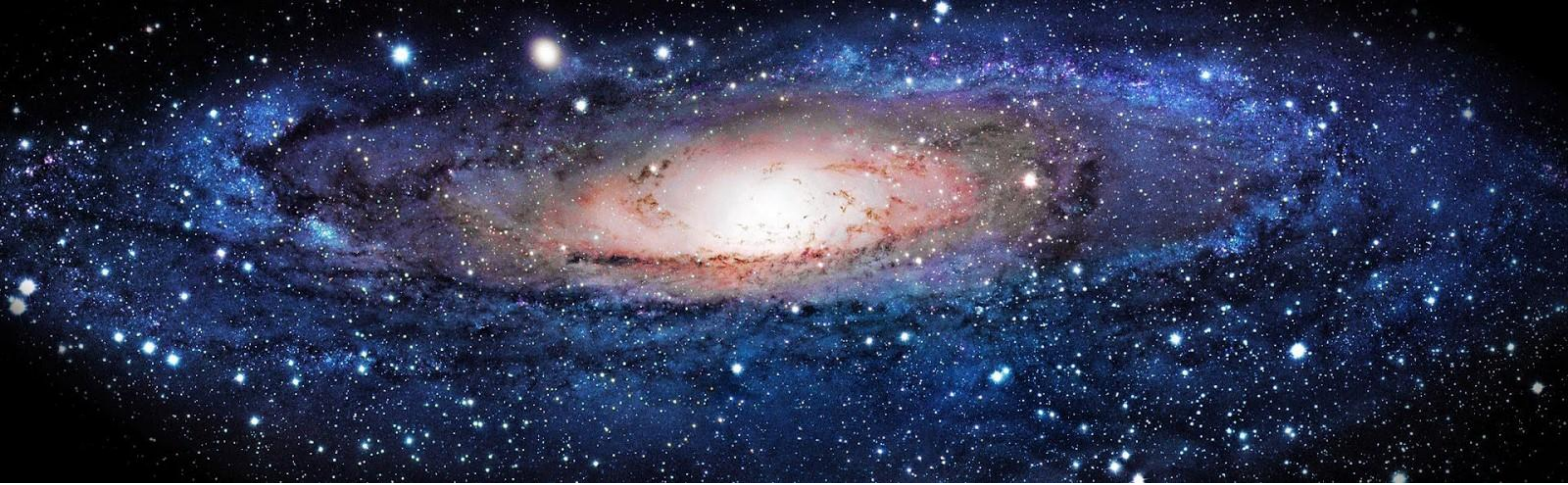


Астрофизика и космология.

Лекция 3. (30.01)

$dp\,dx = h = 2\pi\hbar$

$\underbrace{d^3p}\,\underbrace{d^3x} = (2\pi\hbar)^3$

$\frac{V d^3 p}{(2\pi\hbar)^3}$

ЗБШ 3.2

N – конц-ть $[cm^{-3}]$

→ n – число фот. в клетке фаз. пр.

$n_{фот}$ (в объеме) $= \underset{\text{попер.}}{2} \cdot \int_p n \cdot \frac{V d^3 p}{(2\pi\hbar)^3}$

$\vec{p} = \hbar \vec{k} \qquad |k| = \frac{2\pi}{\lambda}$

$N = \frac{2}{(2\pi\hbar)^3} \int_0^\infty n\, d^3 p$

$N = 2 \int n \frac{d^3 k}{(2\pi)^3}$

$d^3 p = p^2 dp\, d\Omega \qquad p = |\vec{p}| \qquad \int d\Omega = \int \sin\theta\, d\theta\, d\varphi = 4\pi$

$N = \frac{1}{\pi^2 \hbar^3} \int n p^2 dp$

$$E_{ph} = \hbar\omega = h\nu = cp$$

$$\mathcal{E}_r = \frac{c}{\pi^2 \hbar^3}\int n p^3 dp$$ – пл-ть эн. изл.

$$\mathcal{E}_r = \frac{1}{c}\int I_\nu \, d\nu \, d\Omega$$

$I_\nu \to F_\nu$ (3 БМ)

спектр. инт. изл.

