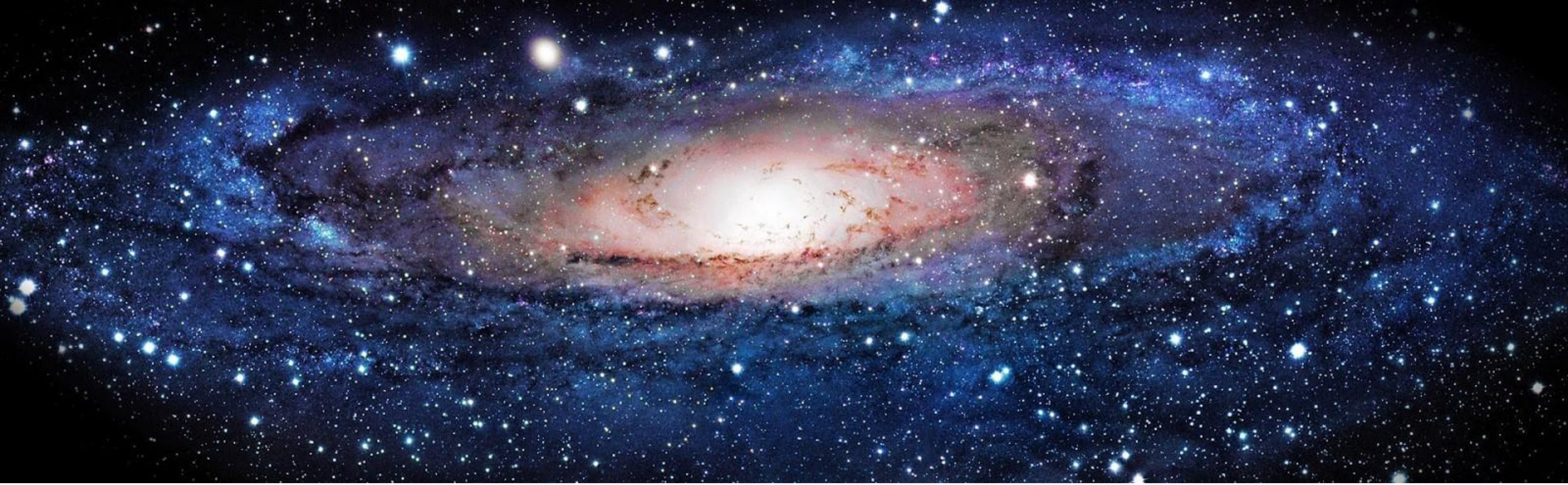


Майнор. Астрофизика.

Семинар 6. (07.10)

Разбор к.р.

$f = \frac{L}{4\pi d^2}$

1. $f_{lim} = f_1 = f_2$

$L_1 = L_0$

$L_2 = 0.1 L_0$

$d_1 = 300$ pc

$d_2 - ?$

$$\frac{L_0}{4\pi d_1^2} = \frac{0.1 L_0}{4\pi d_2^2}$$

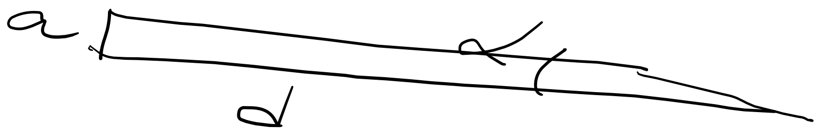
$d_2 = 300 \cdot \sqrt{0.1}$ pc ≈ 95 pc

2. $a = 10^5$ св. лет $\approx 10^{23}$ см

$D = 30$ см $\quad \lambda = 0.6 \cdot 10^{-6}$ м $= 0.6 \cdot 10^{-4}$ см

$\theta \approx 1{,}22\, \lambda / D$

$\alpha \approx a/d$



$\theta = \alpha \qquad \frac{a}{d} = 1{,}22 \frac{\lambda}{D} \qquad d = 4 \cdot 10^{28}$ см $= 13$ Gpc

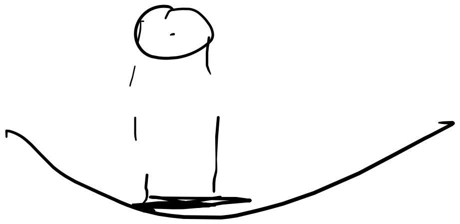
3. $P^2 = \frac{4\pi^2}{G(M_1 + M_2)} a^3 \qquad \frac{P_1^2}{P_2^2} = \frac{(M_1^{(2)} + M_2^{(2)})}{(M_1^{(1)} + M_2^{(1)})} = \frac{4}{2}$

4. $R_* = 0.5 R_\odot$ $\qquad \frac{\Delta F}{F} = 0.01 \qquad R_p - ?$

$$\frac{\Delta F}{F} = \frac{S_p}{S_*} = \frac{R_p^2}{R_*^2} = 0{,}01$$

$$R_p = 0{,}1 R_* = 0{,}05 R_\odot$$

5. $M_* = 0{,}5 M_\odot$ $\qquad a = 0{,}1$ ае.

