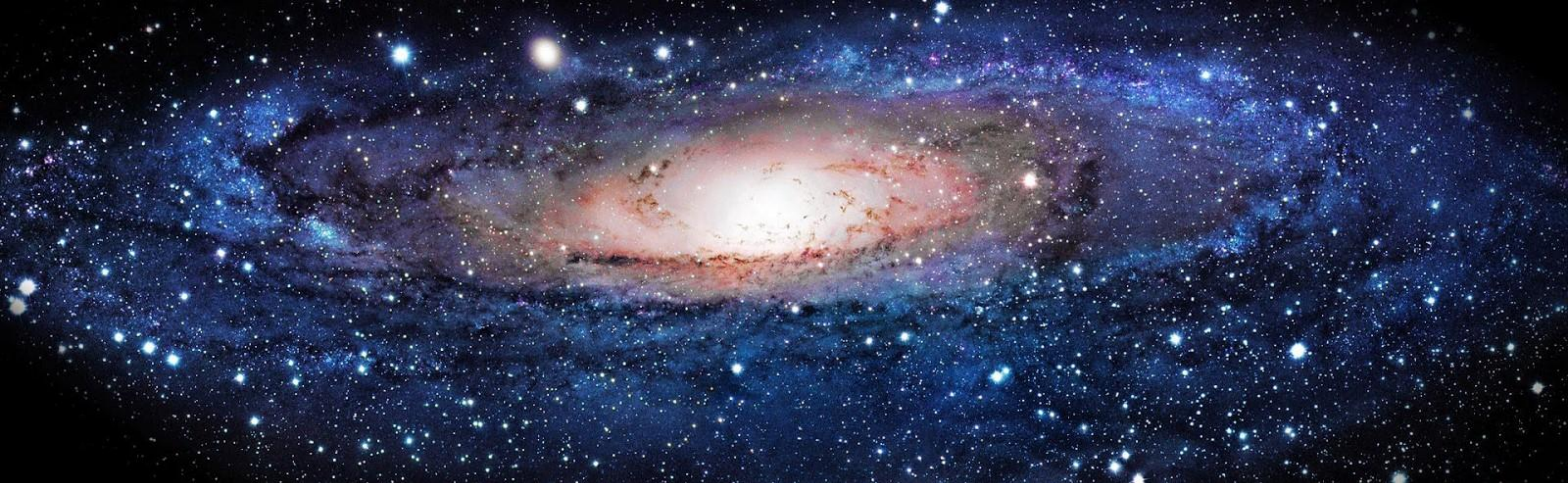


Модуль. Астрофизика.

Семинар 4. (20.11)

(1) <u>Поток ν</u>

p-p cycle

$$\underline{4\,{}_1H} \to \underline{{}_2^4He} + 2e^+ + \underline{2\nu_e}$$

$m_p = 1,00728$ аем   аем $= \frac{1}{12} m_{C12} = 1,66\cdot 10^{-24}$ г

$m_\alpha = 4,001506$ аем

$$\varepsilon = \Delta m c^2 \qquad \eta = \frac{\varepsilon}{mc^2} = \frac{\overbrace{4m_p - m_\alpha}^{\Delta m}}{4m_p} \approx 0,007 \quad 0,7\%$$

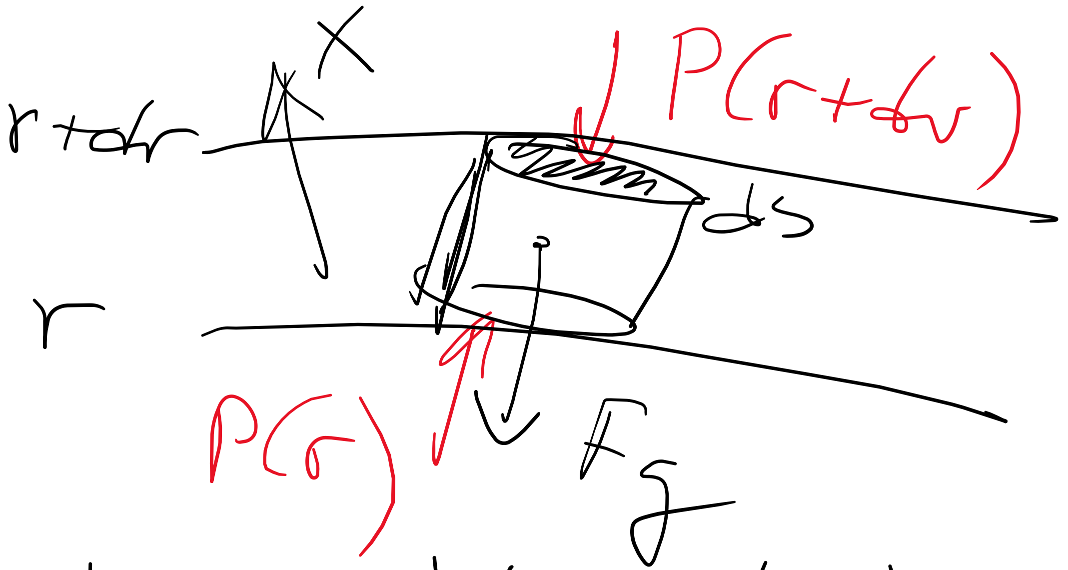
$$\varepsilon = (4m_p - m_\alpha)c^2 \approx 25,7 \text{ МЭВ}$$

