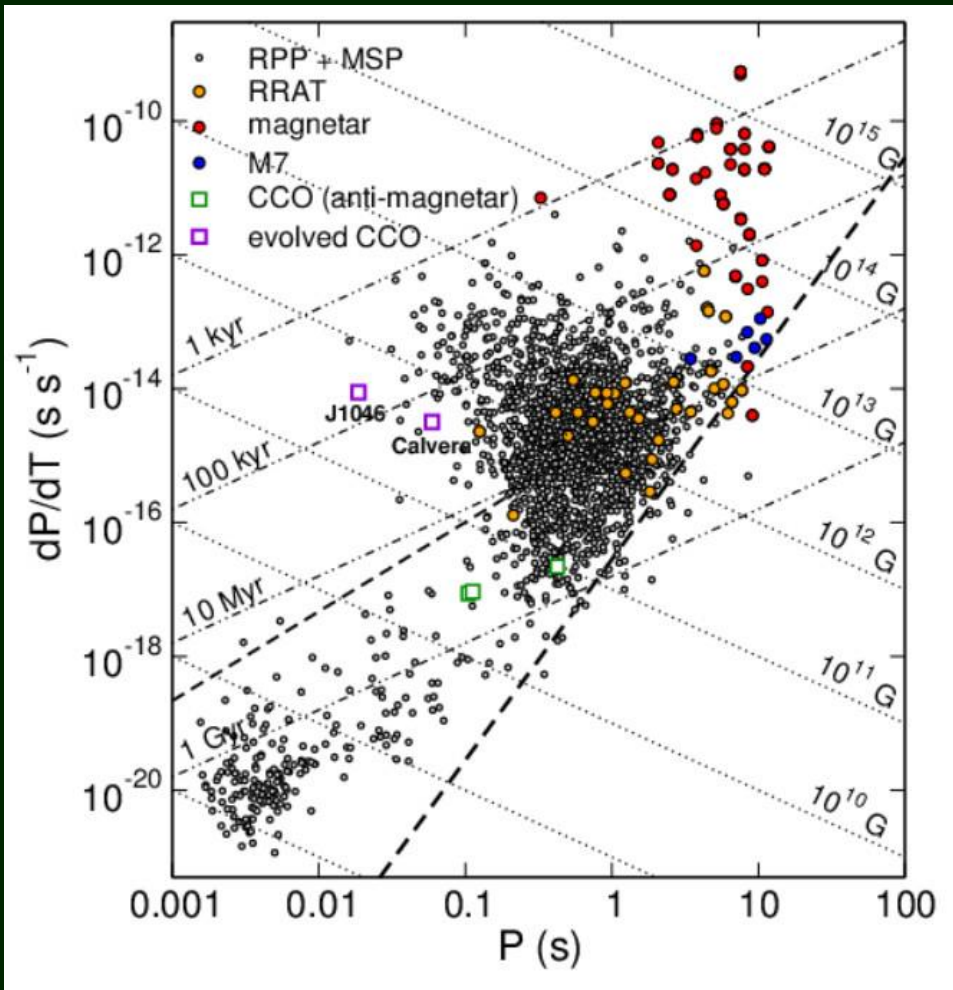

Evolution with decaying and
re-emerging magnetic field

Diversity of young neutron stars



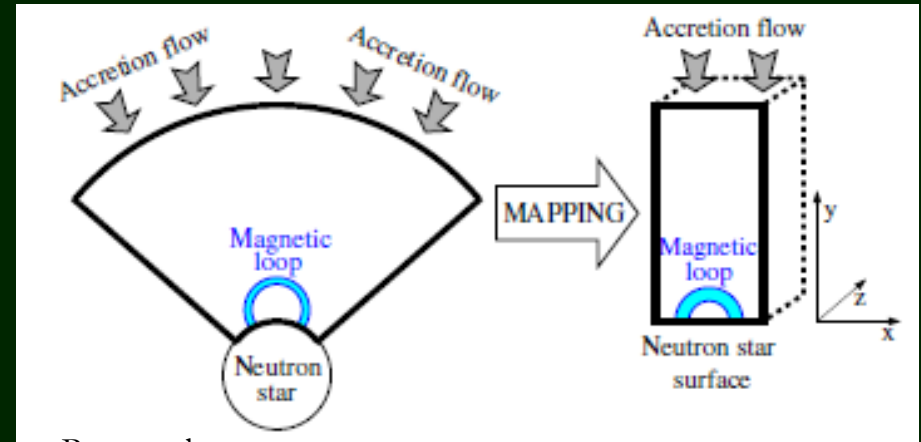
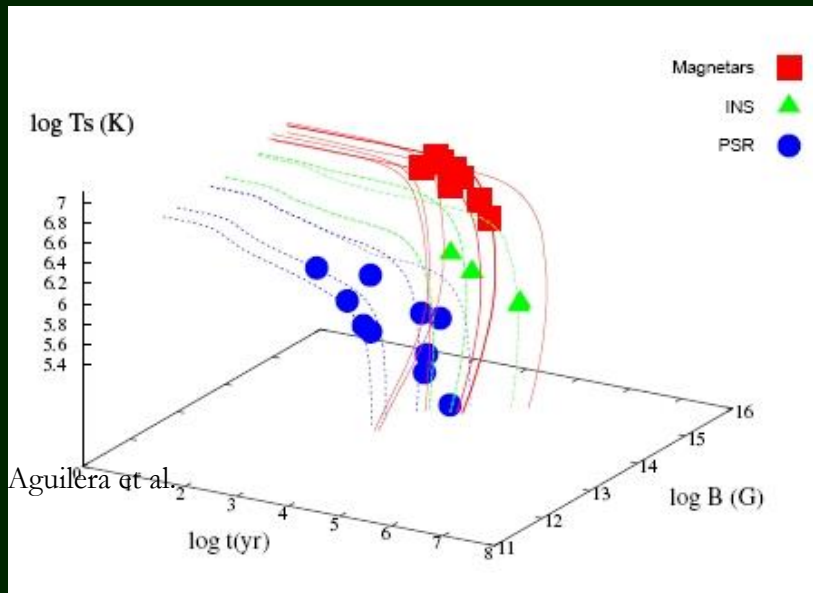
Young isolated neutron stars can appear in many flavors:

- o Radio pulsars
- o Compact central X-ray sources in supernova remnants.
- o Anomalous X-ray pulsars
- o Soft gamma repeaters
- o The Magnificent Seven & Co.
- o Transient radio sources (RRATs)

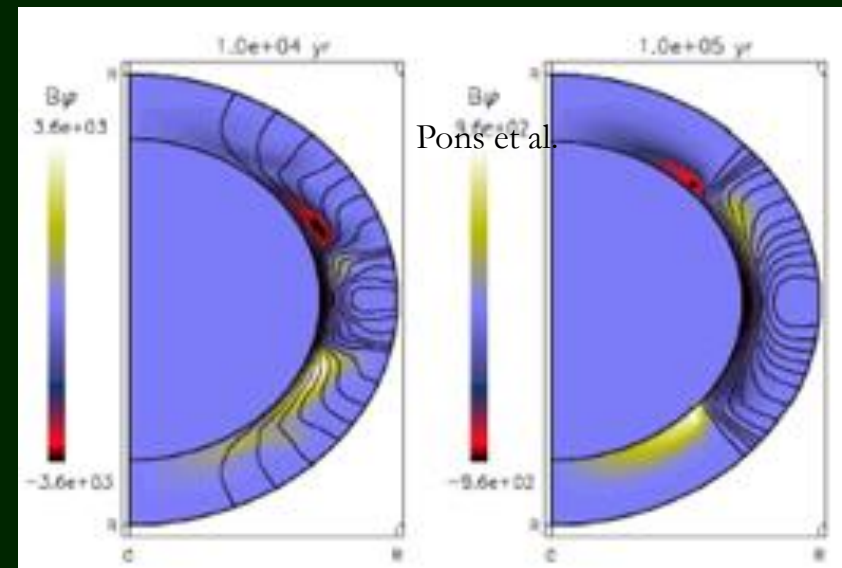
The term
“GRAND UNIFICATION
FOR NEUTRON STARS”
was coined by Kaspi (2010)

PSRs, magnetars and M7
unified in the model by
Popov et al. (2010).

Three main ingredients of a unified model



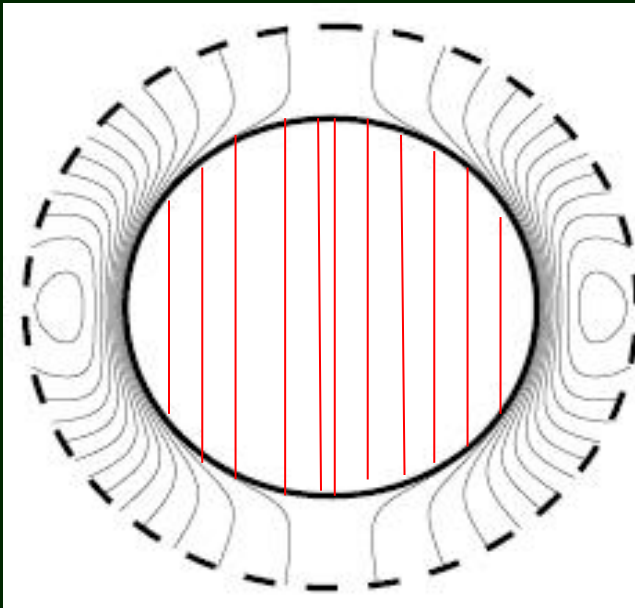
Page et al.



- Field decay
- Emerging magnetic field
- Toroidal magnetic field

Magnetic field decay

Magnetic fields of NSs are expected to decay due to decay of currents which support them.



Crustal field of core field?

It is easy to decay in the crust.

In the core the field is in the form of superconducting vortices.

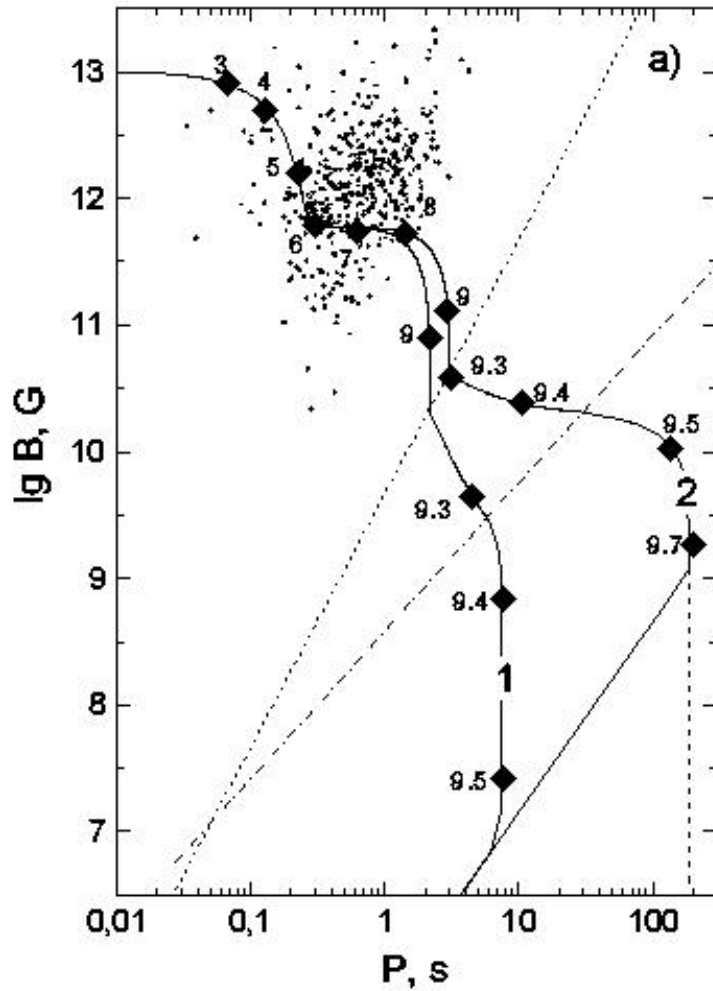
They can decay only when they are moved into the crust (during spin-down).

