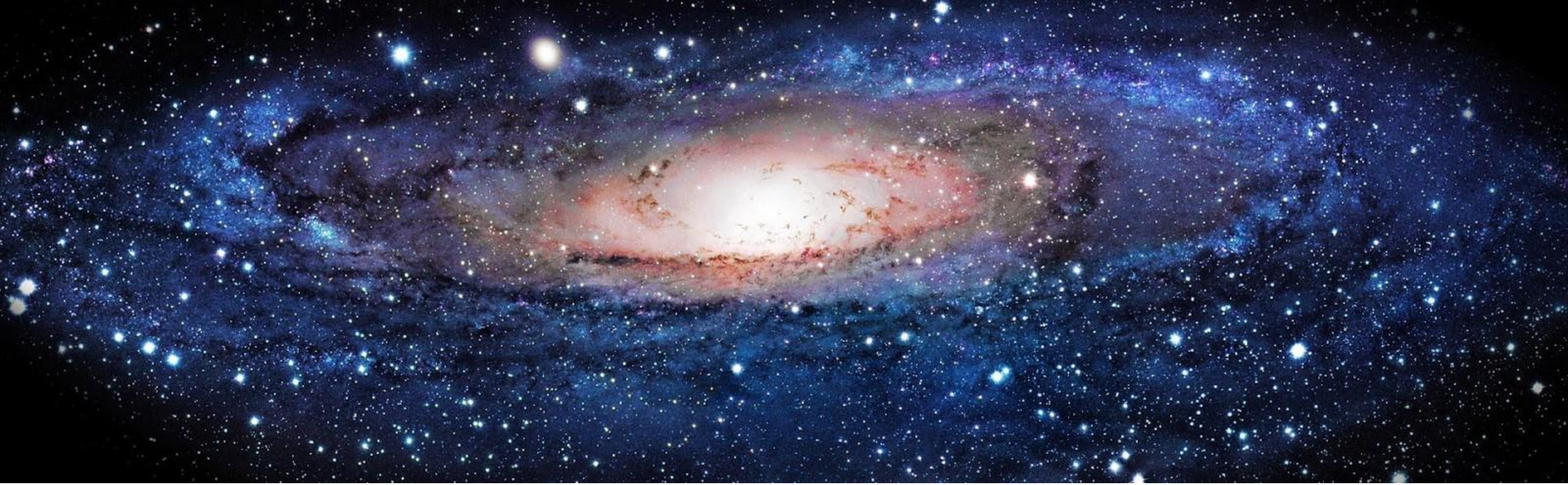
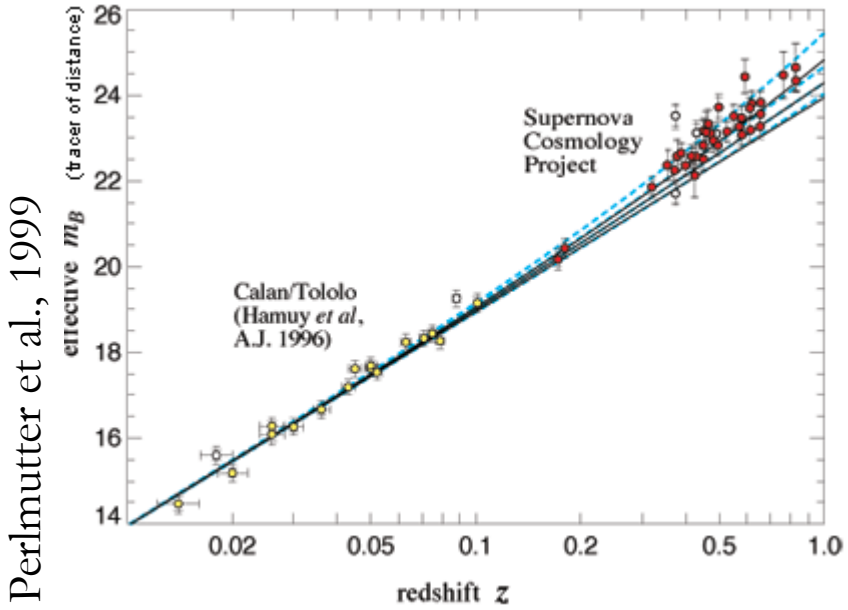
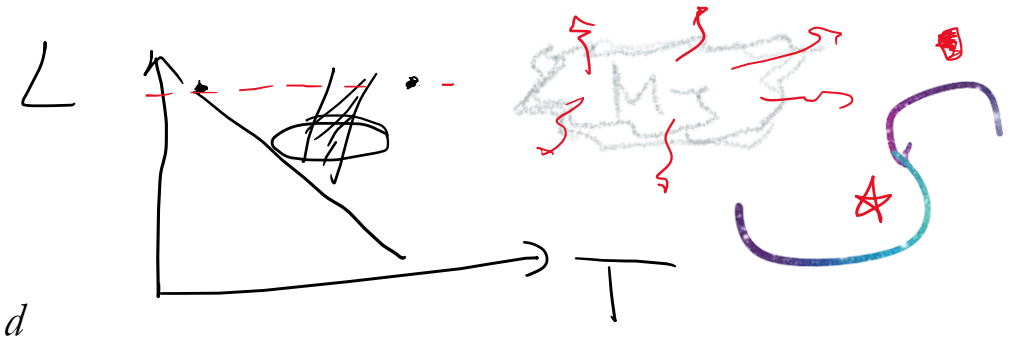