$$f_\nu = \frac{\dot N_\nu}{4\pi d^2} \qquad \dot N_\nu = 2\cdot \dot N_{реак} \qquad \dot N_{реакц} = \frac{L_\odot}{\varepsilon}$$

$$\dot N_\nu = \cancel{\frac{2}{2,57\cdot 10^{7}}} \frac{\cancel{2}\; 10^{33}}{\cancel{1},6\cdot 10^{-12}} \sim 10^{38} \qquad f_\nu = \frac{\dot N_\nu}{4\pi d^2} =$$

$$= \frac{10^{38}}{4\pi(1,5\cdot 10^{13})^2} = \frac{10^{38}}{4\pi\cdot 2,25\cdot 10^{26}} = \frac{100}{4\pi\cdot 2\cdot 2,5}\, 10^{10} \sim 3,5\cdot 10^{10} \frac{шт}{см^2\cdot с}$$

2) Гидростат. равнов. ☉ и звезд.



$$F_g = -G\frac{M(r)\cdot dm}{r^2}$$

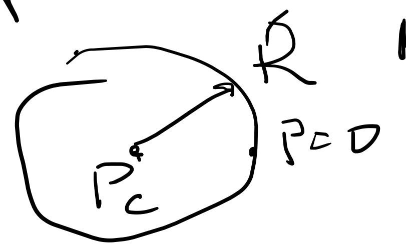
$$F_p = P(r)dS - P(r+dr)dS = -\frac{dP}{dr}\underbrace{dr\,dS}_{dV}$$

$$dm = \rho\, dV = \rho\, dr\, dS$$

$$\vec{F}_g + \vec{F}_p = 0$$

$$\frac{G M(r)\rho\,\cancel{dr\,dS}}{r^2} = -\frac{dP}{dr}\cancel{dr\,dS}$$

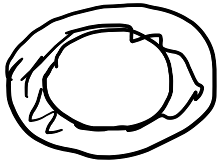
$$\boxed{-\frac{1}{\rho}\frac{dP}{dr} = \frac{G M(r)}{r^2}}$$



$M(R) = M$ $\qquad \rho \sim M/R^3$

$$P_c \sim \frac{GM^2}{R^4}$$

$$E_T = E_K + \underset{<0}{E_p} \qquad E_p = -2E_K \qquad E_T = -E_K$$



$$dm = 4\pi r^2 \rho\, dr \qquad 4\pi r^2\, dP = -\frac{G M(r)}{r^2}\, dm$$

$$U \equiv -\int_0^M \frac{G M(r)}{r}\, dm = \int_0^M \underbrace{4\pi r^3}_{3V} dP = 3\int_0^M V\, dP =$$

= 0 на повърхн.

$$= 3VP\Big|_0^M - 3\int_0^M P\, dV = -3\int P\, dV$$

0 нс нимн. нр.