Still, in most of models strong fields decay.

Large magnetic energy can be stored in the crust: 2201.01881.

Strong field can result in thicker crust layer (in depth, pasta stage): 2202.05595.
Thus, decaying field might modify crust thickness.

Period evolution with field decay



An evolutionary track of a NS is very different in the case of decaying magnetic field.

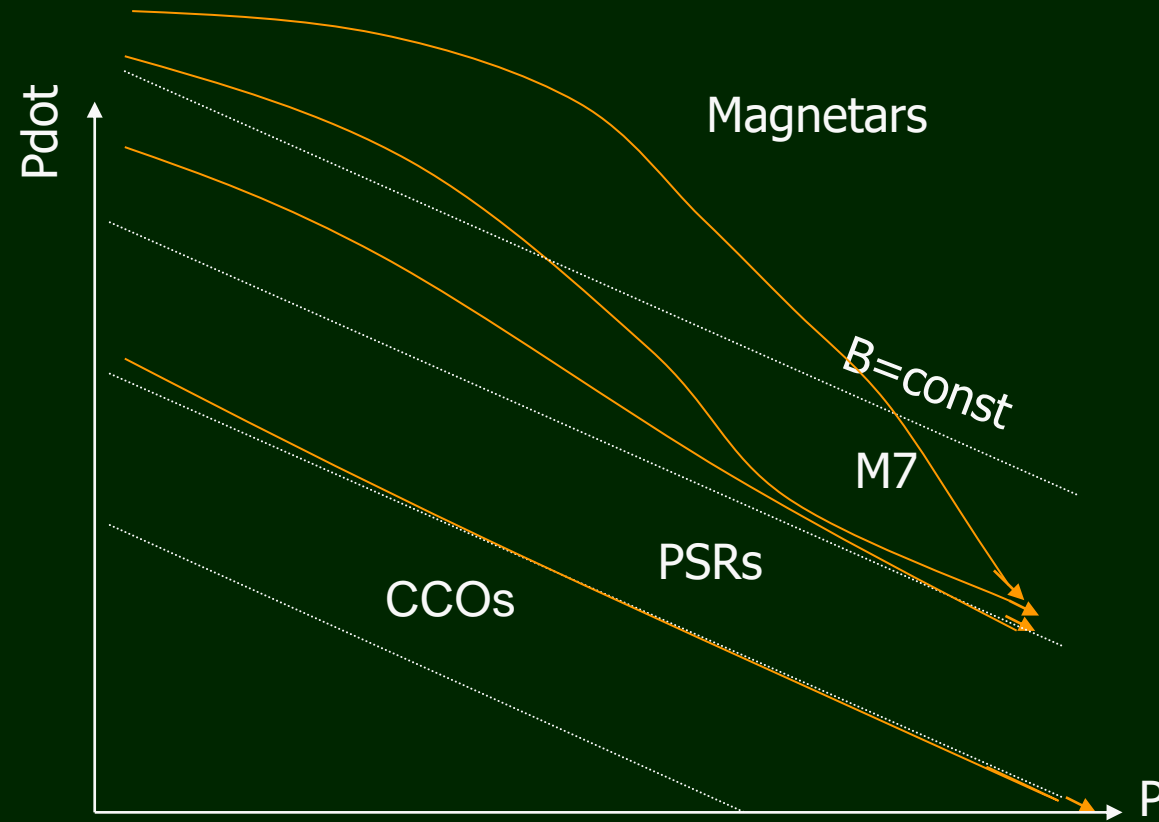
The most important feature is slow-down of spin-down. Finally, a NS can nearly freeze at some value of spin period.

Several episodes of relatively rapid field decay can happen.

Number of isolated accretors can be both decreased or increased in different models of field decay. But in any case their average periods become shorter and temperatures lower.

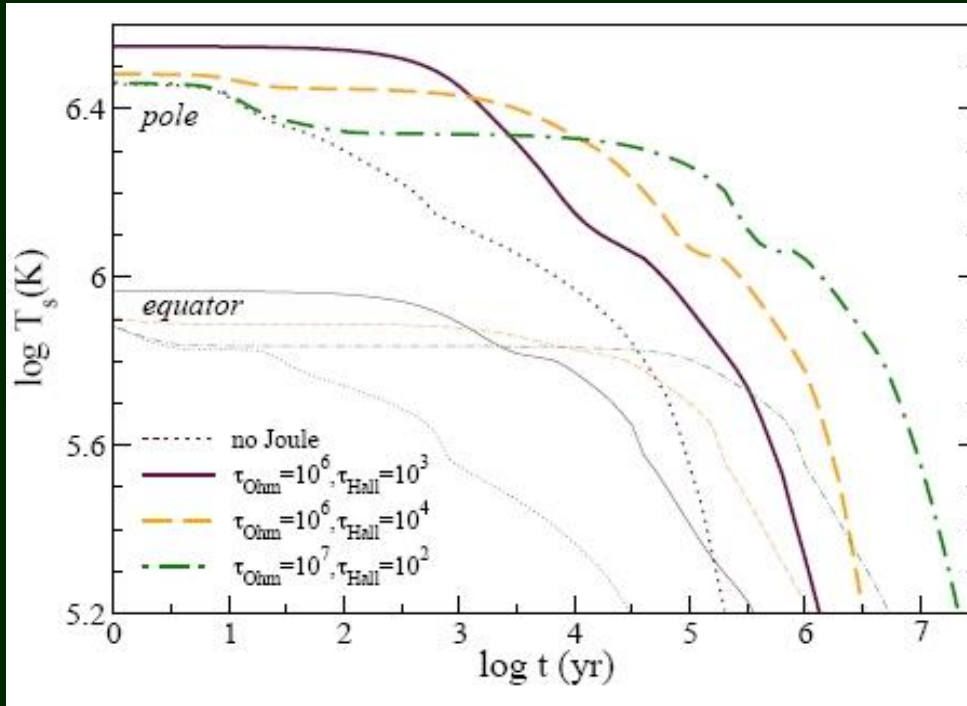
Magnetars, field decay, heating

A model based on field-dependent decay of the magnetic moment of NSs can provide an evolutionary link between different populations (Pons et al.).



Magnetic field decay vs. thermal evolution

Magnetic field decay can be an important source of NS heating.



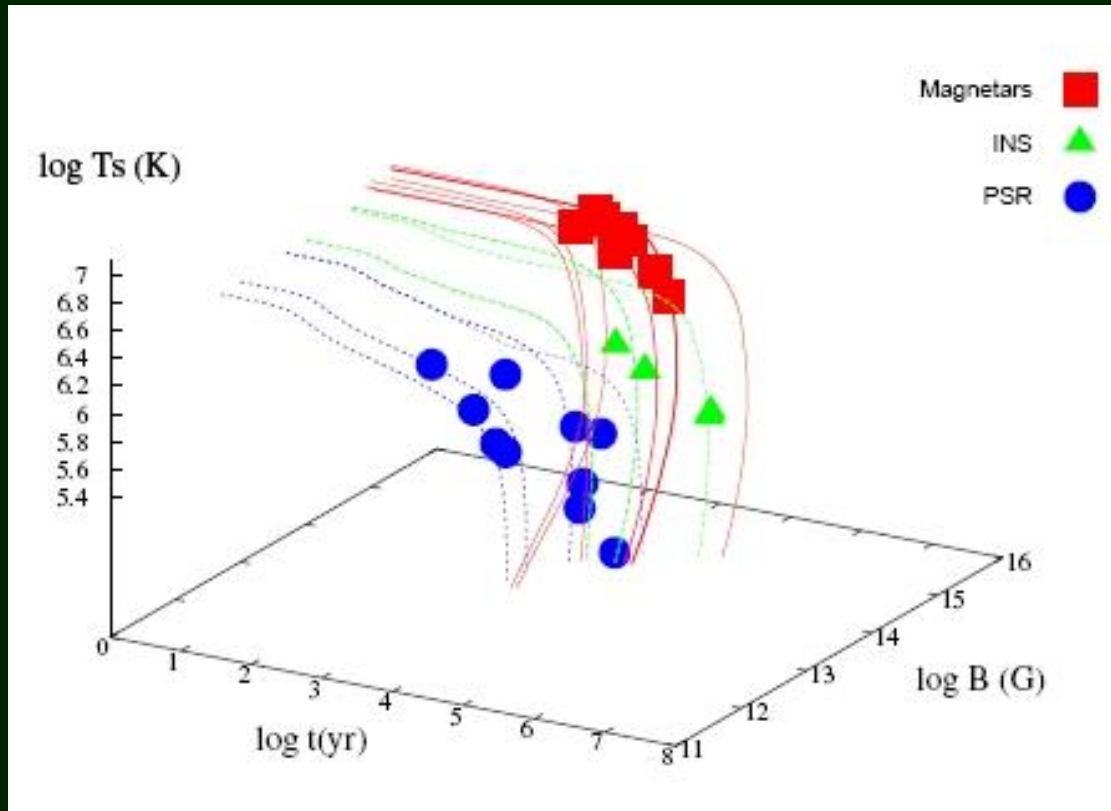
Heat is carried by electrons. It is easier to transport heat along field lines. So, poles are hotter. (for light elements envelope the situation can be different).

Ohm and Hall decay

arxiv:0710.0854 (Aguilera et al.)

$$B = B_0 \frac{\exp(-t/\tau_{\text{Ohm}})}{1 + \frac{\tau_{\text{Ohm}}}{\tau_{\text{Hall}}}(1 - \exp(-t/\tau_{\text{Ohm}}))}$$

Joule heating for everybody?



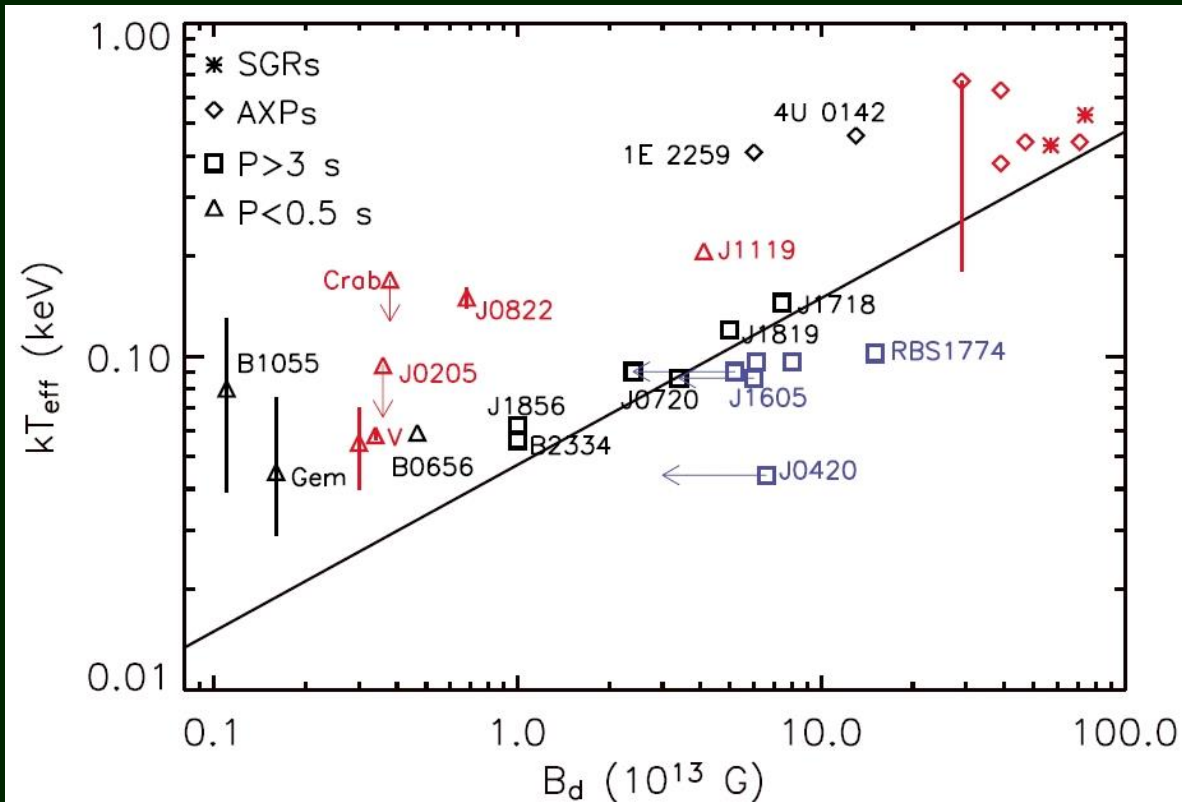
It is important to understand the role of heating by the field decay for different types of INS.