Майнор. Астрофизика.

Семинар 12. (02.12)

Закон Хаббла

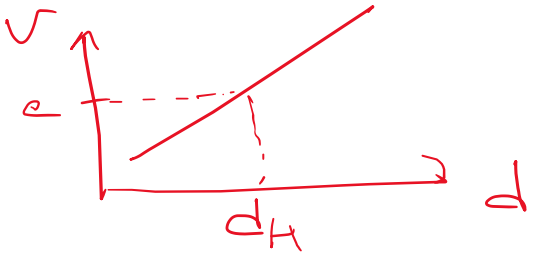




$v = H\,d$

H – постоянная Хаббла.

72 +/- 2 км/с/Мпк



$$ds^2 = c^2dt^2 - a(t)^2dl^2.$$

$$H = \dot{a}/a$$

→ Фридман (1922)

$a^2(t)dl^2 = dd^2$

$dd = a(t)dl$

$d = a(t)\int dl = a\chi$

$v = dd/dt = (da/dt)\chi = [(da/dt)/a](a\chi) = Hd$

Релят. материя: $\rho_r(z) = \rho_r(0)(1+z)^4,$

Нерелят. материя: $\rho_m(z) = \rho_m(0)(1+z)^3,$

Кривизна: $\rho_c(z) = \rho_c(0)(1+z)^2,$

Вакуум: $\rho_\Lambda(z) = const$

$\Rightarrow$

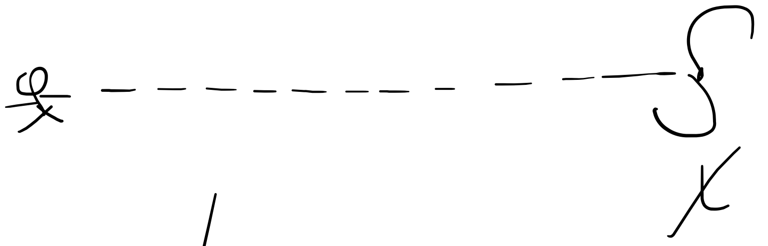
$$H^2(z) = H_0^2\left(\Omega_r(1+z)^4 + \Omega_m(1+z)^3 + \Omega_c(1+z)^2 + \Omega_\Lambda\right)$$

$$0 = ds^2 = c^2 dt^2 - a^2(t)\, d\chi^2$$

(под $0 = ds^2$: свет; под $a^2(t)\,d\chi^2$: dr^2)

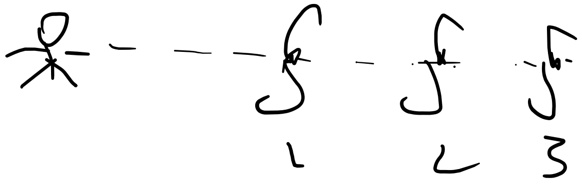
r – собств.

$$r = a\chi \qquad r = \int a\, d\chi = a\int d\chi = a\chi$$



$$v = \frac{dr}{dt} = \frac{d(a\chi)}{dt} = a\frac{d\chi}{dt} + \chi\frac{da}{dt} = \frac{1}{a}\frac{da}{dt}\,\frac{\chi a}{r} = H\cdot r$$

(к dr: наст.; к dt: наше; под $\frac{d\chi}{dt}$: $= 0$; под $\frac{1}{a}\frac{da}{dt}$: H)



$$v_{13} = v_{12} + v_{23} = Hr_{12} + Hr_{23} = Hr_{13}$$

$$v = a\frac{d\chi}{dt} + \chi\frac{da}{dt}$$

(под $a\frac{d\chi}{dt}$: пекуляр; под $\chi\frac{da}{dt}$: косм.)

$v_{M31} < 0$ – синее смещ.

