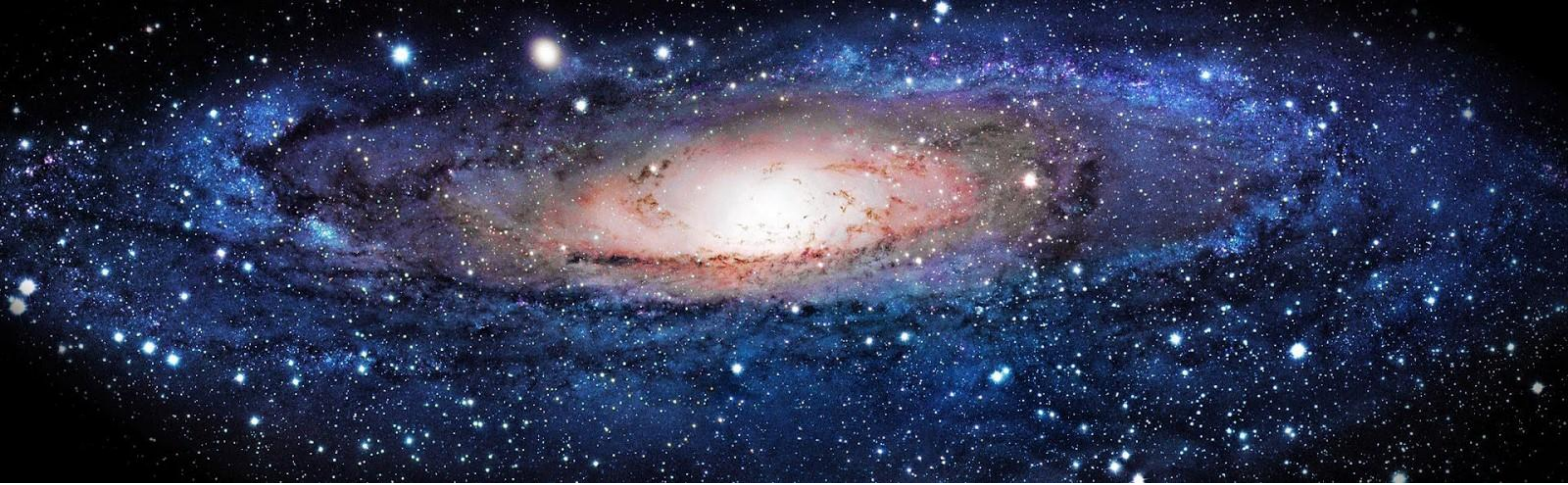
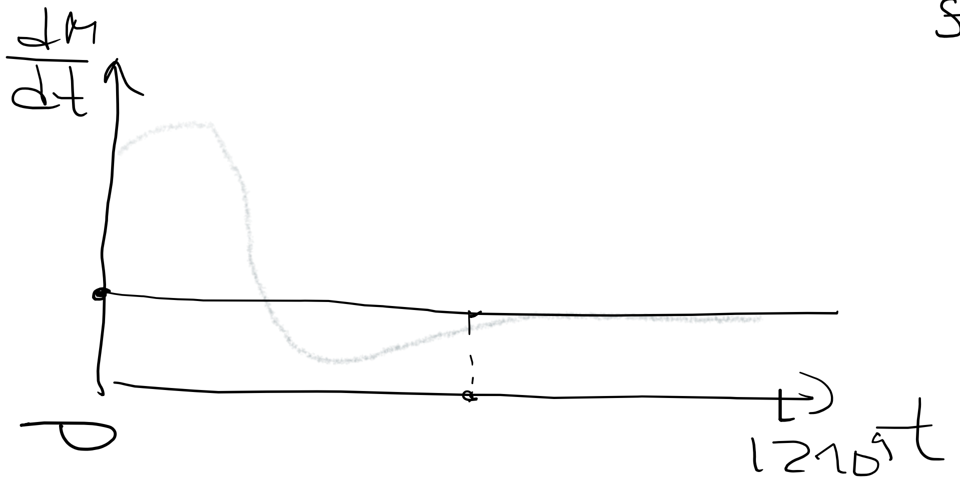


Модуль. Астрофизика.