$$E = h\nu = cp \Rightarrow \frac{dp}{d\nu} = \frac{2\pi\hbar}{c}$$

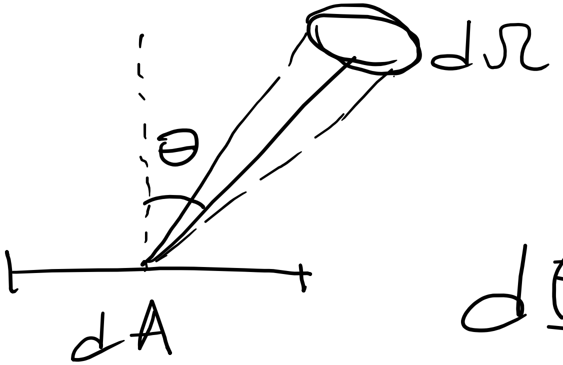
$$I_\nu = \frac{2c^2}{(2\pi\hbar)^3} n p^3 \frac{dp}{d\nu} = \frac{2np^3c}{(2\pi\hbar)^2} = \frac{2n\,h\nu}{\lambda^2}$$

$$\int d\Omega = 4\pi$$

$$[I_\nu] = \frac{\text{эрг}}{\text{см}^2 \cdot \text{с} \cdot \text{стер} \cdot \text{Гц}}$$

$$[I_\nu] = \frac{\text{эрг}}{\text{см}^2 \cdot \text{с} \cdot \text{Гц} \cdot \text{стер}}$$

$$I_\nu = \frac{dE}{dt\, dA \cdot \cos\theta\, d\nu \cdot d\Omega}$$



$$d\Omega = \frac{S_{ист}}{r^2}$$

$$dE = \frac{dL \cdot dt}{4\pi r^2} dA \cdot \cos\theta$$

$$dE = \frac{dL \cdot dt \cdot d\Omega}{4\pi S_{ист}} dA \cdot \cos\theta$$

$$I_\nu = \frac{dL}{4\pi S_{ист}\, d\nu}$$

$$dL = \frac{d\tilde{n}(p) \cdot E_{ph}(p)}{dt}$$

св. пучком спектр. диап.

$$d\tilde{n}(p) = S_{ист} \cdot c \cdot dt \cdot N(p)$$

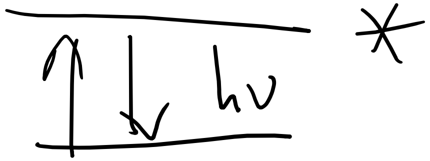
$$N(p) = \frac{1}{\pi \hbar^3} 4 p^3 dp$$

$$I_\nu = \frac{N(p) \cdot E_{ph}(p) \cdot c}{4\pi\, d\nu} =$$

$$I_\nu = \frac{n(p)\, E_{ph}(p) \cdot c}{4\pi\, d\nu} = \frac{1}{4\pi} \frac{1}{\pi^2 \hbar^3}\, pc \cdot n p^2 \frac{dp}{d\nu} \cdot c =$$

$$= \frac{2c^2}{(2\pi\hbar)^3}\, n p^3 \frac{dp}{d\nu} = \frac{2c}{(2\pi\hbar)^2}\, n p^3 = \boxed{\frac{2n\, h\nu}{\lambda^2}}$$

3БШ(3.3) Формула Планка



$N^* \; [\text{атомов}/\text{см}^3]$

$N \; [\text{атомов}/\text{см}^3]$

$$\frac{dn}{dt} = N^* w_{TOT} - N \cdot n \cdot \underbrace{\sigma c}_{w} \qquad [w] = \frac{\text{см}^3}{\text{с}}$$

$$w_{TOT} = w(1+n)$$

$$\frac{dn}{dt} = w\left[N^*(1+n) - Nn\right] = w\left[N^* - n(N - N^*)\right]$$

равн. $N^* = N \cdot e^{-E/kT}$, $E = h\nu$

$$\frac{dn}{dt} = w\left[N \cdot e^{-h\nu/kT} - n\left(N - N \cdot e^{-h\nu/kT}\right)\right] =$$

$$\frac{dn}{dt} = wN\left[e^{-h\nu/kT} - n\left(1 - e^{-h\nu/kT}\right)\right]$$

р-ли. $\frac{dn}{dt} = 0$ $\qquad 0 = e^{-h\nu/kT} - n\left(1 - e^{-h\nu/kT}\right)$

$$n = \frac{1}{e^{h\nu/kT} - 1}$$

$I_\nu \to B_\nu$

$h\nu \ll kT$

Рэлей-Джинс

$h\nu \gg kT$

Вин

1. Р–Д $\qquad n = \dfrac{1}{e^{h\nu/kT}-1}$

$\dfrac{h\nu}{kT} \ll 1$ (длинные волна)