$|v_{пек}| > |v_{косм}|$

$$z_{наб} = z_{косм} + z_{доп.}$$

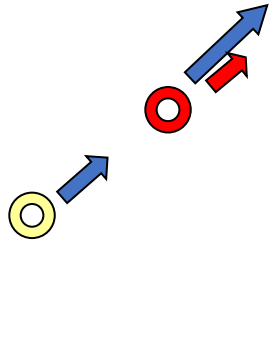
(под $z_{косм}$: крупн.; под $z_{доп.}$: крупн/мал)

Космологическое красное смещение

а) Вблизи

v~d~z Похоже на доплер! Почему?

- Закон Хаббла.
  Пространство расширяется везде одинаково.
  Скорость пропорциональна расстоянию.
- Темп расширения меняется медленно.
  Поэтому для близких галактик он примерно одинаков.

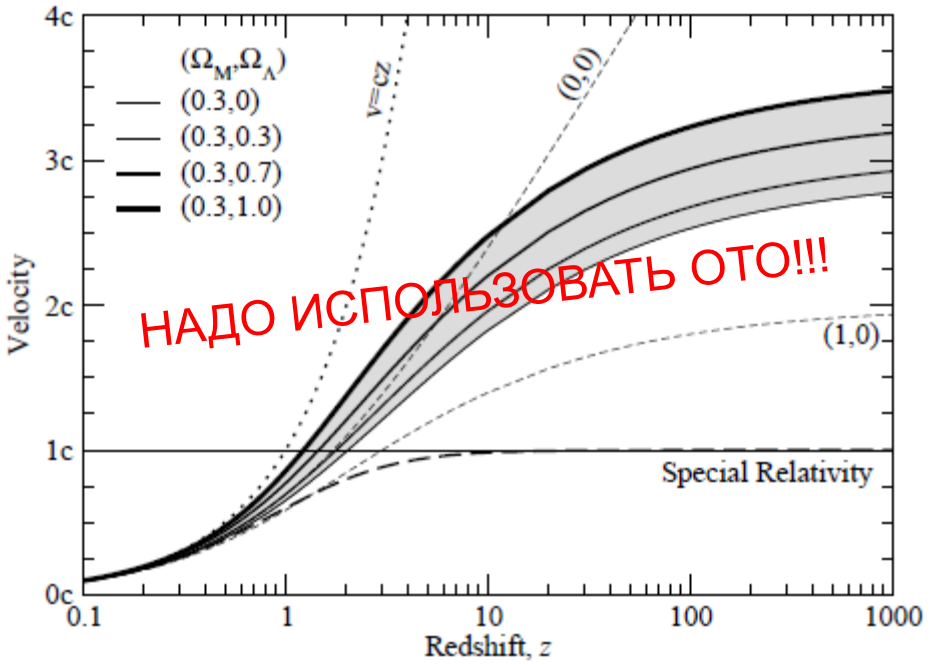


б) Вдали

Важно понимать, что вдали скорость расширения нельзя вычислить по красному смещению, используя релятивистский эффект доплера.

GR $$v_{\rm rec}(t,z) = \frac{c}{R_0}\dot{R}(t)\int_0^z \frac{dz'}{H(z')},$$

SR ~~$$v_{\rm rec}(z) = c\frac{(1+z)^2-1}{(1+z)^2+1}.$$~~



Tamara Davis et al.

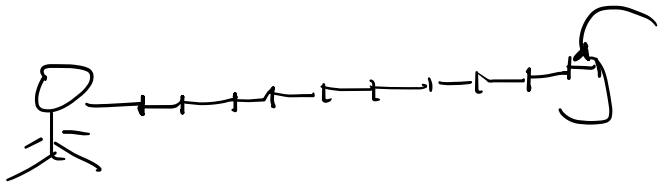
$$d = \frac{c}{(1-\alpha) H_0} \left[(1+z)^{1-\alpha} - 1 \right]$$