Семинар 8. (18.12)

2/3 7 . s2



steady-state

$0 < f_{evol}(M) < 1, M > M_{min}$

① Закон Хаббла

$$ds^2 = c^2 dt^2 - \underbrace{a^2 dx^2}_{dr} \qquad a = a(t)$$

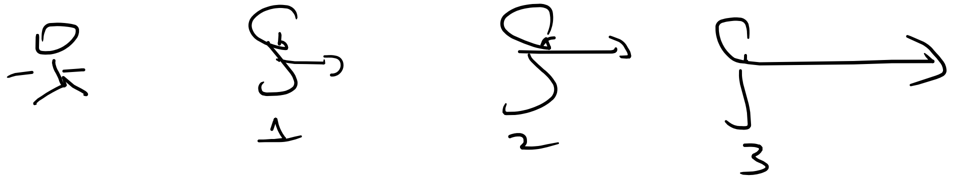
$$r = ax$$

$$r = \int a\,dx \qquad t = t_{obs} \qquad r = a(t_0)\int dx$$

$$x = const$$

$$v = \frac{dr}{dt} = \frac{d(ax)}{dt} = \left(\frac{1}{a}\frac{da}{dt}\right) a x \qquad \frac{1}{a}\frac{da}{dt} = H$$

$$= H \cdot r \qquad \boxed{v = Hr}$$



$$v_2 = v_1 + v_{12}$$

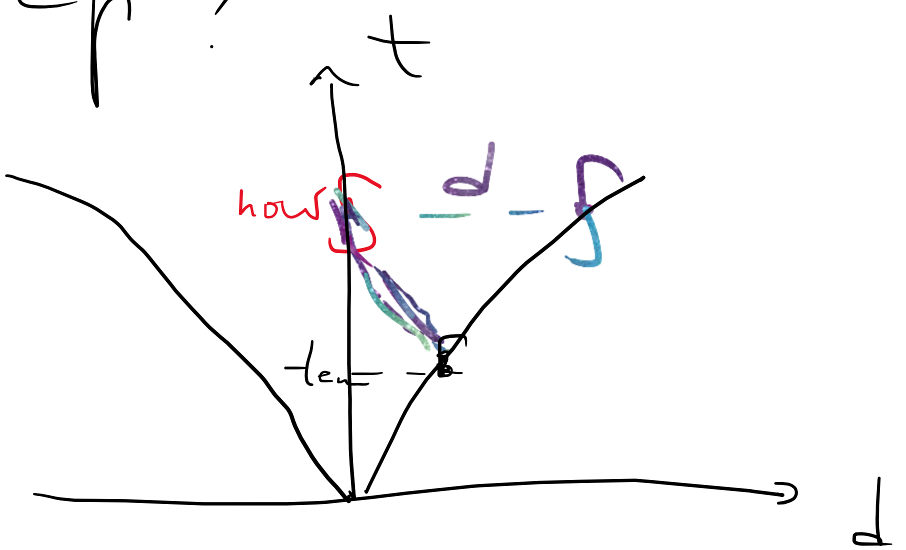
$$v_3 = v_1 + v_{12} + v_{23}$$

$$v_3 = H \cdot r_3 = H\left(r_1 + r_{12} + r_{23}\right)$$

пек. скорость

$$v = \frac{dr}{dt} = \frac{d(ax)}{dt} = (x \neq const) = \underbrace{\dot{a} x}_{\text{косм. расш.}} + \underbrace{\dot{x} a}_{\text{пекул. движ.}}$$

② Почему вблизи Доплер?



