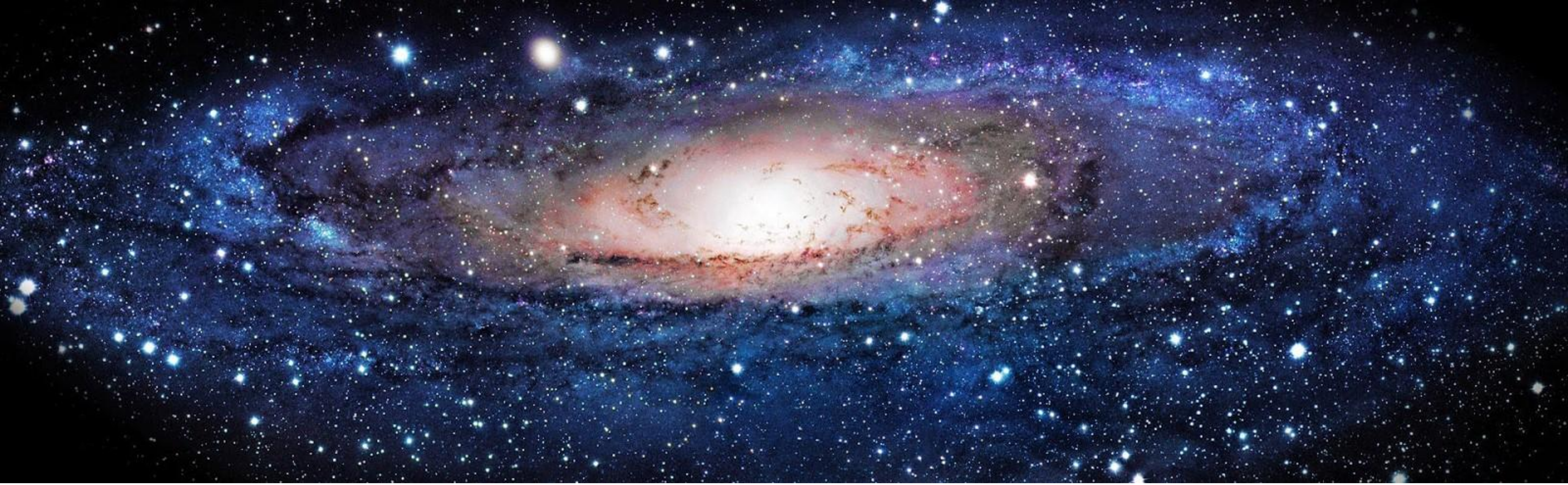
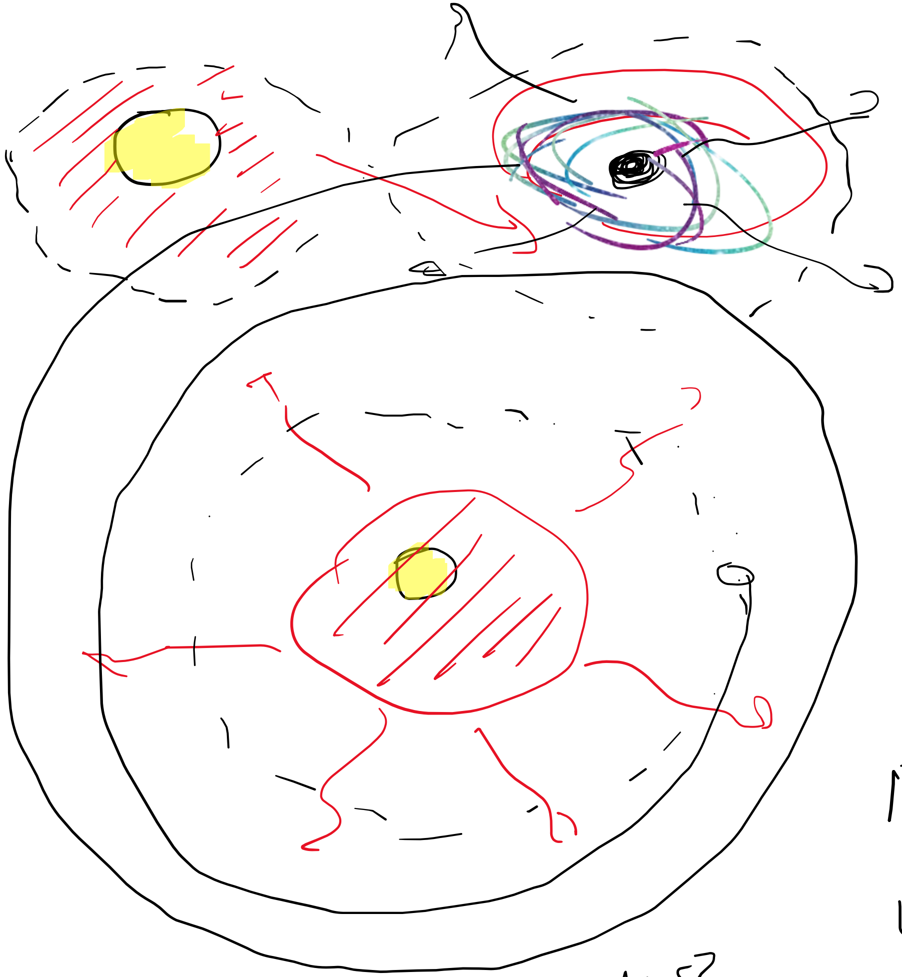


Майнор. Астрофизика.