$z \approx 0$

ряд Тейлора в ϕ

$$f(z \approx 0) = f(0) + \frac{f'(0)}{1!} \cdot z + \dots$$

$$d(z=0) = 0$$



$$d' = \frac{c}{\cancel{(1-\alpha) H_0}\ \cancel{(1-\alpha)}} (1+z)^{-\alpha}$$

$$d'(0) = \frac{c}{H_0}$$

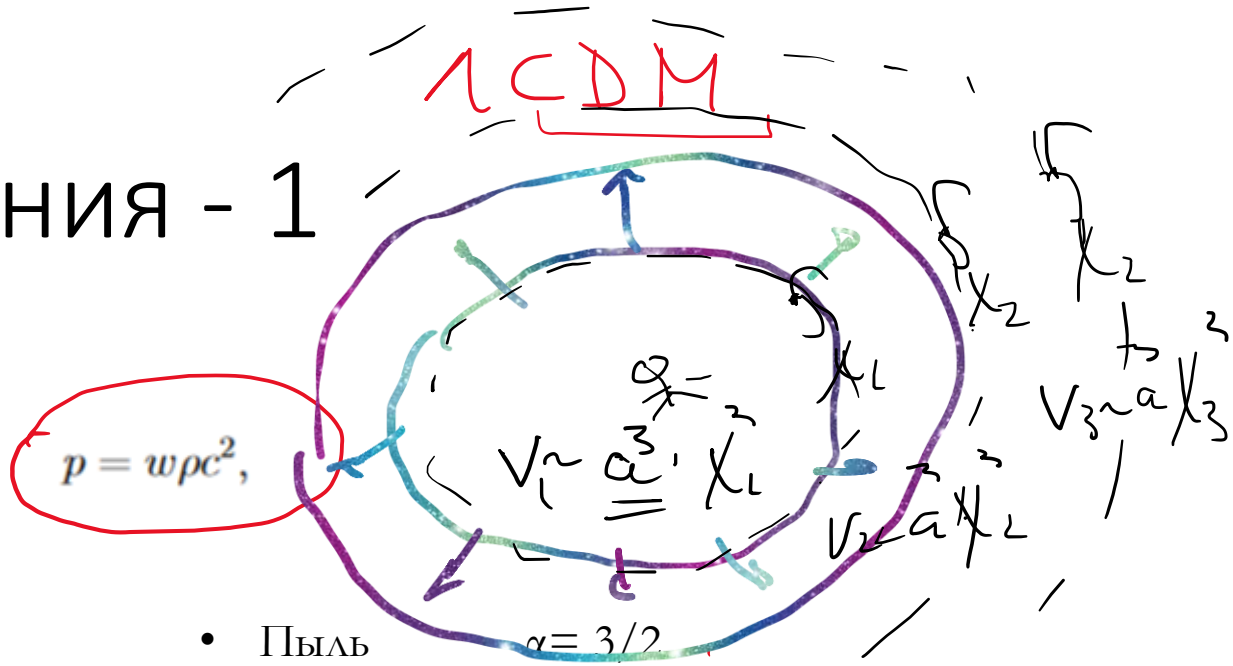
$$d(z \approx 0) = \frac{c}{H_0} \cdot z$$

$$\frac{c \cdot z}{v} = H_0 \cdot d \qquad z \approx 0$$

Формулы для расширения - 1

Разные среды:

- Вещество (пыль, p=0)
- Излучение ($p \sim T^4$)
- Космологическая постоянная ($p=-\rho c^2$)



$$a \sim t^{1/\alpha}, \qquad \alpha = 3(w+1)/2.$$

$$H = \dot{a}/a = 1/(\alpha t)$$

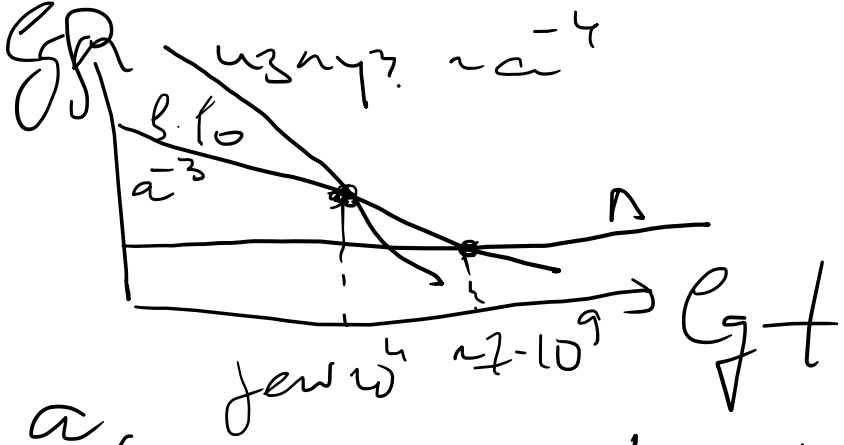
- Пыль $\alpha = 3/2$
- Излучение $\alpha = 2$
- Косм. пост. $\alpha = 0$

