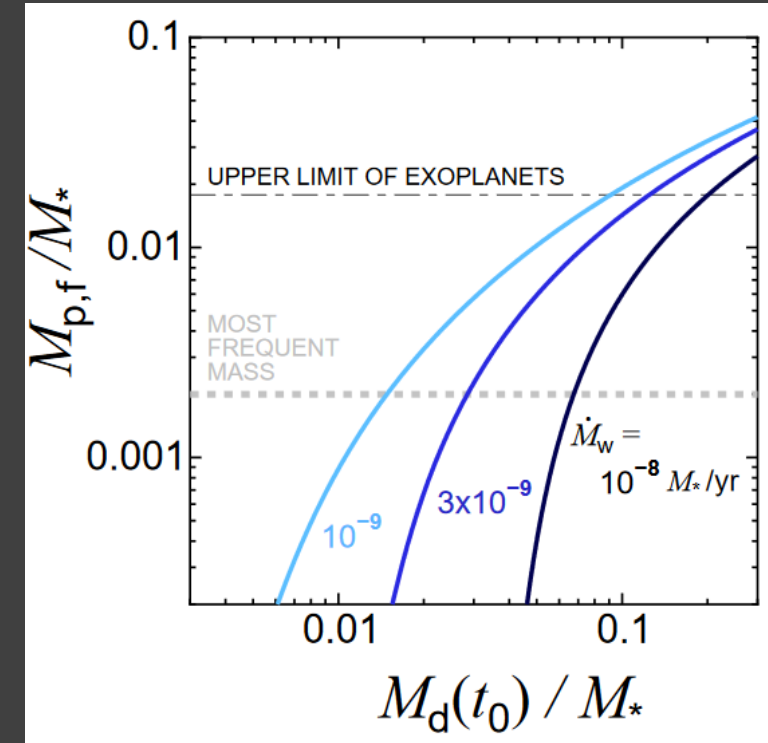
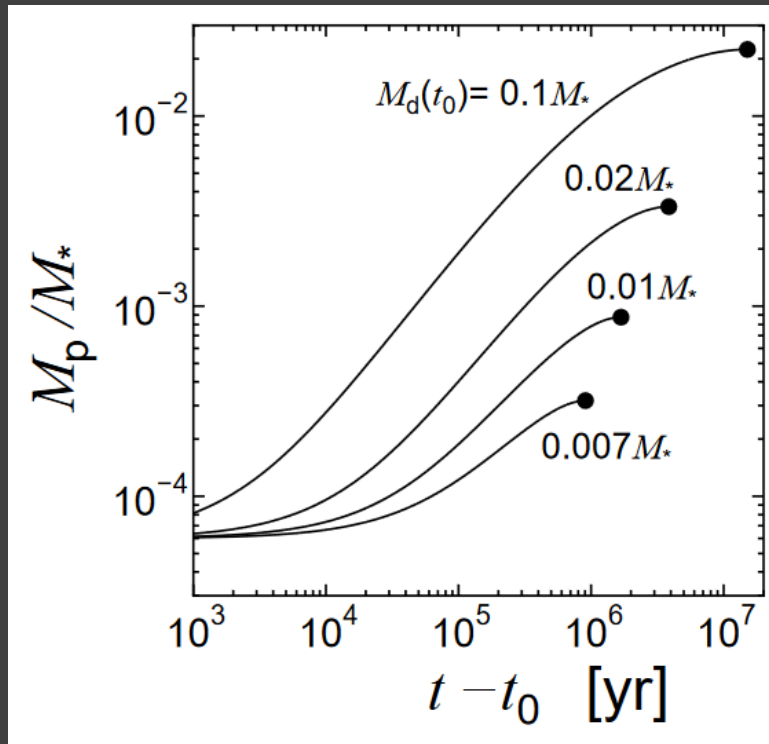
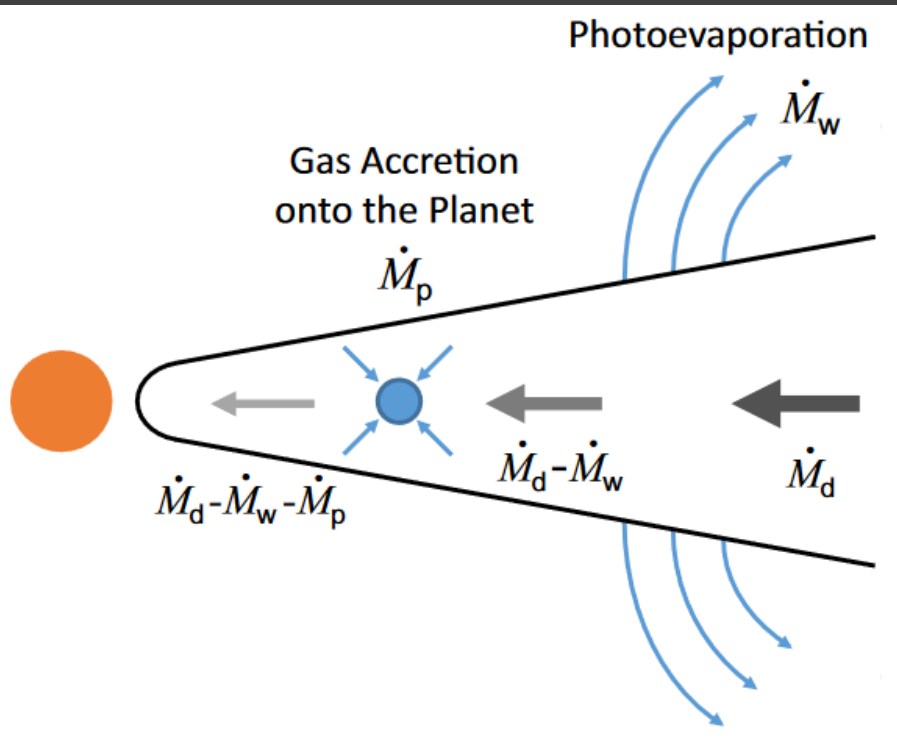
20 planet embryos with  $M=0.1M_{\text{Earth}}$

# Mass growth



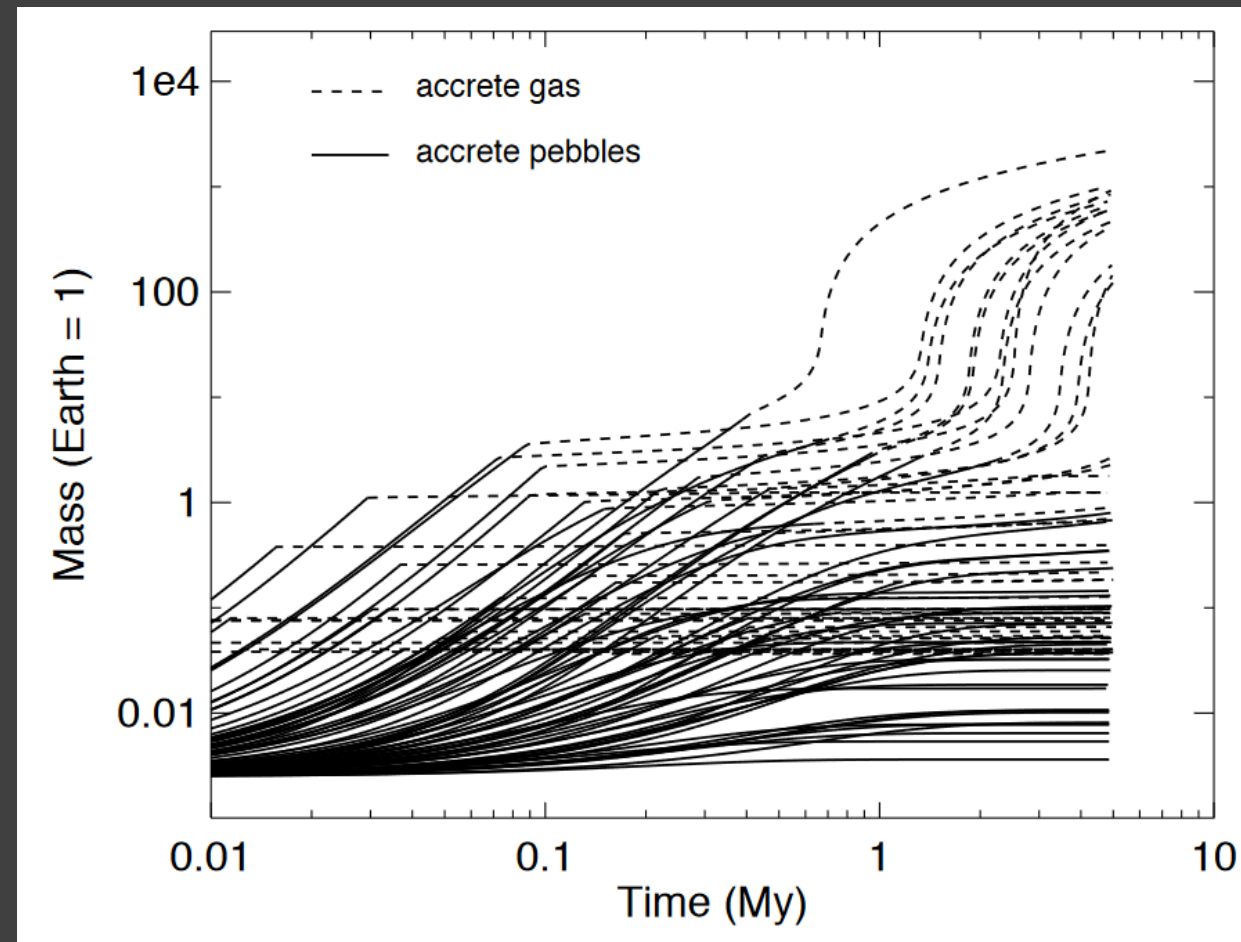
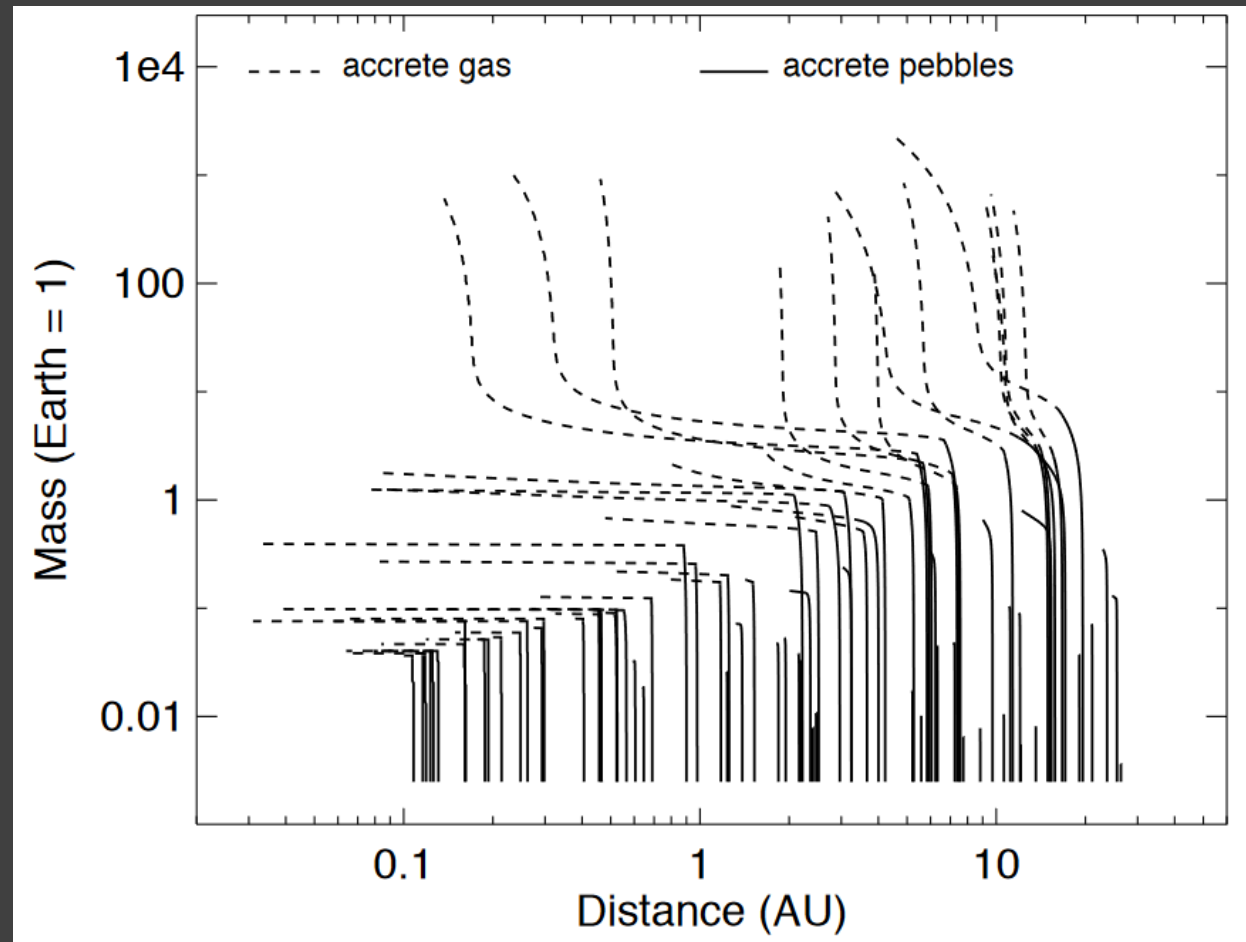
# Mass growth of a single giant planet

Photoevaporation might be important



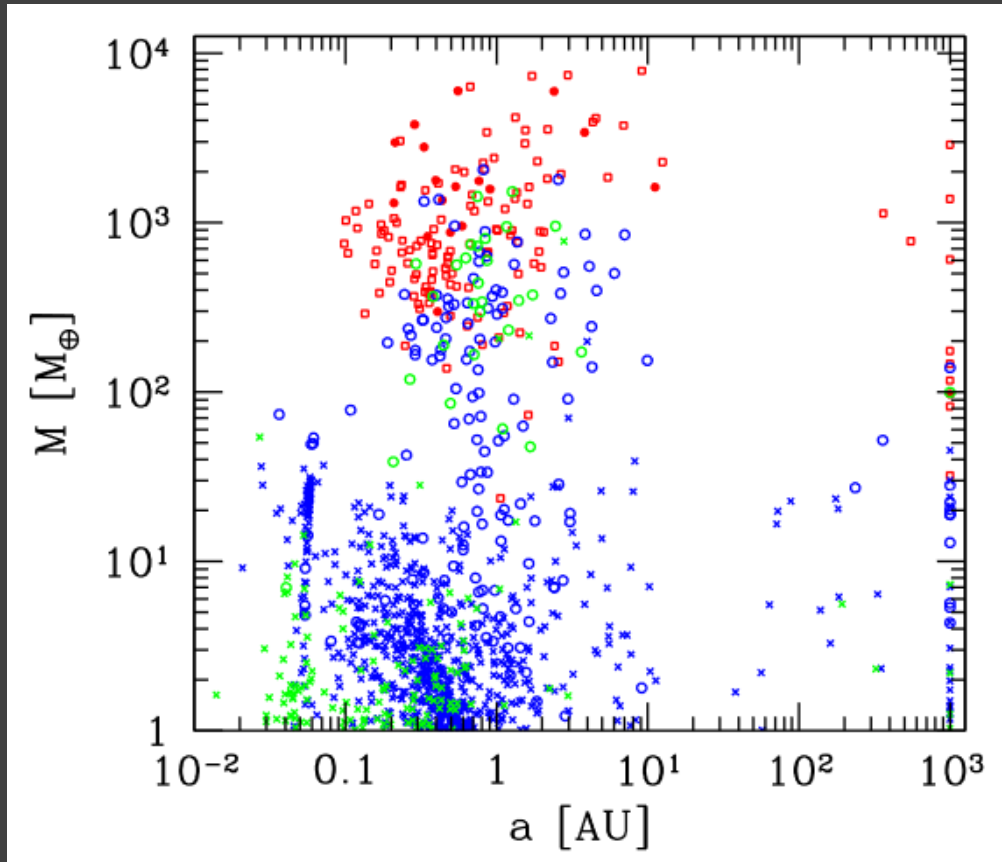
The authors derived universal tracks mass-orbital radius for gas giants.