Семинар 8. (28.10)

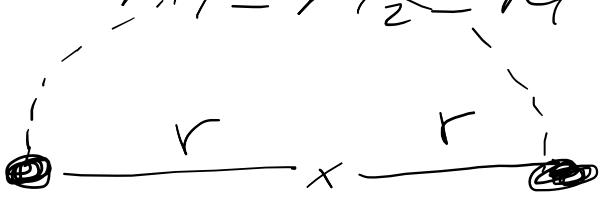


$$E_{TOT} = E_K + E_P < 0$$

$E_K > 0$, $E_P < 0$

① Распад гл. сист.

$M_1 = M_2 = M$



$$2a = r_{min} + r_{max} = 2r + 2r = 4r$$

$$a = 2r \qquad r = \frac{1}{2}a$$

$$P^2 = \frac{4\pi^2}{G(M_1+M_2)}a^3 = \frac{4\pi^2}{G\cdot 2M}\cdot 8r^3$$

$$v_{orb} = \frac{2\pi r}{P} \qquad v_{orb}^2 = \frac{4\pi^2 r^2}{P^2} = \frac{1}{4}\frac{GM}{r}$$

$$E_K = E_K^{(1)} + E_K^{(2)} = 2\cdot\frac{M v_{orb}^2}{2} = \frac{GM^2}{4r}$$

$$E_P = -\frac{GM_1M_2}{2r} = -\frac{GM^2}{2r}$$

$$E_K = \frac{1}{2}|E_P|$$

$$E_T = E_K + E_P = -\frac{1}{4}\frac{GM^2}{r}$$

$v = const$

$$E_{TOT} = -E_{kin}$$

$$E_K = \frac{1}{2}|E_P|$$

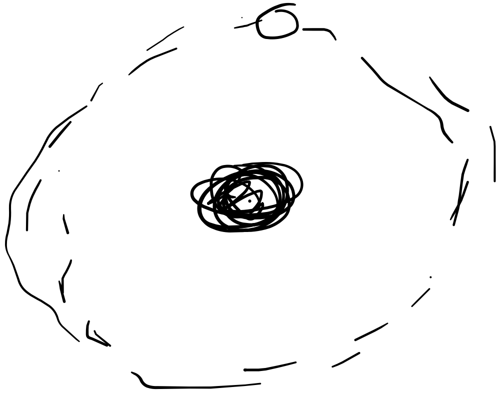
$$E_K \sim \frac{Mv^2}{2} \sim M$$

$$E_P \sim \frac{G\cdot M\cdot M}{a}$$

$$M \to \frac{1}{2}M \qquad E_K \to \frac{1}{2}E_K$$

$$E_P \to \left(\frac{1}{2}\right)^2 E_P \to \frac{1}{4}E_P$$

Теорема Еириала

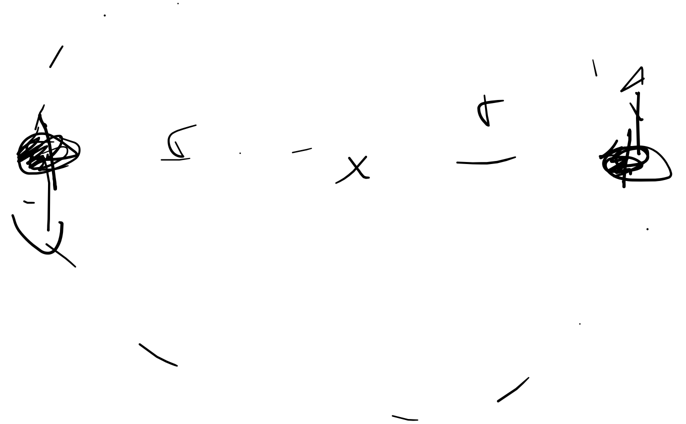


$E_{тот} < 0$

$\Rightarrow$

E_p — грав. поле

E_k



$$\tilde{M} = \frac{1}{2} M$$

$$\tilde{E}_k = \frac{1}{2} E_k$$

$$\tilde{E}_p = \frac{1}{4} E_p$$

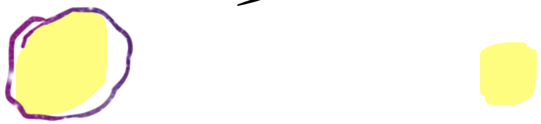
$$\bar{E}_T = \frac{1}{2} E_k + \frac{1}{4} E_p = \frac{GM^2}{8r} - \frac{GM^2}{8r} = 0$$