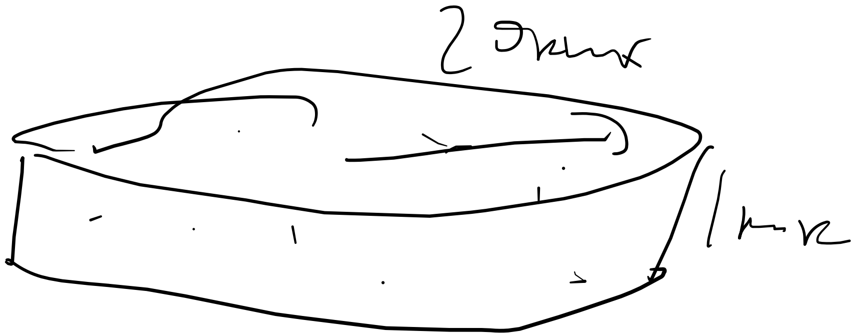


$$v_{orb} = \frac{2R_*}{T} = \sqrt{\frac{GM}{a}}$$

$$T = \sqrt{\frac{a}{GM}}\, 2R_* = 3\cdot 10^{-7} R_* =$$

$$= 5{,}8 \left(\frac{R_*}{R_\odot}\right) \qquad R \sim M^{0{,}8}$$

6.



$$V = \pi R^2 h$$

$$V = N_* \cdot V_*$$

$$V_* = \frac{4}{3}\pi \left(\frac{1}{2} d_*\right)^3$$

$$\pi R^2 h = N_* \frac{4}{3}\pi \frac{1}{8} d_*^3$$

7. $$\frac{2\pi R}{P_{lim}} = \sqrt{\frac{2GM}{R}}$$

$$P_{em} \sim \overset{5}{0},001$$

8. $$\theta = 1{,}22\,\lambda/D$$

$$\alpha = R_*/d$$

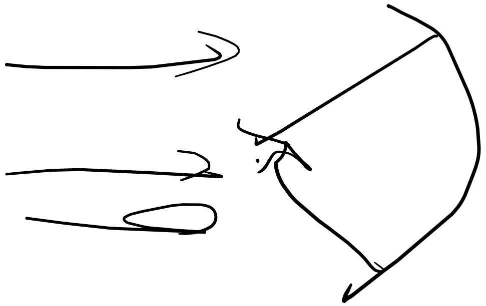
$$\theta = \alpha$$

$$d = \frac{R_* \cdot D}{1{,}22\,\lambda} = 2{,}7 \cdot 10^{17}\ \text{cm}$$

$$\simeq 0{,}1\ \text{pc}$$

9.