# Peebles and gas accretion

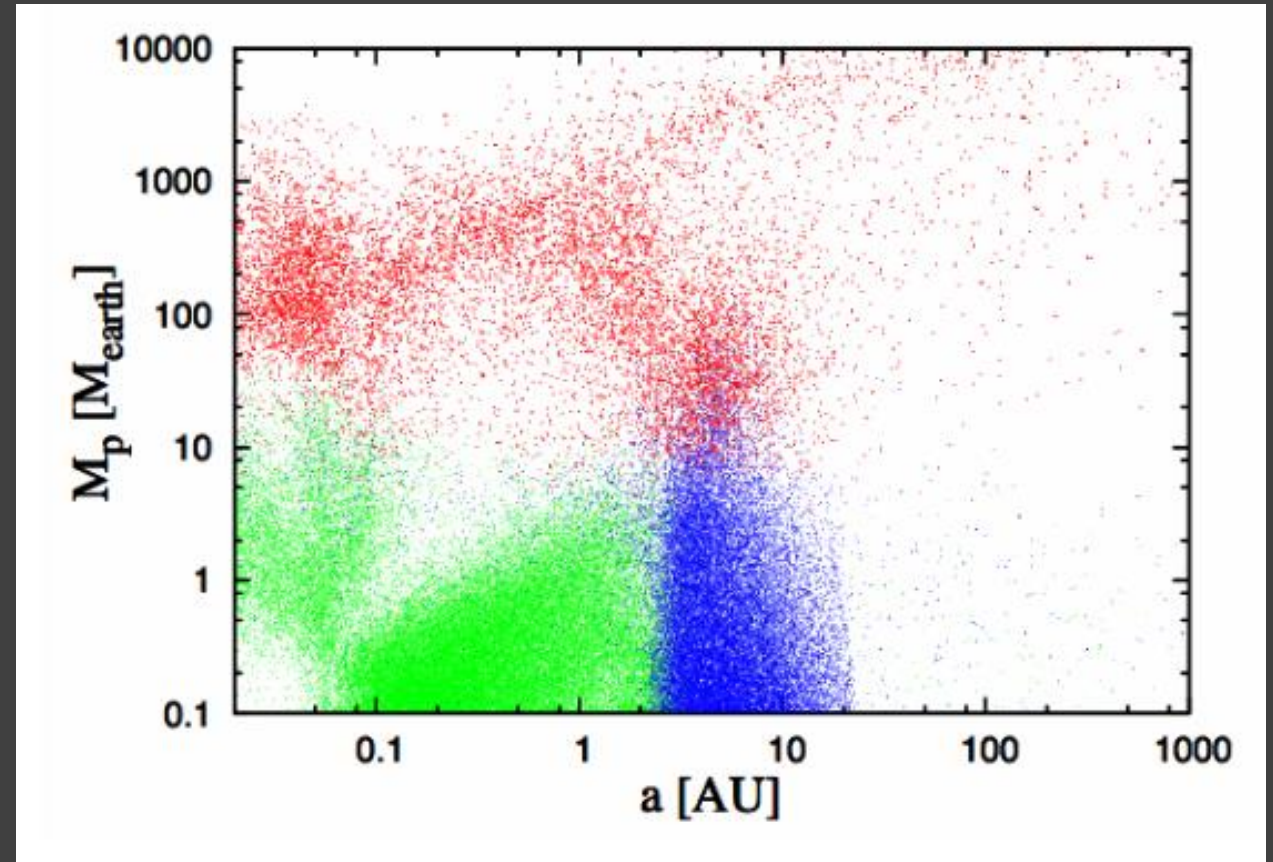




# Mass – semi-major axis distribution

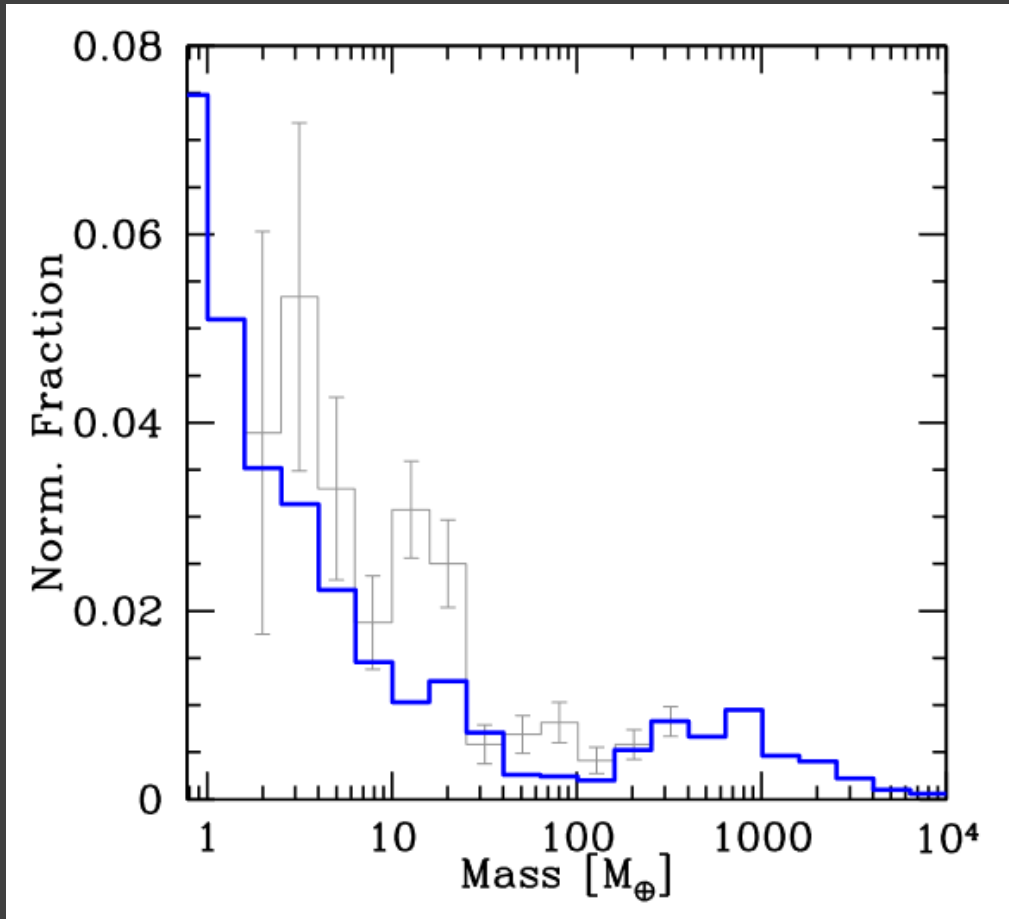


Alibert et al. (2013)



Ida, Lin (2013)

# Mass distribution



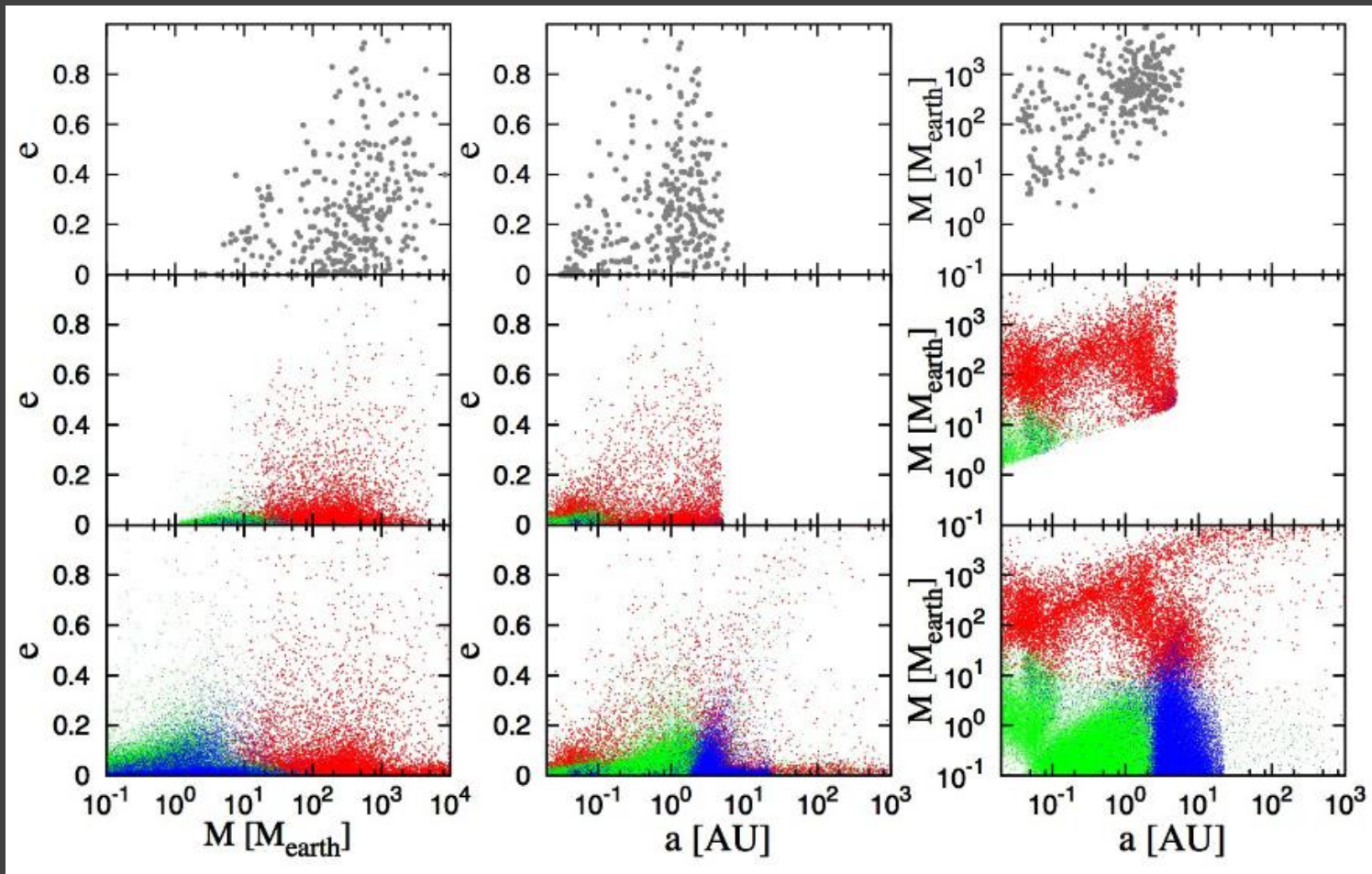
Thick line – computations;

Thin line – bias-corrected data.

Normalization made for  $1M_{\text{Jup}}$

It is still not absolutely clear, if the so-called “planetary desert” exist or not.

# Comparison with observations



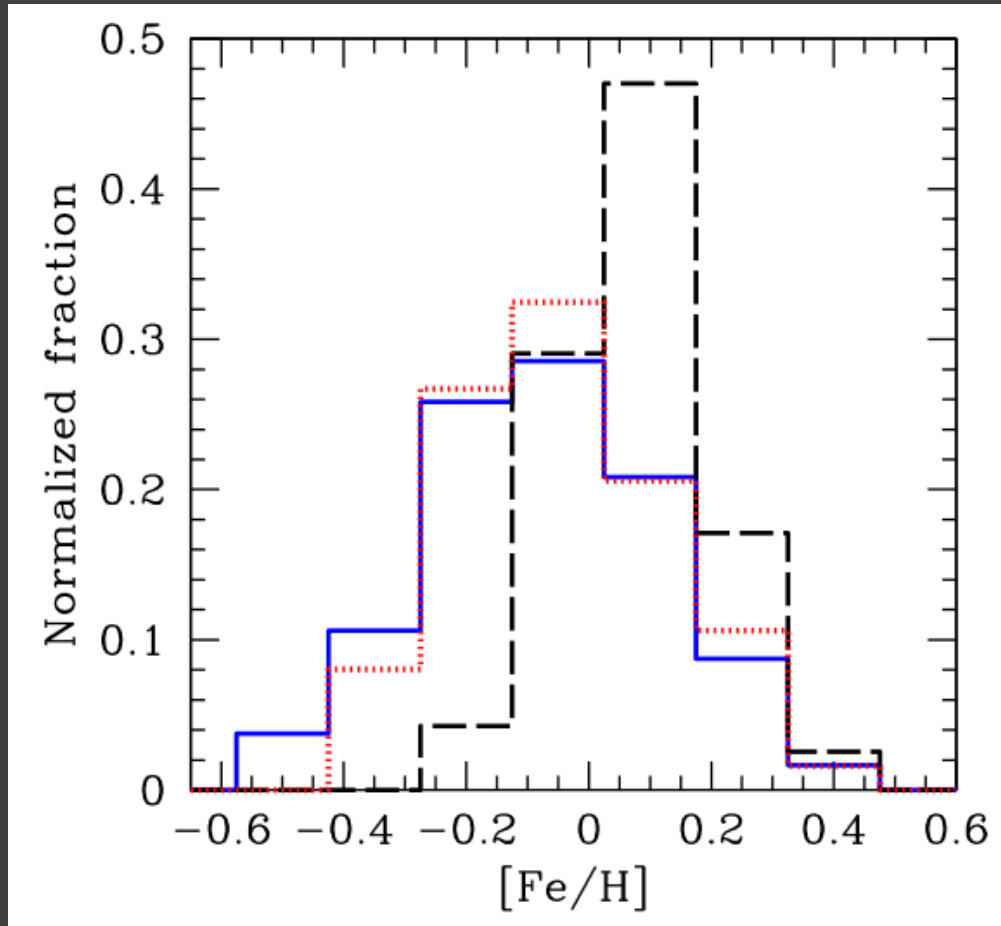
Observations

Calculations for observable planets  
( $P_{\text{orb}} < 10$  yrs;  $v > 1$  m/s)

Calculations

Ida, Lin (2013)

# Metallicity effect

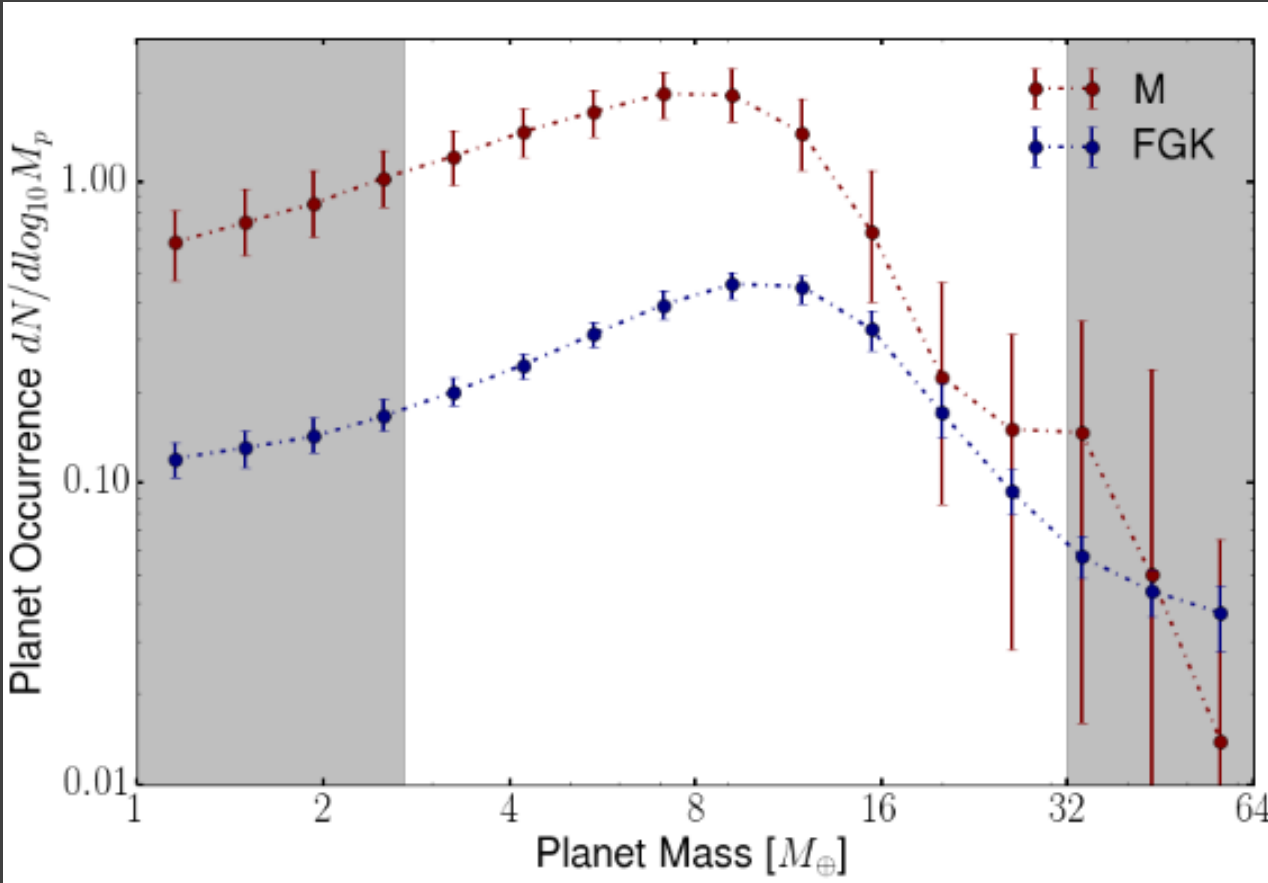


Solid line – all stars.

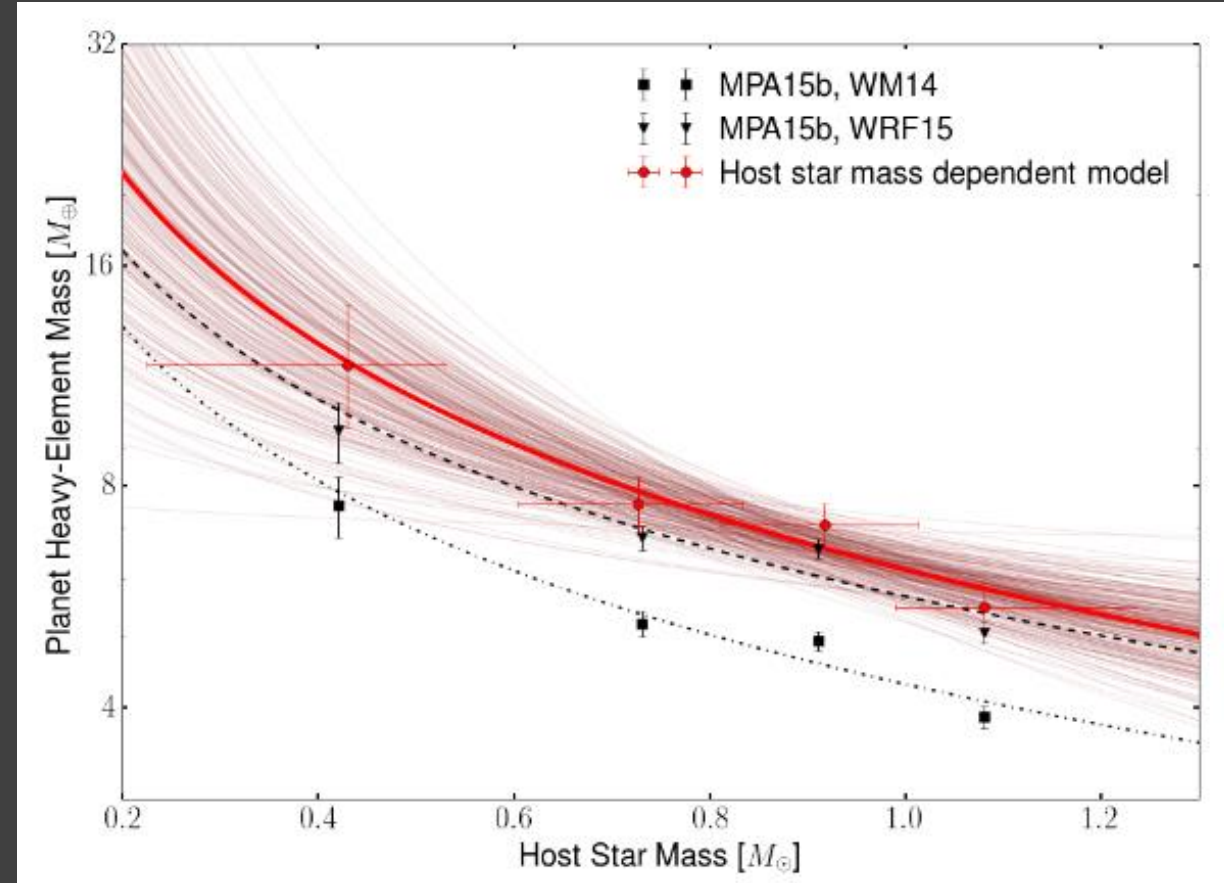
Dashed line – stars with at least one giant planet.