$e^{x} = 1 + x + \ldots \qquad n \simeq \dfrac{1}{x} = \dfrac{kT}{h\nu} \gg 1$

$I_\nu = \dfrac{2n h\nu}{\lambda^2} = \dfrac{2kT}{\lambda^2} \qquad \int I_\nu \, d\nu$ расход. при $\lambda \to 0$

2. Вин $\quad \dfrac{h\nu}{kT} \gg 1 \qquad n \simeq e^{-h\nu/kT} \ll 1$

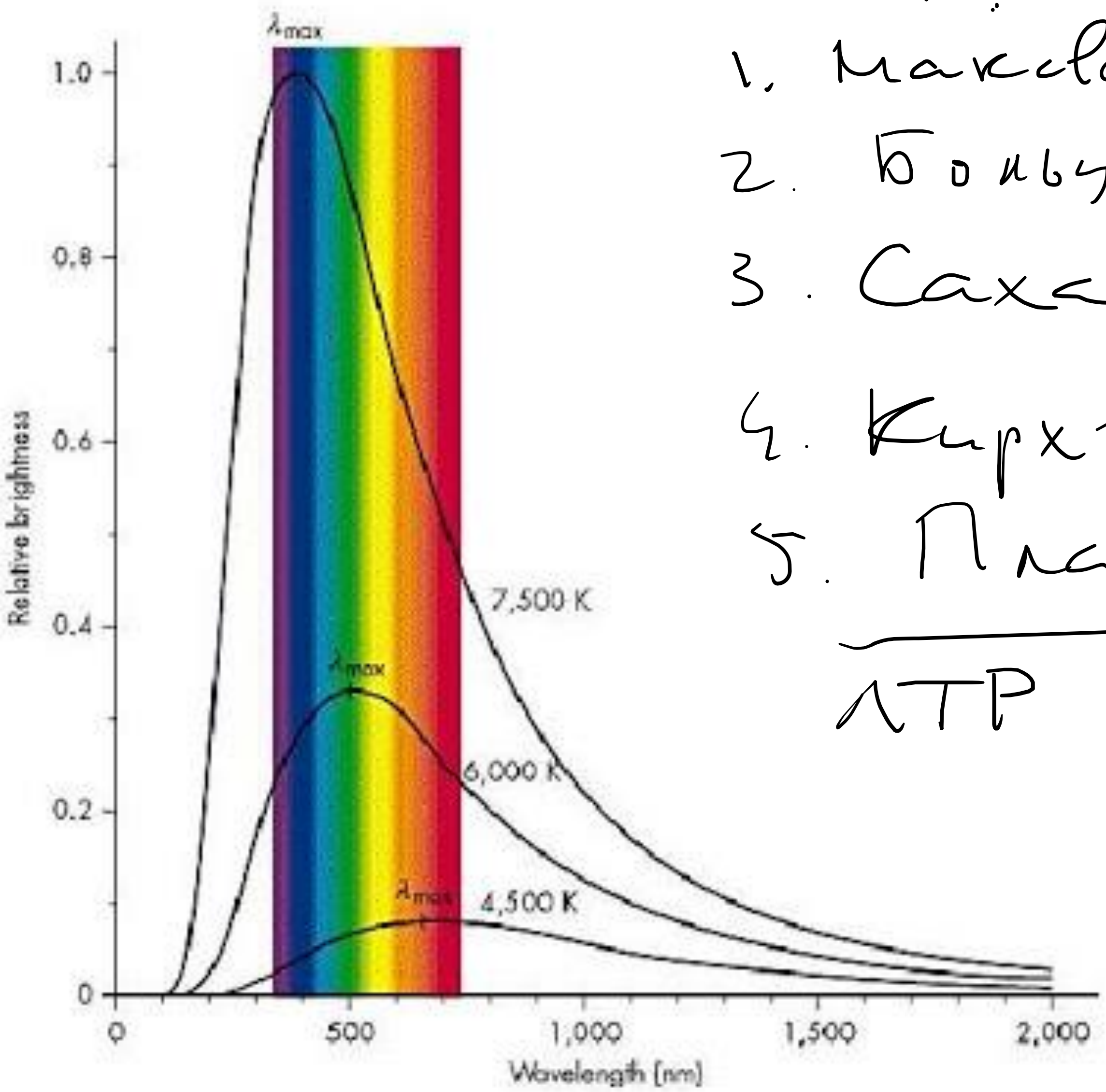
$\varepsilon_r = \dfrac{c}{\pi^2 \hbar^3} \int n p^3 dp = \dfrac{c}{\pi^2 \hbar^3} \int \dfrac{p^3 dp}{e^{+\frac{cp}{kT}}-1} = \left(\dfrac{kT}{c}\right)^4 \dfrac{c}{\pi^2 \hbar^3} \int_0^\infty \dfrac{x^3 dx}{e^{+x}-1} \qquad \left(x = \dfrac{cp}{kT}\right)$