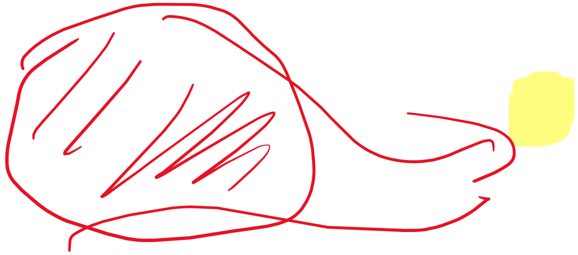
$$\Delta M > \frac{1}{2}(M_1 + M_2) \rightarrow \text{распад}$$

$v_{ws} > v_{orb}$

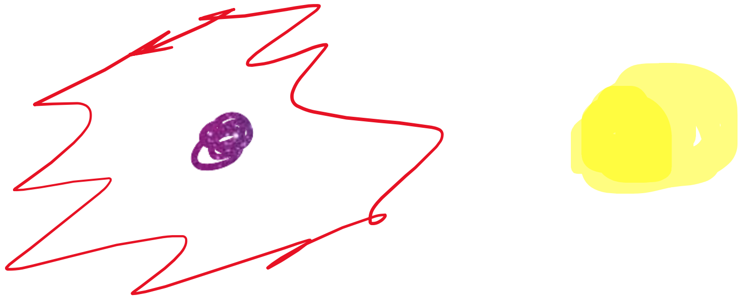
kick

$m_1 > m_2$



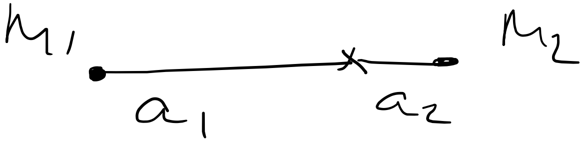


$\tilde{m}_1 < \tilde{m}_2$



$$\Delta M < \frac{1}{2}\left(\tilde{M}_1 + \tilde{M}_2\right)$$

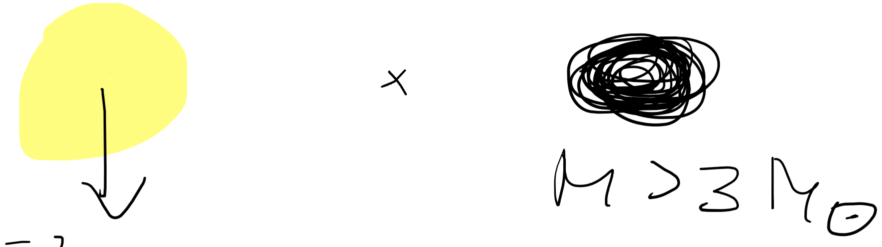
② Функция масс (f)



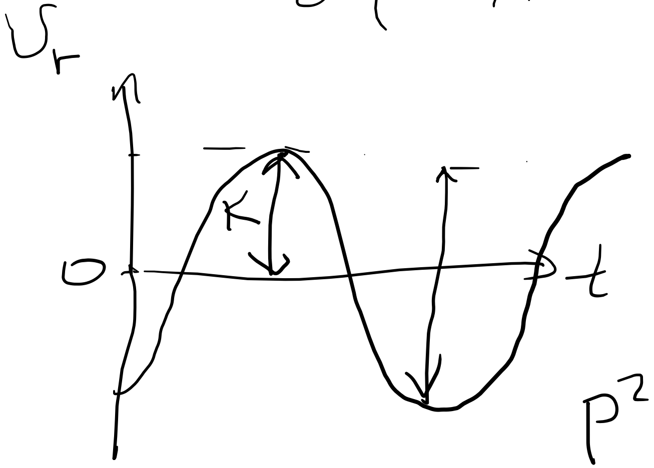
$$a = a_1 + a_2$$

$$a_1 M_1 = a_2 M_2$$

$$a = a_1 + a_2 = a_1\left(1 + \frac{a_2}{a_1}\right) = \frac{a_1}{M_2}\left(M_1 + M_2\right)$$



$$P^2 = \frac{4\pi^2}{G(M_1+M_2)} a^3 = \frac{4\pi^2}{G(M_1+M_2)} \cdot \frac{a_1^3}{M_2^3}(M_1+M_2)^3 = \frac{4\pi^2}{G} a_1^3 \frac{(M_1+M_2)^2}{M_2^3}$$



$$K = v_1 \cdot \sin i \qquad v_1 = \frac{2\pi a_1}{P}$$

$$K = a_1 \cdot \sin i \cdot \frac{2\pi}{P} \qquad a_1 = \frac{P}{2\pi} \frac{1}{\sin i} K$$