Dotted line – stars with at least one low-mass planet.

# Dependence on the stellar mass



Model-dependent analysis of the Kepler data.



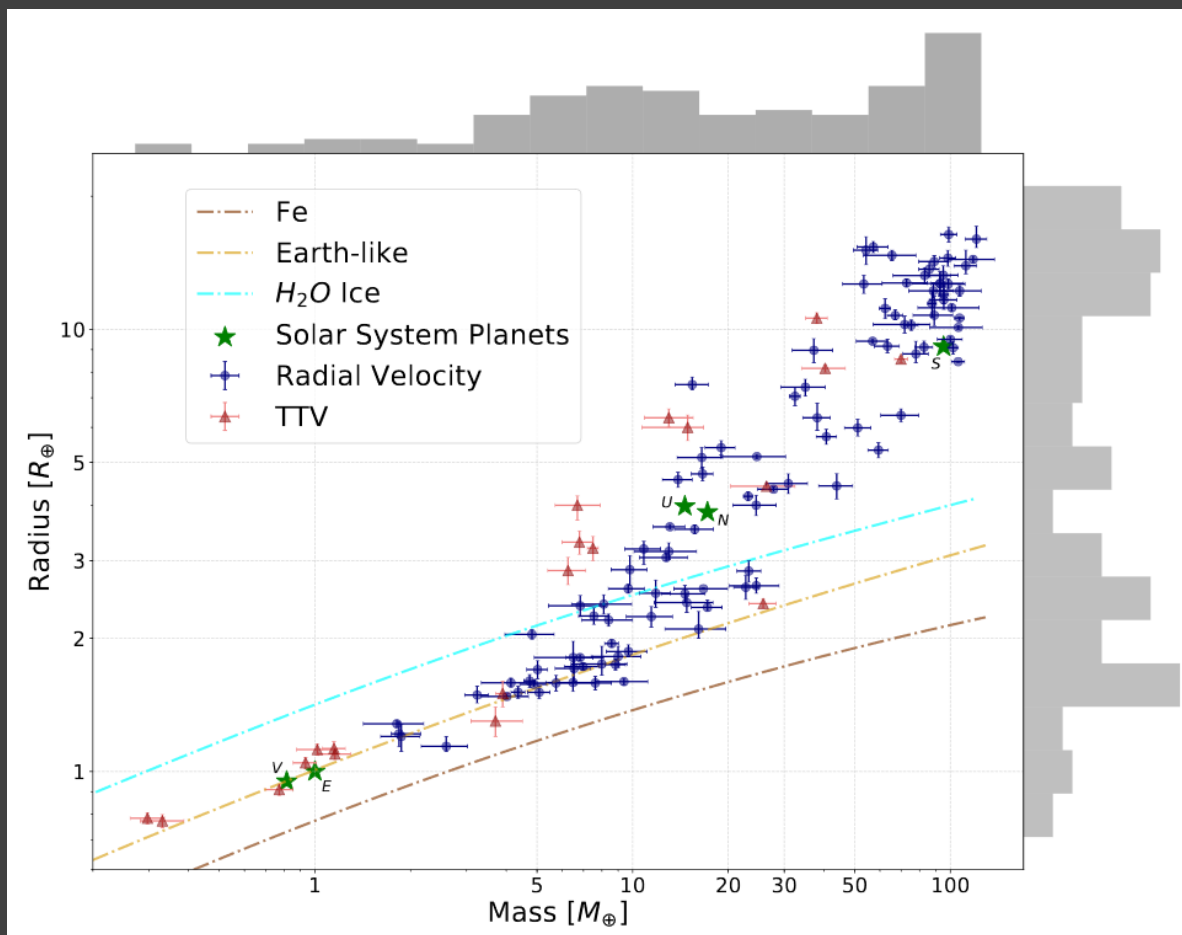
Only near-by planets included. No normal jupiters.



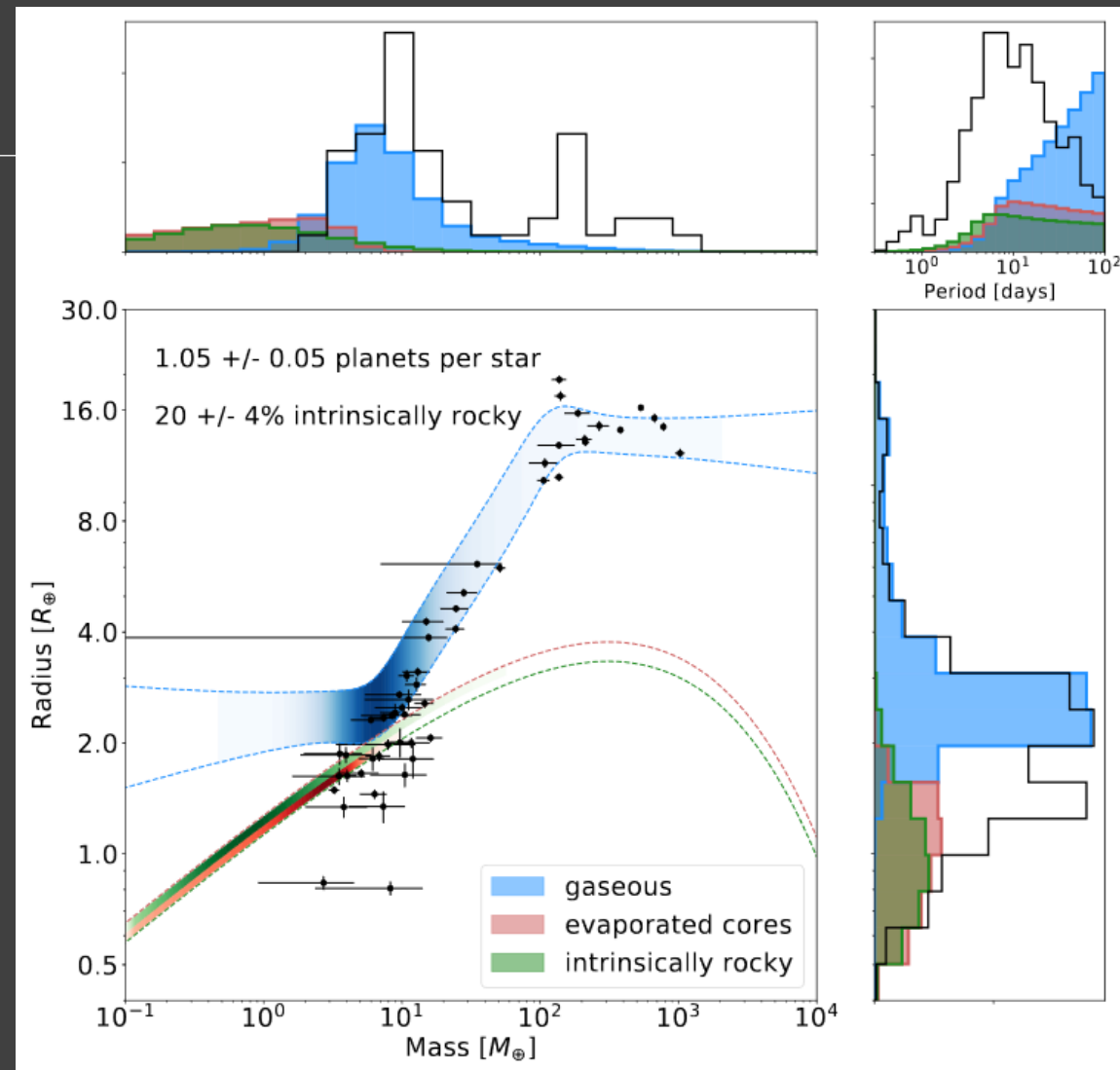




# Mass-radius from data



1911.04745



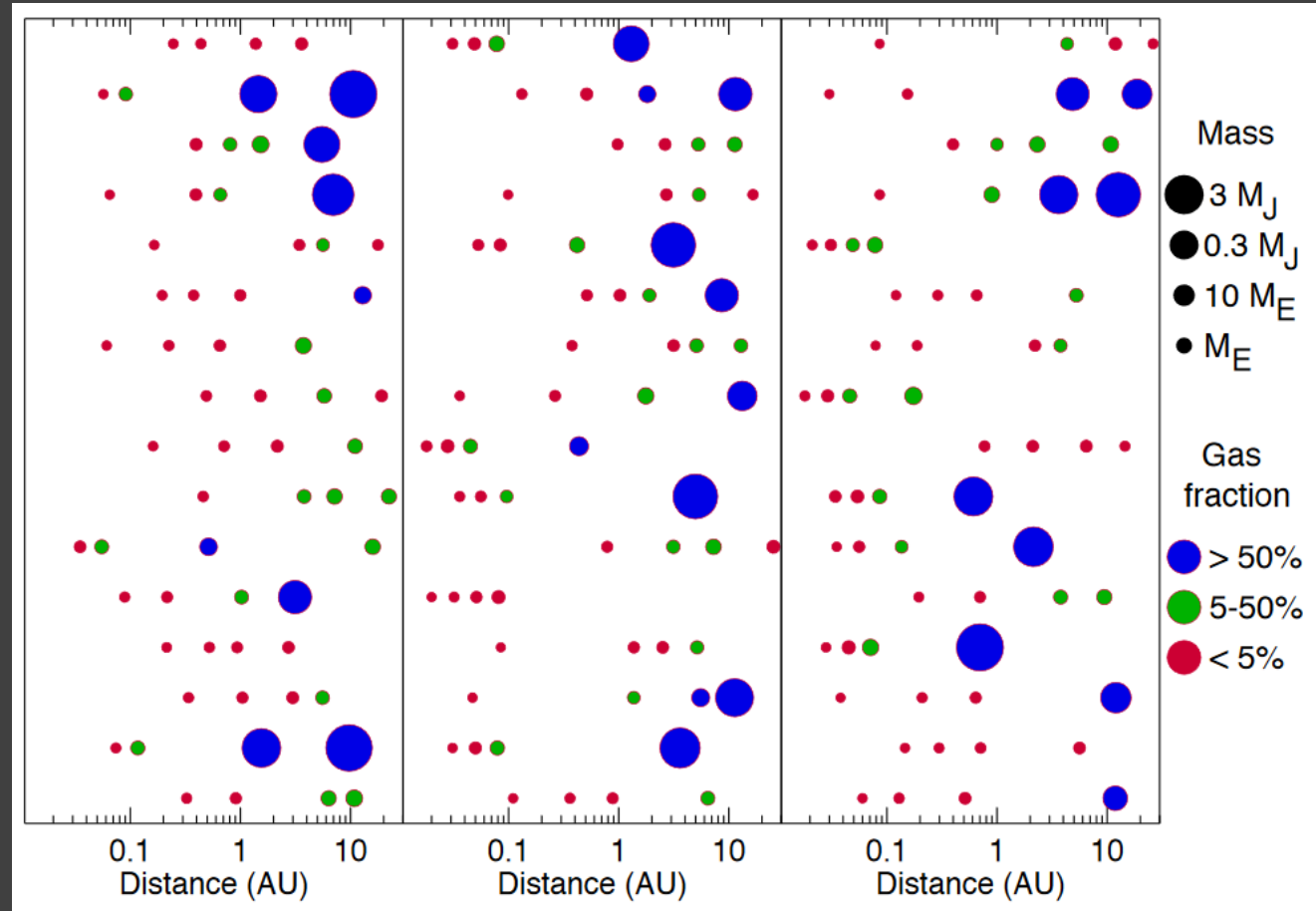
1911.03582

# Another population synthesis model

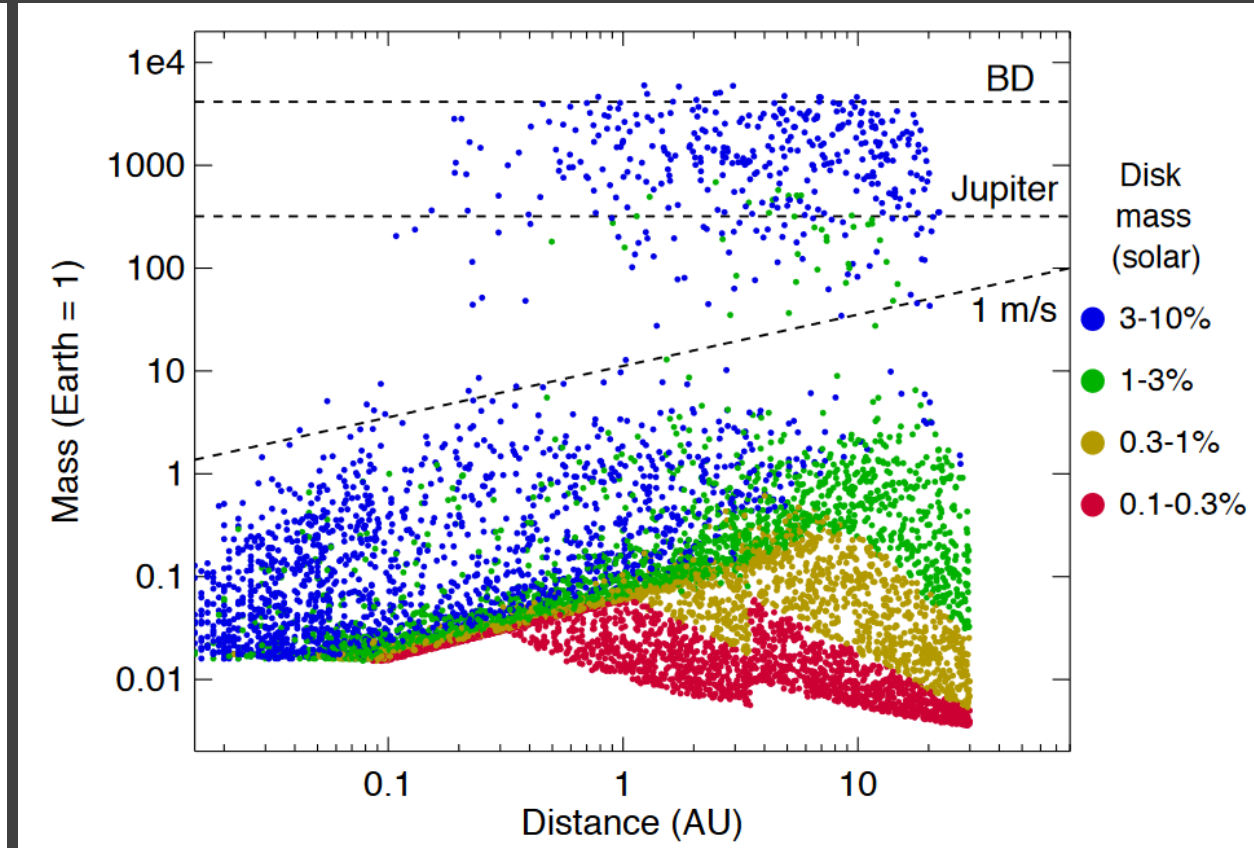
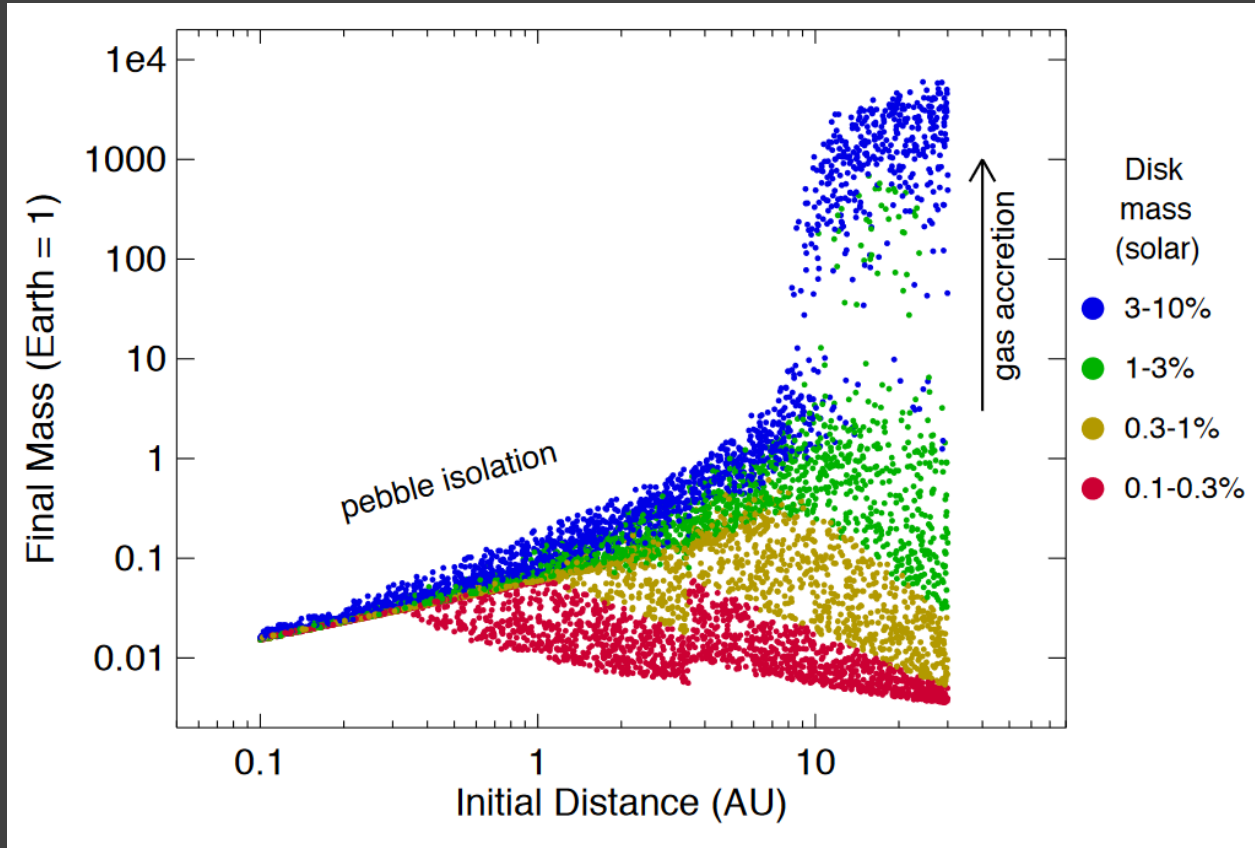
Simple model  
with analytical equations.