$$\boxed{U = -3\int P\, dV}$$

$$U = -3\int P\,dV$$

$$P = K\rho^{\gamma} \qquad \varepsilon = \frac{P}{(\gamma-1)\rho}$$

$$Q = \int \varepsilon \rho\, dV$$

$$\boxed{\begin{matrix}\gamma = 5/3 \\ 2Q = -U\end{matrix}}$$

$$-3\int P\,dV = -3(\gamma-1)\int \frac{P}{\gamma-1}\,dV =$$

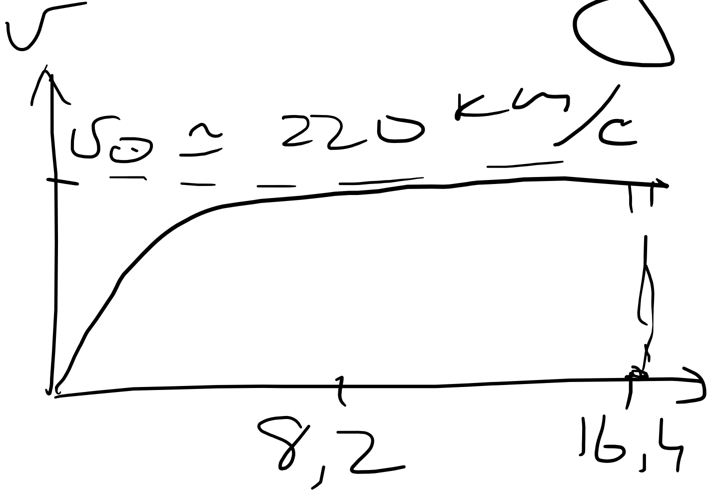
$$= -3(\gamma-1)\int \frac{P}{(\gamma-1)\rho}\,\rho\,dV = -3(\gamma-1)\int \varepsilon\rho\,dV = -3(\gamma-1)Q$$

$$\gamma = 5/3 \qquad Q = \frac{3}{2}NkT = \frac{3}{2}\frac{M}{\mu}RT$$

$$U = -\frac{3}{5}\frac{GM^2}{R} \qquad \mu = 0.6$$

$$\langle T \rangle = \mu\,\frac{GM}{5RR} \sim 10^7\,K$$

③ Число звёзд в Г.



