

Lectures on relativistic gravity and cosmology.

Lectures 11-12

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Present matter content of the Universe

Dark matter

Dark energy

Three fundamental cosmological constants

The standard LambdaCDM cosmological model

Determining partial terms in the Friedmann equation

$$\sum_i \Omega_i + \Omega_{curv} + \Omega_\Lambda = 1,$$

$$\Omega_i = \frac{\rho_i}{\rho_c} = \frac{8\pi G \rho_i}{3H_0^2}, \quad \Omega_{curv} = -\frac{\mathcal{K}}{a^2(t_0)H_0^2}, \quad \Omega_\Lambda = \frac{\Lambda}{3H_0^2}.$$

1. Spatial curvature:

$$\Omega_{curv} = 0.001 \pm 0.002$$

- arXiv:1807.06209 (Planck 2018 results) - from CMB angular temperature and polarization fluctuations (mostly from the position of the first acoustic peak) + BAO (baryon acoustic oscillations) - to be derived later.

2. Visible matter: stars, galaxies (optical data), hot gas in rich galaxy clusters (X-ray data): $\Omega_{vis} \lesssim 0.01$.

3. Total amount of baryons (p,n) and electrons (e^-):

$$\Omega_b = (0.0457 \pm 0.0002) \left(\frac{70}{H_0} \right)^2$$

- arXiv:1807.06209 (Planck 2018 results).

From two independent sources leading to the same result:

1) CMB angular temperature and polarization fluctuations (mostly from the distance between acoustic peaks and the height of the third acoustic peak) - to be derived later;

2) abundance of light chemical elements (He^4 , D, He^3 , Li^7), which were produced during the Big Bang nucleosynthesis (BBN) in the early Universe - to be derived later.

No primordial antimatter.

The most part of baryonic matter ('dark baryons') is in the intergalactic medium (IGM).

4. Kinetic energy of baryons - very small compared to their rest-mass energy. Typical peculiar velocities of galaxies with respect to the FLRW background are ~ 500 km/s.

The velocity of the Solar system with respect to the reference frame where CMB is at rest on average is ≈ 370 km/s. It is determined from the dipole anisotropy of the CMB temperature. In turn, CMB on average is practically at rest with respect to the FLRW background with $\sim 10^{-5}$ accuracy. The velocity of the Milky Way with respect to CMB is 627 ± 22 km/s.

5. CMB photons (restoring c):

$$\rho_\gamma c^2 = \frac{\pi^2 (k_B T_\gamma)^4}{15 \hbar^3 c^3},$$

$$\Omega_\gamma = 5.045 \cdot 10^{-5} \left(\frac{T_\gamma}{2.725} \right) \left(\frac{70}{H_0} \right)^2$$

6. 3 families of neutrinos (ν_e, ν_μ, ν_τ) from which at least two are massive.

$$\Delta m_{31}^2 \approx \Delta m_{32}^2 \approx 2.5 \cdot 10^{-3} \text{ eV}^2, \quad \Delta m_{21}^2 \approx 0.75 \cdot 10^{-4} \text{ eV}^2.$$

The BBN theory predicts that $T_\nu = \left(\frac{4}{11}\right)^{1/3} T_\gamma$ for $T_\nu \gg m_\nu$ - to be derived later. Then

$$\sum_{i=1}^3 m_{\nu i} = 46 \Omega_\nu \left(\frac{H_0}{70}\right)^2 \text{ eV}.$$

Observations (arXiv:1807.06209 - Planck 2018 results, model dependent however):

$$\sum_{i=1}^3 m_{\nu i} < 0.12 \rightarrow \Omega_\nu < 0.003.$$

Thus, about **95%** of the total present energy density of the Universe is due to 'dark entities' not known from terrestrial experiments. We divide these dark entities into dark matter and dark energy.

Dark matter

Dark matter and dark energy are seen through gravitational interaction only – we know the structure of their effective energy-momentum tensor.

DM - non-relativistic, gravitationally clustered.

DE - relativistic, unclustered.

Definition of their effective EMT – through equations (chosen conventionally). We choose what follows from GR.