$$P^2 = \frac{4\pi^2}{G} \frac{(M_1+M_2)^2}{M_2^3} \frac{P^3}{8\pi^3} \cdot \frac{1}{\sin^3 i} K^3$$

$$\underbrace{\frac{P}{2\pi} K \frac{1}{G}}_{f} = \frac{M_2^3}{(M_1+M_2)^2} \cdot \sin^3 i$$

$$q = \frac{M_1}{M_2} \qquad f = \frac{M_2}{(1+q)^2} \cdot \sin^3 i$$

$$M_2 = f \cdot (1+q)^2 \cdot (\sin i)^{-3}$$

$$>1 \qquad\qquad >1$$

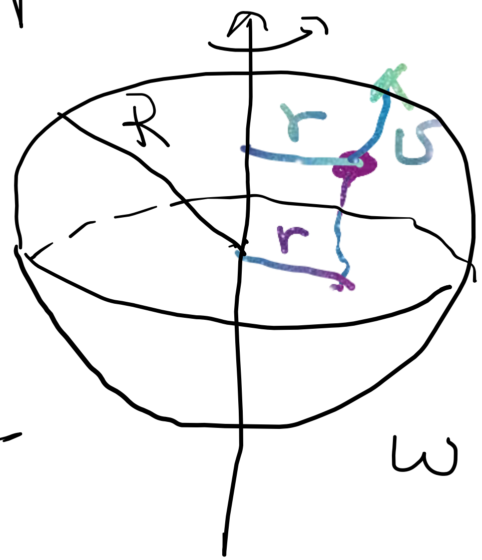
$$\boxed{M_2 > f}$$

нижн.
предел.

③ Энергия НЗ

а) вращ.
б) аккреция
в) маг. поле
г) тепловая эн.

а) вращение



dm

$r < R$

$$E_{k0} = \frac{dm}{2} v^2$$

$$v = \frac{2\pi r}{P} = \omega r$$

$$\omega = \frac{2\pi}{P}$$

$E_{вращ} \ (\equiv E_к) \Rightarrow$

$\sim M$

$\sim \omega^2$

$\sim R^2$

$$E_{вращ} \sim MR^2 \omega^2$$

Момент инерции $I \sim MR^2$

$$E_{вращ} = \frac{I\omega^2}{2}$$

$$E_{вращ} = \frac{I\omega^2}{2}$$

$$I \approx MR^2 = 1,5 \cdot M_\odot \cdot (10\,км)^2 = 1,5 \cdot 2 \cdot 10^{33}\,г \cdot (10^6\,см)^2 = 3 \cdot 10^{45}\,г \cdot см^2$$

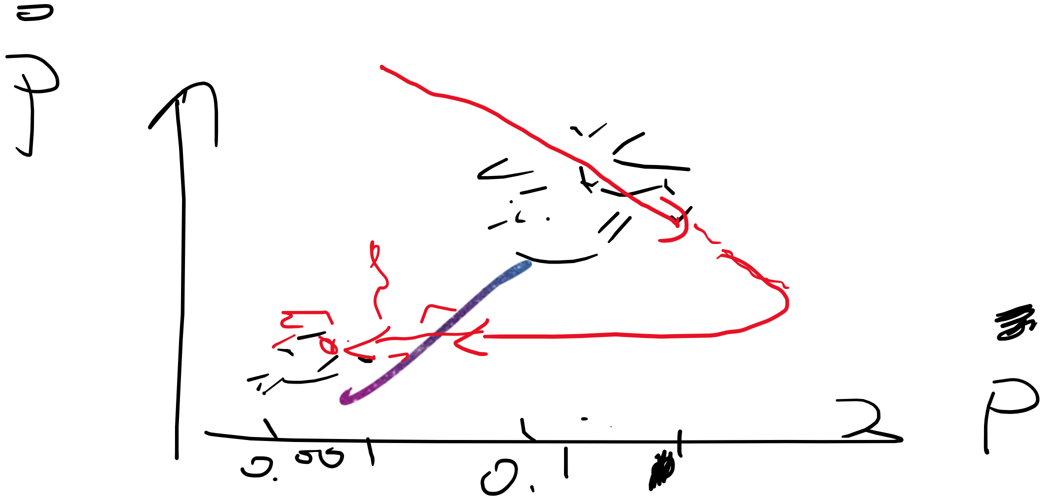
$$I_{NS} \simeq 10^{45}\,г \cdot см^2$$

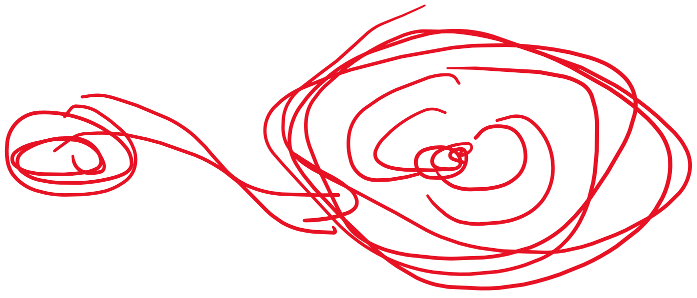
$$\omega = \frac{2\pi}{P} \qquad \frac{2\pi R}{P_{кр}} < \sqrt{\frac{GM}{R}}$$