$$d = \frac{c}{(1-\alpha)H_0}\left[(1+z)^{1-\alpha} - 1\right]$$

Ряд Тейлора в 0 по z

$$f(0) + \frac{f'(0)}{1!}z + \ldots \qquad d(0) = 0$$

$$d' = \frac{c}{\cancel{(1-\alpha)}H_0}\cancel{(1-\alpha)}(1+z)^{-\alpha} = \frac{c}{H_0(1+z)^{\alpha}}$$

$$d'(0) = \frac{c}{H_0}$$

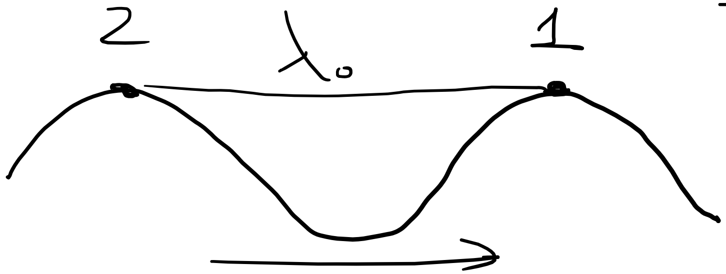
$$d(z \simeq 0) = \frac{c}{H_0}\cdot z$$

$$\left.\begin{array}{l} d \sim z \\ v \sim d \end{array}\right\} \Rightarrow v \sim z \quad (z \simeq 0)$$

③ Крас. смещ. $\quad ds^2 = c^2 dt^2 - a^2(t)\, d\chi^2$

Для света $\quad ds^2 = 0 \qquad c^2 dt^2 = a^2(t)\, d\chi^2$

$$d\chi = \frac{c\,dt}{a(t)} \qquad \int_{t_{em}}^{t_{obs}} \frac{c\,dt}{a(t)} = \int_{0}^{\chi_{em}} d\chi = \chi_{em}$$



$$\int_{t_{em}+\Delta t_{em}}^{t_{obs}+\Delta t_{obs}} \frac{c\,dt}{a(t)} = \chi_{em}$$

$$\frac{c\,\Delta t_{em}}{a(t_{em})} - \frac{c\,\Delta t_{obs}}{a(t_{obs})} = 0 \Rightarrow \frac{\Delta t_{em}}{\Delta t_{obs}} = \frac{a(t_{em})}{a(t_{obs})}$$

$$c\,\Delta t = \lambda \qquad z = \frac{\lambda_{obs} - \lambda_{em}}{\lambda_{em}} = \frac{a(t_{obs})}{a(t_{em})} - 1$$

$$z + 1 = \frac{a(t_{obs})}{a(t_{em})}$$

(5) Параметр замедления

$$q = -\frac{\ddot{a}a}{\dot{a}^2} = \left[H = \frac{\dot{a}}{a} \right] = -\left(1 - \frac{\dot{H}}{H^2}\right)$$

при $q = const$ $\quad \dot{a}(t) \sim a^{-q}$

$$H(t) = a(t)^{-q-1}$$

при $q = -1 \Leftrightarrow H = const \quad a(t) = a_0 e^{Ht}$

$q > 0$, то $\ddot{a} < 0$, т.е. $\dot{a}$ уменьш.

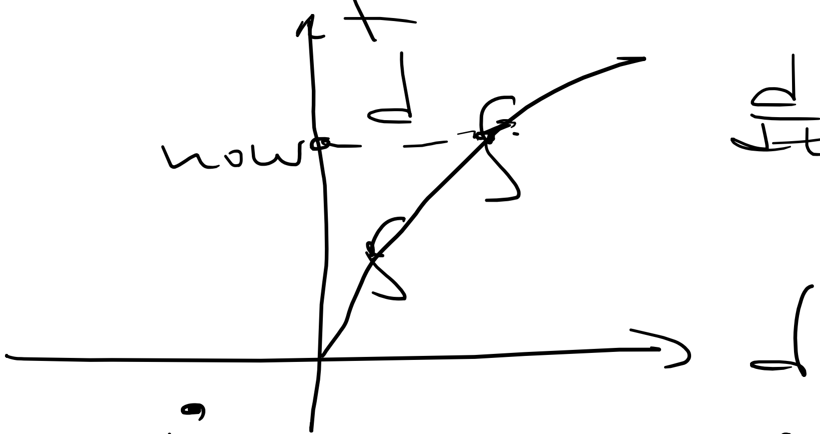
$q < 0$, то $\ddot{a} > 0$, т.е. $\dot{a}$ растет