$\int_0^\infty \dfrac{x^3 dx}{e^x - 1} = \dfrac{\pi^4}{15} \simeq 6.5$

$\varepsilon_r = \text{const} \cdot T^4 \qquad \left[\dfrac{\text{эрг}}{\text{см}^3}\right]$

Максимум на $x = 2{,}7 \Rightarrow h\nu_{max} = 2{,}7\,kT$

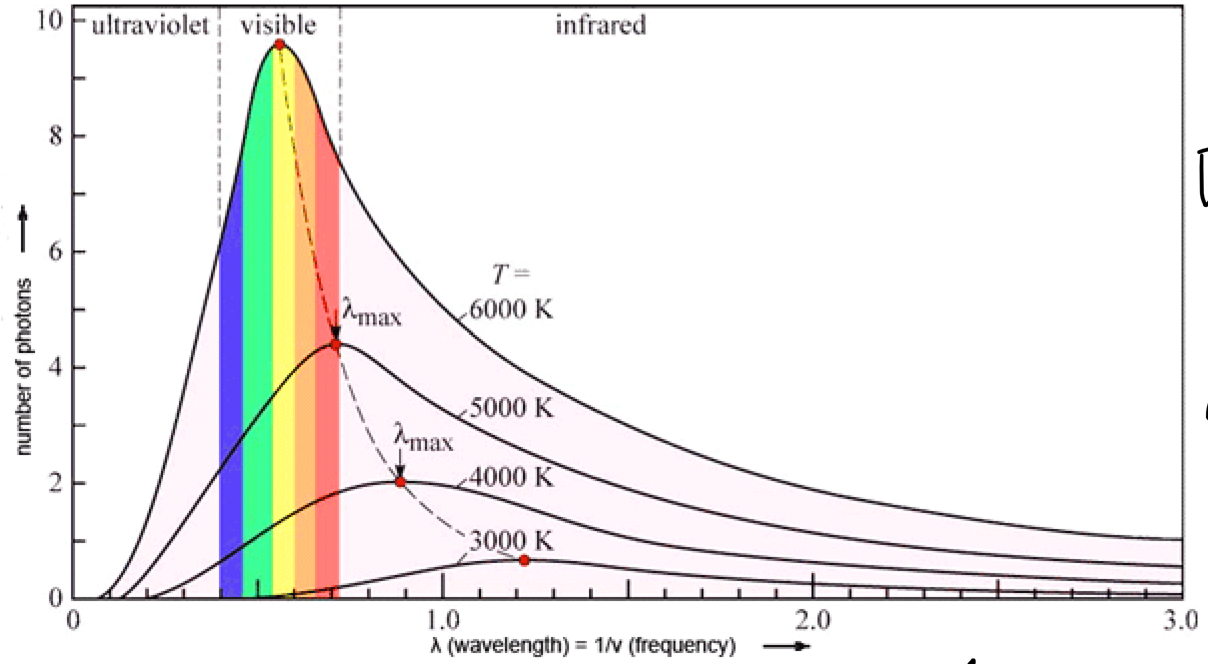
Часто шсп. $\displaystyle\int_0^\infty \frac{x^3\,dx}{e^x} = 3! = 6 \Rightarrow 3kT$