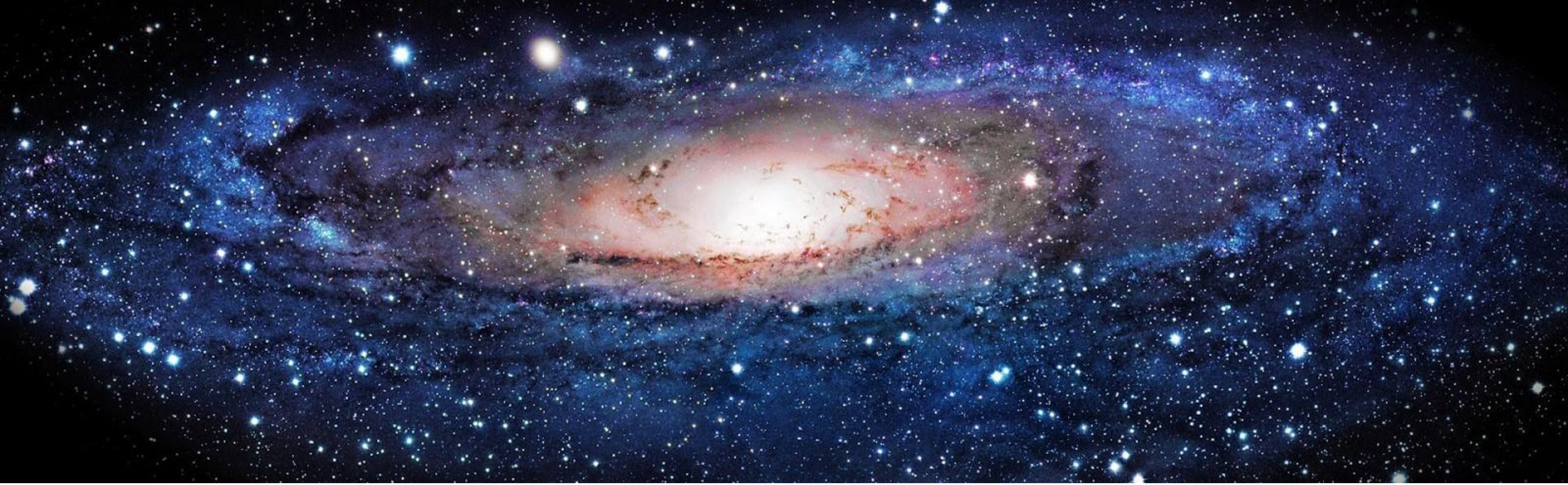
$$P_{кр} \sim \frac{1}{\sqrt{\rho}} \sim 0.001\,с \Rightarrow \omega_{max} \simeq 10^4\,с^{-1}$$

$$E^{max}_{вращ} = \frac{I_{NS} \cdot \omega^2_{max}}{2} \sim 10^{53}\,эрг \quad (!!)$$