в совр. В. $\quad q \approx -0.55$

(6) Сфера Хаббла

$v = H \cdot d \qquad H = \dot{a}/a$

$c = H \cdot d_H \qquad d_H = c/H$



$\frac{d}{dt}(d)$

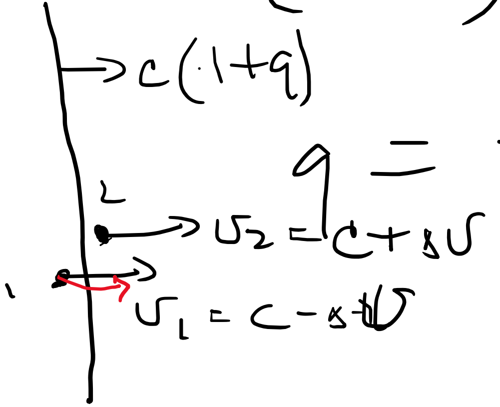
$(d_H)' = \left(\frac{c}{H}\right)' = \left(\frac{ca}{\dot{a}}\right)' = -\frac{c\dot{H}}{H^2} =$

$\dot{H} = \left(\frac{\dot{a}}{a}\right)' = \frac{\ddot{a}a - \dot{a}^2}{a^2}$

$(d_H)' = v_H = -c\,\frac{\ddot{a}a - \dot{a}^2}{a^2} \cdot \frac{a^2}{\dot{a}^2} =$

$= c\,\frac{\dot{a}^2 - \ddot{a}a}{\dot{a}^2} = c(1+q)$

$q = -\frac{\ddot{a}a}{\dot{a}^2}$



$\delta v \ll c$

Е. $q < 0$, то $v_H < c$

Е. $q > 0$, то $v_H > c$

$d_H \uparrow$
$\chi_H \downarrow$

Hubble sphere $d_H = a\chi_H$

7) Четыре скорости.

$$D_{now} = a_{now} \cdot \chi = \left[a_{now} = a_{em}(1+z)\right] = a_{em}(1+z)\chi = D_{em}(1+z)$$

D_{now} — собств. расст.

D_{em} — угловое расст.

$$D_{light} = ct = \int_{t_{em}}^{t_{obs}} c\,dt$$

$$D_{em} < D_{light} < D_{now}$$

$$v_{now} = \dot{D}_{now} = \dot{a}_{now}\chi$$

$$v_{em} = \dot{D}_{em} = \dot{a}_{em}\chi \text{ — по } t_{em}$$

$$v_{app} = v_{em}/(1+z) \text{ — по } t_{obs} \text{ (по часам наблюд.)}$$

$$v_e = \frac{dD_{light}}{dt} \leftarrow \text{now}$$

$$dD_{light} = c\left(dt_{obs} - dt_{em}\right)$$

$$\frac{dD_{light}}{dt} = c\left[\underbrace{\frac{dt_{obs}}{dt}}_{1} - \frac{dt_{em}}{dt}\right]$$