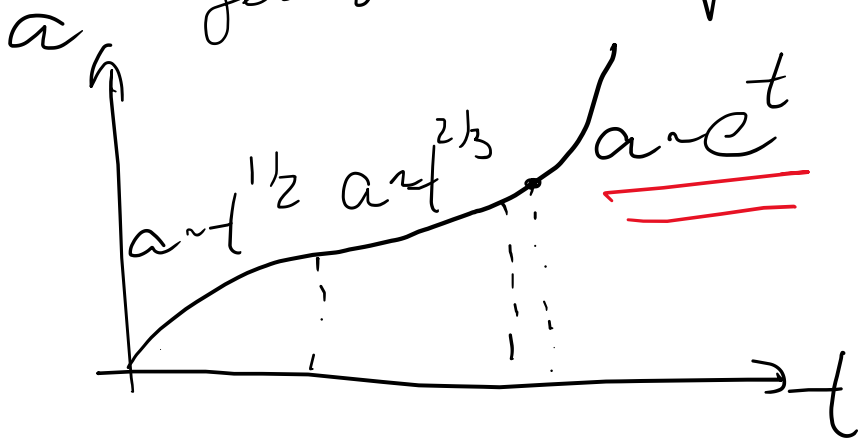
$$1 + z(t) = a(t_0)/a(t), \qquad H = H_0(1+z)^\alpha$$

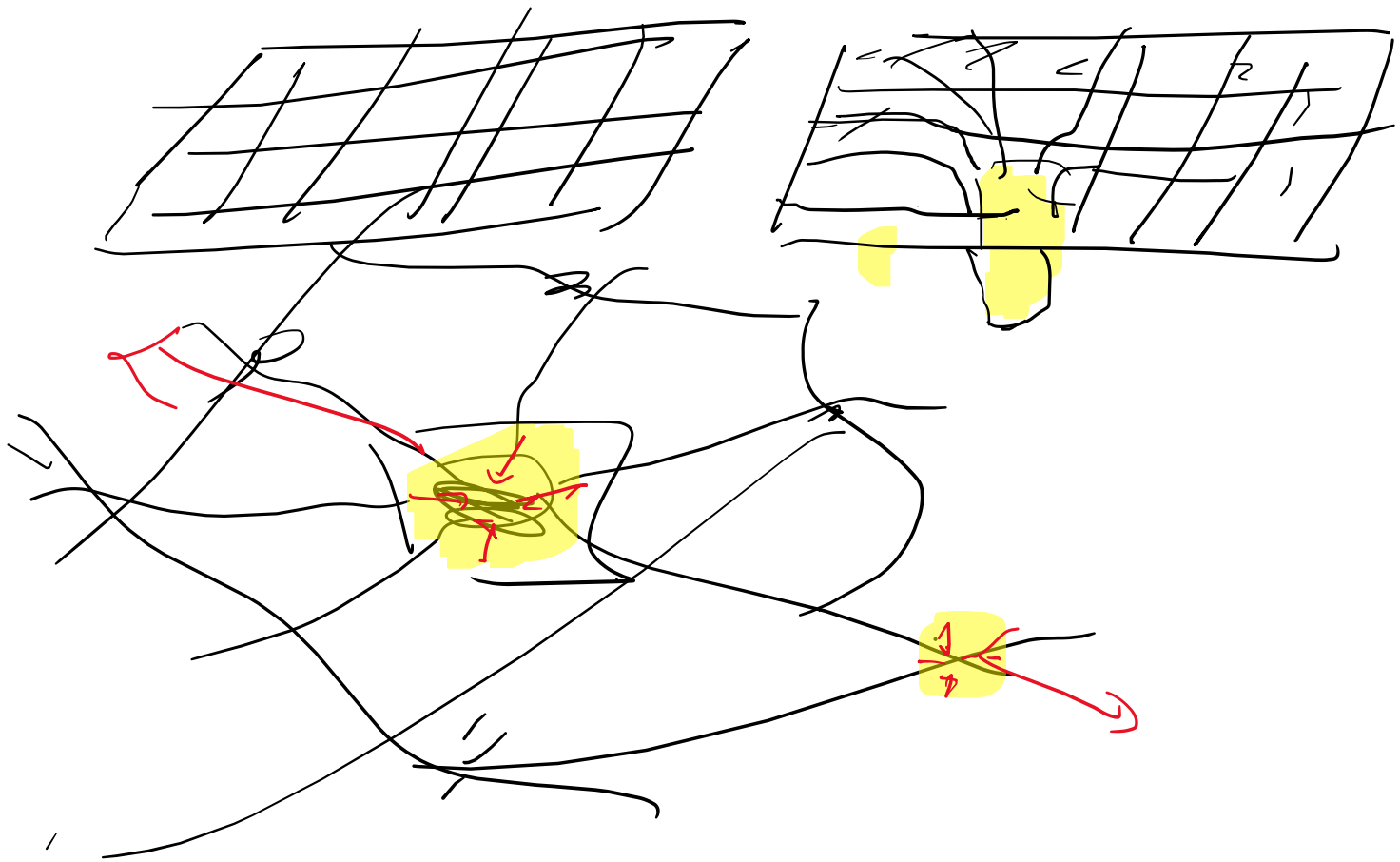
Для света ($ds^2=0$):

$$\chi = \frac{c}{a(t_0)H_0}\int_0^z \frac{dz}{H(z)} = \frac{c}{a(t_0)H_0}\frac{1}{1-\alpha}[(1+z)^{1-\alpha} - 1].$$