In the model by Pons et al. the effect is more important for NSs with larger initial B .

Note, that the characteristic age estimates ($P/2 \dot{P}$) are different in the case of decaying field!

arXiv: 0710.4914 (Aguilera et al.)

Magnetic field vs. temperature

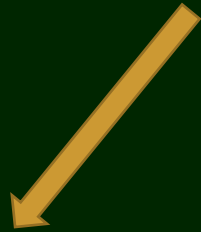


The line marks balance between heating due to the field decay and cooling. It is expected that a NS evolves downwards till it reaches the line, then the evolution proceeds along the line:

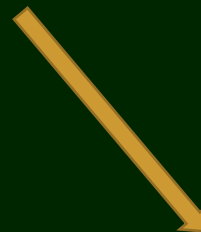
$$T_{\text{eff}} \sim B_d^{1/2}$$

Selection effects are not well studied here. A kind of population synthesis modeling is welcomed.

What kind of decay do we see?



Ohmic decay due to phonons



Hall cascade

Both time scales fit, and in both cases, we can switch-off decay at $\sim 10^6$ yrs either due to cooling or due to the Hall attractor.

Hall cascade and field evolution

$$\frac{\partial B}{\partial t} = -c \nabla \times E,$$

advection

Ohm

Hall

$$E = -\frac{1}{c} v \times B + \frac{J}{\sigma} + \frac{J \times B}{n_e e c},$$

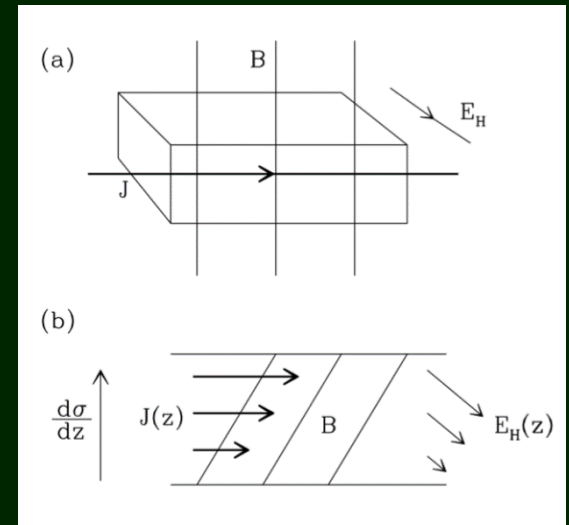
$$\tau_{\text{Ohm}} = \frac{4\pi\sigma L^2}{c^2}.$$

$$J = (c/4\pi)(\nabla \times B)$$

With only Hall term we have:

$$\frac{\partial B}{\partial t} = -\nabla \times \left(\frac{J \times B}{n_e e} \right),$$

$$t_{\text{Hall}} = \frac{n_e e L}{J} = \frac{4\pi n_e e L^2}{c B},$$



The Hall term tends to push the electric currents toward the crust-core boundary, where the high impurity content and pasta phases could cause a fast dissipation of the magnetic field and therefore much less spin-down.

Characteristic timescales

$$\tau_{\text{Hall}} = \frac{4\pi en_e L^2}{cB(t)},$$

Hall time scale strongly depends on the current value of the field.

$$\tau_{\text{Hall}} = \tau_{\text{Hall},0} \frac{B_0}{B(t)}.$$

$$\tau_{\text{Ohm}} = \frac{4\pi\sigma L^2}{c^2},$$

Ohmic decay depends on the conductivity

$$\sigma = \frac{\sigma_Q \sigma_{\text{ph}}}{\sigma_Q + \sigma_{\text{ph}}}.$$

$$\tau_{\text{Ohm}}^{-1} = \tau_{\text{Ohm,ph}}^{-1} + \tau_{\text{Ohm,Q}}^{-1}.$$

Resistivity can be due to

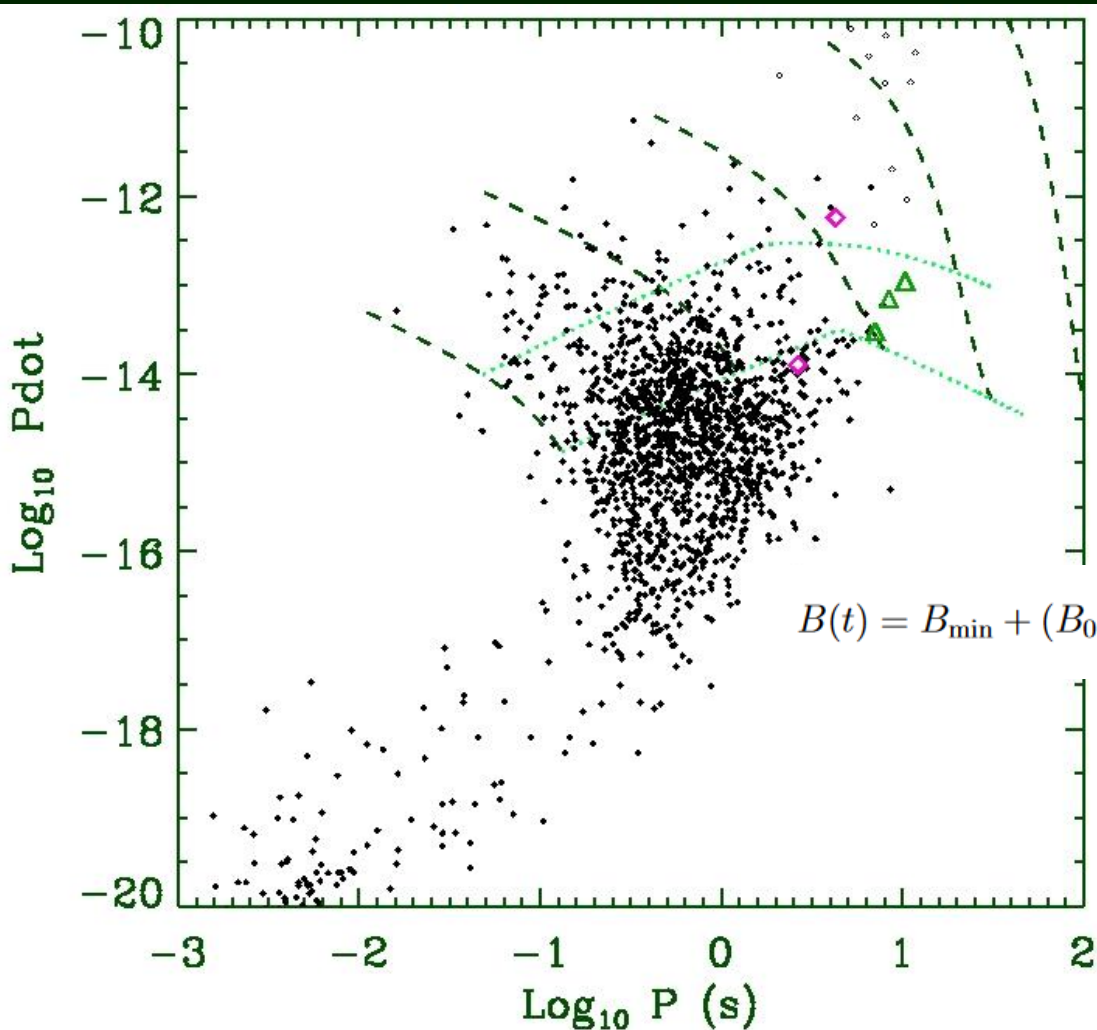
- Phonons
- Impurities

$$\sigma_Q = 4.4 \times 10^{25} \text{s}^{-1} \left(\frac{\rho_{14}^{1/3}}{Q} \right) \left(\frac{Y_e}{0.05} \right)^{1/3} \left(\frac{Z}{30} \right),$$

$$Q = n_{\text{ion}}^{-1} \sum_i n_i \times (Z^2 - \langle Z \rangle^2).$$

$$\sigma_{\text{ph}} = 1.8 \times 10^{25} \text{s}^{-1} \left(\frac{\rho_{14}^{7/6}}{T_8^2} \right) \left(\frac{Y_e}{0.05} \right)^{5/3},$$

P-Pdot diagram and field decay

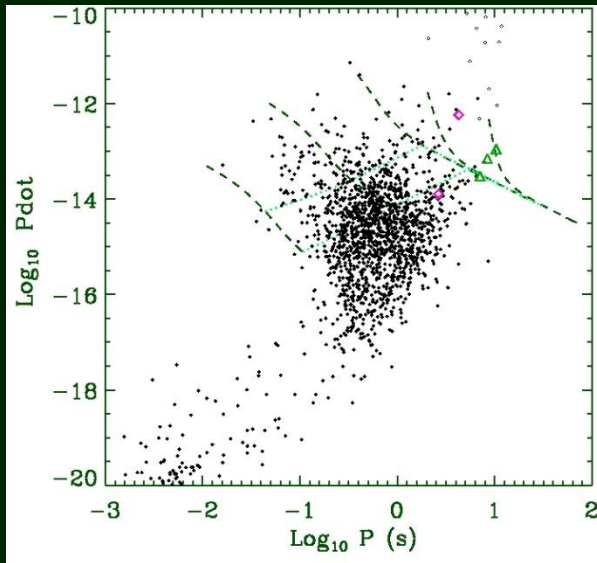


$$B(t) = B_{\min} + (B_0 - B_{\min}) \frac{\exp(-t/\tau_{\text{Ohm}})}{1 + \tau_{\text{Ohm}}/\tau_{\text{Hall}}(1 - \exp(-t/\tau_{\text{Ohm}}))}$$