now

$$\frac{a_{em}}{a_{obs}} = \frac{1}{(1+z)}$$

$$= cz\frac{dt_{em}}{dt} = c\frac{z}{z+1} < c$$

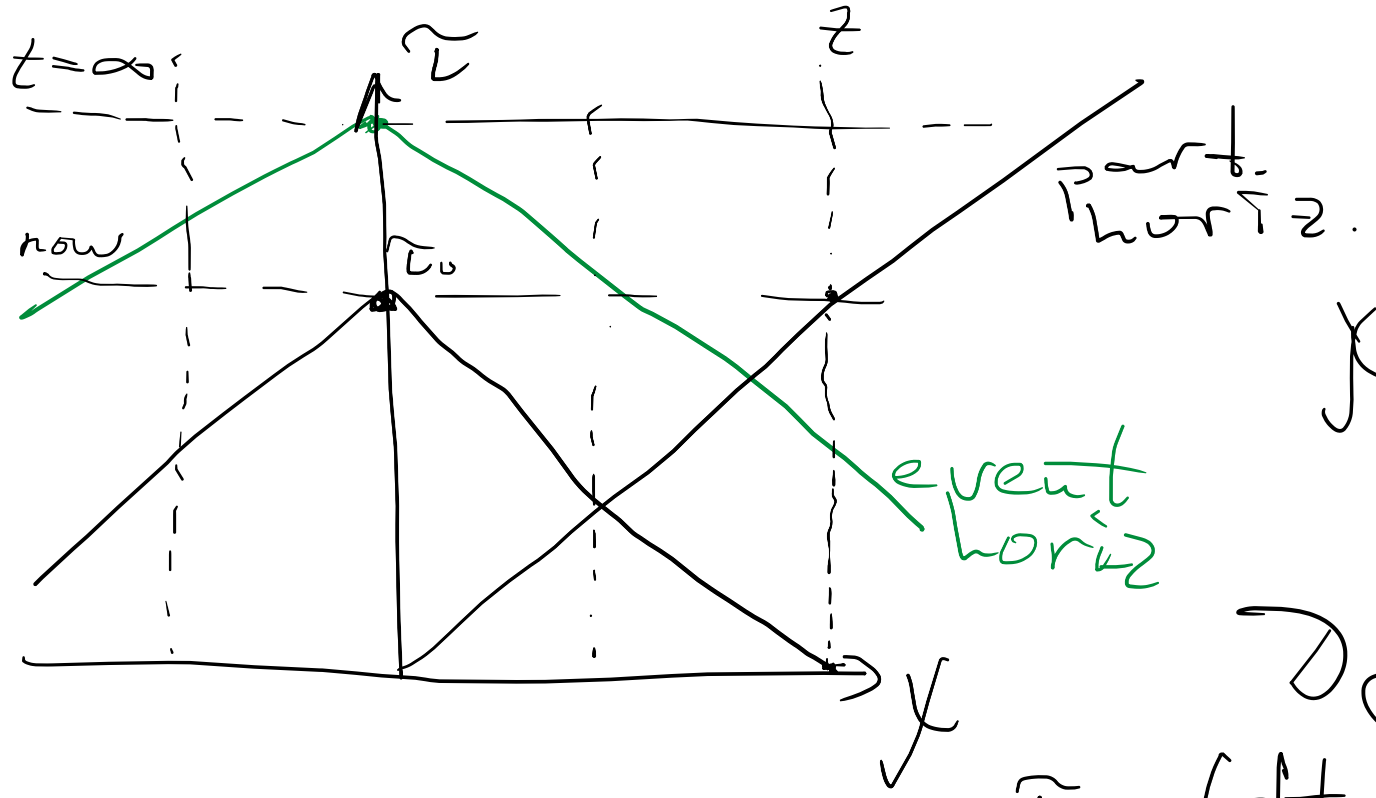
8 Конформное время

$$d\tau = \frac{dt}{a(t)} \Rightarrow \underline{\underline{d\chi = c\, d\tau}}$$

$$ds^2 = c^2 dt^2 - a^2 d\chi^2 =$$

$$= a^2(t)\left[\frac{c^2 dt^2}{a^2} - d\chi^2\right]$$

$ds^2 = 0$ $\quad c\,dt = a\,d\chi$



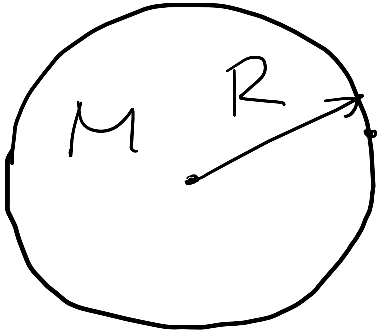
$$\chi_{ph} = \int_0^t \frac{c\,dt'}{a(t')} = c\int_0^{now} d\tau$$

$$D_{pl} = a\chi_{ph} = a_0 \cdot c \cdot \tau_0$$

$$\tau = \int \frac{dt}{a} = \int \underbrace{\frac{dt}{da}}_{\frac{1}{\dot a}} \frac{da}{a} = \int \frac{da}{a\dot a} = \int \frac{da}{a^2 H}$$