$$f = \frac{L}{4\pi d^2}$$

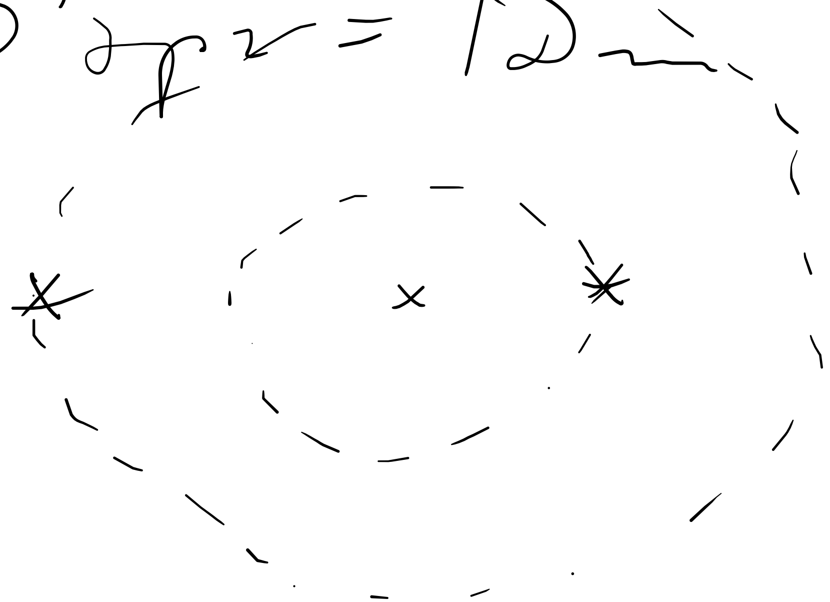


$$[f] = \frac{\text{эрг}}{\text{см}^2 \cdot \text{с}}$$

$$W = f \cdot S = \frac{L}{4\pi d^2}\pi R^2 = 0.38\ \text{эрг}/\text{с}$$

10^7 эрг = 1 Дж $\qquad 0.38 \cdot 10^{-7}$ Вт

10.



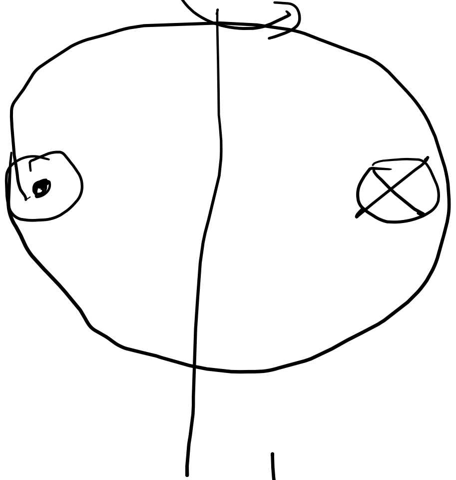
$$a_1 = a_2 \cdot 4$$

$$a_1 = \frac{1}{5} a = 0{,}4 \text{ ае}$$

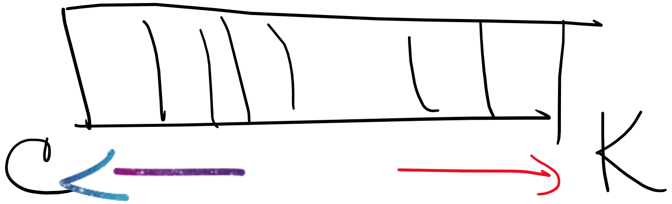
$$a_2 = 1{,}6 \text{ ае}$$

$$\Delta_{1,2} = 2a_{1,2}\,\alpha_{1,2} = \frac{2a_{1,2}}{20 \text{ пк}} = \begin{matrix} 0{,}04 \\ 0{,}16 \end{matrix}$$

1.



$$v = \frac{2\pi R}{P} = \frac{2\pi \; 6{,}96\cdot 10^{10}\,\text{см}}{30\cdot 3600\cdot 24} = 1{,}7\cdot 10^{5}\,\frac{\text{см}}{\text{с}} = 1{,}7\,\frac{\text{км}}{\text{с}}$$

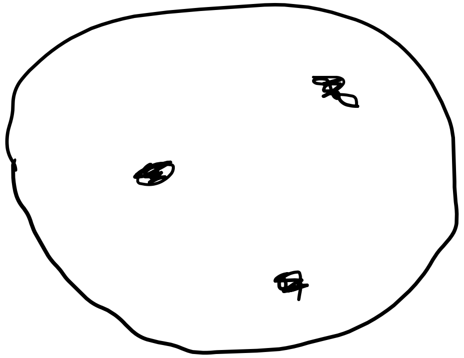


$$\frac{\Delta\lambda}{\lambda_0} \approx \frac{v}{c}$$

$$\lambda_0 \approx 5500\,\text{Å}$$

$$\Delta\lambda = \lambda_0 \frac{v}{c} \approx 0.03\,\text{Å}$$

2.



$$L = S \cdot \sigma T^4, \quad S = 4\pi R^2$$

$$\sigma = 5{,}67\cdot 10^{-5}\ \text{СГС}$$

$$\left(\frac{T_p}{T_d}\right)^4 = \left(\frac{4300\,K}{5800\,K}\right)^4 = \frac{1}{2{,}76}$$