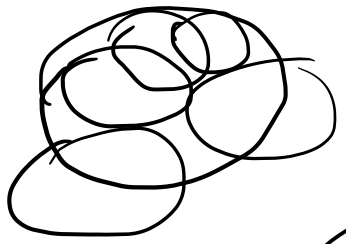
$$H^2(z) = H_0^2\left(\Omega_r(1+z)^4 + \Omega_m(1+z)^3 + \Omega_c(1+z)^2 + \Omega_\Lambda\right)$$



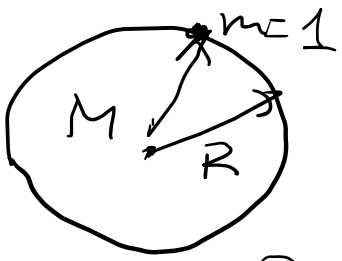




Засов, Постнов сер ~ 460+



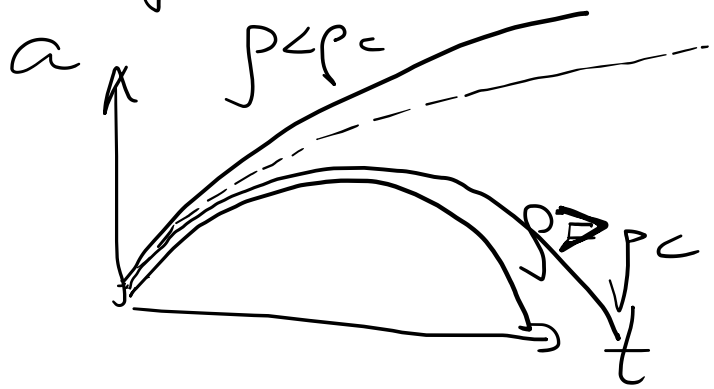
$$\frac{1}{2}\left(\frac{dR}{dt}\right)^2 - \frac{GM}{R} = \text{const}$$



$$\frac{1}{2}\,\frac{1}{R^2}\left(\frac{dR}{dt}\right)^2 - \frac{GM}{R^3} = \frac{\text{const}}{R^2} \quad \rightarrow \text{кривизна}$$

$M = \frac{4}{3}\pi R^3 \rho$

$R \sim a \rightarrow \frac{1}{2}H^2 - \frac{4}{3}\pi G\rho_c = 0 \quad - - - \quad \rho_c = \frac{3H^2}{8\pi G}$