(9) Ω (Засов, Постнов стр 480+)

$R = a R_0$



$$\frac{1}{2}\left(\frac{dR}{dt}\right)^2 - \frac{GM}{R} = \text{const} = 0$$

$$H = \frac{1}{R}\frac{dR}{dt} \qquad M = \frac{4}{3}\pi R^3 \rho$$

$$\frac{1}{2}H^2 - \frac{4}{3}\pi G \rho_c = 0 \qquad \rho_c = \frac{3H^2}{8\pi G}$$

$H \neq \text{const}$

$H = H(t)$

$$\Omega = \rho / \rho_c$$

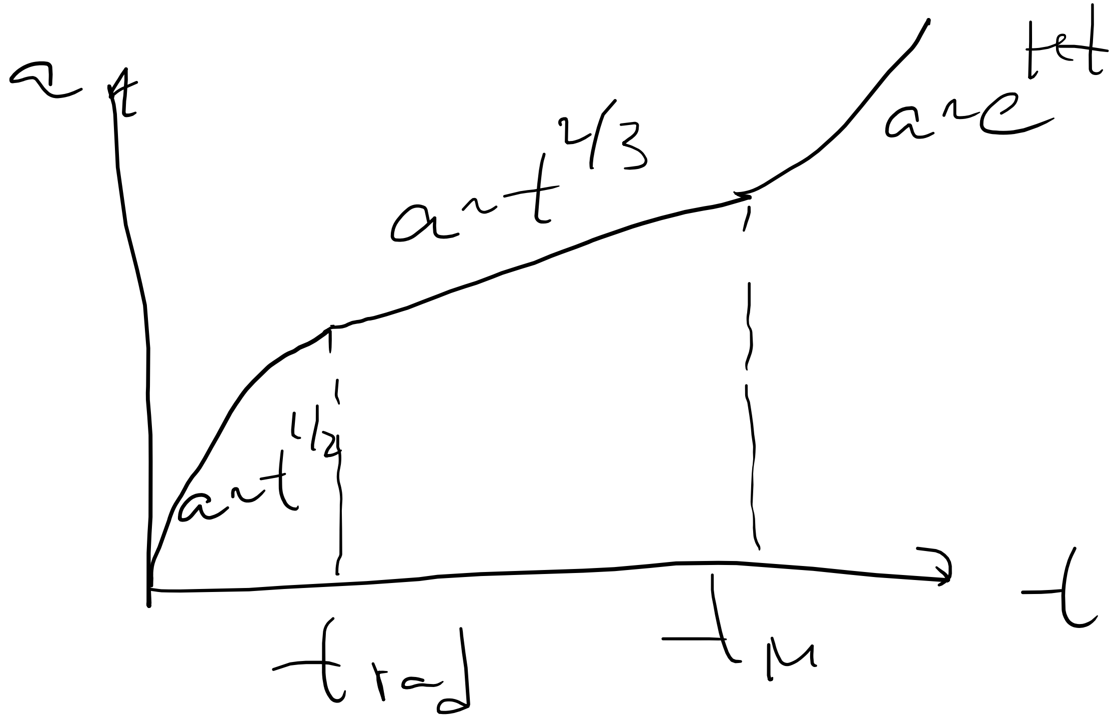
Изл.	ρ_r	Ω_r
В-во	ρ_m	Ω_m
ТЭ	ρ_Λ	Ω_Λ

$k = \{-1, 0, 1\}$

$$1 = \Omega_r + \Omega_m + \Omega_\Lambda + \Omega_c$$

$$H^2 = \frac{8\pi G\rho}{3} + \frac{\Lambda c^2}{3} + \Omega_{[illegible]} - \left(\frac{kc^2}{a^2}\right)$$

lg ρ
изл.
Λ
t_rad
t_M
≈ few · 10^4 yrs
≈ few · 10^9 yrs
lg t



a
a ~ t^{1/2}
a ~ t^{2/3}
a ~ e^{Ht}
t_rad
t_M
t