ТДР:

1. Максвелл по скор.
2. Больцман по нас. уровней
3. Саха для иониз.
4. Кирхгоф
5. Планк

ЛТР 1÷4



Планк. ф-л.

$$B_\nu(T) = \frac{2h\nu^3}{c^2}\left(e^{\frac{h\nu}{kT}}-1\right)^{-1}$$

$$[B_\nu] = \frac{\text{эрг}}{\text{см}^2\cdot\text{с}\cdot\text{Гц}\cdot\text{стер.}}$$

$$I_\nu d\nu = I_\lambda d\lambda \qquad \left|\frac{d\lambda}{d\nu}\right| = \frac{\lambda}{\nu}$$

$$B_\lambda(T) = \frac{2hc^2}{\lambda^5}\left[e^{\frac{hc}{\lambda kT}}-1\right]^{-1}$$

$$B_\nu^{RJ}(T) = \frac{2\nu^2}{c^2}kT$$

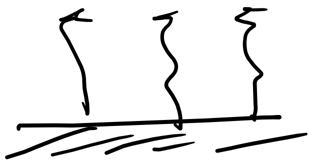
$$B_\nu^{W}(T) = \frac{2h\nu^3}{c^2}e^{-h\nu/kT}$$

Закон смещ. Вина

$$\frac{\partial B_\nu}{\partial \nu} = 0 \rightarrow h\nu_{max} \simeq 3kT$$

$$\frac{\partial B_\lambda}{\partial t} = 0 \rightarrow \lambda_{max} \simeq \frac{0{,}29\,(\text{см})}{T}$$

Закон Стефана-Больцмана



$$F = \iint B_\nu \cdot \cos\theta \, d\Omega \, d\nu = \pi \int_0^\infty B_\nu d\nu = \sigma_B T^4$$

$$\sigma_B = \frac{2\pi^6 k}{15 c^5 h^3}$$

Пл-ть энергии (Закон Больцмана)

$$u_{bb} = \frac{1}{c}\int_0^\infty B_\nu(T)\,d\nu\,d\Omega = \frac{4\pi}{c}\int_0^\infty B_\nu\,d\nu = \underbrace{\frac{4\sigma_B}{c}}_{= a_T} T^4$$

$$p_{bb} = \frac{a_T T^4}{3}$$ — давл. изл.

$$\langle \varepsilon_\gamma \rangle = u / \langle n_\gamma \rangle$$

$$\langle n_\gamma \rangle = \frac{4\pi}{c}\int_0^\infty \frac{B_\nu(T)}{h\nu}\,d\nu \simeq 20\cdot T^3$$

$$\langle \varepsilon_\gamma \rangle = kT\,\frac{\int_0^\infty \frac{x^3\,dx}{e^x-1}}{\int_0^\infty \frac{x^2\,dx}{e^x-1}} \simeq 2{,}7\,kT$$