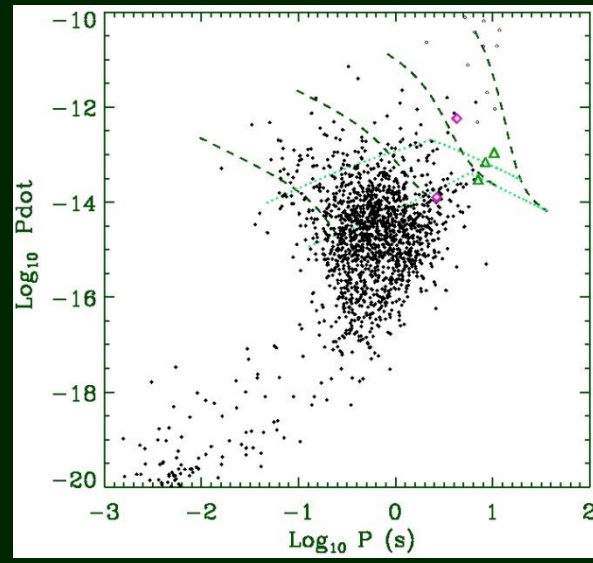
$$\tau_{\text{Ohm}} = 10^6 \text{ yrs}$$

$$\tau_{\text{Hall}} = 10^4 / (B_0 / 10^{15} \text{ G}) \text{ yrs}$$

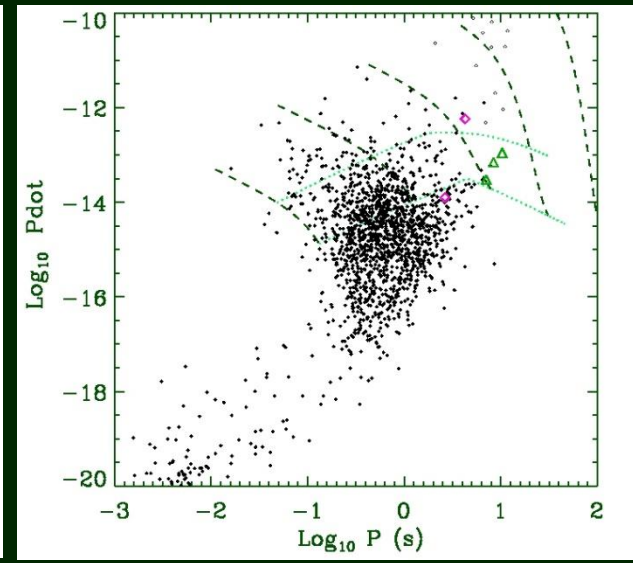
Decay parameters and P-Pdot



$$\begin{aligned} T_{\text{Ohm}} &= 10^7 \text{ yrs} \\ T_{\text{Hall}} &= 10^2 / (B_0 / 10^{15} \text{ G}) \end{aligned}$$



$$\begin{aligned} T_{\text{Ohm}} &= 10^6 \text{ yrs} \\ T_{\text{Hall}} &= 10^3 / (B_0 / 10^{15} \text{ G}) \end{aligned}$$



$$\begin{aligned} T_{\text{Ohm}} &= 10^6 \text{ yrs} \\ T_{\text{Hall}} &= 10^4 / (B_0 / 10^{15} \text{ G}) \end{aligned}$$

Longer time scale for the Hall field decay is favoured.

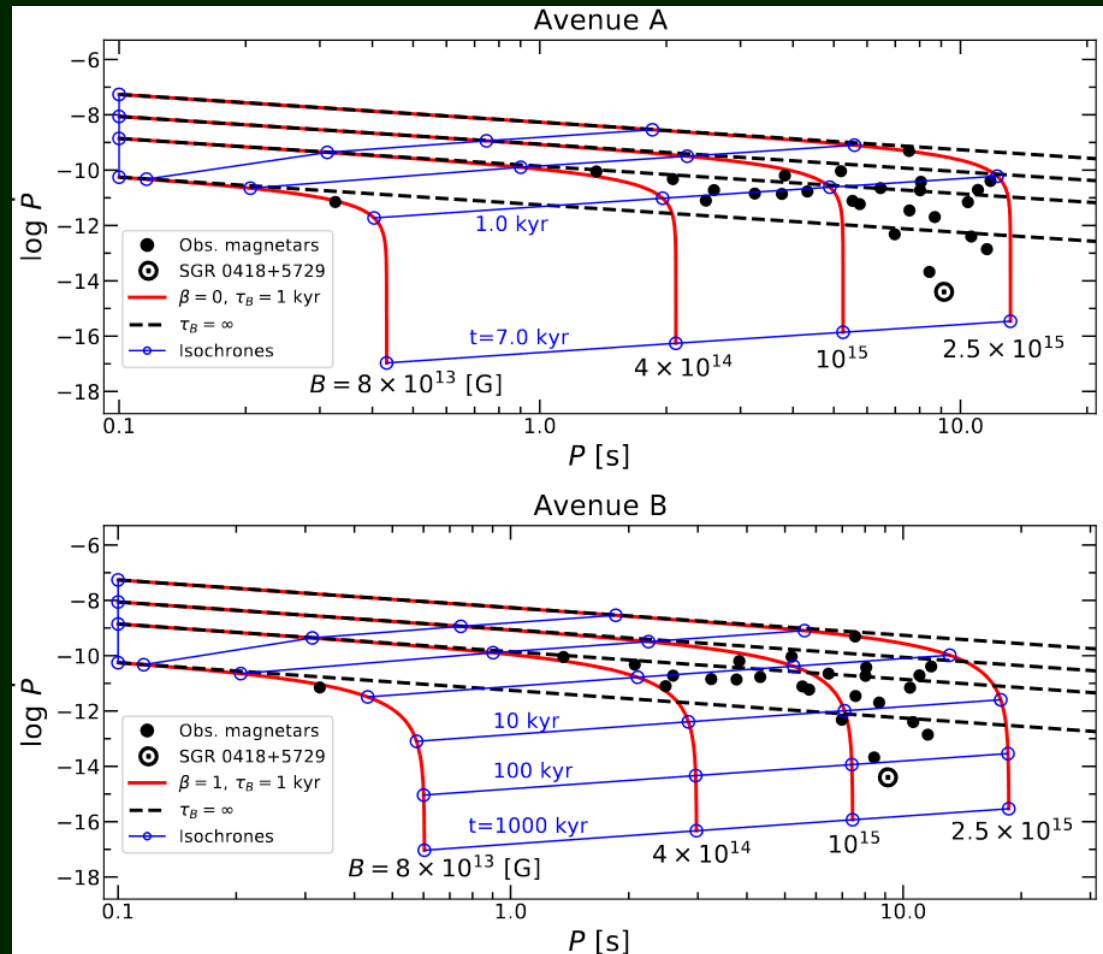
It is interesting to look at HMXBs to see if it is possible to derive the effect of field decay and convergence.

Analytical models of decay

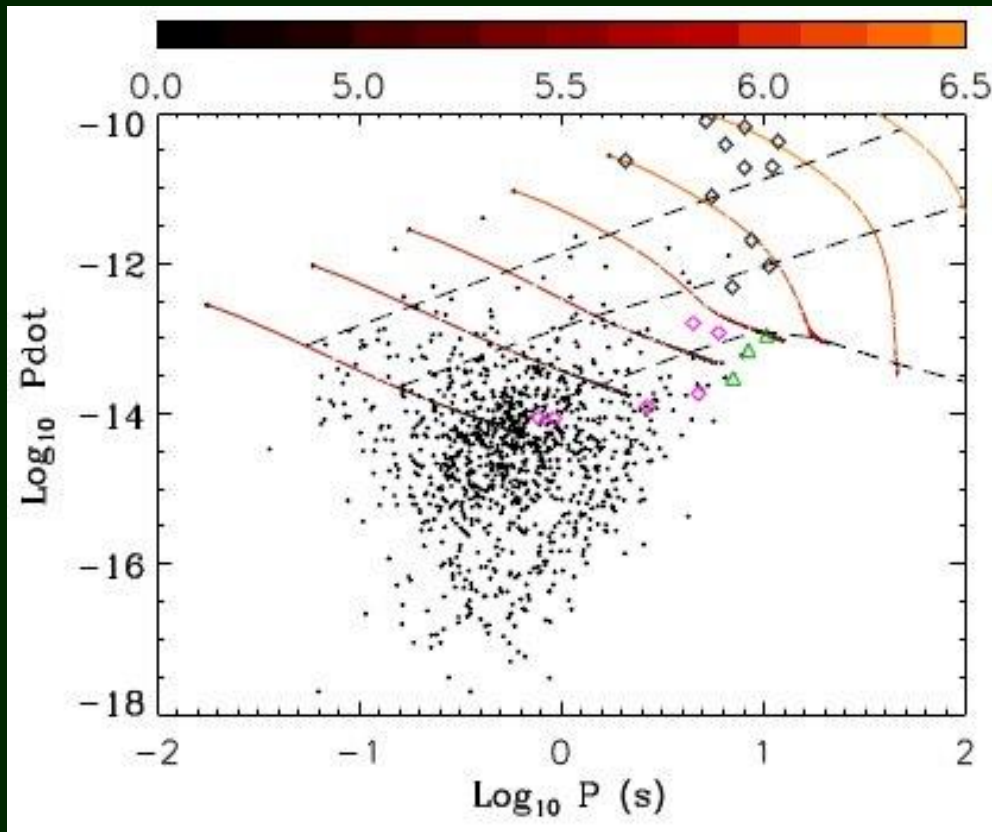
$$\frac{dB(t)}{dt} = -aB(t)^{1+\beta}$$

$$B(t) = \begin{cases} B_0 \left(1 + \frac{\beta t}{\tau_B}\right)^{-\frac{1}{\beta}} & \beta \neq 0, \\ B_0 e^{-t/\tau_B} & \beta = 0. \end{cases}$$

$$P(t) = \begin{cases} \sqrt{\frac{2KB_0^2 \sin^2 \alpha_0 \tau_B}{\beta - 2} \left[\left(1 + \frac{\beta t}{\tau_B}\right)^{\frac{\beta-2}{\beta}} - 1 \right] + P_0^2} & \beta \neq 0 \\ \sqrt{KB_0^2 \sin^2 \alpha_0 \tau_B \left(1 - \exp\left(\frac{-2t}{\tau_B}\right)\right) + P_0^2} & \beta = 0 \end{cases}$$



Realistic tracks



Using the model by Pons et al. (arXiv: 0812.3018) we plot realistic tracks for NS with masses 1.4 Msolar.

Initial fields are:
 $3 \cdot 10^{12}$, 10^{13} , $3 \cdot 10^{13}$, 10^{14} ,
 $3 \cdot 10^{14}$, 10^{15}

Color on the track encodes surface temperature.

Tracks start at 10^3 years, and end at $2 \cdot 10^6$ years.

(Popov et al. MNRAS 2010)

Joint description of NS evolution with decaying magnetic field

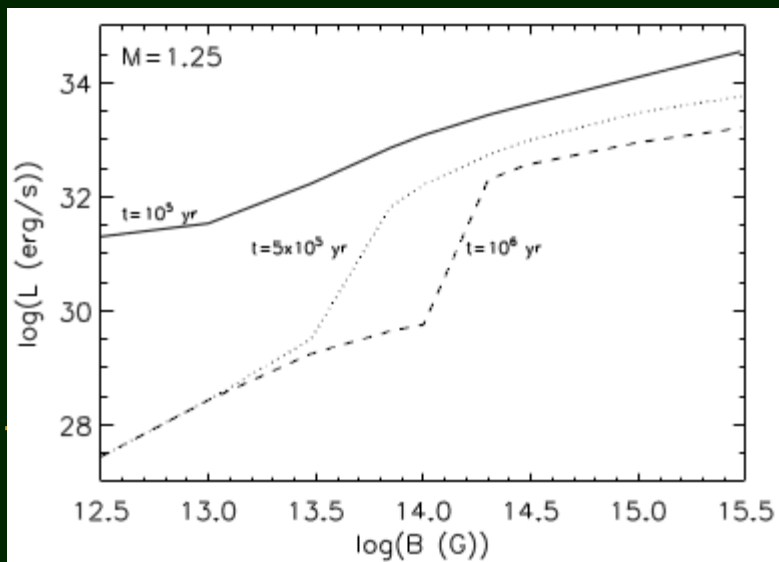
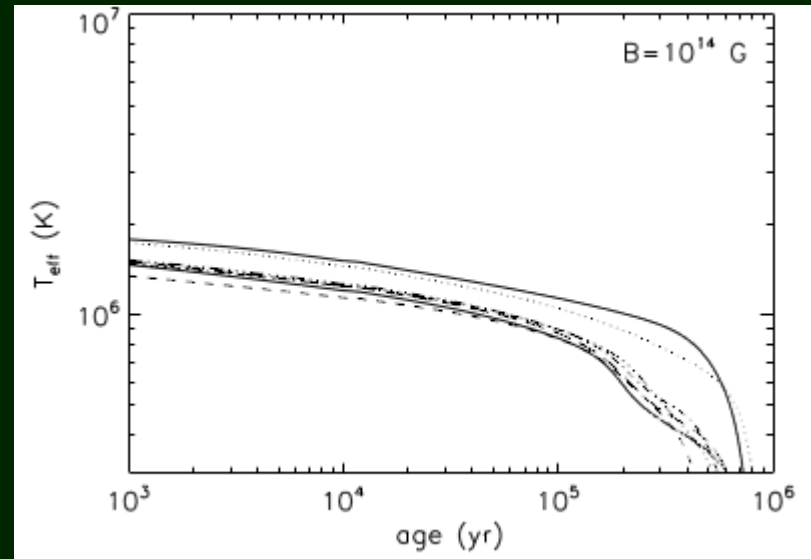
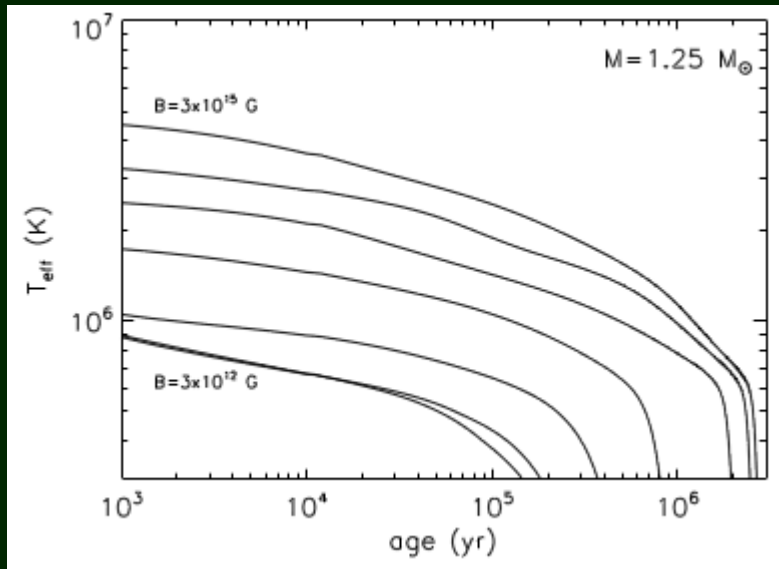
The idea to describe all types of NSs with a unique model using one initial distribution (fields, periods, velocities) and to compare with observational data, i.e. to confront vs. all available observed distributions:

- P - \dot{P} for PSRs and other isolated NSs
- Log N – Log S for cooling close-by NSs
- Luminosity distribution of magnetars (AXPs, SGRs)
-

The first step is done in Popov et al. (2010)

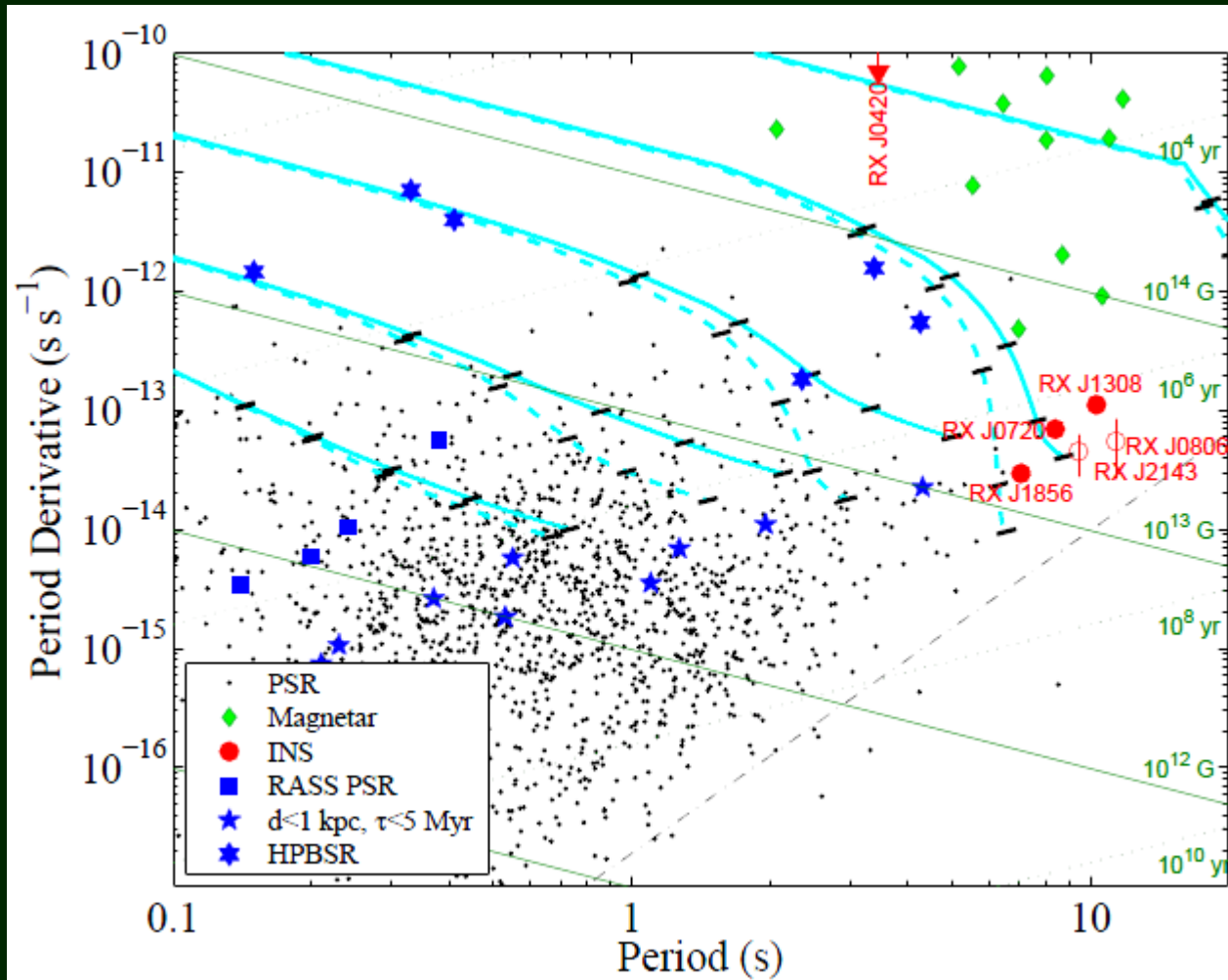
The initial magnetic field distribution with $\langle \log B_0 \rangle \sim 13.25$ and $\sigma \sim 0.6$ gives a good fit. $\sim 10\%$ of magnetars.

Cooling curves with decay

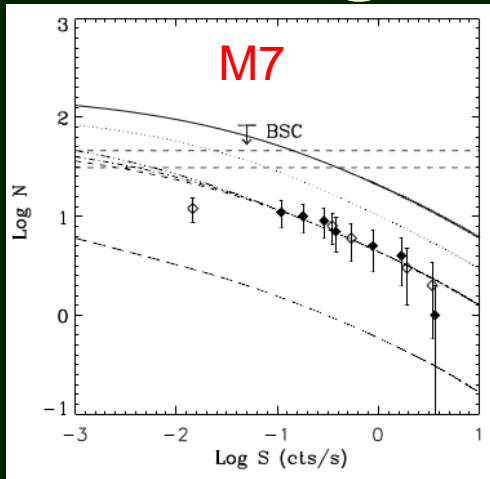


Magnetic field distribution is more important than the mass distribution.

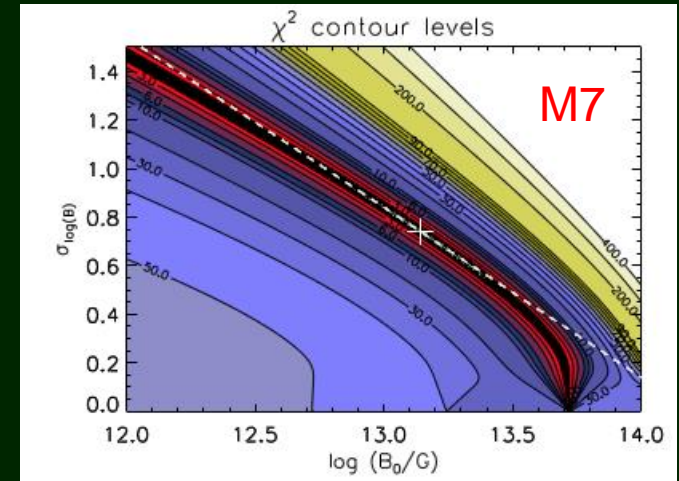
Observational evidence?



Extensive population synthesis: M7, magnetars, PSRs

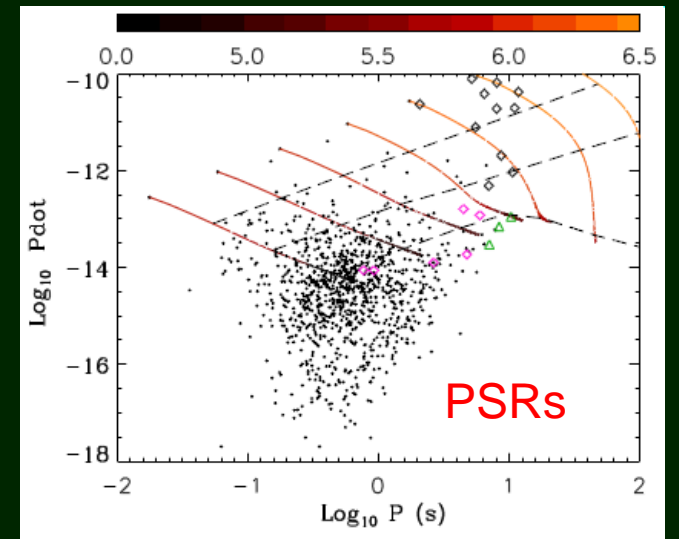
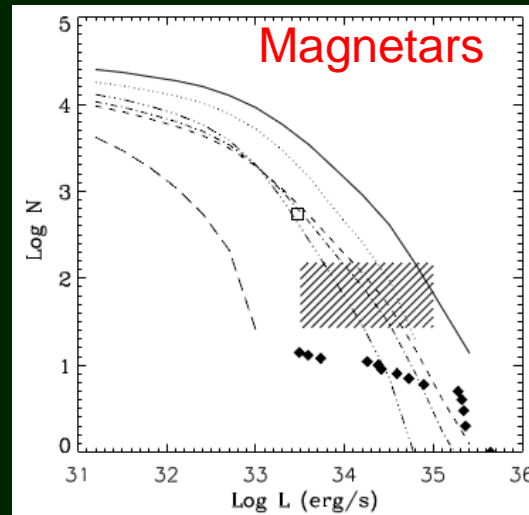


Using one population it is difficult or impossible to find unique initial distribution for the magnetic field

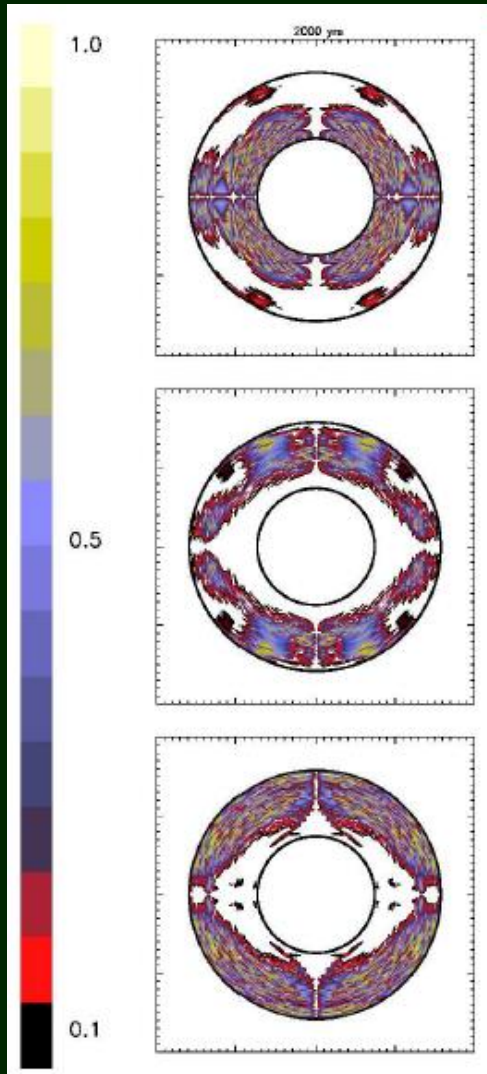


All three populations are compatible with a unique distribution.