Model parameters are optimized  
to fit known data.

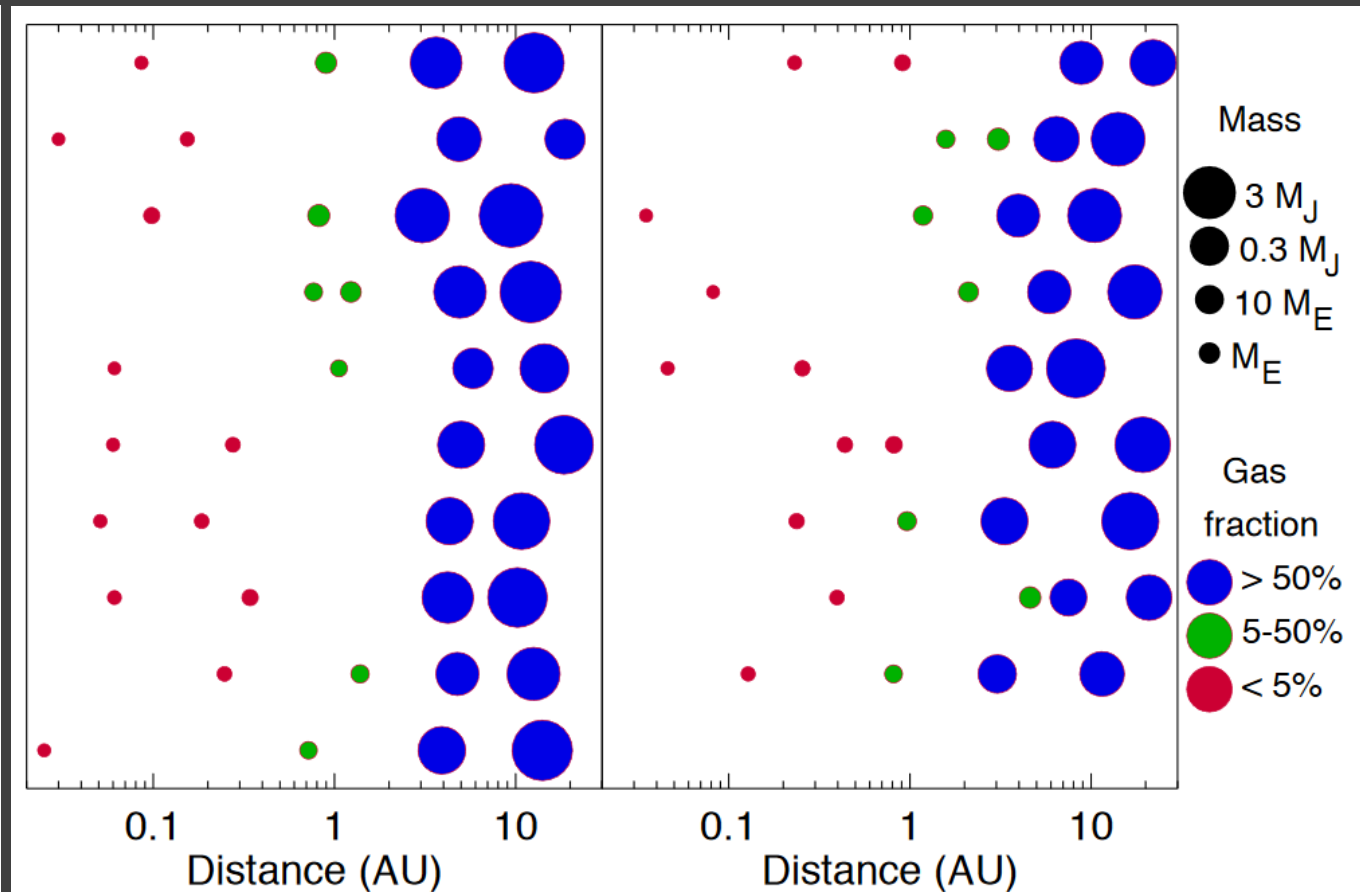
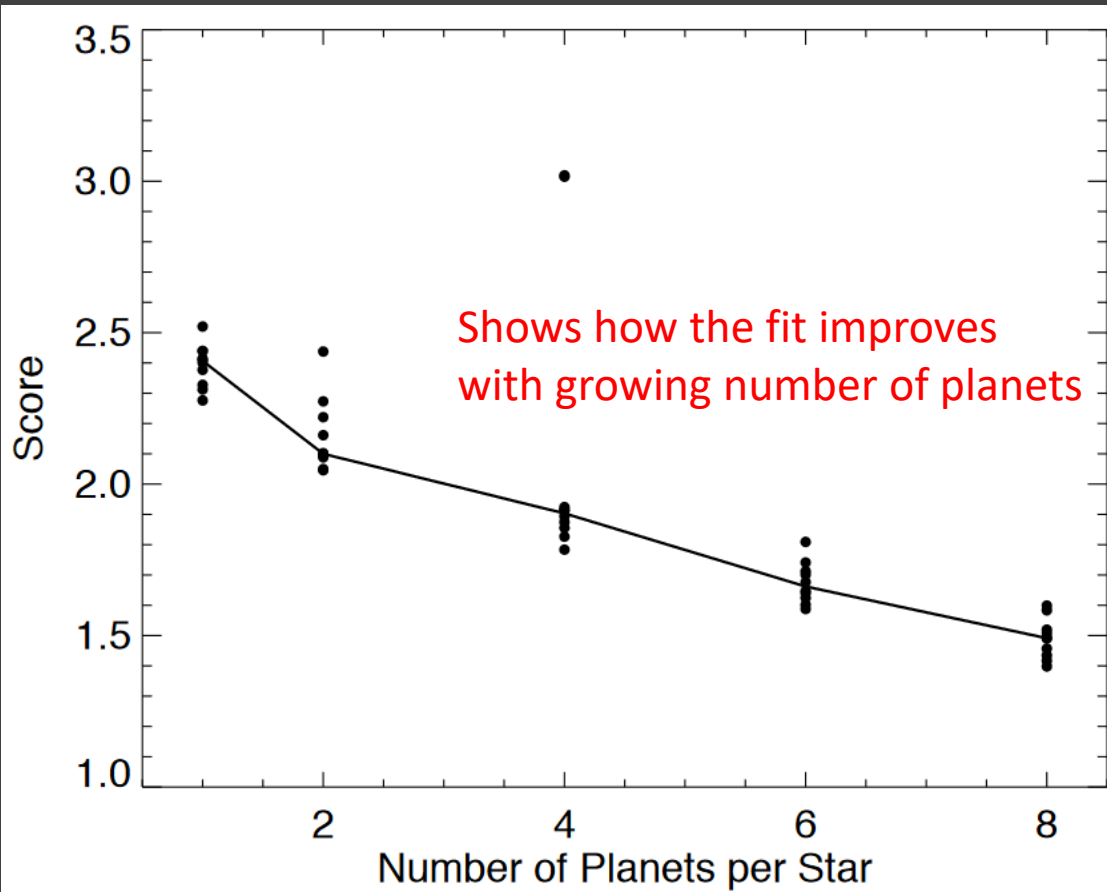
Single and four planet cases  
were studied.



# Mass- distance distribution



# Number of planets and SoSys analogues



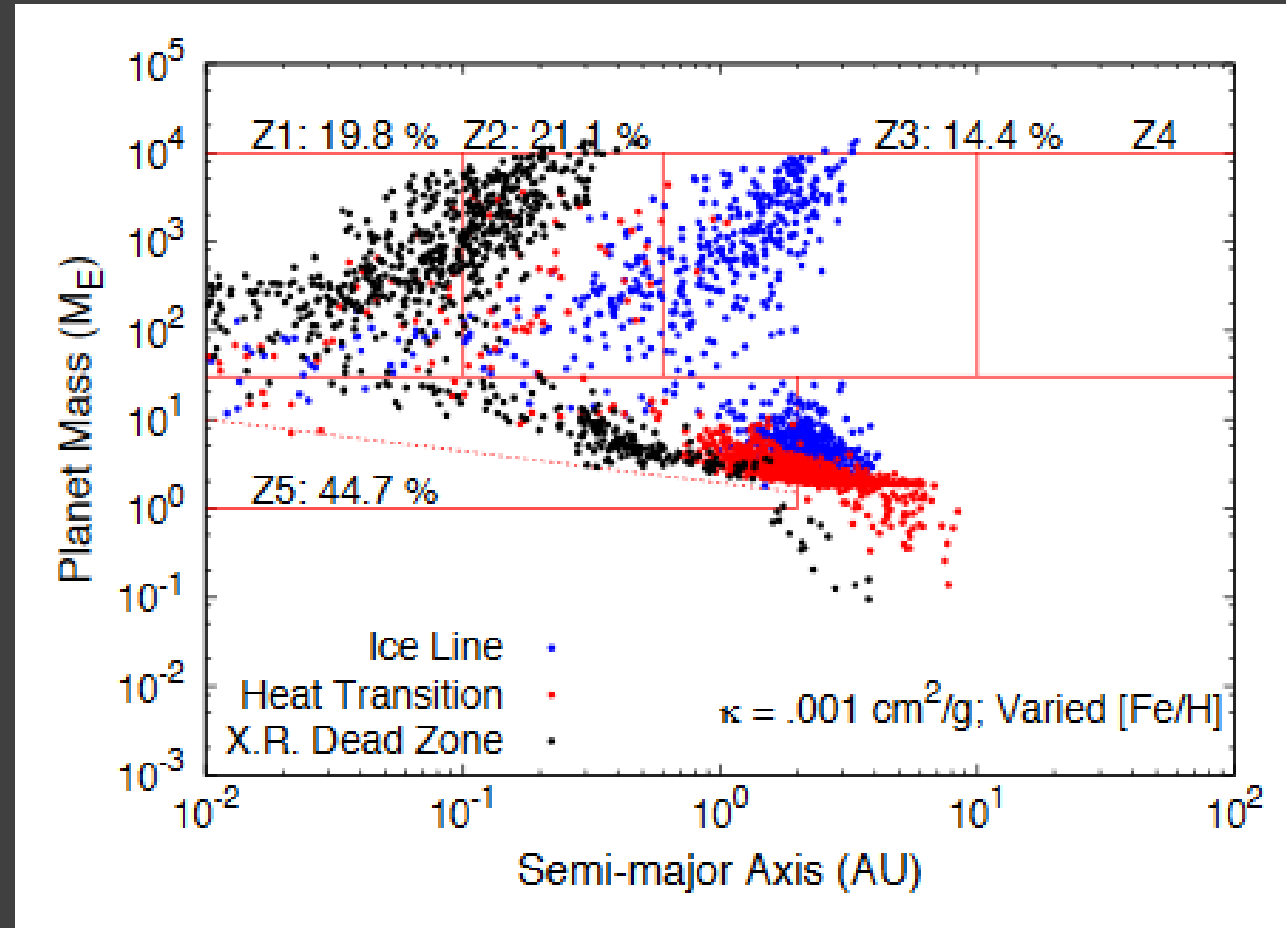
# Role of more complicated migration models

Traps (regions of zero net torque) can slow planet migration (type I).

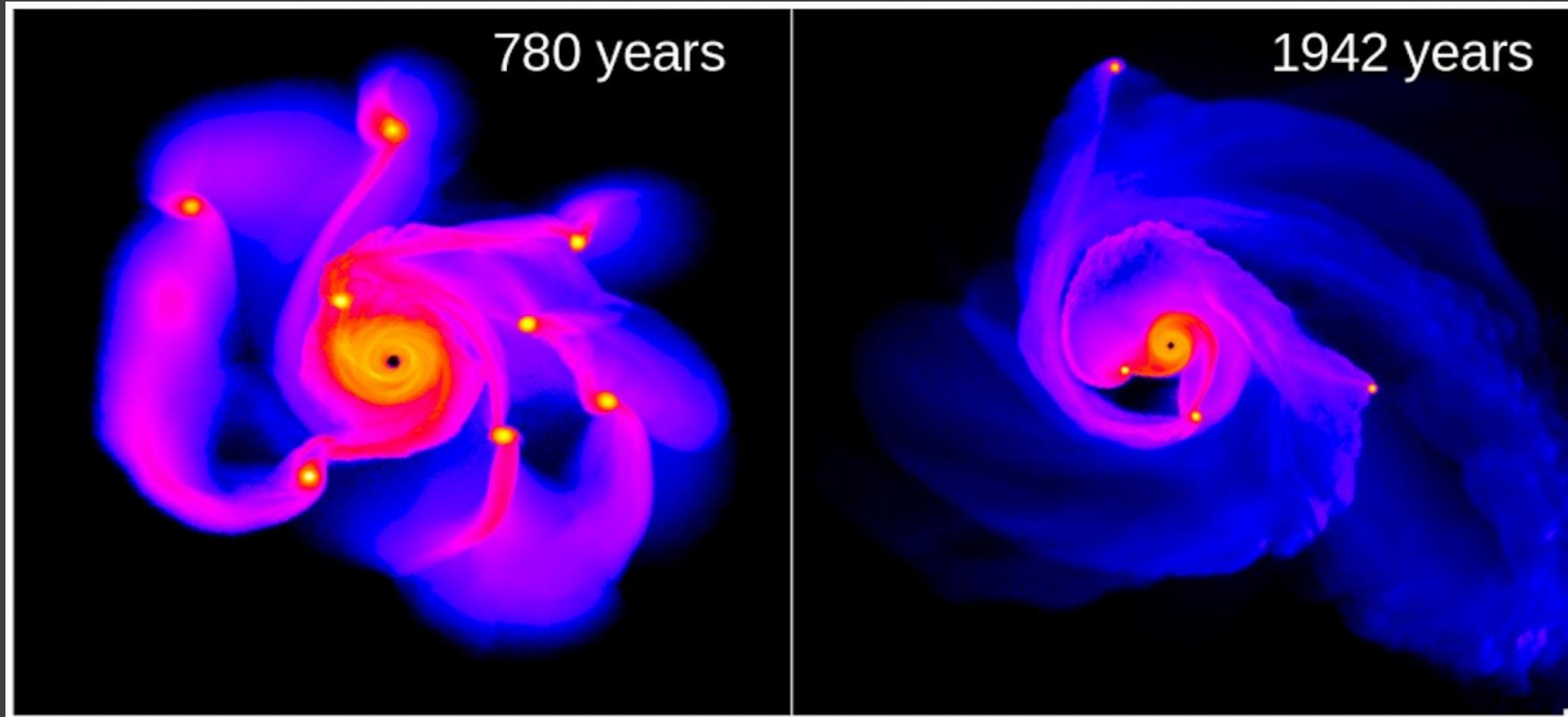
Traps can be related to peculiarities in density or/and temperature profiles. For example, an ice line can be such critical distance, at which planets are trapped.

Heat transition zone – is another trap. There viscous heating (inside) is changed by irradiation by the star (outer zone).

X-rays due to magnetospheric accretion and cosmic rays ionize the disk. Low ionization produces dead zones in the disk.



# Another way to form planets



Gravitational instability in the outer parts of the disc.

Allows to form massive planets out to few tens AU.

Might also work for brown dwarfs and very light stars.

$$Q = \frac{c_s \kappa e}{\pi G \Sigma} < 1.5 - 1.7,$$



# Tidal downsizing

Hypothesis by Nayakshin (2010).

It is possible to make solid planets at low orbits

$$M = M_J = \frac{4\sqrt{2}\pi^3}{3G} \frac{Q^{1/2} c_s^2 H}{\left(1 + \frac{\Delta\Sigma}{\Sigma}\right)} = \frac{4\sqrt{2}\pi^3}{3G} \frac{Q^{1/2} c_s^2 H}{(1 + 4.47\sqrt{\alpha})}$$

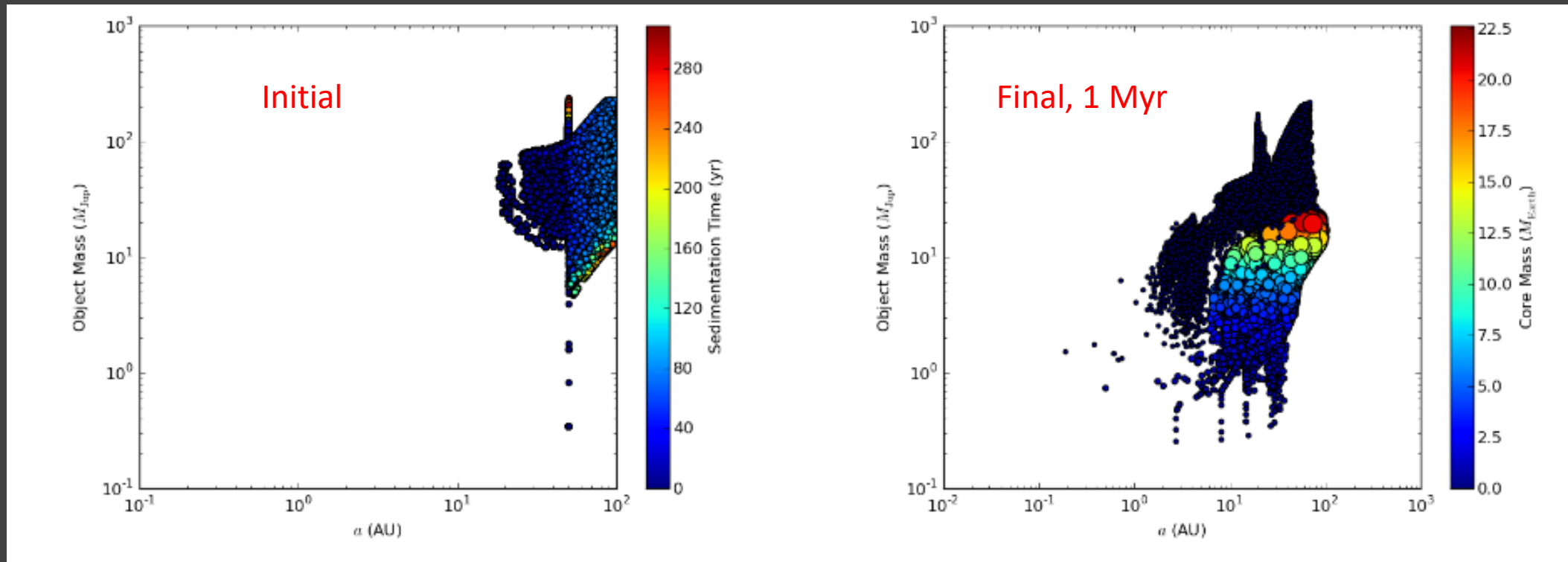
Fragment mass  
just after  
fragmentation

$$R_H = a_p \left(\frac{q}{3}\right)^{1/3}$$

Hill radius becomes smaller  
as a planet migrates towards the star.