$$E = mc^2 = 3 \cdot 10^{33} \cdot 10^{21} = 3 \cdot 10^{54}\,эрг$$







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б) Аккреция

$E_p^{\infty} \approx 0$

$|E_p^{\infty}| \ll |E_p(R)|$

$E_p(R) < 0$

$|E_p(R)| = \frac{GMm}{R}$

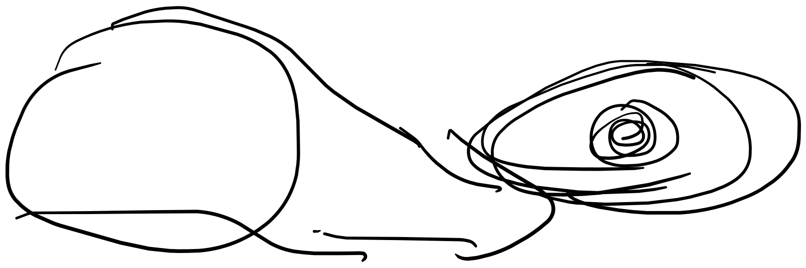
$m = 1$ г

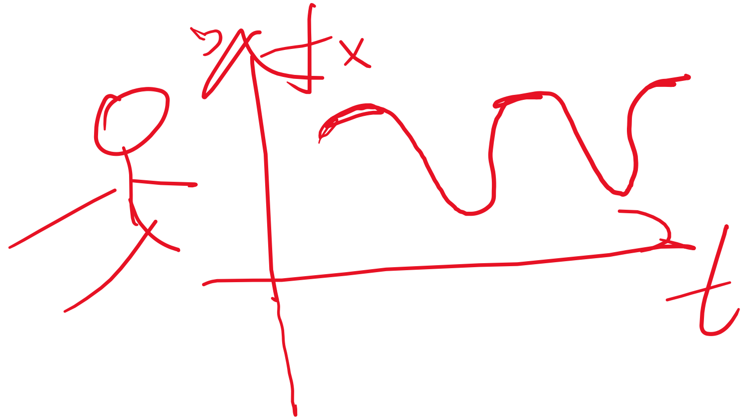
$E_0 = m_p c^2 = 10^{21}$ эрг

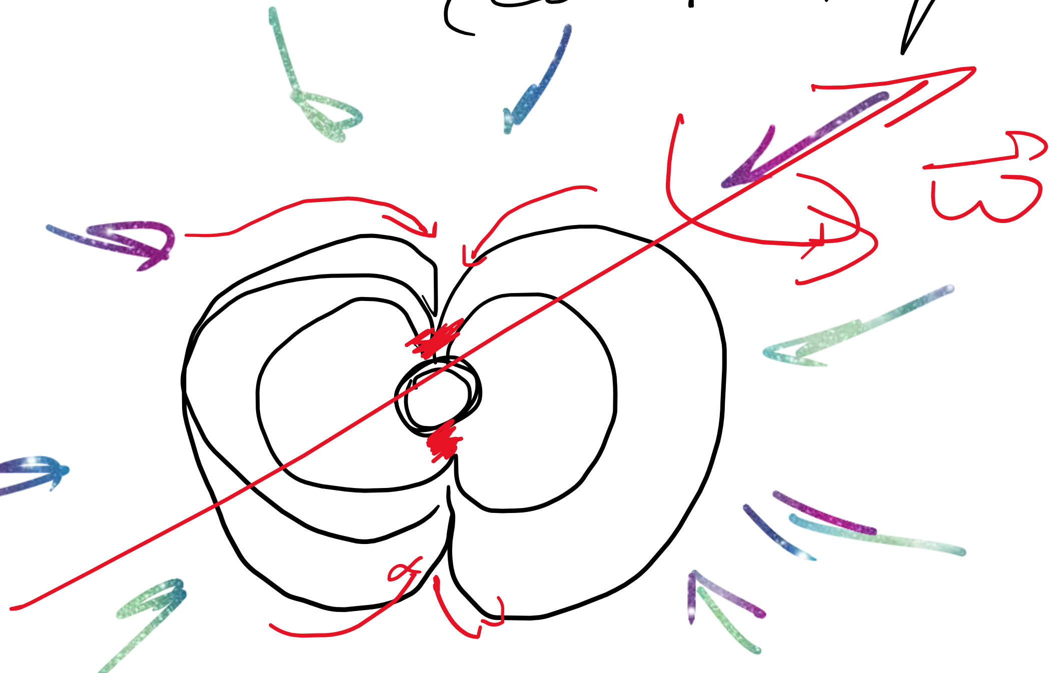
$$\frac{GM}{R} = \frac{10^{-7} \cdot 1.5 \cdot 2 \cdot 10^{33}}{1.5 \cdot 10^{6}} = 2 \cdot 10^{20} \text{ эрг}$$

$> 10\% \, mc^2$

(Солнце термояд 0.7%)







4) маг. поле.