Of course, the result is model dependent.

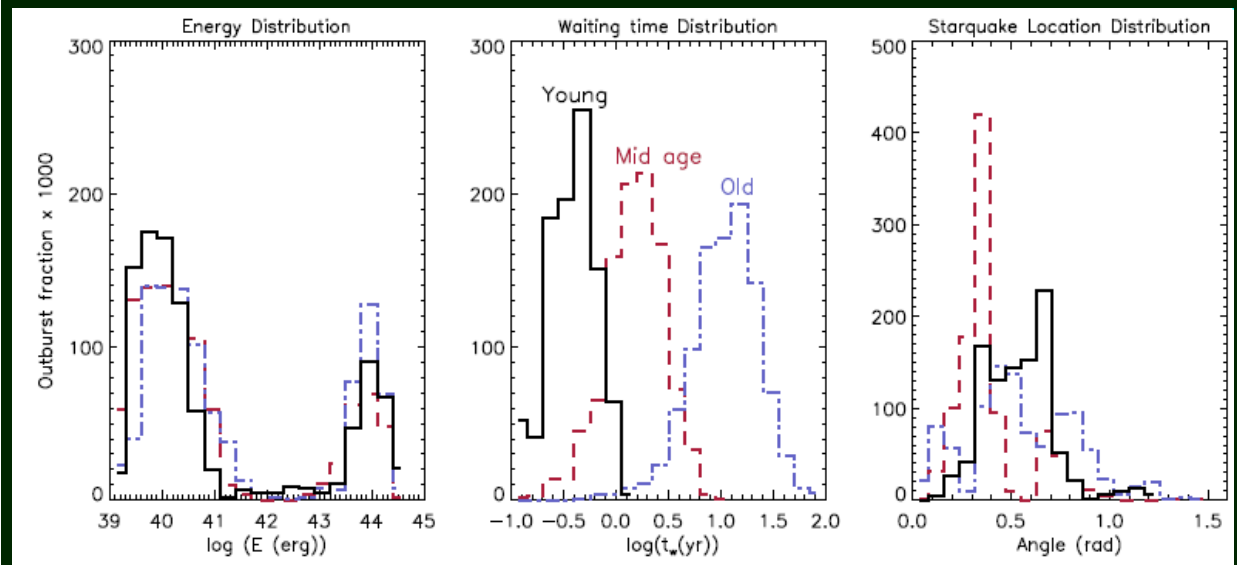


Magnetars bursting activity due to decay

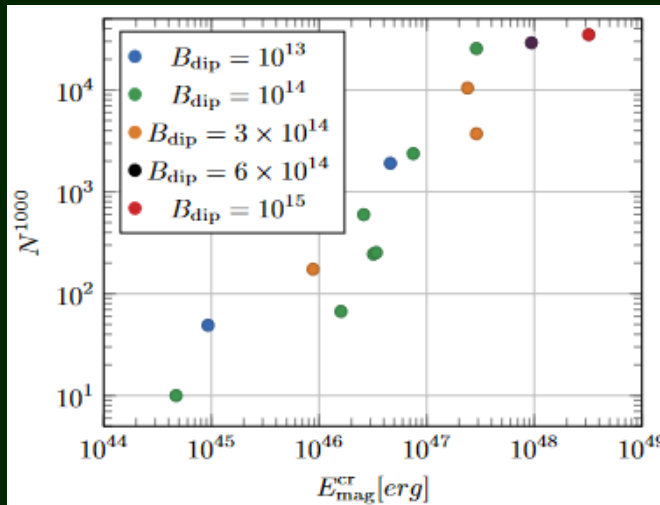


In the field decay model it is possible to study burst activity. Bursts occur due to crust cracking. The decaying field produce stresses in the crust that are not compensated by plastic deformations. When the stress level reaches a critical value the crust cracks, and energy can be released.

At the moment the model is very simple, but this just the first step.

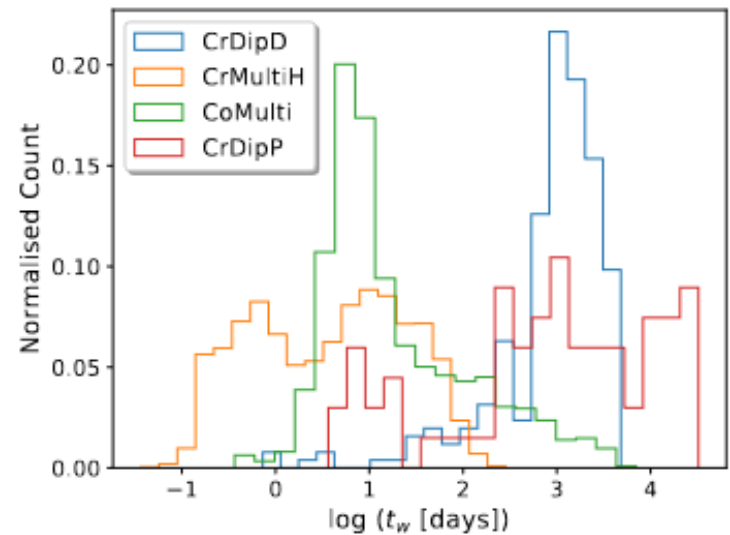
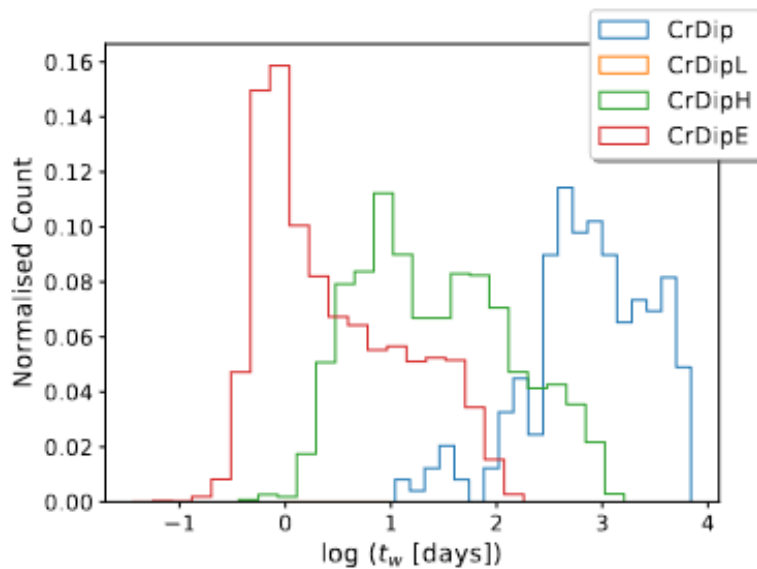


Rate of crustal failures in young magnetars

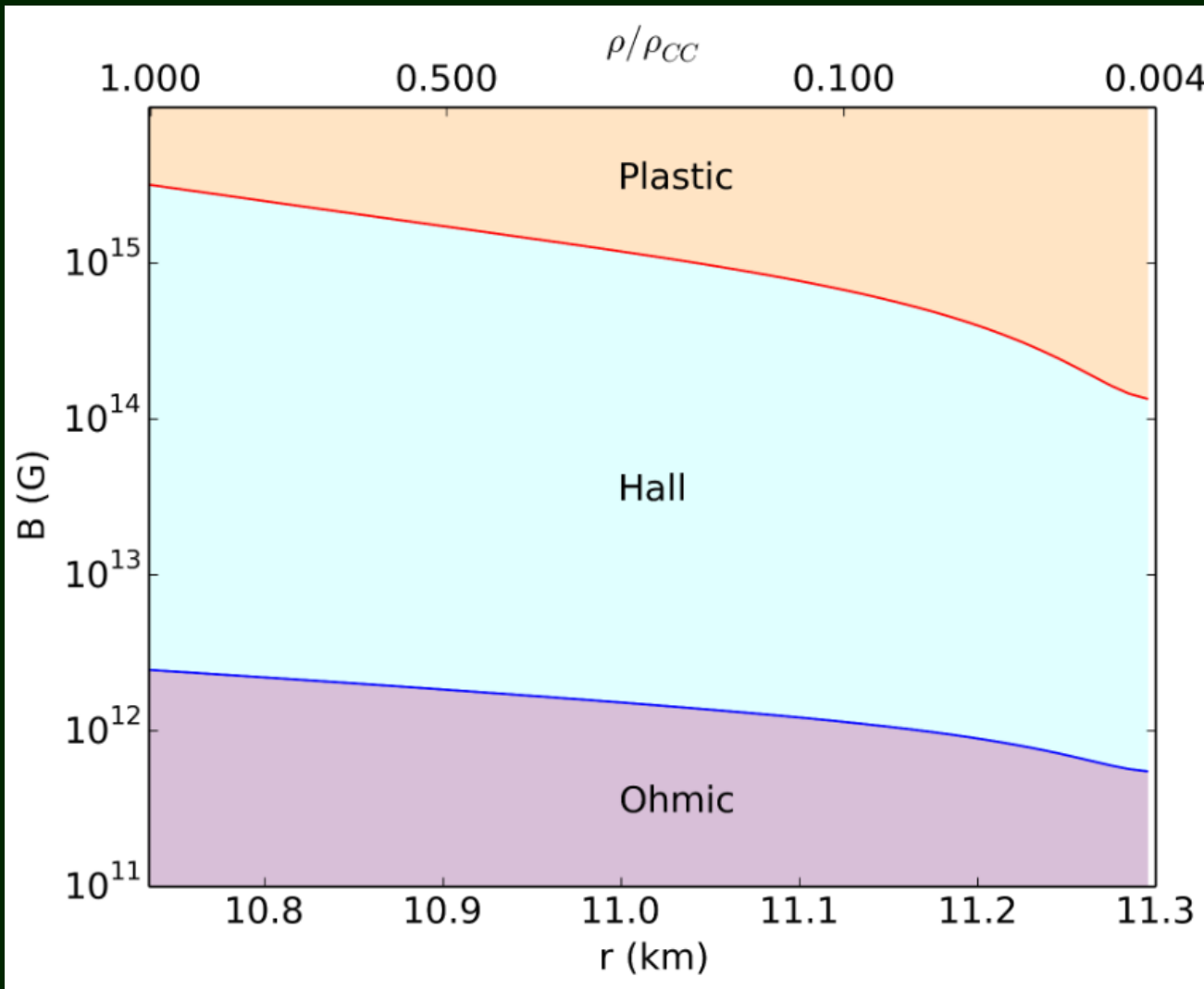


$$\sigma^{\text{max}} = \left(0.0195 - \frac{1.27}{\Gamma - 71} \right) n_i \frac{Z^2 e^2}{a}$$

Different models provide very different results. Various curves are for different values and combinations of dipolar and toroidal fields. The blue curve is for the reference model.