3. R, M, G

$$P = P(R, M, G) \qquad E = \underset{\sim 1}{\underbrace{\ }} G^x M^y R^z$$

$$F = G\frac{M_1 M_2}{r^2} \qquad \left(\frac{г \cdot см}{с^2}\right)$$

$$[E] = [G]^x [M]^y [R]^z$$

$[E]$: $г \cdot см^2 \cdot с^{-2}$, $[M]$: $г$, $[R]$: $см$

$$[G] = г^{-1} см^3 с^{-2}$$

$$г^1 \cdot см^2 \cdot с^{-2} = г^{-x} см^{3x} с^{-2x} г^y см^z = г^{y-x} см^{3x+z} с^{-2x}$$

$$\begin{aligned} 1 &= y - x \\ 2 &= 3x + z \\ -2 &= -2x \end{aligned} \qquad \begin{cases} y = 2 \\ z = -1 \\ x = 1 \end{cases}$$

$$E = \frac{GM^2}{R} \qquad P = \frac{E}{V} \qquad V \sim R^3$$

$$P = \frac{E}{V} = \frac{E}{R^3} = \frac{GM^2}{R^4} \qquad P_{\odot} \simeq 10^{10}\ атм$$

$$PV = \frac{M}{\mu}RT \qquad \mu = 10^{-3} \; [Cu]$$

$$R = 8.34$$

$$V \sim R^3 \qquad \rho \sim \frac{M}{R^3} \qquad P = \frac{GM^2}{R^4}$$

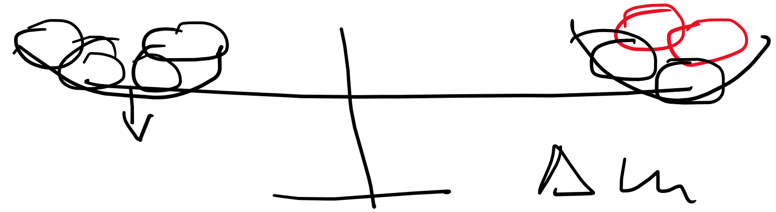
$$T = \frac{PV\mu}{MR} = G\frac{M\mu}{RR} \sim 2 \cdot 10^7 K$$

4. $\nu_{\odot}$ p-p cycle $4\,{}^1_1H \rightarrow {}^4_2He + 2e^+ + 2\nu_e$

$m_p = 1{,}00728$ a.e.m

$aem = \frac{1}{12} m_{C^{12}} = 1.66 \cdot 10^{-24}$ г

$m_\alpha = 4{,}001506$ aem



Δm

$$\mathcal{E} = \Delta m c^2$$

$$\eta = \frac{\mathcal{E}}{mc^2} = \frac{4m_p - m_\alpha}{4m_p} = 0,007$$

$$\eta_{\%} = 0,7\%$$

$$\mathcal{E} = (4m_p - m_\alpha)\,c^2 = 25,7\ \text{МэВ}$$

$$1\ \text{эВ} = 1,6 \cdot 10^{-12}\ \text{эрг}$$

$$f_\nu = \dot{N}_\nu / (4\pi r^2)$$

?

$$\dot{N}_\nu = 2 \cdot \dot{N}_{\text{реак}}$$

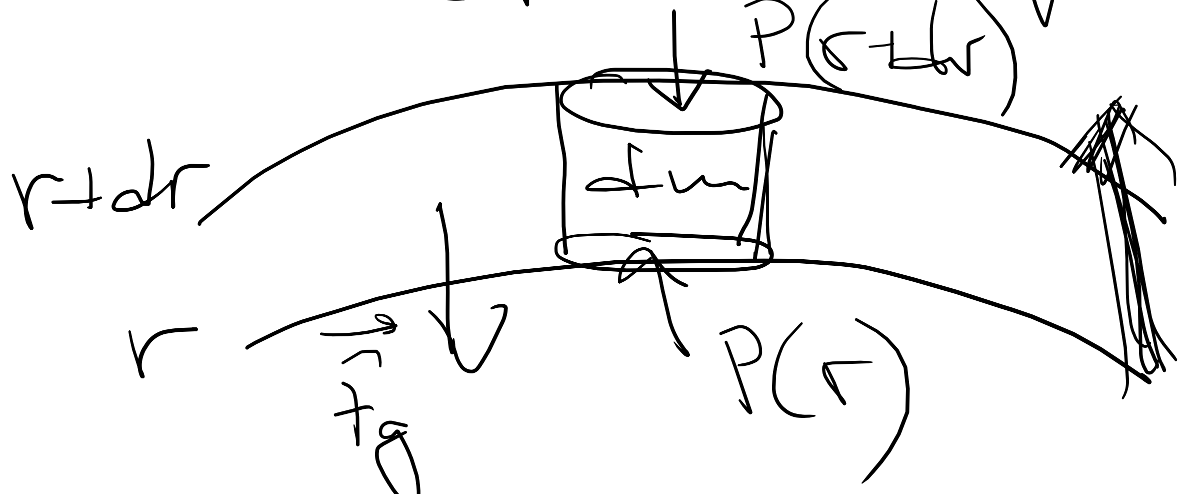
$$\dot{N}_{\text{реак}} = \frac{L_\odot}{\mathcal{E}} = \frac{2 \cdot 10^{33}\ \text{эрг}}{25,7 \cdot 10^{6} \cdot 1,6 \cdot 10^{-12}} = \ldots$$