For non-relativistic matter in the longitudinal gauge (valid in the first order in $|\Phi|$ only):

$$ds^2 = (1 + 2\Phi) dt^2 - (1 - 2\Phi) a^2(t)(dx^2 + dy^2 + dz^2),$$

we get the generalized Poisson equation (to be derived later):

$$\frac{\Delta\Phi}{a^2} = 4\pi G(\rho - \rho_0(t)).$$

Observational determination of Φ

To determine the amount of DM and its properties, we consider scales less than ~ 100 Mpc at which gravitational matter clustering is seen.

$\Phi(\mathbf{r}, t)$ is measured using the motion of 'test particles' in it.

a) Stars in galaxies \rightarrow rotation curves.

b) Galaxies \rightarrow peculiar velocities.

Peculiar velocity is $\mathbf{v} - H_0 \mathbf{r}$.

c) Hot gas in galaxies \rightarrow X-ray profiles.

d) Photons \rightarrow gravitational lensing (strong and weak).

Strong lensing - multiple images of a source.

Weak lensing - astigmatism, distortion of the shape of galaxies.

Amount and properties of DM

Observations: DM is non-relativistic, has a dust-like EMT – $p \ll \rho$, $p > 0$, collisionless in the first approximation – $\sigma/m < 1 \text{ cm}^2/\text{g}$, and has the same spatial distribution as visible matter for scales exceeding a few Mpc. However, its amount is about 5 times more than that of baryonic matter.

$$\Omega_{DM} \approx 0.25, \quad \Omega_m = \Omega_{DM} + \Omega_b \approx 0.3$$

(arXiv:1807.06209: $\Omega_m = 0.315 \pm 0.007$, model dependent).

In addition, DM is **cold** - CDM: relative (e.g., thermal) velocities of particles which constitute it (if exist) are much less than the velocity $v = |-\nabla\Phi|$ acquired in the cosmic gravitational field. This follows from the absence of damping of matter perturbations at galaxy and smaller scales (to be discussed later).

At smaller scales, DM is less clustered than baryonic matter.

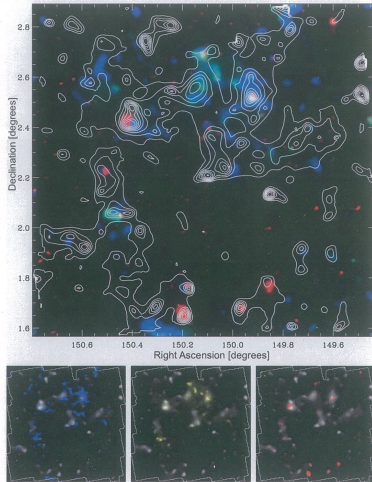


Figure 3 | Comparison of baryonic and non-baryonic large-scale structure. The total projected mass from weak lensing, dominated by dark matter, is shown as contours in panel a and as a linear grey scale in panels b, c and d. Independent baryonic tracers comprise (i) stellar mass (blue, colour scale peaks at $2.3 \times 10^{11} M_{\odot} \text{ deg}^{-2}$ within $\Delta z=0.1$), (ii) galaxy number density (yellow, peak at $1.4 \times 10^5 \text{ deg}^{-2}$ within $\Delta z=0.1$) seen in optical and near-IR light (adjusted to the redshift sensitivity function of the lensing mass), and (iii) hot gas (red, peak at $2.6 \times 10^{11} \text{ erg/s/cm}^2/\text{arcmin}^2$) seen in x-rays after removal of point sources.

Possible candidates for DM

Purely hypothetical at present.

1. Weakly interacting massive particles (WIMPs) which were in thermal equilibrium with CMB during BBN or earlier.

Appear in extensions of the Standard Model of elementary particles (e.g. in its supersymmetric extensions).

Ground experiments: $\sigma < 10^{-43} \text{ cm}^2$ for $m \sim (50 - 100) \text{ GeV}$.

2. Light cold particles which were never in thermal equilibrium with CMB and form a Bose-Einstein condensate. A popular example: an axion with the mass in the range $(10^{-6} - 10^{-3}) \text{ eV}$.