$B_{NS} \sim 10^{8} \div 10^{15}$ Гс

$B_{\oplus} \sim 0,5$ Гс

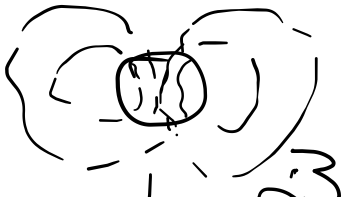
$B_{\odot} \lesssim 1$ Гс

$B_{SS} \sim few \cdot 10^{3}$ Гс

Объёмная плотность энергии маг. поля

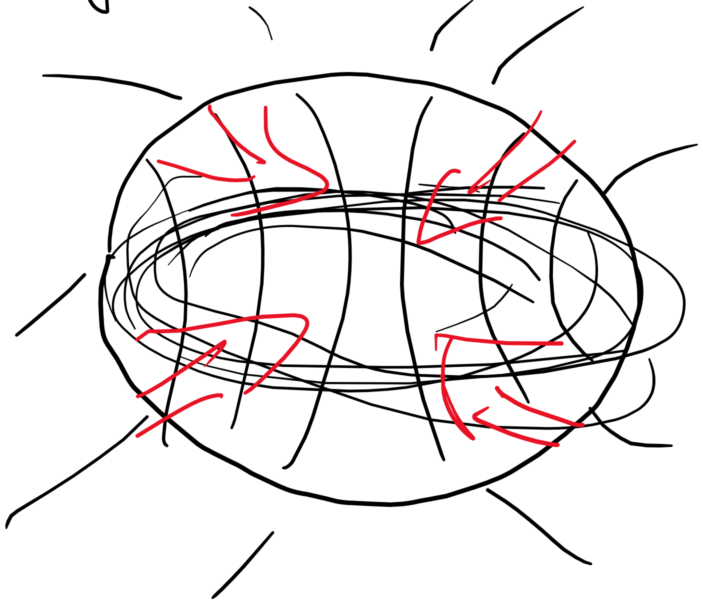
$$\varepsilon = \frac{B^2}{8\pi}$$

$$B \sim B_0 \cdot \left(\frac{r}{R_0}\right)^{-3}$$



$$E_{mag} = \varepsilon \cdot V = \frac{B^2}{8\pi} \cdot \frac{4}{3}\pi R^3 = \frac{B^2 R^3}{6} \approx$$

$$\approx \frac{(10^{15})^2 \cdot (10^6)^3}{6} B_{15}^2 R_6^3 = 10^{47} \text{ эрг } B_{15}^2 R_6^3$$

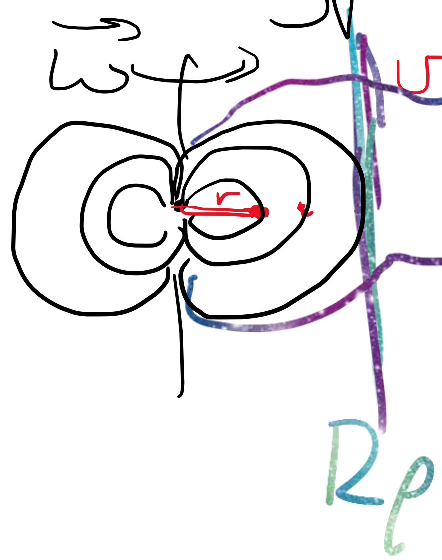


$$\varepsilon \cdot V = \frac{GM^2}{R} \qquad \frac{B^2 R^3}{6} = \frac{GM^2}{R}$$

$$B_{max} = \sqrt{6G}\,\frac{M}{R^2} = 2 \cdot \sqrt{10^{-7}} \cdot \frac{3 \cdot 10^{33}}{10^{12}} =$$

$$= 6 \cdot \sqrt{10^{-7}} \cdot 10^{21} = 6 \cdot 3,2 \cdot 10^{-4} \cdot 10^{21} = 2 \cdot 10^{18} \text{ Гс}$$

4. Возраст PSR



$v = \omega r$

$\omega = \frac{2\pi}{P}$

$v = c = \omega R_\ell$

$R_\ell = c/\omega$

$B \sim r^{-3}$

$B = B_0 \left(\frac{r}{R_0}\right)^{-3}$

$E \sim \frac{B^2(R_\ell)}{8\pi} \cdot \frac{4}{3}\pi R_\ell^3$

$\dot{E} \sim E/\Delta t$ $\quad \Delta t \sim R/c \sim \frac{1}{\omega} \sim P$

$\dot{E}_{dip} = \frac{B_0^2}{8\pi} \frac{R_0^6}{R_\ell^6} \frac{4}{3}\pi R_\ell^3 \omega \sim \frac{B_0^2 R_0^6}{R_\ell^3}\omega = k \frac{B_0^2 R_0^6}{R_\ell^3}\omega = k \frac{B_0^2 R_0^6}{c^3}\omega^4$

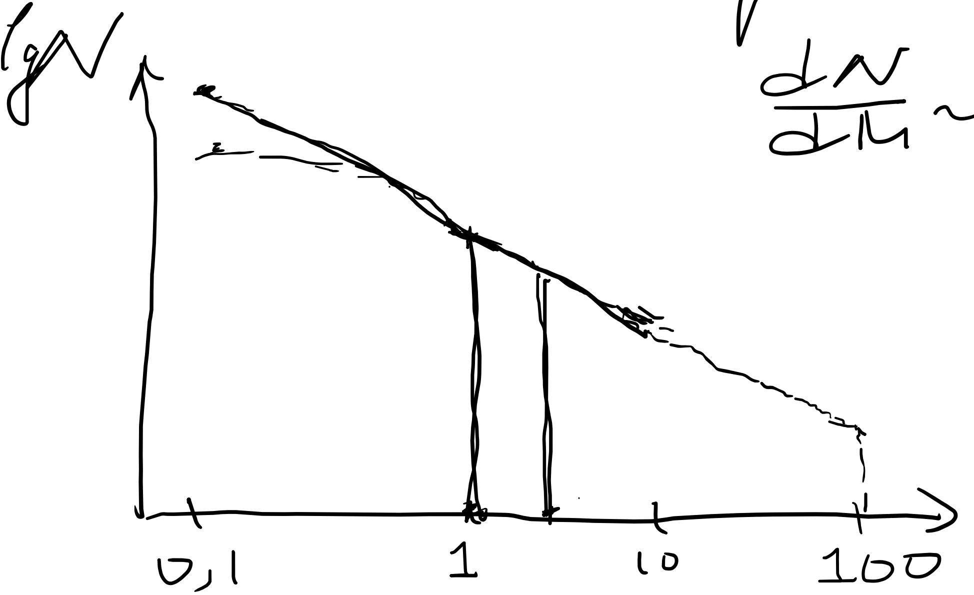
$E_{rot} = \frac{I\omega^2}{2}$ $\quad \dot{E} = \frac{I}{2} 2\omega\dot{\omega} = I\omega\dot{\omega}$