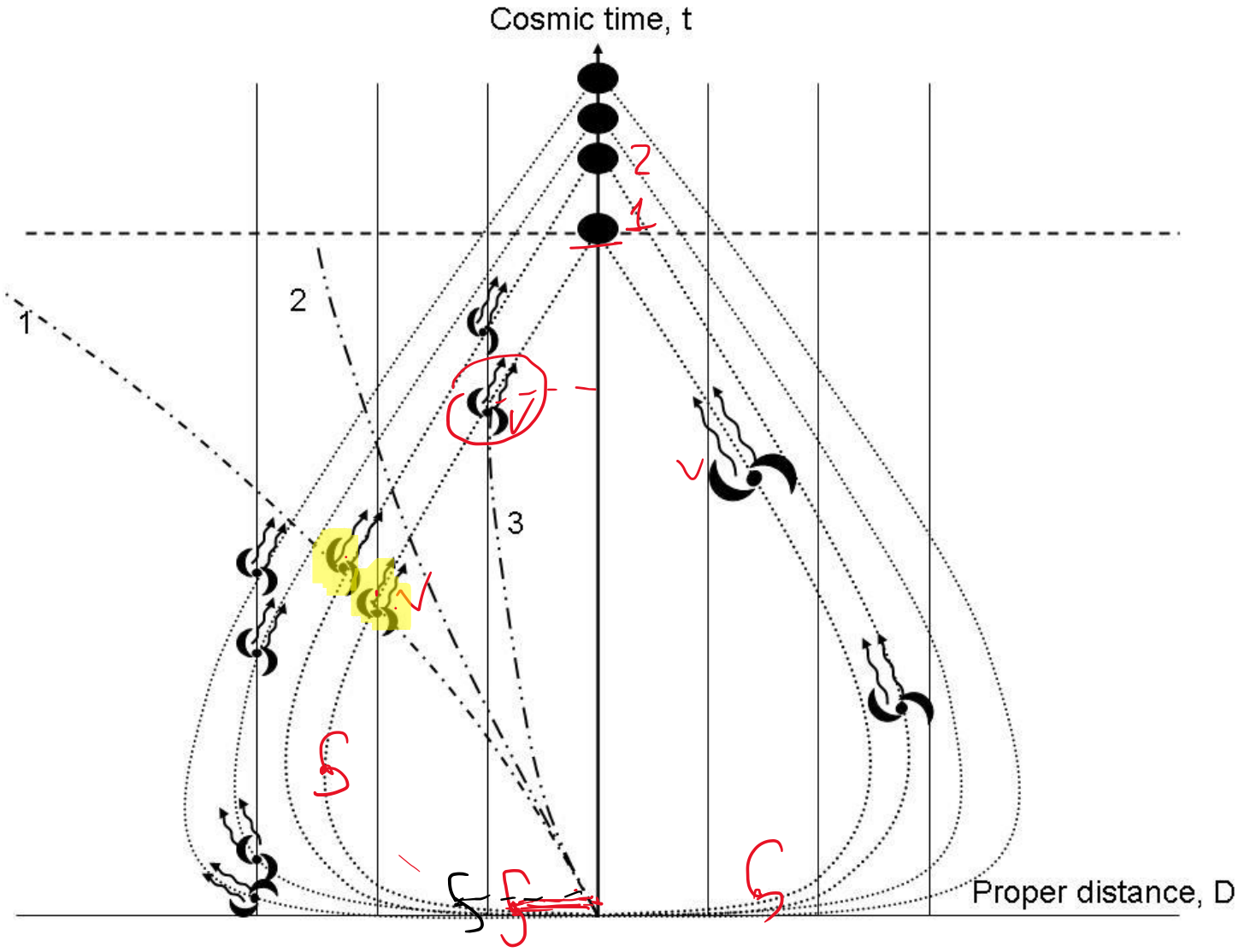
$$\Omega_m = \rho_m / \rho_c$$

$$H^2 = H_0^2\left[\Omega_m^{(1)} + \Omega_r^{(1)} + \Omega_\Lambda^{(1)} + \Omega_k^{(1)}\right]$$

$$1 = \Omega_m + \Omega_\Lambda + \Omega_c \qquad \Omega_c = 1 - \Omega_m - \Omega_\Lambda - \Omega_r$$

$$H^2 = \frac{8\pi}{3}G\rho + \frac{\Lambda c^2}{3} - \left(\frac{kc^2}{a^2}\right)$$

Смотрим вдоль светового конуса, а движемся по мировым линиям



Формулы для расширения - 2

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2} + \frac{\Lambda c^2}{3},$$

$$\rho = \rho_m(a) = \frac{\rho_{m_0}}{a^3}, \qquad \rho_c = \frac{3H^2}{8\pi G};$$

$$\Omega_m \equiv \frac{\rho_{m_0}}{\rho_c} = \frac{8\pi G}{3H_0^2}\rho_{m_0};$$

Кривизна: k= -1, 0, 1

$$dl^2 = \frac{dr^2}{1-kr^2} + r^2(\sin^2\theta d\varphi^2 + d\theta^2)$$

$$\Omega_k \equiv \frac{-kc^2}{(a_0 H_0)^2}$$

$$\Omega_\Lambda \equiv \frac{\Lambda c^2}{3H_0^2},$$

$$H^2(z) = H_0^2\left(\Omega_M(1+z)^3 + \Omega_k(1+z)^2 + \Omega_\Lambda\right).$$

Параметр замедления q

$$q = -\left(1 + \frac{\dot{H}}{H^2}\right).$$

q=0 t=1/H

q=1/2 t=(2/3H)

Сейчас q<0.

$$q = -\frac{\ddot{a}a}{\dot{a}^2} = \left[H = \frac{\dot{a}}{a}\right] = -\left(1 - \frac{\dot{H}}{H^2}\right)$$

$q = const \quad \ddot{a}(t) \sim a^{-q}$

$$H(t) = a(t)^{-q-1}$$

$q = -1 \Rightarrow H = const \quad \underline{a(t) = a_0 \cdot e^t}$

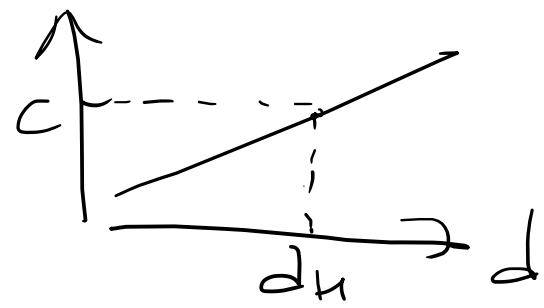
$q > 0 \quad \ddot{a} < 0 \quad \dot{a}$ - уменьш.