Plastic flow



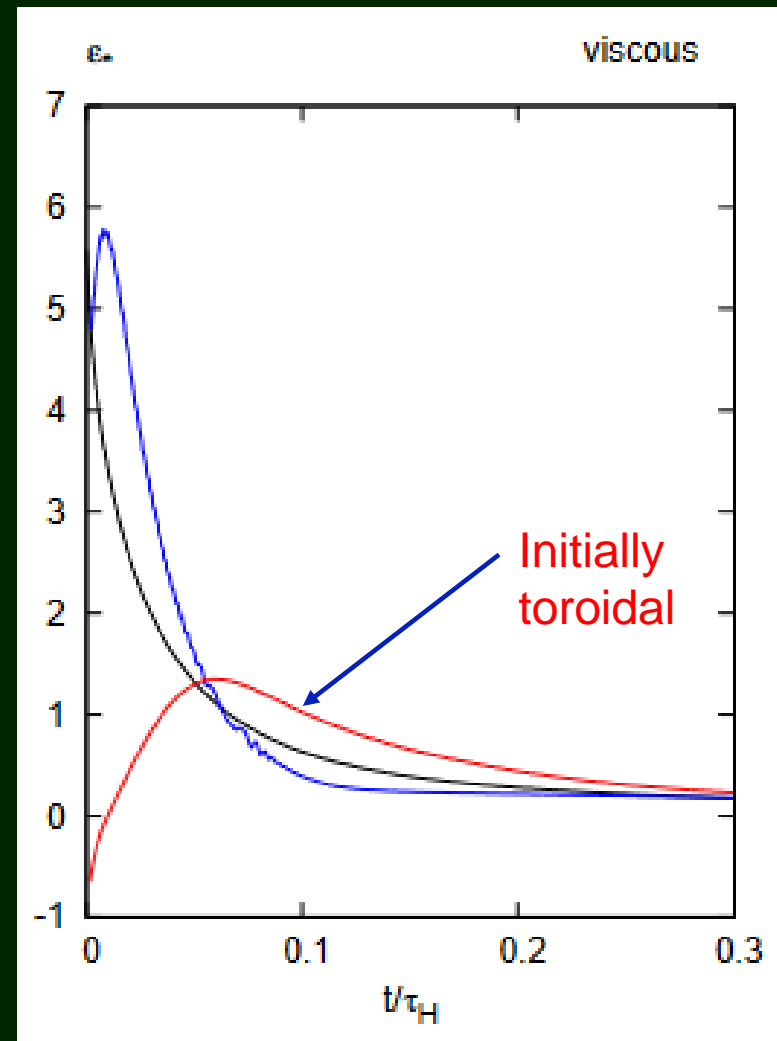
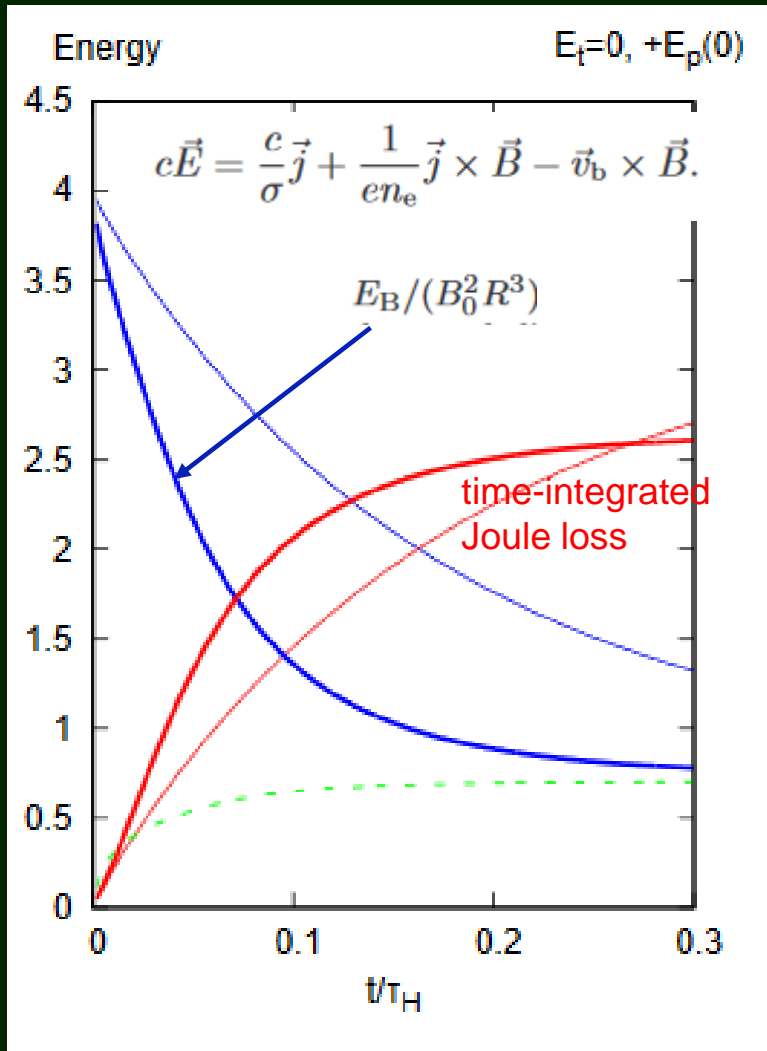
With strong magnetic field the crust does not crack due to the Lorentz force (see, Levin, Lyutikov 2012)

10 – 100 cm/yr

Plastic flow works against formation of small-scale field.

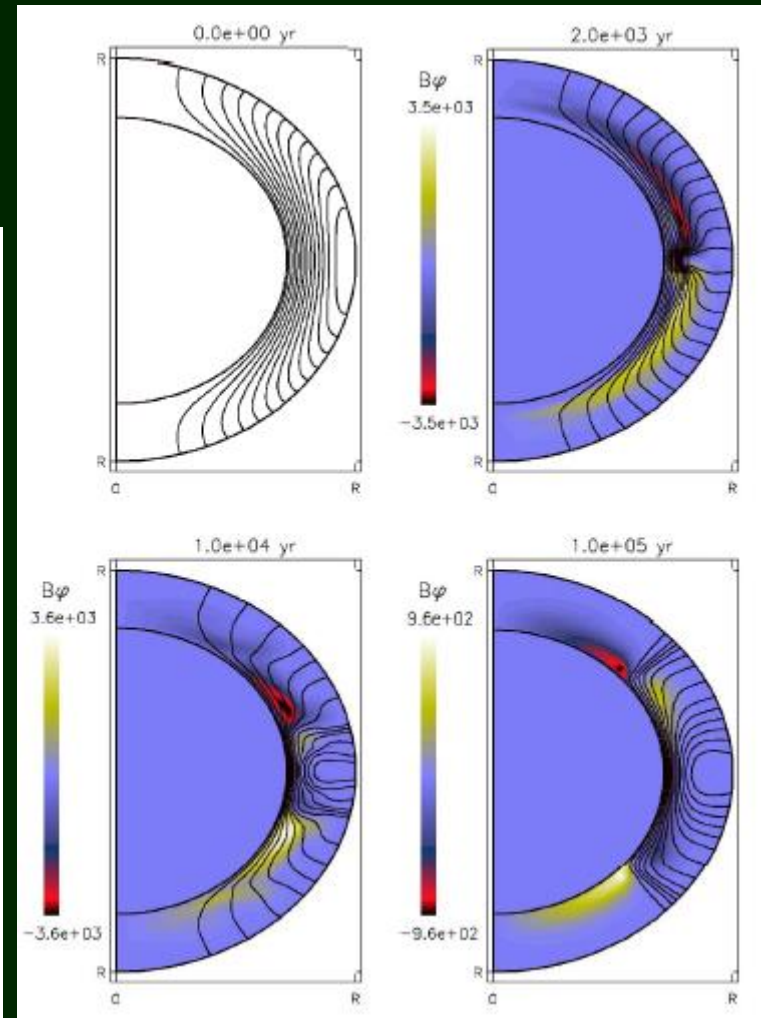
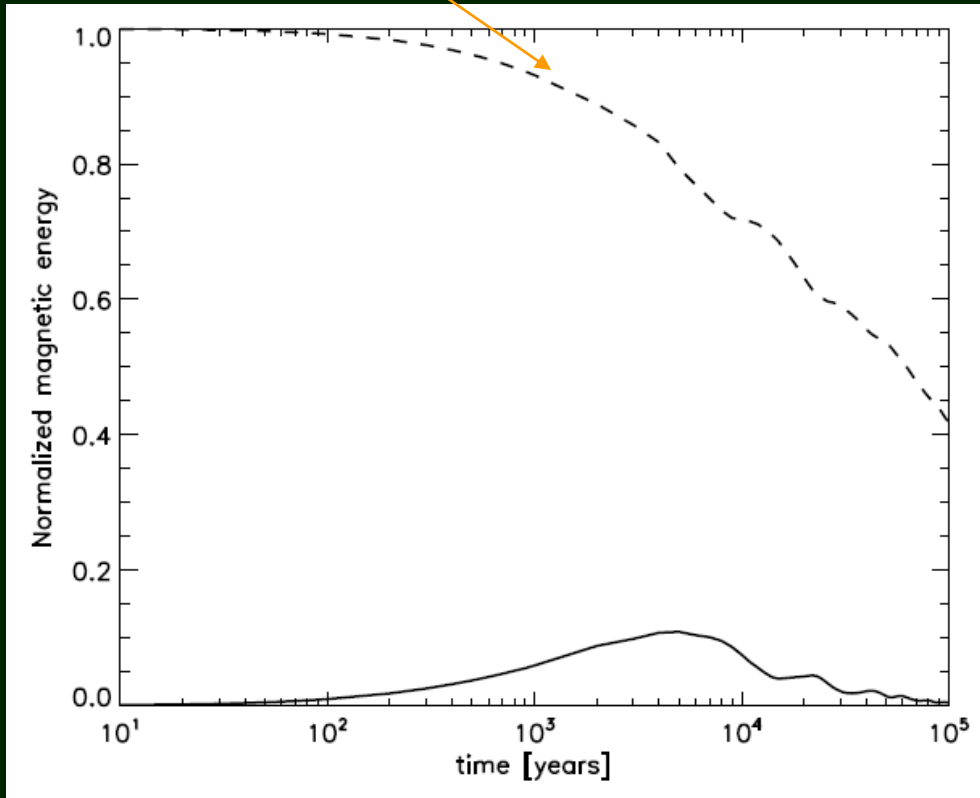
By itself, the flow is a dissipative process.

Field evolution and ellipticity



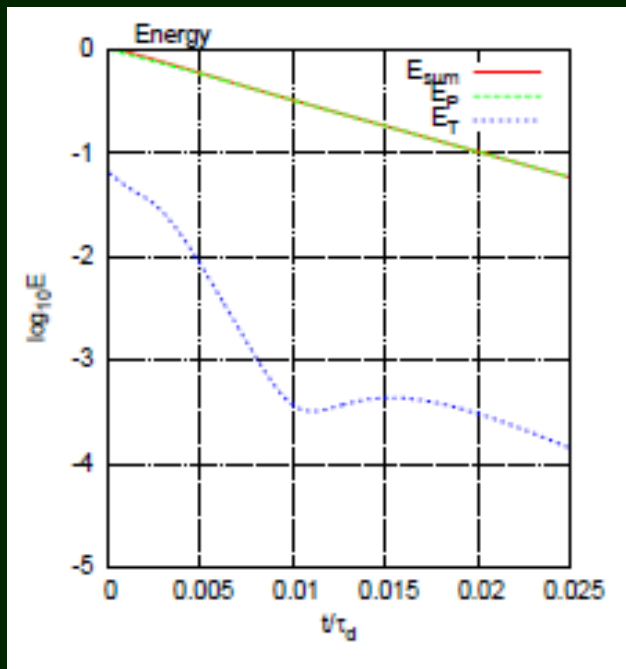
An illustrative model

Poloidal

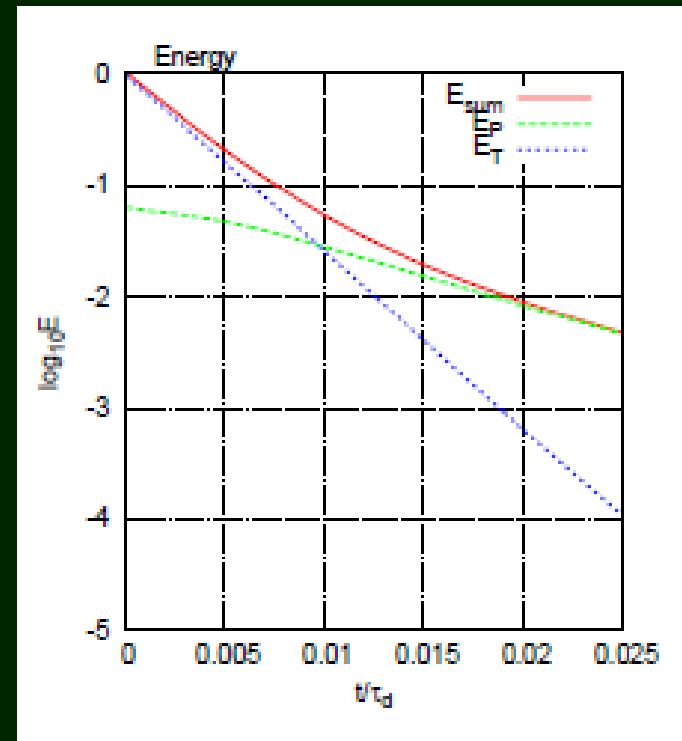


Test illustrates the evolution of initially purely poloidal field

Another model



Initially the poloidal field is large.