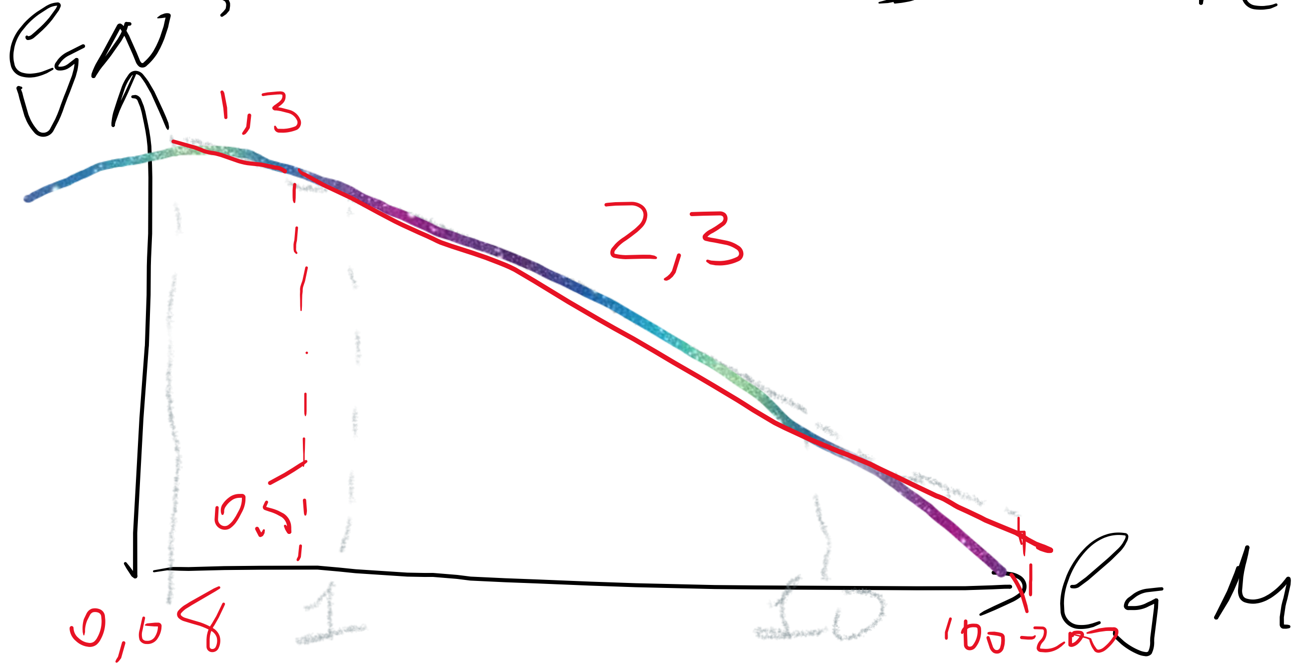
$$v = \sqrt{\frac{GM(r)}{r}}$$

$d = 8{,}2$ кпк

$$M_T = M_* + M_{ISM} + M_{DM}$$

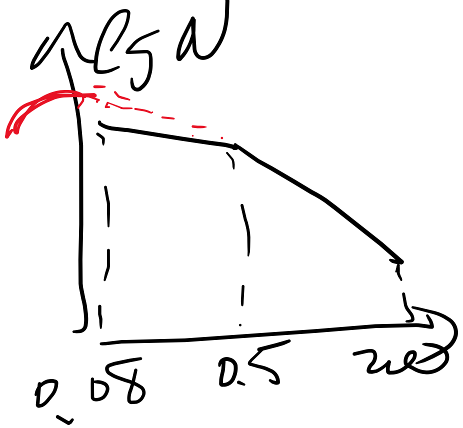
$$M_* \simeq \frac{2 v_\odot^2 \cdot d}{G} = 3{,}7\cdot 10^{44}\ \text{г} = 1{,}85\cdot 10^{11} M_\odot$$

$$N_* = \frac{M_*}{\langle m \rangle}$$



IMF Salpeter (1955)

$$\frac{dN}{dm} \sim M^{-\alpha}, \quad \alpha = 2{,}35$$



$$\langle M \rangle = \frac{M_*}{N_*} \qquad \frac{dN}{dM} \sim M^{-\alpha}$$

$$dN = A M^{-\alpha} dM$$

$$N_* = A \int_{0.08}^{0.5} M^{-1.3} dM + A \int_{0.5}^{200} M^{-2,3} dM$$

$$M_* = A \int_{0.08}^{0.5} M \cdot M^{-1,3} dM + A \int_{0.5}^{200} M \cdot M^{-2,3} dM$$

$$\langle M \rangle = \frac{\int_{0.08}^{0.5} M^{-0.3} dM + \int_{0.5}^{200} M^{-1,3} dM}{\int_{0.08}^{0.5} M^{-1.3} dM + \int_{0.5}^{200} M^{-2,3} dM} = 0,82 M_\odot$$

$$N_* = \frac{M_*}{\langle M \rangle} = \frac{1,85 \cdot 10^{11} M_\odot}{0.82 M_\odot} = 225 \text{ млрд.}$$

4) M − L (маломасс.)

$$\frac{dP}{dr} = -\frac{GM(r)\rho(r)}{r^2}$$

$$\langle P\rangle = -\frac{1}{3}\frac{E_p}{V} \text{ (вириал.)}$$

$$E_p \cong -\frac{GM^2}{R} \Rightarrow \langle P\rangle \simeq \frac{GM^2}{4\pi R^4}$$

$$P = nkT \qquad n = \langle\rho\rangle/\bar{m} \qquad R = \left(\frac{3}{4\pi}\frac{M}{\rho}\right)^{1/3} \sim M^{1/3}$$

$$\langle P\rangle = \frac{\langle\rho\rangle}{\bar{m}}kT \Rightarrow kT = \frac{GM\bar{m}}{3R}$$