$$\dot{N}_\nu = \frac{2 \cdot 2 \cdot 10^{33}}{2,57 \cdot 1,6 \cdot 10^{-5}} \simeq 10^{38}\ \frac{\text{шт}}{c}$$

$$f_\nu = \frac{\dot{N}_\nu}{4\pi (1\ \text{ае})^2} = 3,5 \cdot 10^{10}\ \frac{\text{шт}}{\text{см}^2\,\text{с}}$$

5. Гидростат. равн.

$$-\frac{1}{\rho}\frac{dP}{dr} = \frac{GM(r)}{r^2}$$



$$F_g = -\frac{GM(r)\,dm}{r^2}$$

$$F_p = P(r)\,dS - P(r+dr)\,dS = -\frac{dP}{dr}\,dr\,dS$$

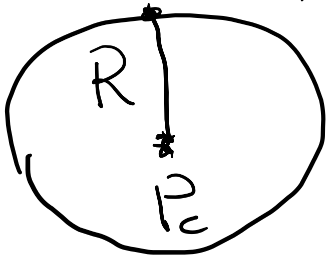
$$F_g + F_p = 0$$

$dm = \rho \cdot dV$

$dV = dS \cdot dr$

$dm = \rho\, dr\, dS$

$$\frac{GM(r)\rho\, dr\, dS}{r^2} = -\frac{dP}{dr}\,dr\,dS$$



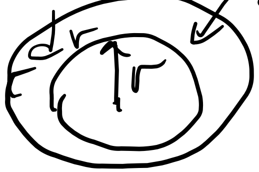
$M(R) = M$

$$\rho \sim \frac{M}{R^3}$$

$$\frac{dP}{dr} = -\frac{GM(r)}{r^2}\rho \Rightarrow \frac{P}{R} = \frac{GM}{R^2}\rho$$

$$P \approx \frac{GM^2}{R^4}$$

6. Вириал для звёзд



$$dm = \rho \, 4\pi r^2 dr$$

$$4\pi r^2 \frac{dP}{dm} = -\frac{GM(r)}{r^2}$$

$$4\pi r^3 dP = -\frac{GM(r)\,dm}{r}$$

$$U \equiv -\int_0^M \frac{GM(r)}{r} dm = \int_0^M \underbrace{4\pi r^3}_{3V} dP = \int_0^M \underbrace{3V}\, d\underbrace{P}$$

по частям

$$= 3\underset{=0}{V}\underset{=0}{P}\Big|_0^M - \int_0^M P\,d(3V) = -3\int_0^M P\,dV$$

$$\boxed{U = -3\int P\,dV}$$

$$P = K\cdot\rho^{\gamma}$$

ε — уд. вн-я энерг. (на ед. массы)

$$\varepsilon = \frac{P}{(\gamma-1)\rho}$$

$$Q = \int \varepsilon\,dm = \int \varepsilon\rho\,dV$$

$$-3\int P\,dV = -3(\gamma-1)\int \frac{P}{\gamma-1}\,dV = -3(\gamma-1)\int \frac{P}{(\gamma-1)\rho}\,\rho\,dV$$

$$= -3(\gamma-1)\underbrace{\int \varepsilon\rho\,dV}_{Q} = -3(\gamma-1)\,Q$$

$$U = -3(\gamma - 1)Q$$

Ид. одноат. газ $\quad \gamma = 5/3$

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$$U = -2Q \qquad E_p = -2E_k$$

↑ гр. пот. $\qquad$ ↑ тепл. эн.

СГС; $\mu = 0.6$

7: $$Q = \frac{3}{2}kT \cdot N = ~~\frac{3}{2}kT \frac{M}{\mu}~~ \frac{3}{2}\frac{M}{\mu}RT$$

$$U = -\frac{3}{5}\frac{GM^2}{R}$$

$$\frac{3}{5}\frac{GM^2}{R} = 2 \cdot \frac{3}{2}\frac{M}{\mu}RT$$

$$\langle T \rangle = G \frac{M\mu}{\mathcal{R} R} \frac{1}{5} \sim 10^7 K$$