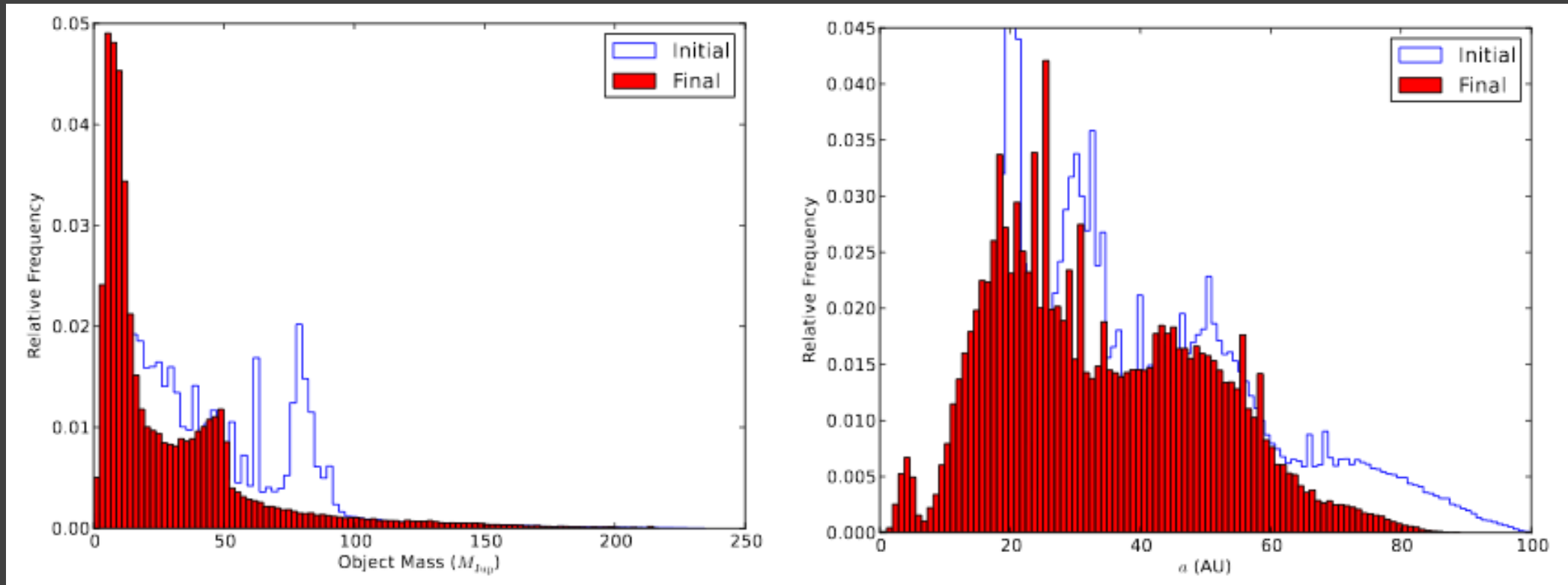
Evolution of a fragment in a disc can result in appearance  
of a low-mass planet closer to the star, or in appearance  
of a belt of particles.

# Initial and final semi-major axis distribution

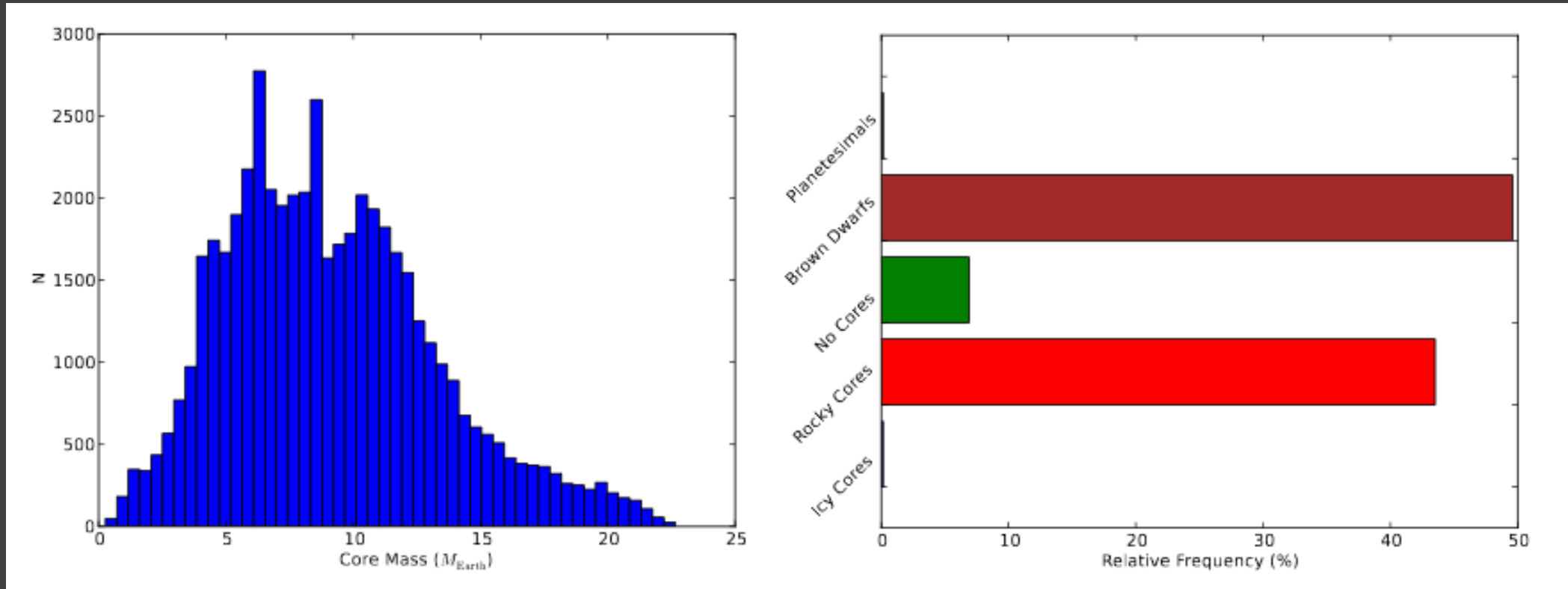


45% survived  
20% formed solid cores

# Mass and semi-major axis distribution

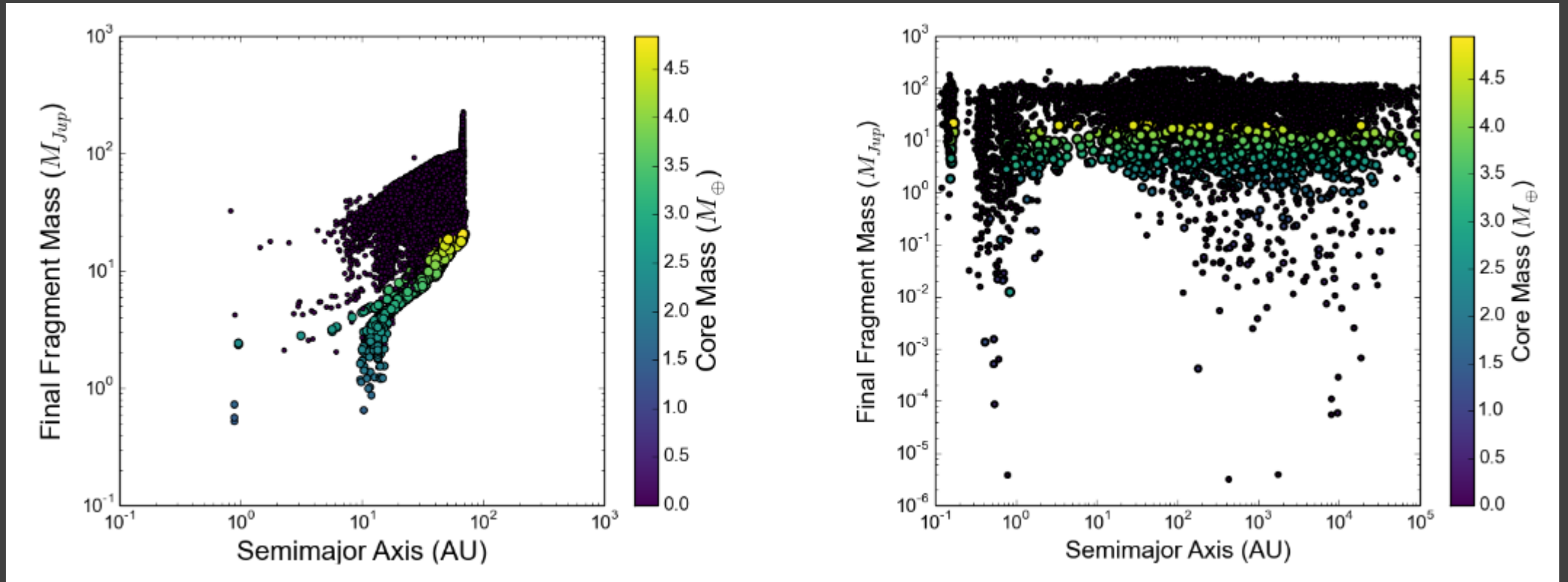


# Mass distribution and planet types



Many brown dwarfs (and even low-mass stars for some parameters) can be produced via this channel.

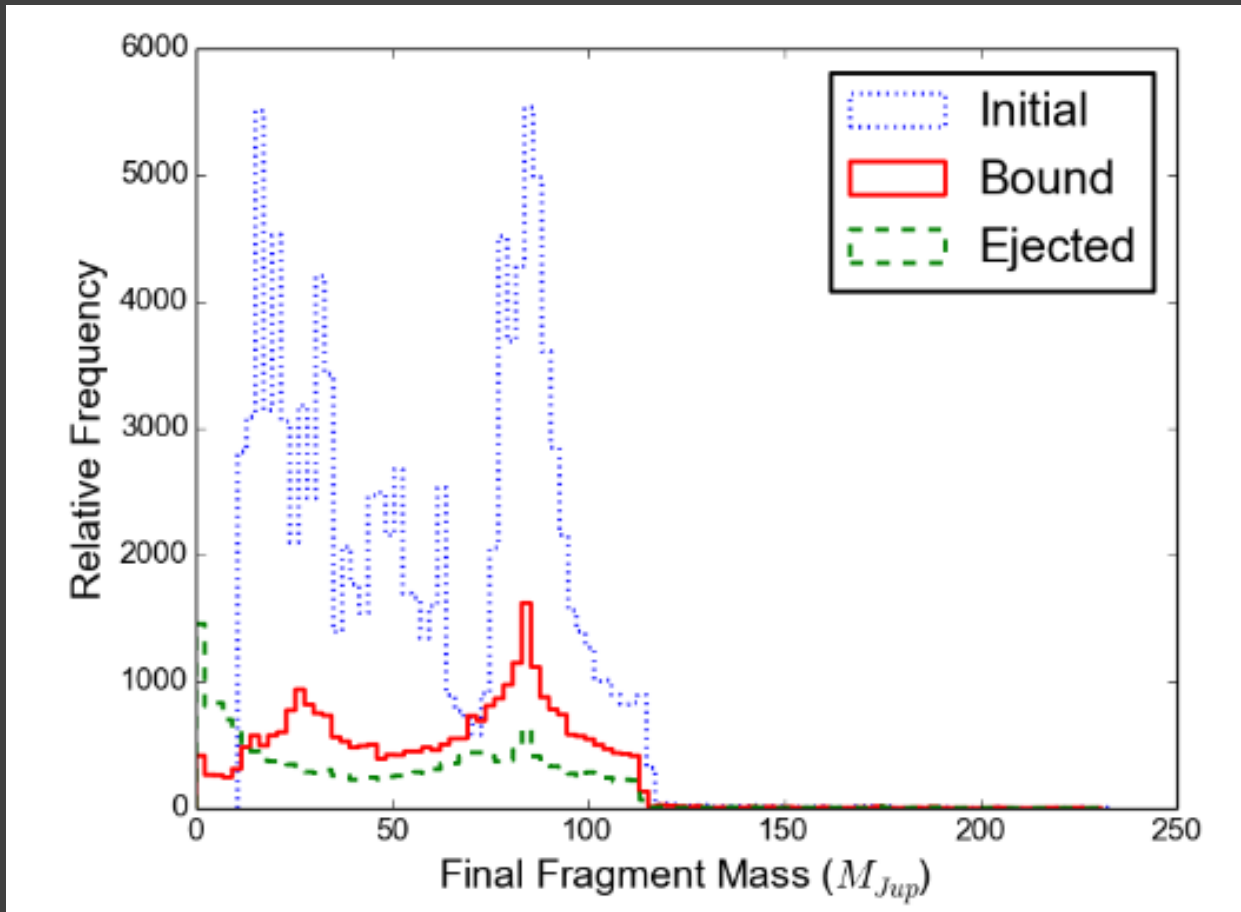
# Role of fragment-fragment interaction



Interaction off

Interaction on

# Ejection

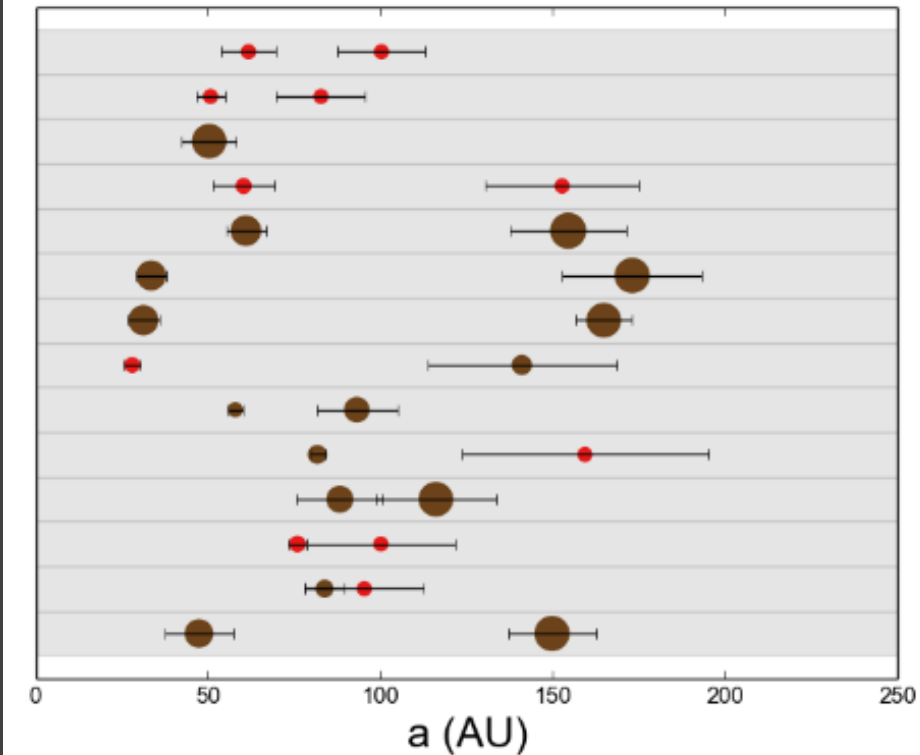


Many fragments are ejected.  
So, this mechanism of planet formation  
can be an important contributor  
to the population of free-floating  
planets and brown dwarfs.

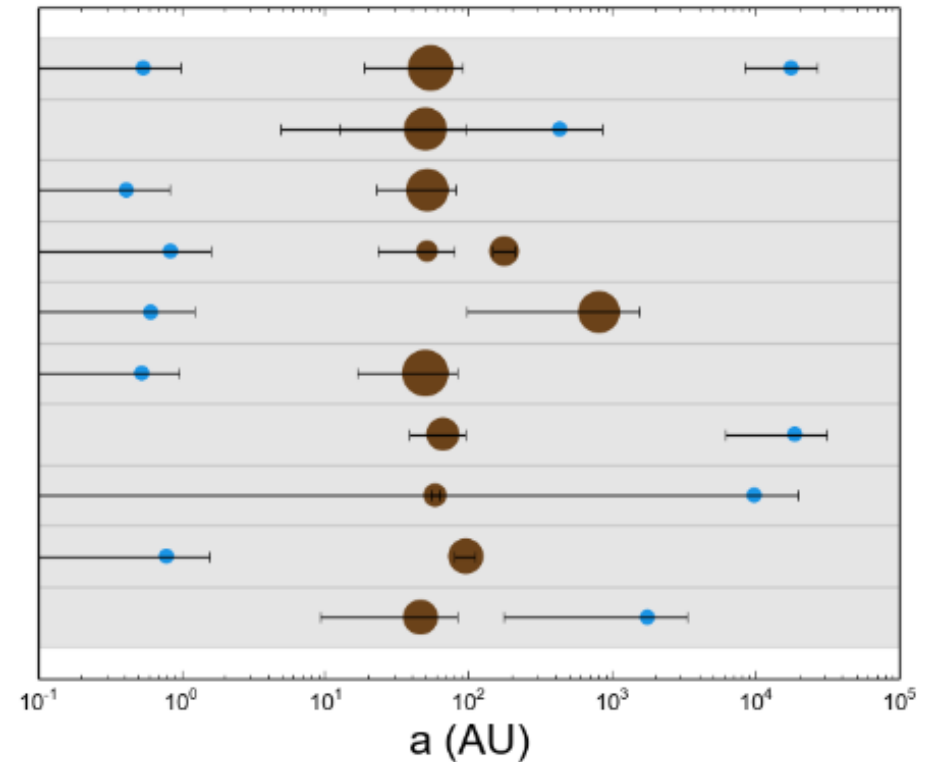


# System architecture

Brown – brown dwarfs;  
Red – gas giants;  
Blue – rocky (>50%).