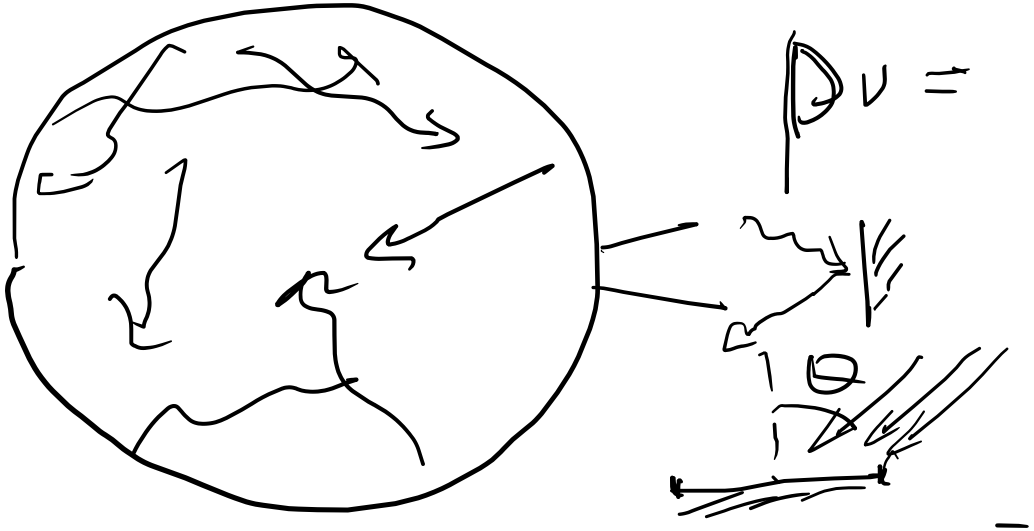
Постнов (Астронет) гл 2.

$f(\nu, \vec{r}, \vec{\Omega}, t)$ – функция распределения фотонов

$$I_\nu(\vec{r}, \vec{\Omega}, t)\, d\nu\, d\vec{\Omega} = h\nu \cdot c \cdot f(\ldots)\, d\nu\, d\Omega$$

$$u_\nu = h\nu \int f \cdot d\vec{\Omega} = \frac{1}{c}\int I_\nu\, d\vec{\Omega} = \frac{4\pi}{c} J_\nu$$

$$J_\nu = \frac{1}{4\pi}\int I_\nu\, d\Omega$$



$$p_\nu = \frac{2}{c}\int I_\nu \cdot \cos^2\theta \, d\Omega$$

$$p = \int p_\nu \, d\nu =$$

$$= \frac{2}{c}\int I_\nu \, d\nu \int_0^{2\pi}\int_0^{\pi/2} \cos^2\theta \, d\Omega$$

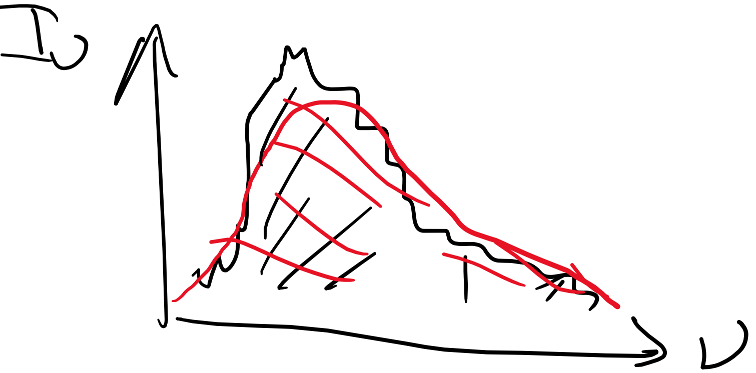
$$\iint \cos^2\theta \, d\Omega = \iint \cos^2\theta \sin\theta \, d\theta \, d\varphi =$$

$$= -2\pi \int_0^{\pi/2} \cos^2\theta \, d(\cos\theta)$$

$$p = \frac{2}{4\pi} u \int_0^{2\pi}\int_0^{\pi/2} \cos^2\theta \, d\Omega = \frac{2\cdot 2\pi}{4\pi} u \frac{1}{3} = \frac{u}{3}$$

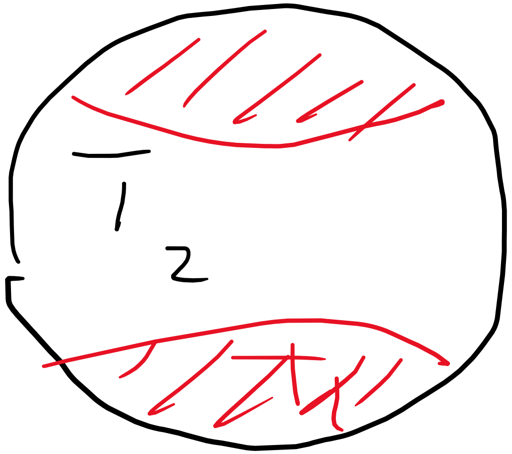
Температуры (Засоль, Потоше)

1. Эффективная



$$F = \int\limits_{2\pi}\int\limits_0^\infty I_\nu \cos\theta \, d\Omega \, d\nu = \sigma_B T_{eff}^4$$

реал.