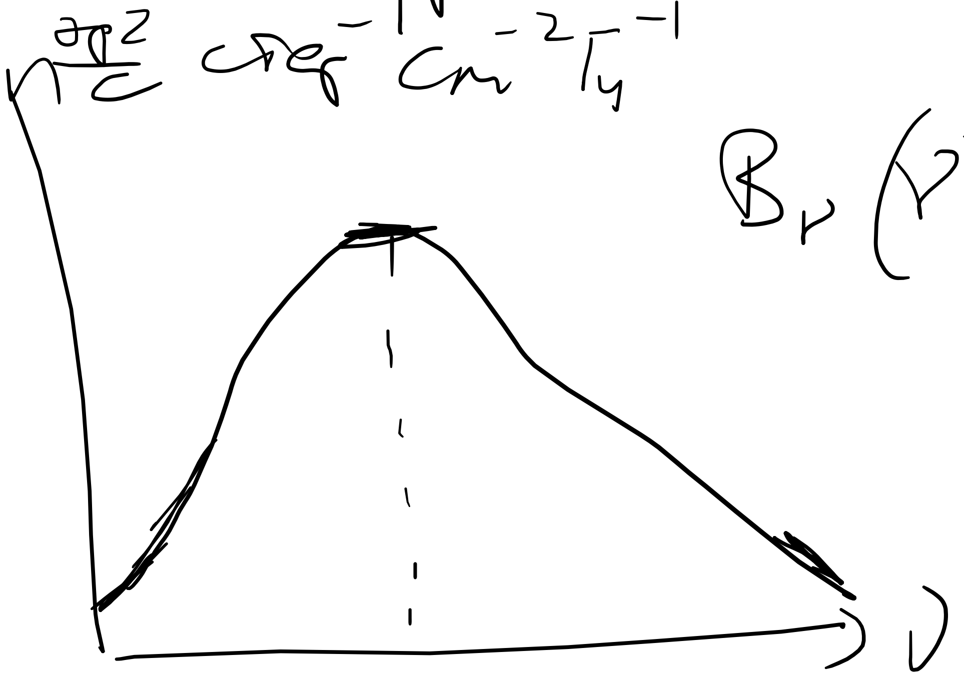
$$L = 4\pi R^2\sigma T^4 = 4\pi\left(\frac{3}{4\pi}\frac{M}{\rho}\right)^{2/3}\sigma\left(\frac{GM\bar{m}}{3kR}\right)^4 \sim$$