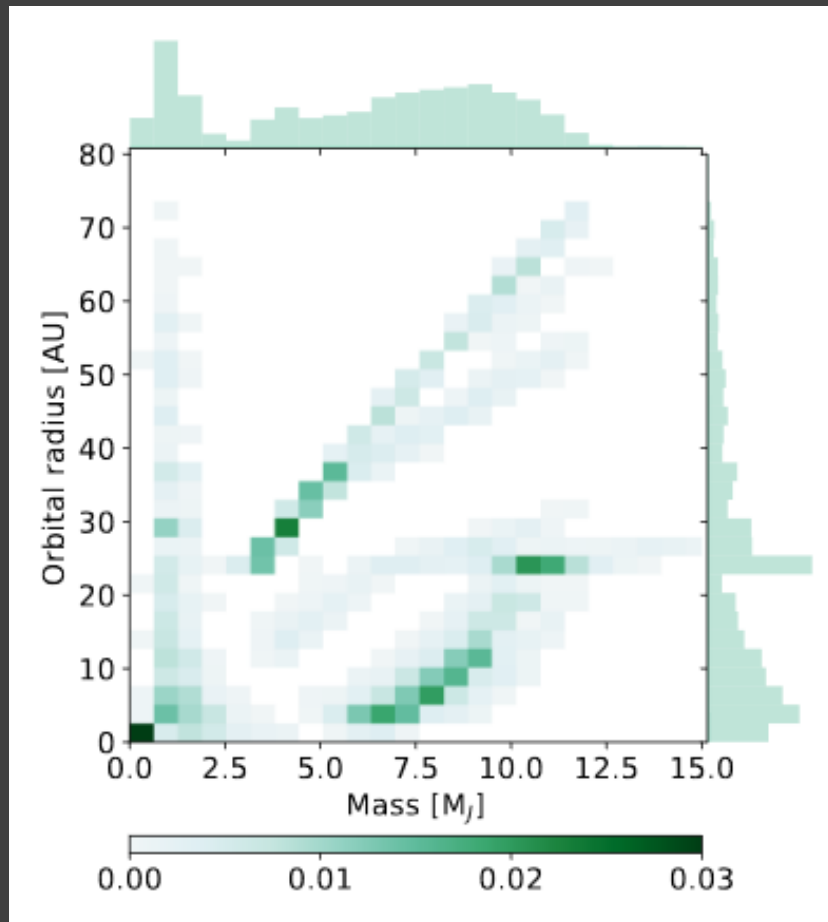


Typical systems



Non-typical systems

# Another example of population synthesis of planets formed by instability

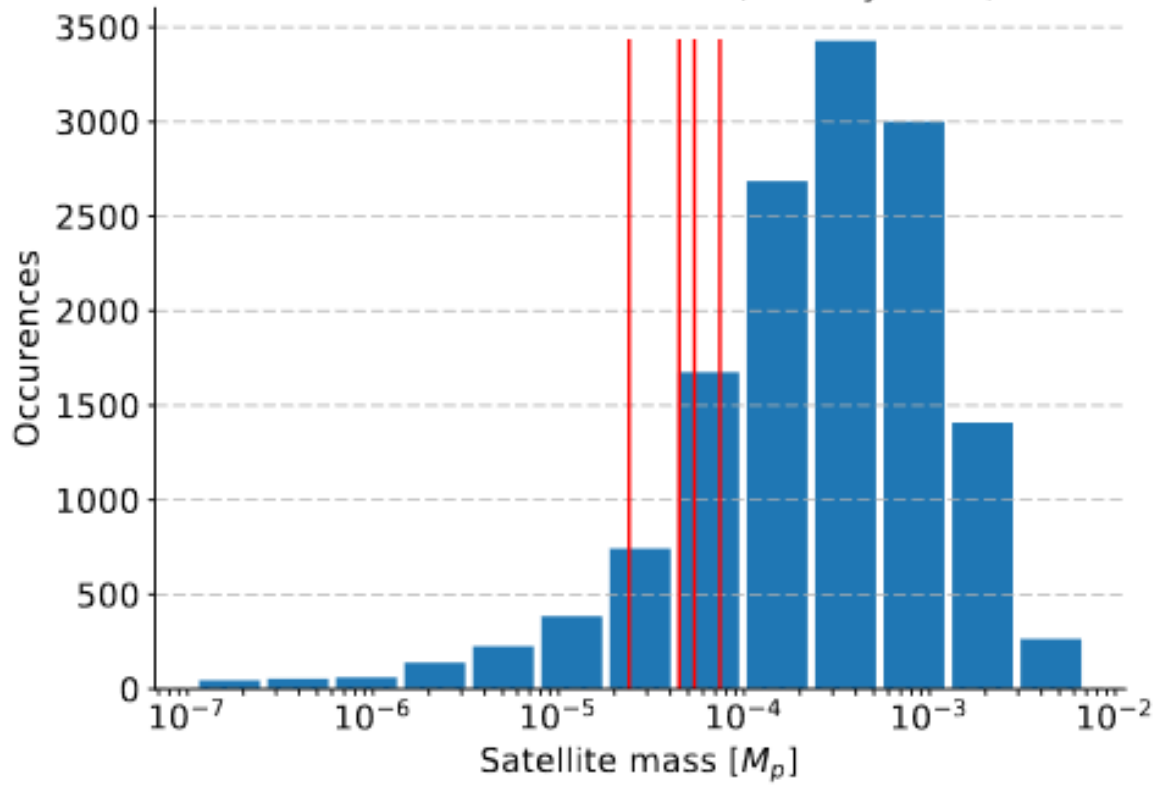


Many uncertainties.

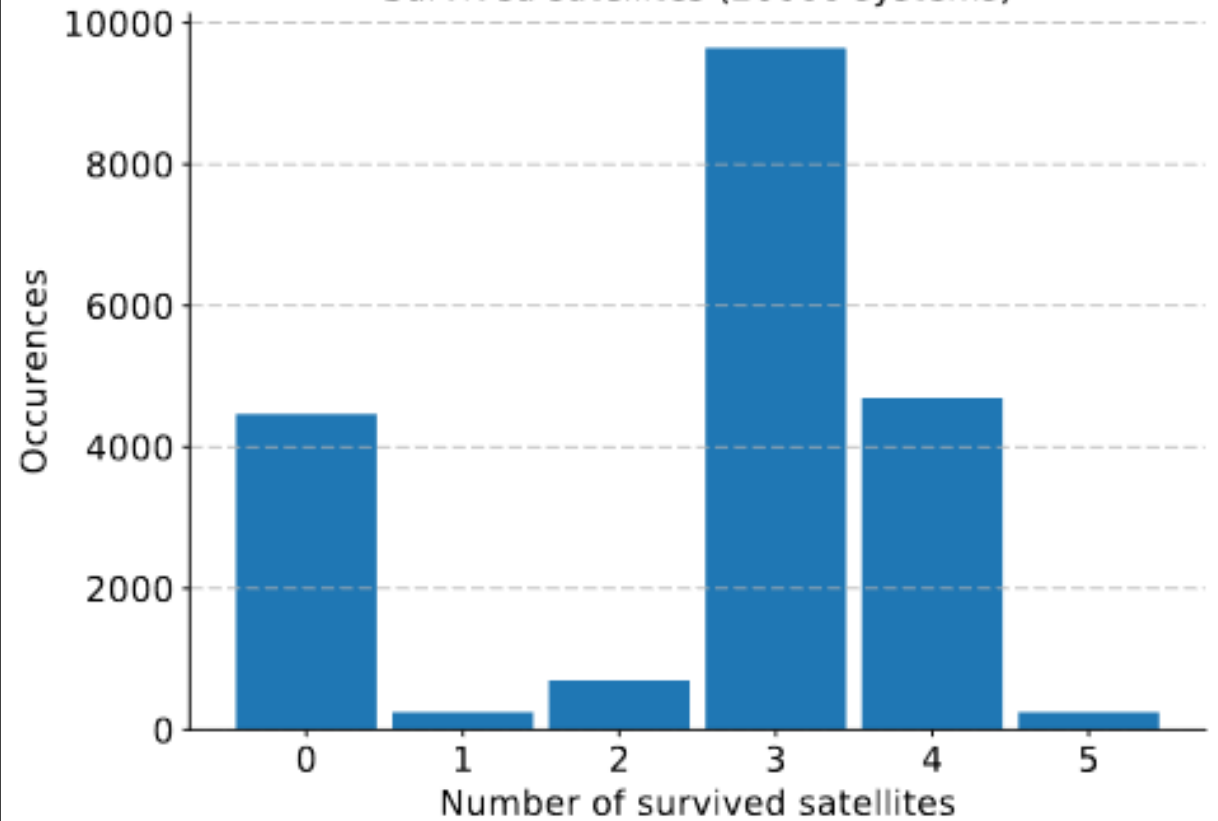
This picture summarizes all the models, calculated for different assumptions and parameters.

# Population synthesis of Galilean satellites

Satellite mass distribution (7500 systems)



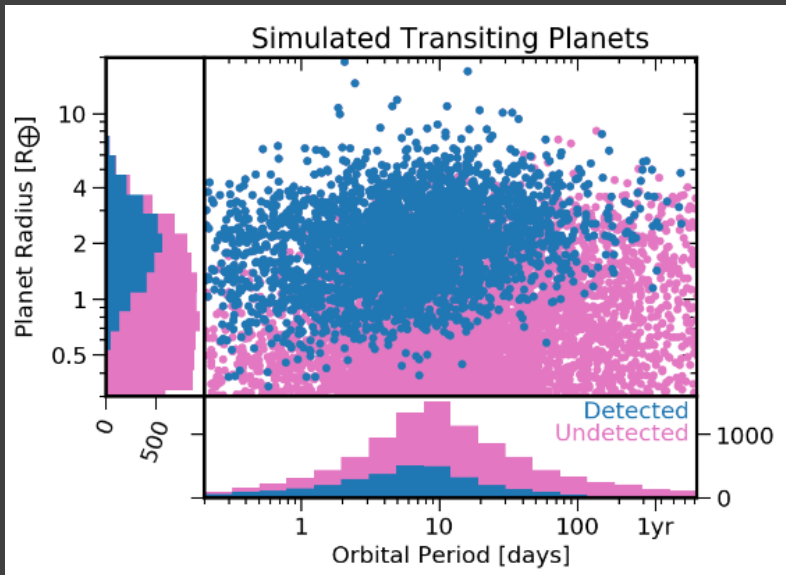
Survived satellites (20000 systems)



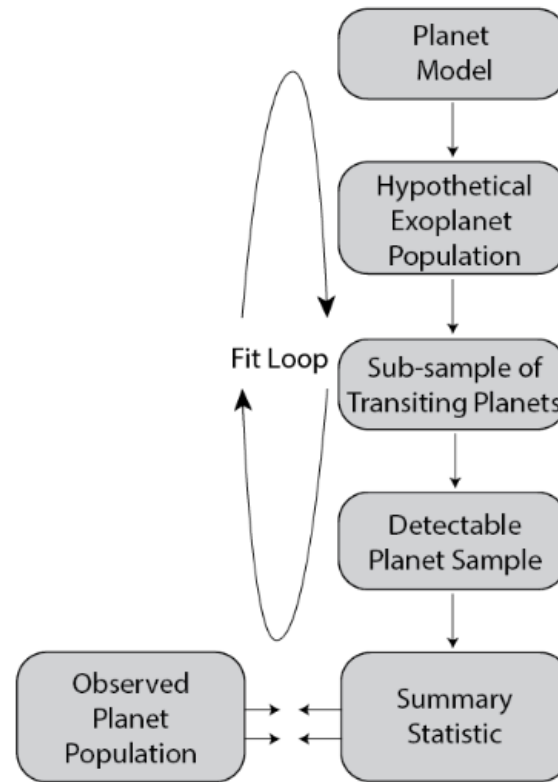
# How to compare calculations with data

After calculations are made it is necessary to compare it with observational data.

For the case of transiting planets a special script was written.



## Logical Steps



Figs. 9, 16

## Quantitative Implementation

### Occurrence Rate Mode

$$f_{pl}(R,P)$$

Eq. (1) Fig. 2

$$\{R, P\}_p$$

Eq. (13)

$$\{R, P\}_t$$

Fig. 4

$$\{R, P\}_d$$

Fig. 4

$$N, \{R\}, \{P\}$$

Eq. (18) Figs. 10, 14

assign planets to stars

viewing geometry

detection efficiency  
Eq. (16) Fig. 5

### Multi-planet Mode

$$f_k(R,P,i)$$

Eq. (5) Fig. 3

$$\{R, P, i, ID\}_s$$

Eq. (15)

$$\{R, P, ID\}_t$$

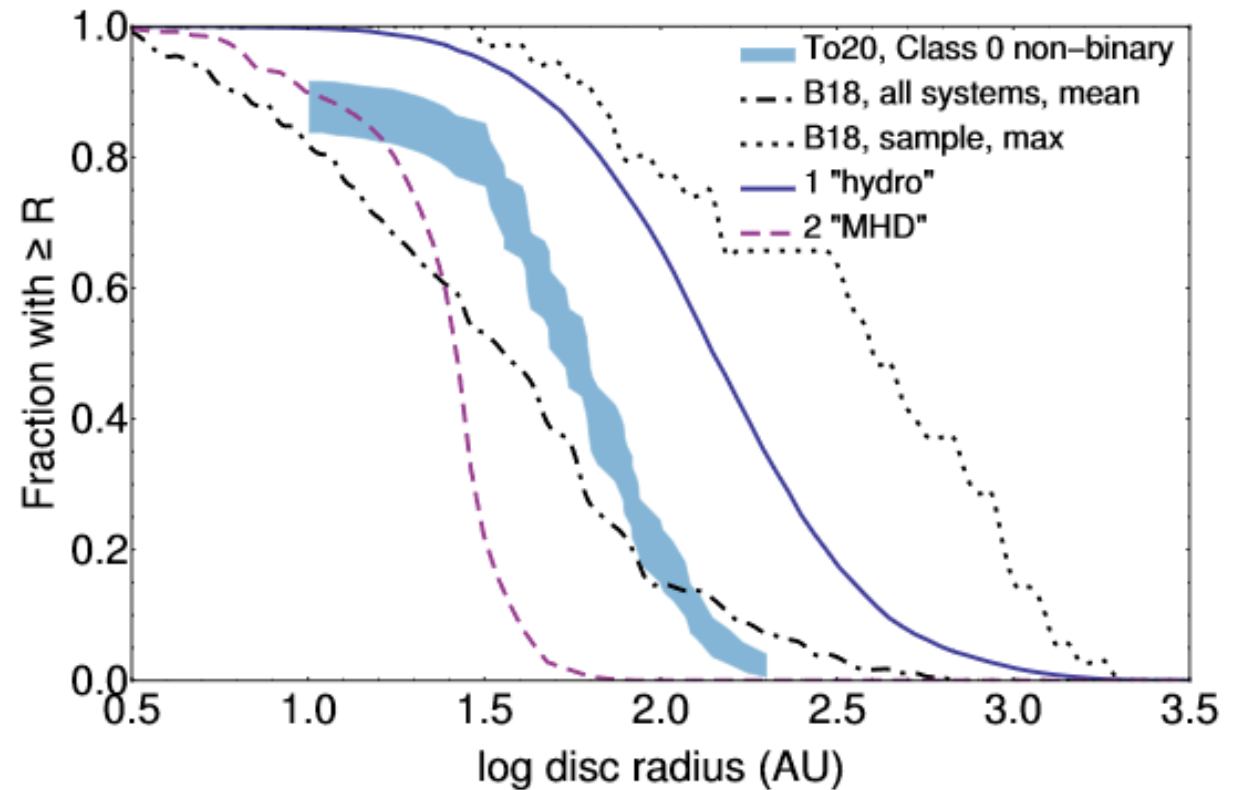
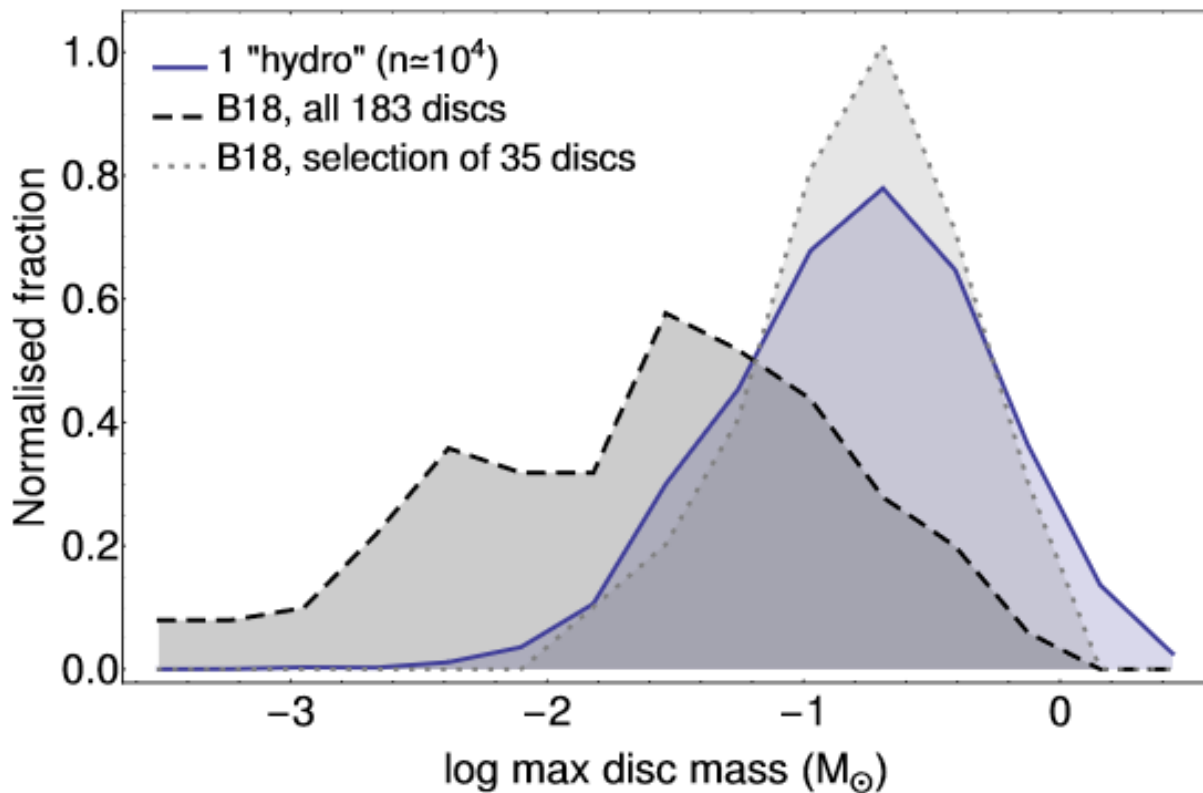
$$\{R, P, ID\}_d$$