$q < 0 \quad \ddot{a} > 0 \quad \dot{a}$ - растет

$$\boxed{q = -0{,}55}$$

≈ Плоская
$\Omega \cong 1$
$\Omega_m = 0{,}3 \downarrow$
$\Omega_\Lambda = 0{,}7 \uparrow$

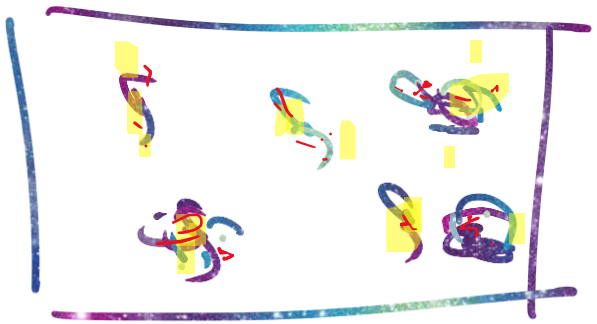
Сфера Хаббла



$$v = H \cdot d$$

$$c = H \cdot d_H$$

$$d_H = c/H$$



$$q = -\frac{\ddot{a}\,a}{\dot{a}^2}$$

$$H = \dot{a}/a$$

$$(d_H)' = \left(\frac{c}{H}\right)' = c\left(\frac{1}{H}\right)' = -\frac{c\dot{H}}{H^2}$$

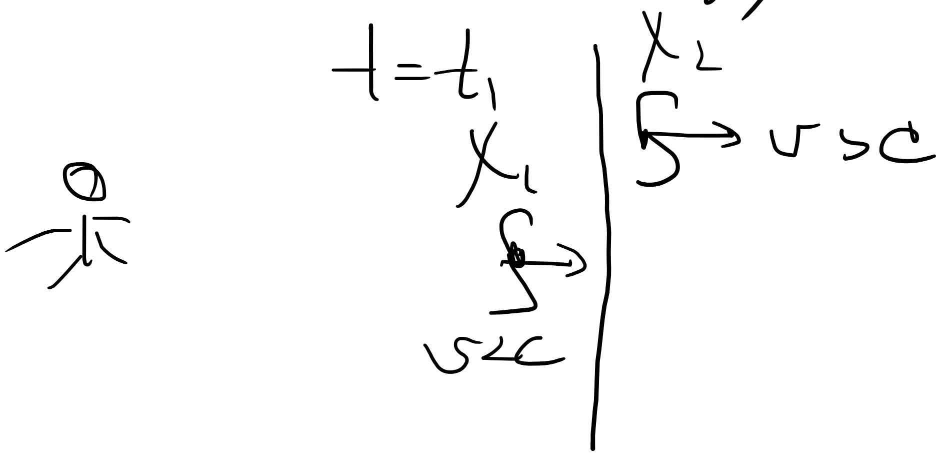
$$\dot{H} = \left(\frac{\dot{a}}{a}\right)' = \frac{\ddot{a}a - \dot{a}\dot{a}}{a^2} = \frac{\ddot{a}a - \dot{a}^2}{a^2}$$

$$v_H = (d_H)' = -c\,\frac{\ddot{a}a - \dot{a}^2}{a^2} \cdot \frac{a^2}{\dot{a}^2} = c\,\frac{\dot{a}^2 - \ddot{a}a}{\dot{a}^2} = c\left(1 - \frac{\ddot{a}a}{\dot{a}^2}\right) =$$

$$= c(1+q)$$

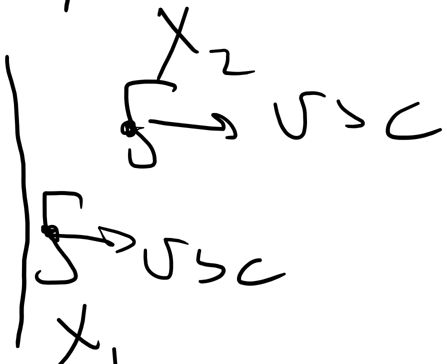
$$U_H = c(1+q)$$

$t = t_1$



$$d_H = c/H = a\chi_H$$

$t = t_2 > t_1$



1) Собств. расст. до сферы Х растет

Наша В.

$$q = -0.55$$

2) Галактики вылетают за сферу Хаббла

Т.е. сопутств. расст. уменьш.