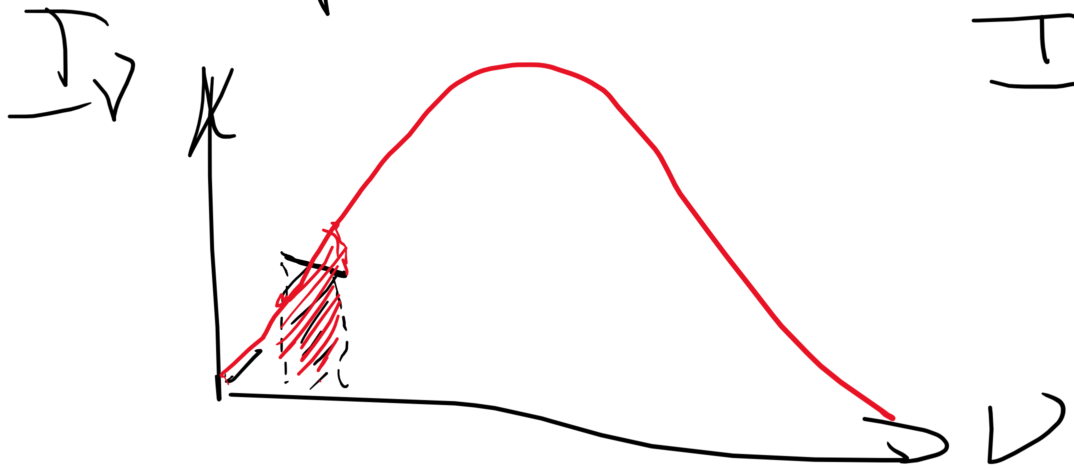


$$L = L_1 + L_2 = \sigma \cdot T_{eff}$$

$$L_1 = S_1 \cdot \sigma T_1^4$$

$$L_2 = S_2 \cdot \sigma T_2^4$$

2. Яркостная темп.



$$I_\nu = B_\nu(T_b)$$

$$T_b = \frac{c^2}{2\nu^2 k} I_\nu$$

(где Р-Д т.е. где радио)

Brightness temperature

Black-body radiation

$$I_\nu = \frac{2h\nu^3}{c^2}\frac{1}{e^{\frac{h\nu}{kT}}-1}$$

Brightness temperature

$$T_b = \frac{h\nu}{k}\ln^{-1}\left(1+\frac{2h\nu^3}{I_\nu c^2}\right)$$

For $h\nu \ll kT$ we have

$$T_b = \frac{I_\nu c^2}{2k\nu^2}$$

$$2\pi k T_b = \frac{S_\nu D^2}{(W\nu)^2} \qquad W \sim l/c,$$

$$T_b \approx 10^{35.8}[\text{K}]\left(\frac{S_\nu}{1\ \text{Ян}}\right)\left(\frac{(D/1\ \text{Гпк})}{(\nu/1\ \text{ГГц})(W/1\ \text{мс})}\right)^2$$