Figs. 6,7,8

$$N, \{R\}, \{P\}, N_k, \{P_{in}\}, \{P\}$$

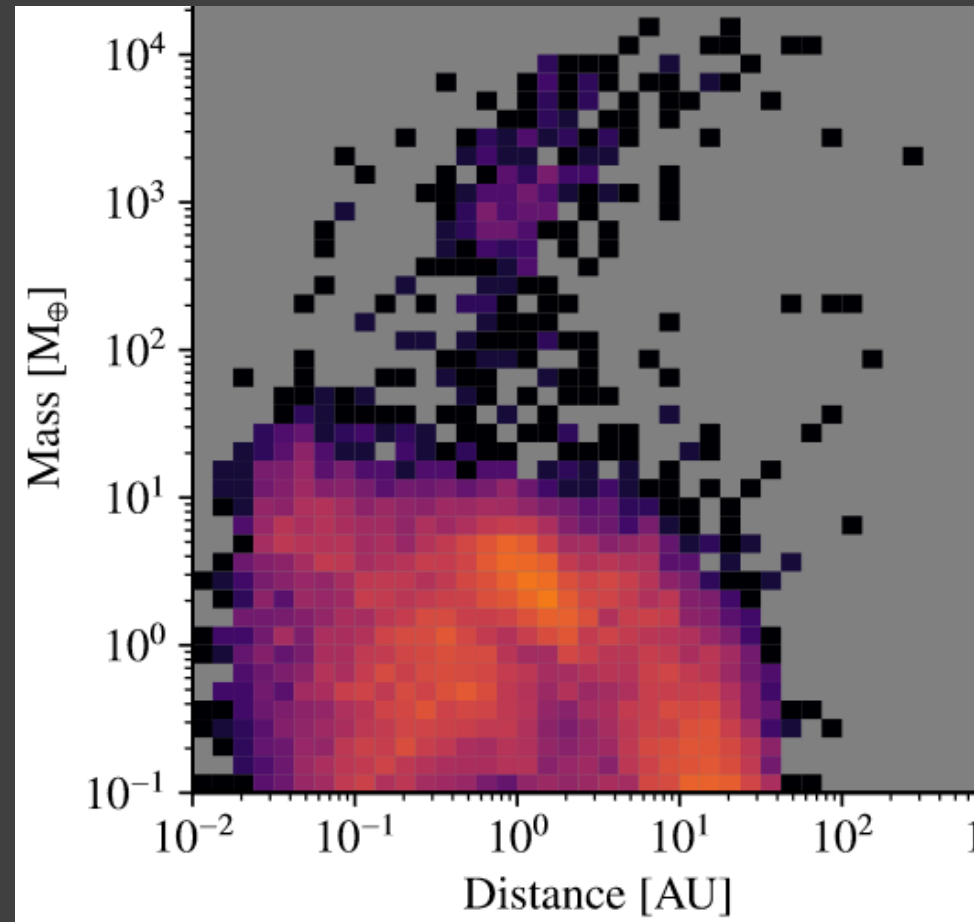
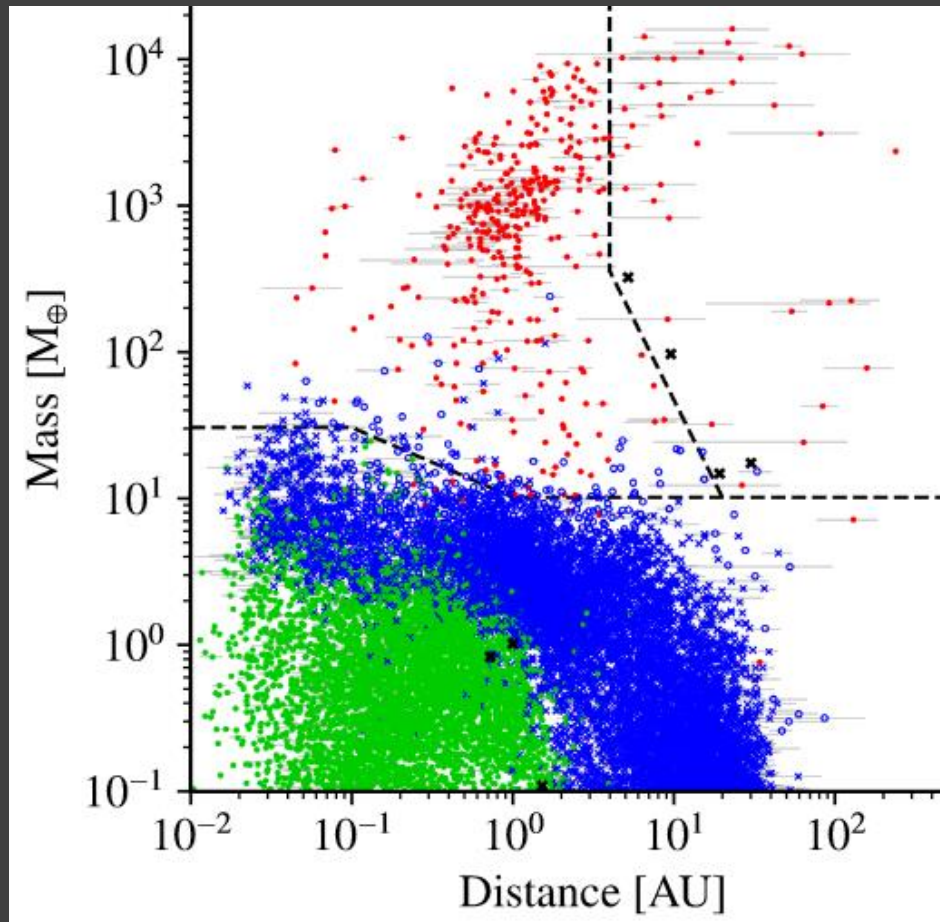
Eq. (1) Figs. 11,12, 13,17

# Population synthesis of protoplanetary discs



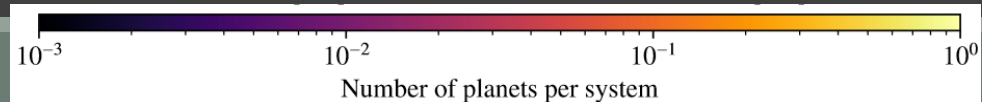
Disc formation is modeled. The authors obtain that that protoplanetary discs are very massive early in their lives.

# New population synthesis models

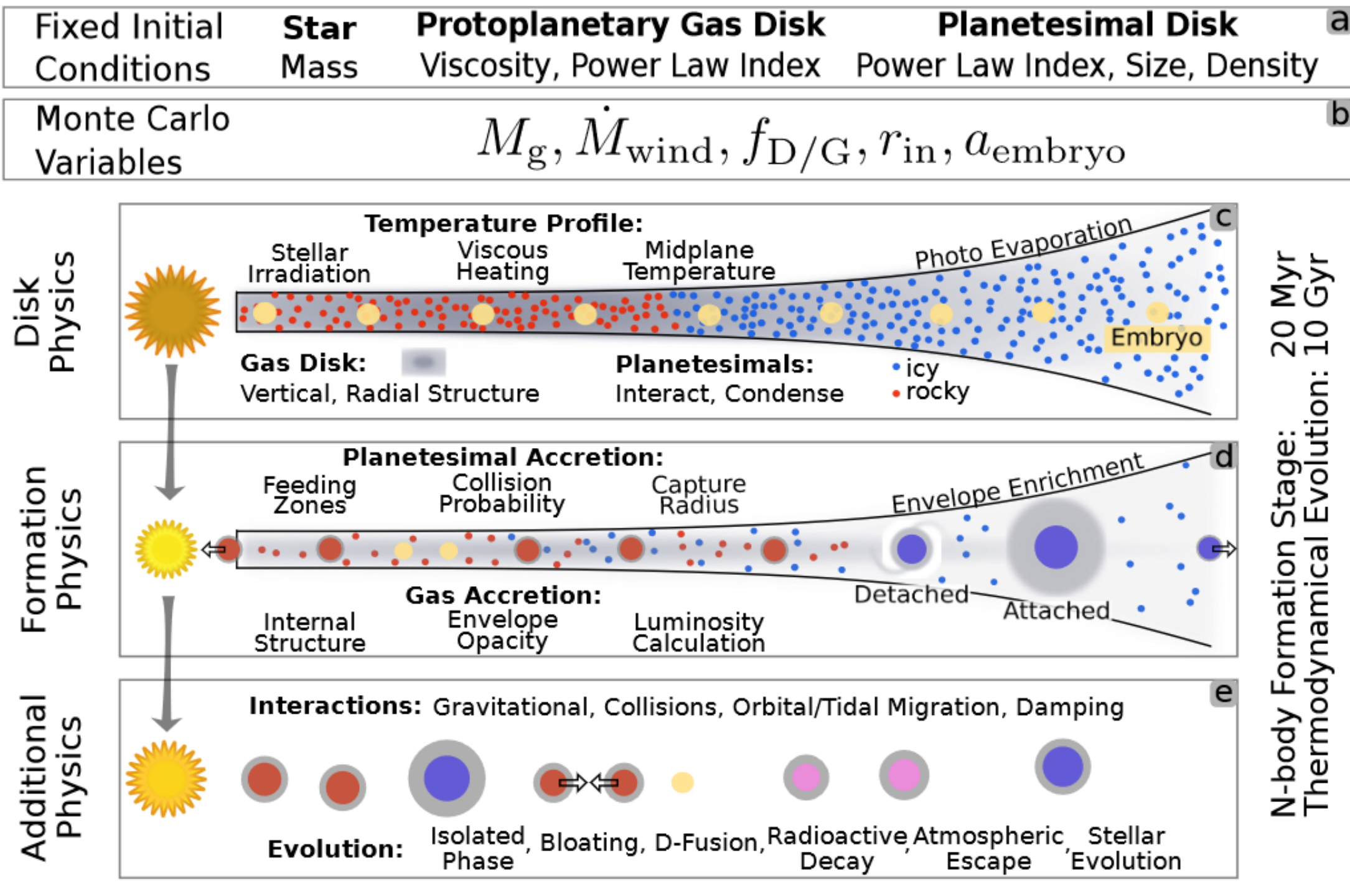


All at 5 Gyrs.

50 embryos

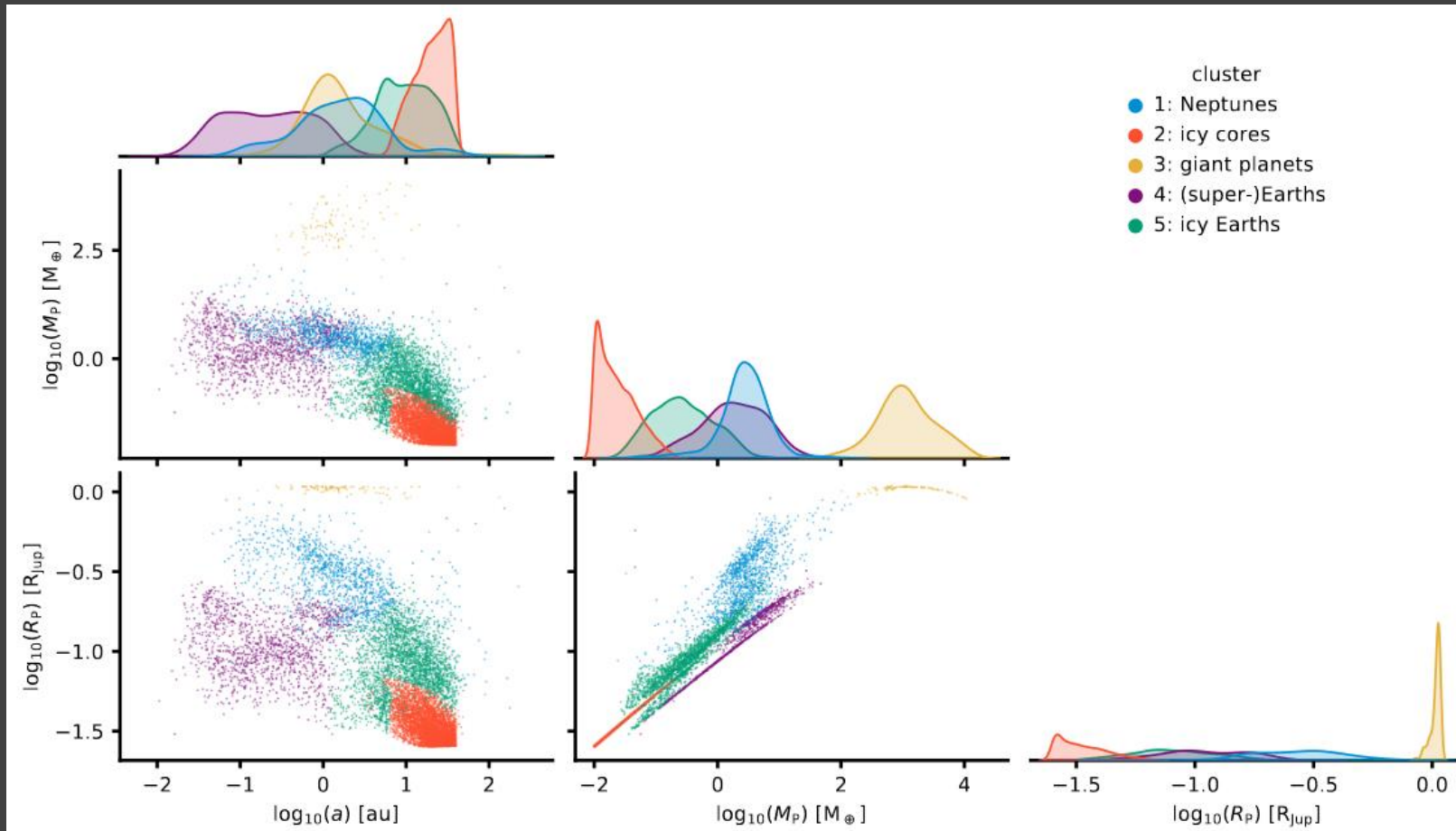






N-body Formation Stage: 20 Myr  
 Thermodynamical Evolution: 10 Gyr

# Types of planets in the model



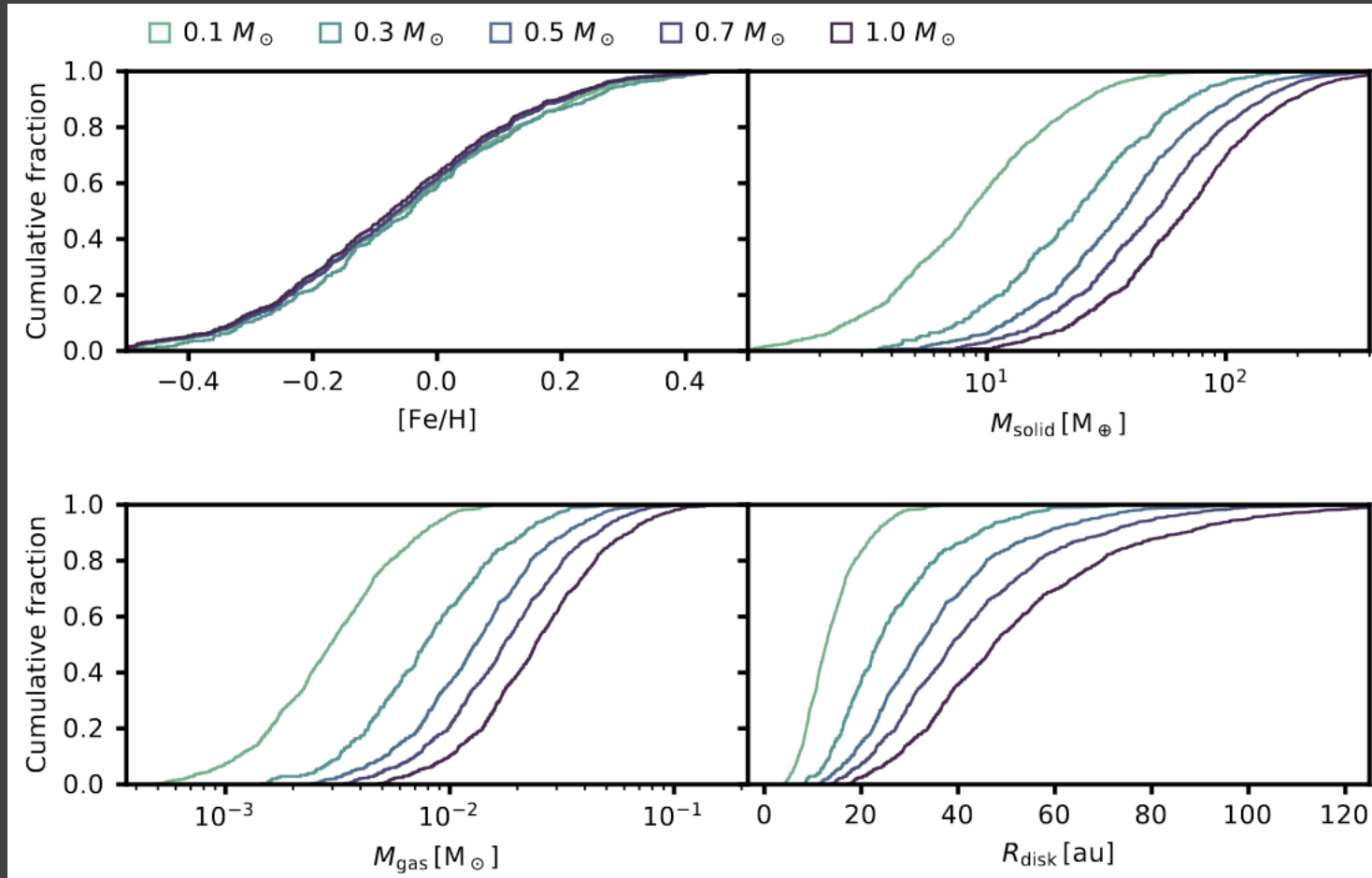
Solar type stars.