3. Primordial black holes (PBHs) with masses in the range $(10^{17} - 10^{23}) \text{ g}$ (to be discussed later). PBH with larger masses are possible, too, but they may not constitute the whole DM.

Dark energy

DE is more general than Λ , it provides the remaining part in the Friedmann equation.

$$\Omega_{DE} \approx 0.7.$$

Now the whole FRLW universe is used as a test particle. Quantitative and internally self-consistent definition of its effective EMT - through gravitational field equations conventionally written in the Einstein form:

$$\frac{1}{8\pi G} \left(R^\nu_\mu - \frac{1}{2} \delta^\nu_\mu R \right) = T^\nu_{\mu(vis)} + T^\nu_{\mu(DM)} + T^\nu_{\mu(DE)},$$

$G = G_0 = \text{const}$ - the Newton gravitational constant measured in laboratory. In the absence of direct interaction between DM and DE: $T^\nu_{\mu(DE); \nu} = 0$.

The left side can be determined from observations, e.g. from $D_L(z)$. Then, after subtracting $T^\nu_{\mu(vis)}$ and $T^\nu_{\mu(DM)}$ determined at the previous steps, $T^\nu_{\mu(DE)}$ follows.

Possible forms of DE

1. Physical DE.

New non-gravitational field of matter. DE proper place – in the **rhs** of gravity equations.

2. Geometrical DE.

Modified gravity. DE proper place – in the **lhs** of gravity equations.

3. Λ - intermediate case.

Observations: $T_{\mu}^{\nu}(DE)$ is very close to $\Lambda\delta_{\mu}^{\nu}$ for the concrete solution describing our Universe.

$$\langle w_{DE} \rangle = -1.03 \pm 0.03$$

(arXiv:1807.06209), where $w_{DE} \equiv p_{DE}/\rho_{DE}$.

$w_{DE} > -1$ – normal case,

$w_{DE} < -1$ – phantom case,

$w_{DE} \equiv -1$ – the exact cosmological constant (“vacuum energy”).

Three fundamental cosmological constants

A fundamental theory beyond each of these constants.

1. Baryon to photon ratio.

$$\frac{n_b}{n_\gamma} = 6.01 \times 10^{-10} \frac{\Omega_b}{0.0045} \left(\frac{2.725}{T_\gamma(\text{K})} \right)^3 \left(\frac{70}{H_0} \right)^2.$$

Theory of baryogenesis.

2. Baryon to total non-relativistic matter density.

$$\frac{\rho_b}{\rho_m} = 0.167 \frac{\Omega_b}{0.05} \frac{0.3}{\Omega_m}.$$

Theory of dark matter.

3. Energy density of present dark energy.

$$\rho_{DE} = 6.44 \times 10^{-30} \frac{\Omega_{DE}}{0.7} \left(\frac{H_0}{70} \right)^2 \text{ g/cm}^3,$$

$$\frac{G^2 \hbar \rho_{DE}}{c^5} = 1.25 \times 10^{-123} \frac{\Omega_{DE}}{0.7} \left(\frac{H_0}{70} \right)^2.$$

The standard LambdaCDM cosmological model

The standard Λ CDM model (neglecting radiation).

$$a(t) = a_1 \left(\sinh \frac{3H_1 t}{2} \right)^{2/3}, \quad \Lambda = 3H_1^2 = 3H_0^2 (1 - \Omega_m)$$

with $\Omega_m \approx 0.3$, $H_1 \approx 0.84H_0 \approx 60 \text{ km/s/Mpc}$.

$$H^2(z) = H_0^2 [1 - \Omega_m + \Omega_m(1+z)^3].$$

The stage of acceleration begins at $t = t_{ac}$ when

$$\ddot{a} = 0, \quad z = \left(\frac{2(1 - \Omega_m)}{\Omega_m} \right)^{1/3} - 1 \approx 0.67, \quad \Omega_m(z) = 2/3.$$

The age of the Universe: $t(H) = \frac{1}{3H_1} \ln \frac{H+H_1}{H-H_1}$. For $H_0 = 70$ km/s/Mpc, $t_0 \approx 0.96H_0^{-1} \approx 13.5$ byl. y, $t_{ac} \approx 7.3$ byl. y.