<u>Inverse Compton catastrophe</u>

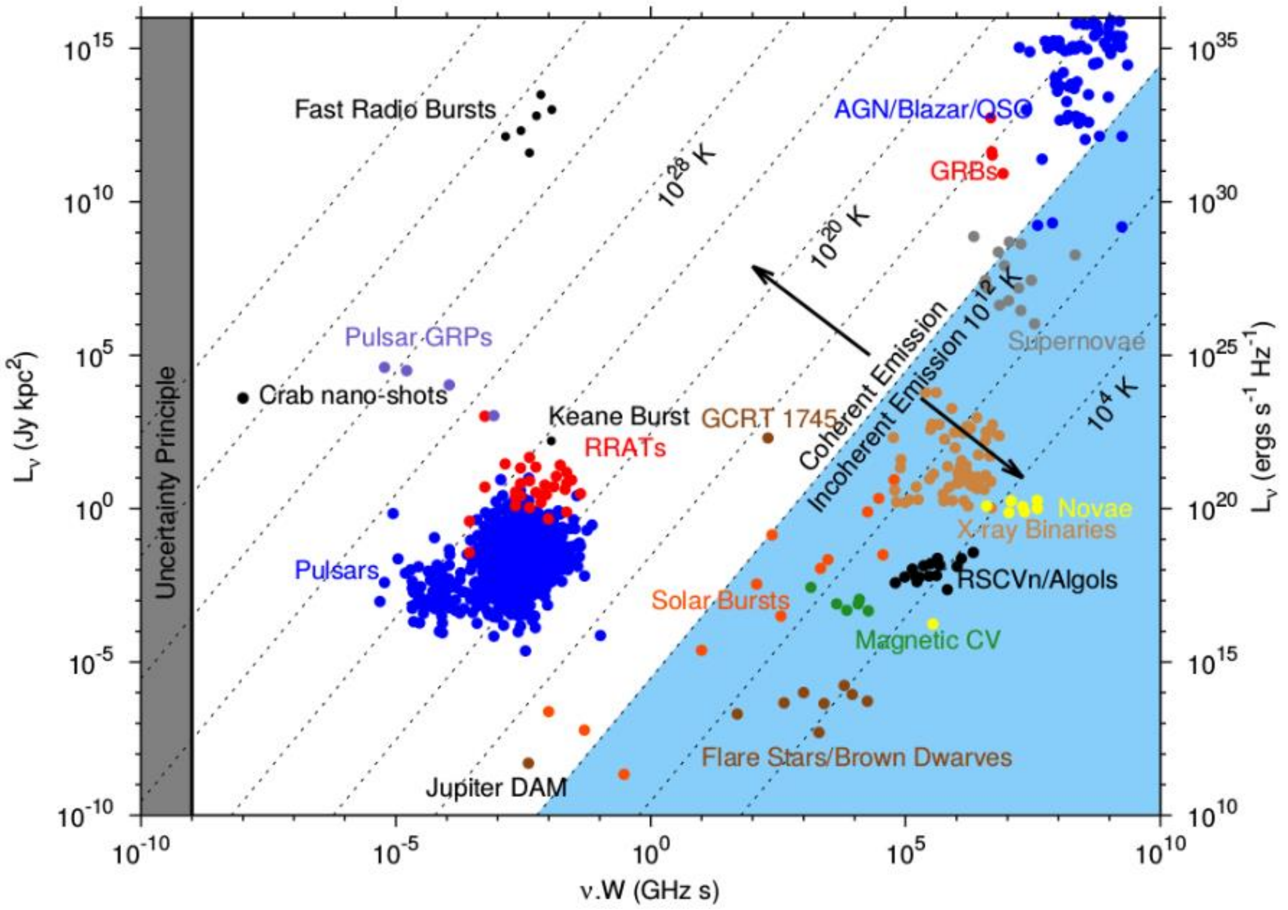
Inverse-Compton losses
very strongly cool the relativistic electrons
if the source brightness temperature
exceeds T_b ~10^{12} K in the rest frame of the source

see astro-ph/0611667

$$\frac{L_{\rm IC}}{L_{\rm s}} = \left(\frac{T_{\rm B}}{T_{\rm thresh}}\right)^5\left[1+\left(\frac{T_{\rm B}}{T_{\rm thresh}}\right)^5\right]$$

$$\left(\frac{\nu_m}{10^9 Hz}\right)\left(\frac{T_B}{10^{12} K}\right)^5 = 1$$

Brightness temperature



Many different types of transient sources are already detected at radio wavelengths.

However, detection of very short and non-repeating flares of unknown sources without identification at other bands is a very complicated task.

Rotating Radio Transients (RRATs) – millisecond radio bursts from neutron stars, - have been identified in 2006.

In 2007 the first example of a new class of millisecond radio transients have been announced: the first extragalactic millisecond radio burst.

1507.00729, 1411.1067

Учетная темпе.

$$\text{Учет} = \frac{\text{поток}\,(V_1 \div V_2)}{\text{поток}\,(V_3 \div V_4)}$$