Interaction between planets is very important for calculations of the main parameters.

Ice earths are not observed, yet.

# Modeling planets around low-mass stars

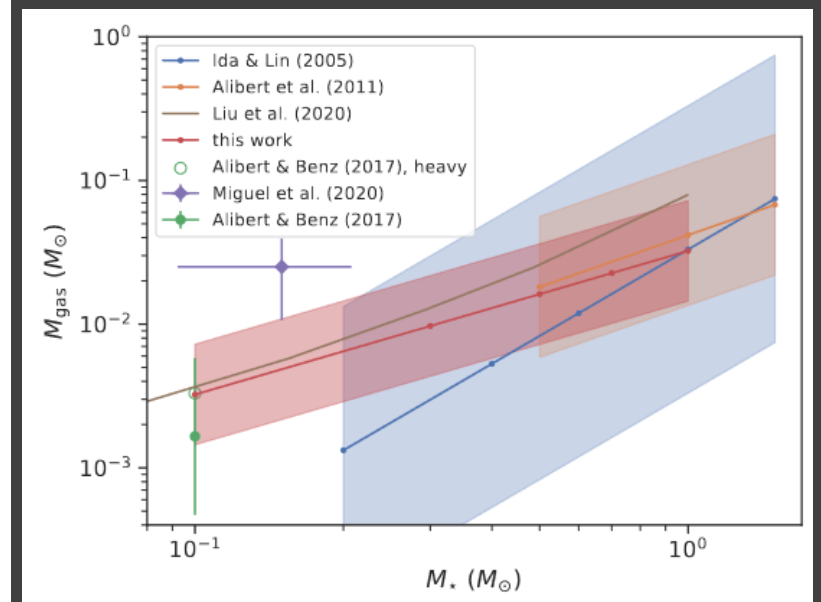
## What is different about discs?



$$R_{\text{disk}} \propto M_{\text{gas}}^{-1.6}$$

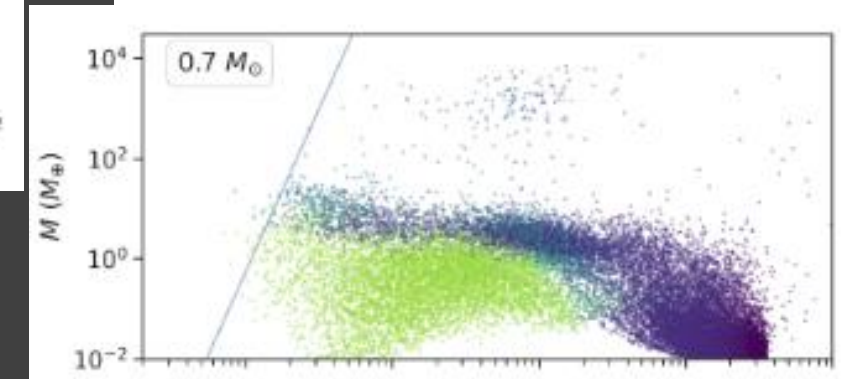
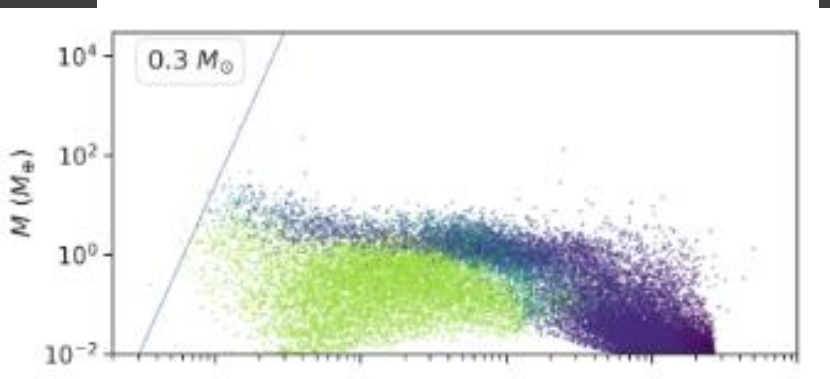
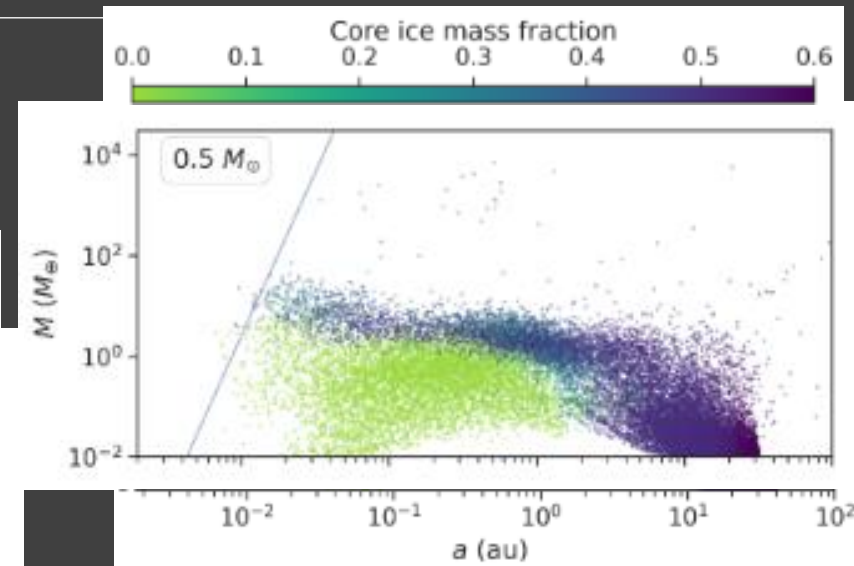
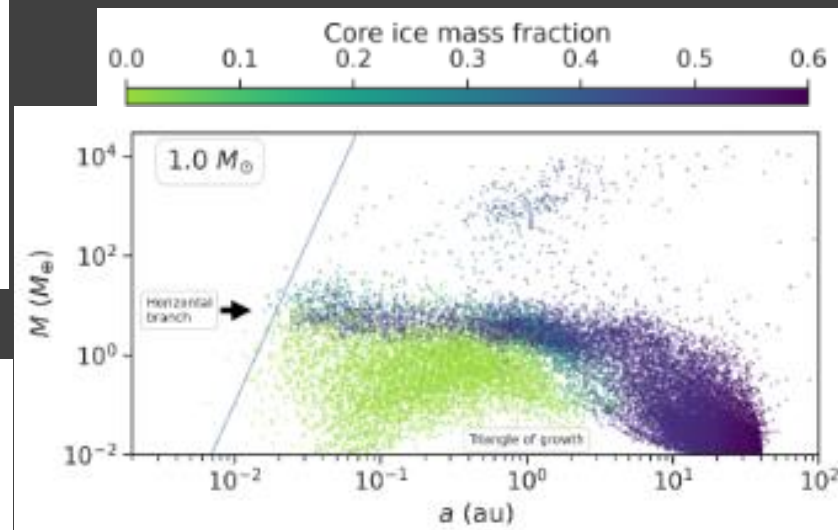
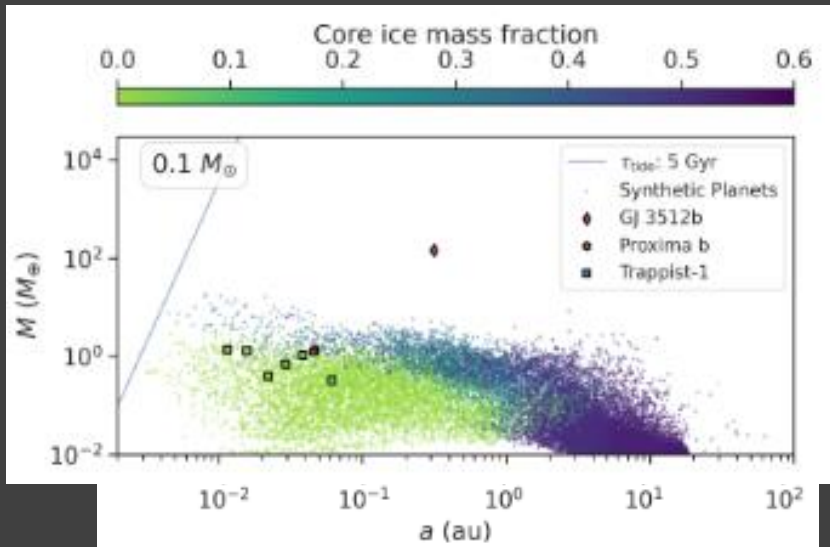
$$M_{\text{solid}} = 10^{[\text{Fe}/\text{H}]} f_{\text{dg},\odot} M_{\text{gas}}$$

$$f_{\text{dg},\odot} = 0.0149$$



# Modeling planets around low-mass stars

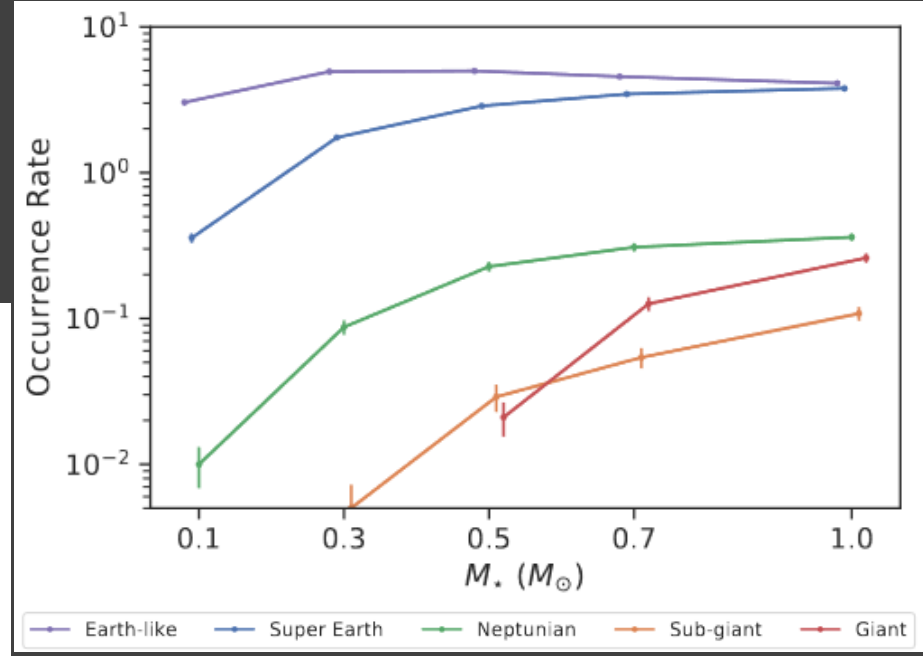
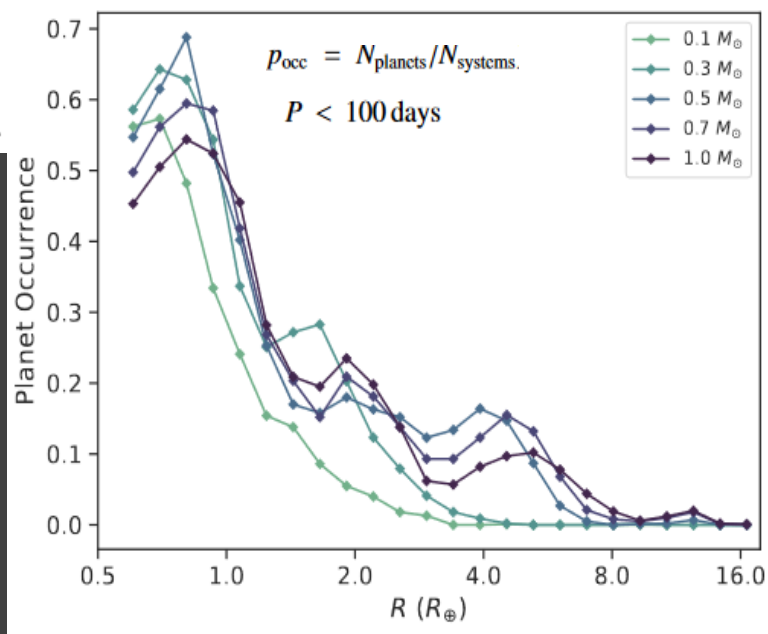
## Mass – large semiaxis diagrams.



# Modeling planets around low-mass stars

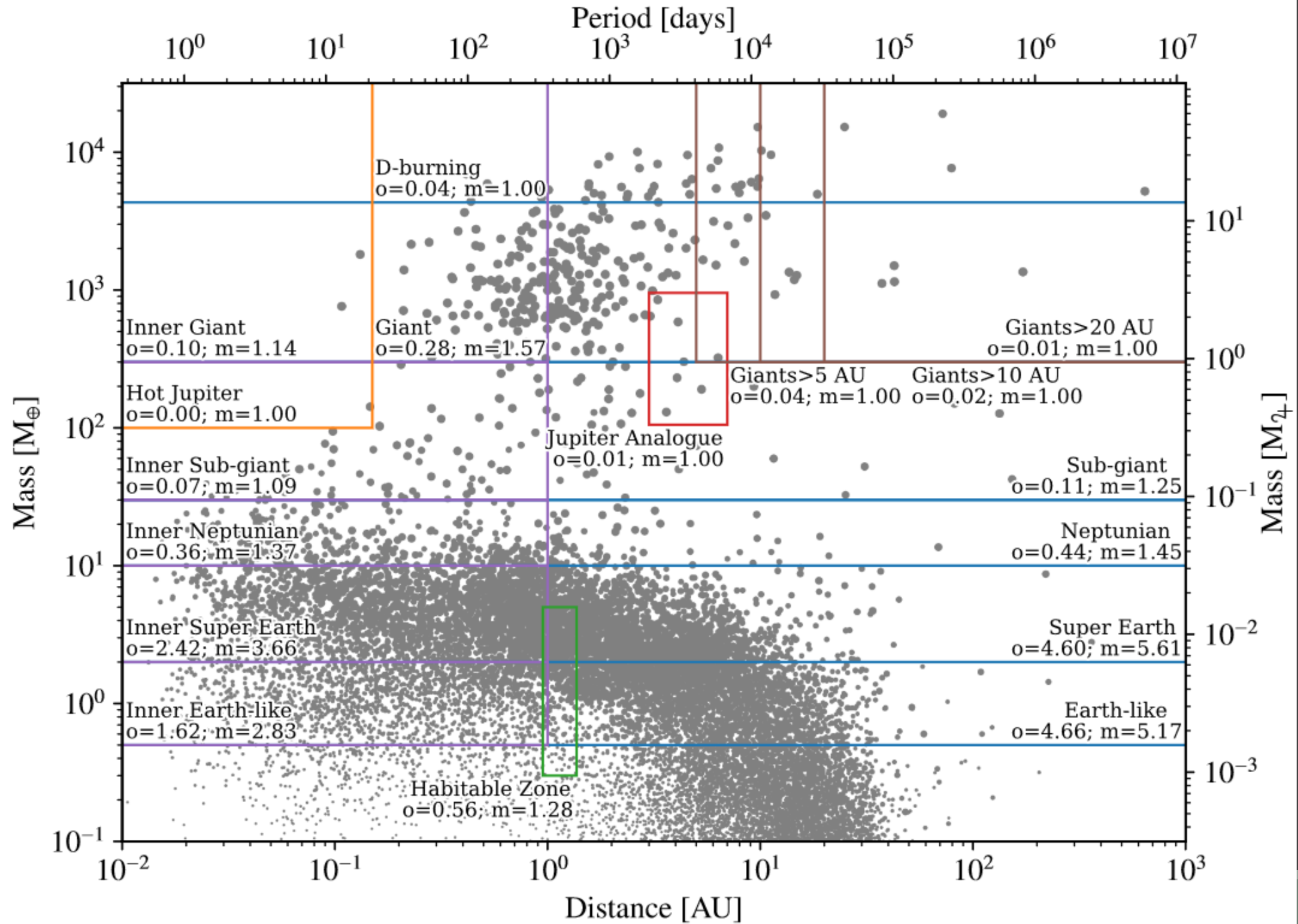
## Planet statistics for different masses.

Type	Stellar mass ( $M_{\odot}$ )				
	0.1	0.3	0.5	0.7	1.0
$M > 1 M_{\oplus}$	0.44	0.77	0.88	0.91	0.95
Earth-like	0.70	0.88	0.89	0.89	0.84
Super Earth	0.19	0.54	0.71	0.78	0.79
Neptunian	0.01	0.08	0.17	0.22	0.27
Sub-giant	0.00	0.00	0.02	0.03	0.05
Giant	0.00	0.00	0.02	0.09	0.19
Temperate zone	0.35	0.66	0.70	0.66	0.57





100 embryos





# Literature

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- 1402.7086 Planet Population Synthesis. W. Benz et al.
- 1804.01532 Planetary population synthesis. C. Mordasini
- 2007.05561-3 New Mordasini et al. synthesis  
2104.11750, 2105.04596, 2105.12745