$I\omega\dot{\omega} = k \frac{B_0^2 R_0^6}{c^3}\omega^4$ $\quad \dot{\omega} = k \frac{B_0^2 R_0^6}{I c^3}\omega^3$ $\quad \omega = \frac{2\pi}{P}$ $\quad \dot{\omega} = -\frac{2\pi}{P^2}\dot{P}$

$P\dot{P} = \frac{2}{3} \frac{B_0^2 R_0^6}{I c^3} (2\pi)^2$ $\quad P\,dP = (\ldots)\,dt$

$P^2 - P_0^2 = \tau \cdot (\ldots)$ $\quad P \sim \sqrt{t}$, при $P \gg P_0$

5. Темп звездообразования. Доли НЗ и ЧД.



$$\frac{dN}{dM} \sim M^{-2,35} \quad \text{Salpeter (1955)}$$

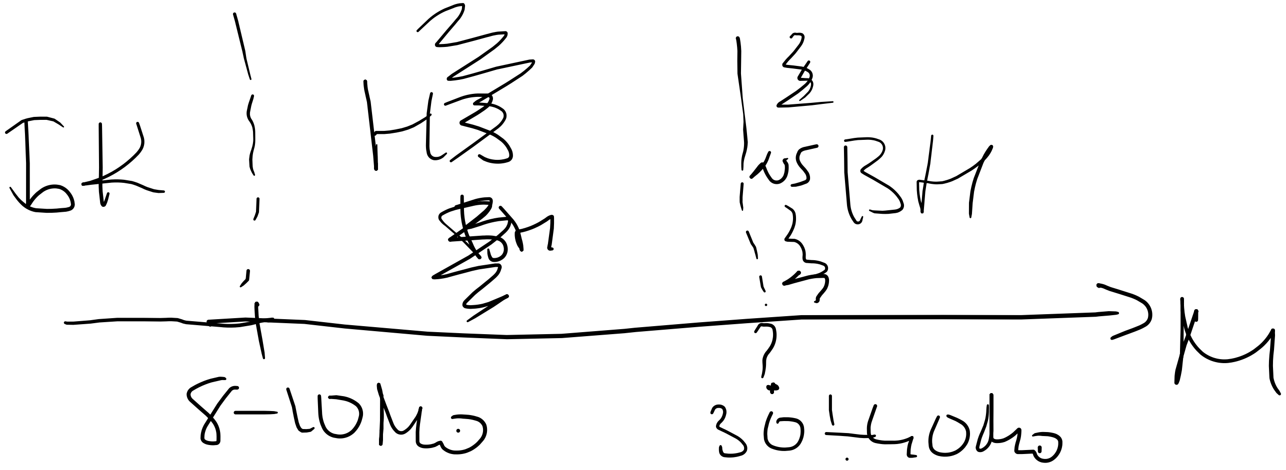
$$SFR \sim 3\ M_\odot/\text{год}$$

$$\langle M \rangle = \frac{\int_{0,1}^{100} A \cdot M^{-2,35} \cdot M\, dM}{\int_{0,1}^{100} A \cdot M^{-2,35}\, dM} = \frac{\int M^{-1,35}\, dM}{\int M^{-2,35}\, dM} \approx 0,4\ M_\odot \Rightarrow \frac{3}{0,4} \approx 7,5\ \text{звезд/год}$$

$M > 10 M_\odot$

$$\eta_{10} = \frac{\int_{10}^{100} A \cdot M^{-2,35}\, dM}{\int_{0,1}^{100} A \cdot M^{-2,35}\, dM} = \left(\frac{10}{0,1}\right)^{-1,35} \sim \frac{1}{500}$$

$$\frac{500}{7,5} \approx 70\ \text{лет}$$



$$\frac{N_{NS}}{N_{BH}} = \frac{\int_{8}^{30} A\cdot M^{-2,35}\, dM}{\int_{30}^{200} A\cdot M^{-2,35}\, dM} \sim \left(\frac{30}{8}\right)^{1,35} \sim 6$$

SN: 30 лет

$T_{Gal} \sim 10^{10}$ лет

$N_{NS}\,(+N_{BH}) = \frac{10^{10}}{30} \simeq 3 \cdot 10^{8}$ штук

$N_{NS} + N_{BH} \sim 10^{9}$