$$\sim M^{2/3} M^4 M^{-4/3} \sim M^{3\frac{1}{3}}$$

$$L \sim M^{3\frac{1}{3}}$$

⑤ Спектры

$1\,eV \sim 10^4\,K$



$$B_\nu(\nu, T) = \frac{2h\nu^3}{c^2}\frac{1}{e^{h\nu/kT}-1}$$

$E_{peak} \sim 3kT$

$\lambda_{peak} \sim \frac{1}{T}$

$\nu_{peak} = T \cdot 5{,}879 \cdot 10^{11}\,\frac{Гц}{К}$

Рэлей-Джинс

$\nu \to 0$

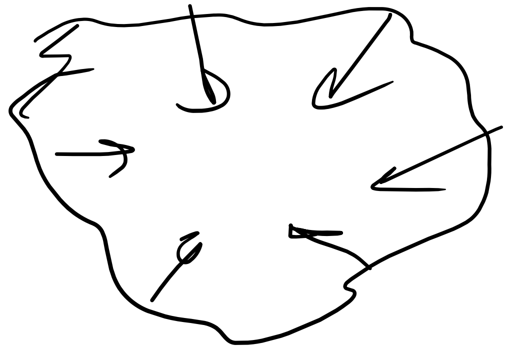
$$B_\nu = \frac{2\nu^2 kT}{c^2}$$

$$e^x = 1 + \frac{x}{1!} + \ldots$$

Вин $B_\nu = \frac{2h\nu^3}{c^2} e^{-h\nu/kT}$

$e^{h\nu/kT} \gg 1$

6) Масса Джинса



а) $2E_k + U = 0$

$$2\,\frac{3}{2}kT\cdot N = \frac{3}{5}\frac{GM^2}{R}$$

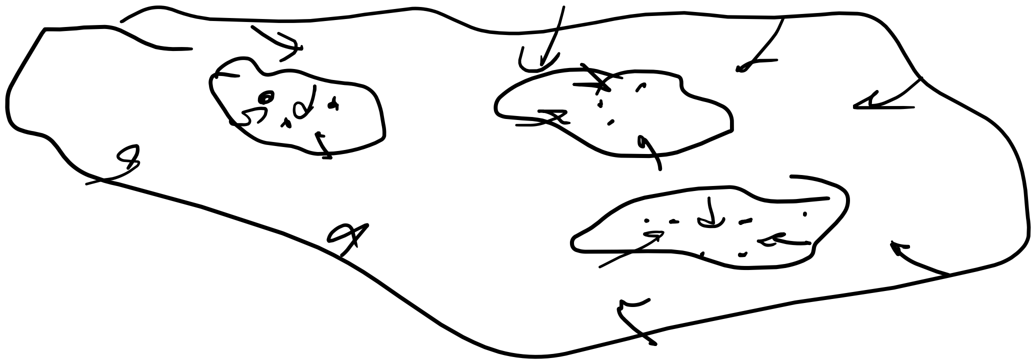
$$N = \frac{M}{m_0}$$

$$M = \frac{4}{3}\pi R^3 \rho$$

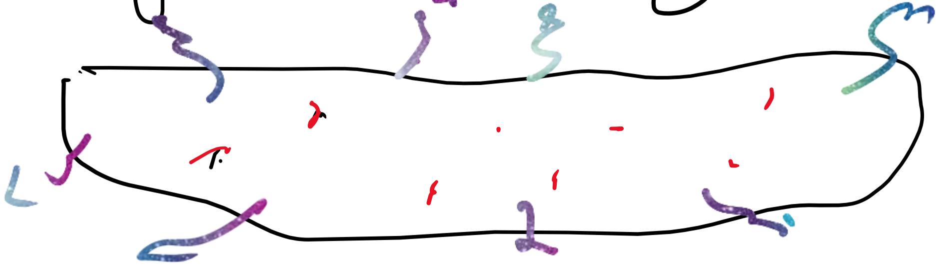
$$\frac{1}{5}\frac{GM}{R} = \frac{kT}{m_0} \Rightarrow \frac{1}{5}\frac{G}{R}\frac{4}{3}\pi R^3 \rho = \frac{kT}{m_0}$$

$$R_J = \left(\frac{15kT}{4\pi G m_0 \rho}\right)^{1/2}$$

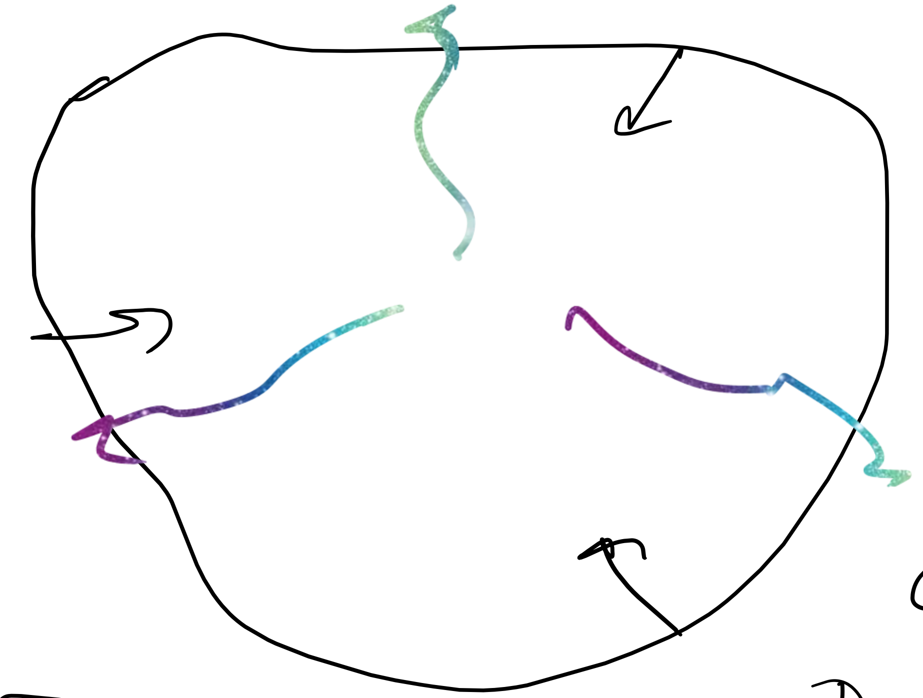
$$M_J = \frac{5kT}{G m_0} R_J = 3M_\odot \frac{T}{10K}\frac{R_J}{10^{15}\,\text{cm}}$$



$$M_J \sim \text{few}\cdot 10^3 M_\odot$$



5).



$$t_{ff} \sim \frac{R}{\sqrt{GM/R}} \sim \sqrt{\frac{R^3}{GM}} \sim \frac{1}{\sqrt{G\rho}}$$

$$t_s = R/c_s$$

$$c_s^2 = \frac{\partial P}{\partial \rho}$$

$$PV = \frac{M}{\mu} RT$$

$$P \sim \frac{R\rho T}{\mu}$$

$$c_s = \sqrt{\frac{RT}{\mu}} \sim \sqrt{T}$$

$$c_s = \sqrt{\frac{kT}{m_0}}$$

$$t_{ff} = t_s$$

$$\frac{1}{\sqrt{G\rho}} = \frac{R_J}{\sqrt{kT/m_0}} \Rightarrow R_J = \frac{c_s}{\sqrt{G\rho}} = \sqrt{\frac{kT}{G m_0 \rho}}$$

$$M_J = \frac{4}{3}\pi R_J^3 \rho$$