Initially the toroidal field is large.

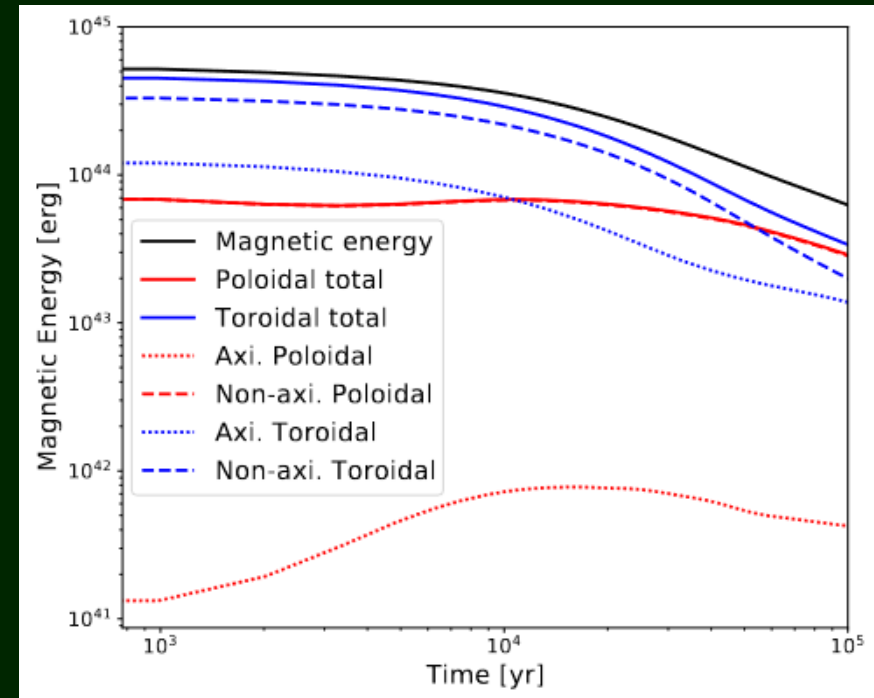
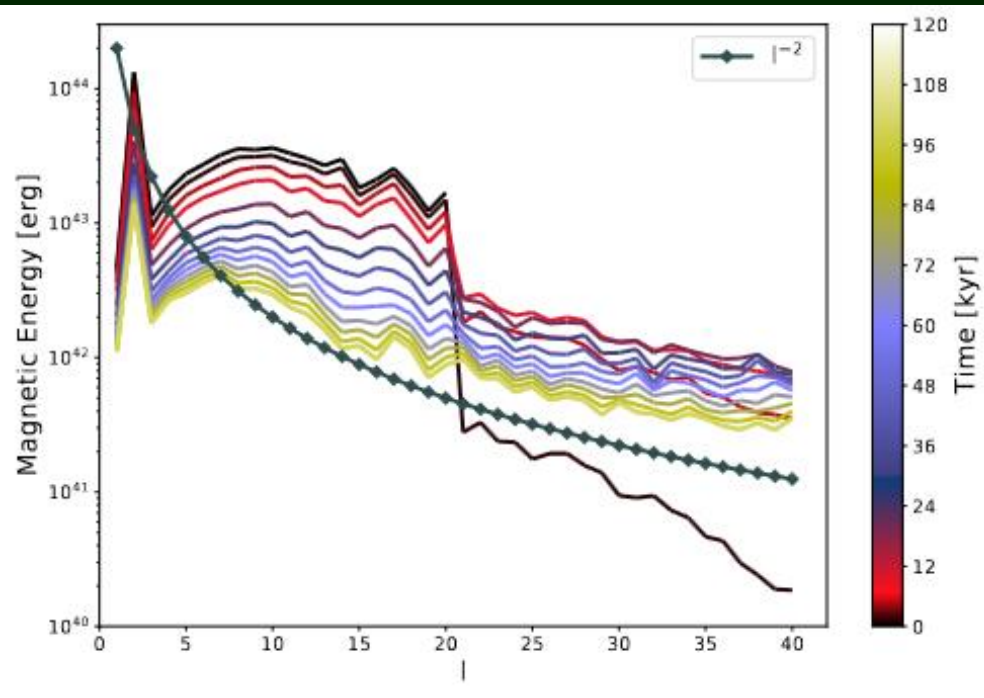
If the toroidal field dominates initially then significant energy is transferred to the poloidal component during evolution.

In the opposite case, when the poloidal component initially dominates, energy is not transferred. The toroidal component decouples.

1201.1346, toroidal might also rapidly appear in young NSs at a time scale < 1 sec (2108.11858).

Realistic initial conditions

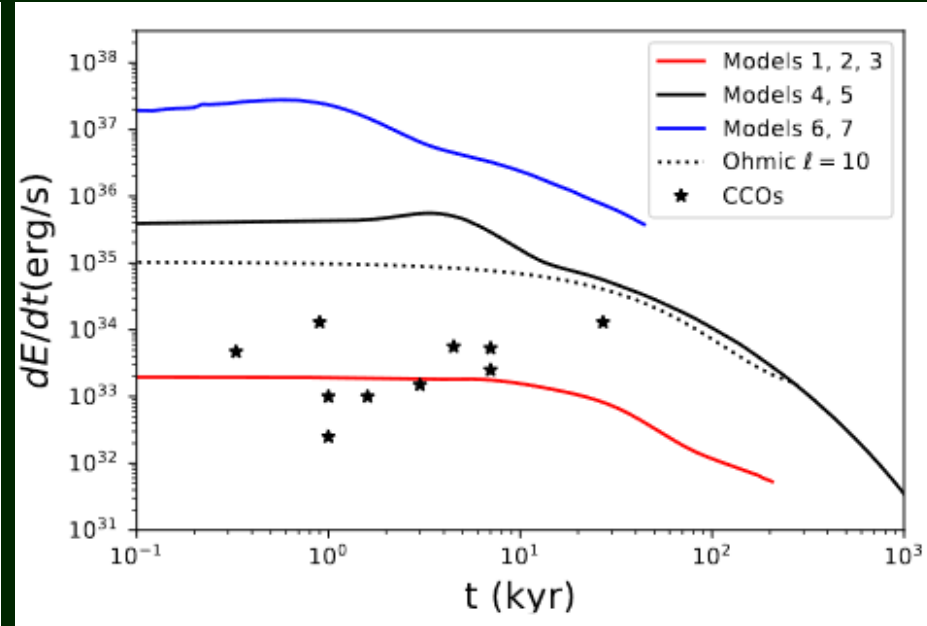
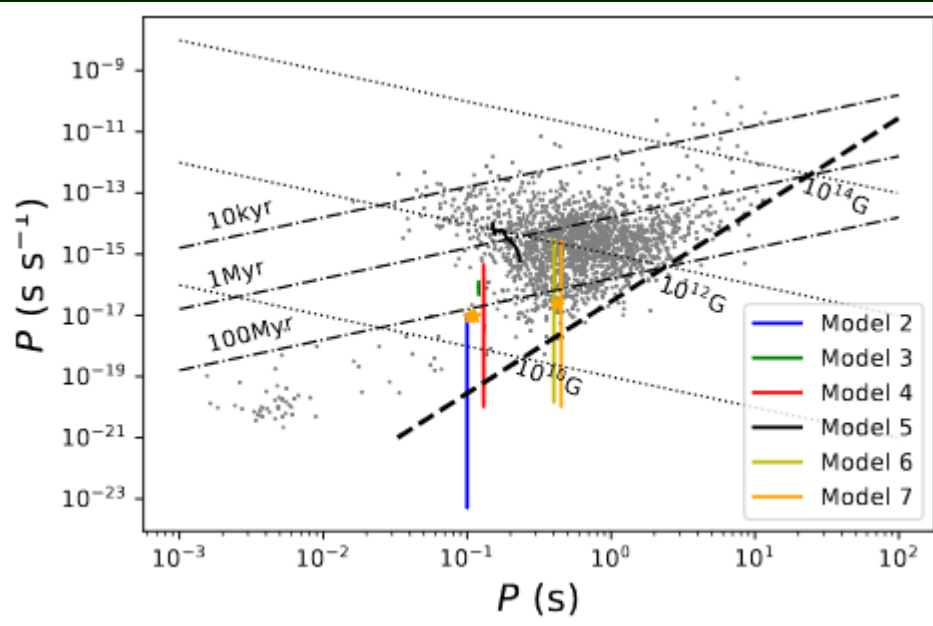
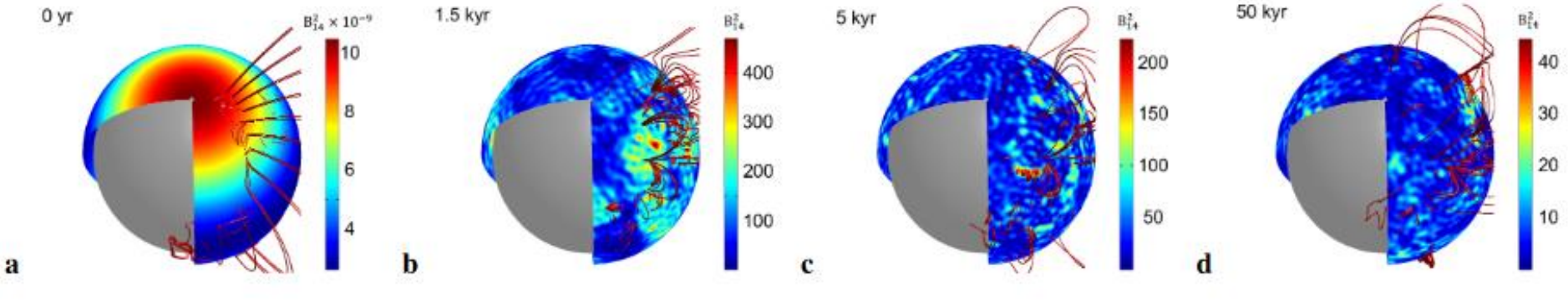
The initial field corresponds to expectations from the SN and NS formation models.



See the code description in 2209.12920

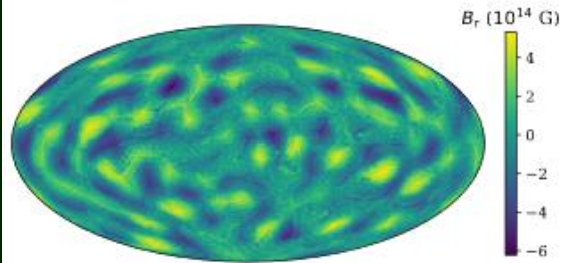
Tangled initial field

Can be important to explain CCOs

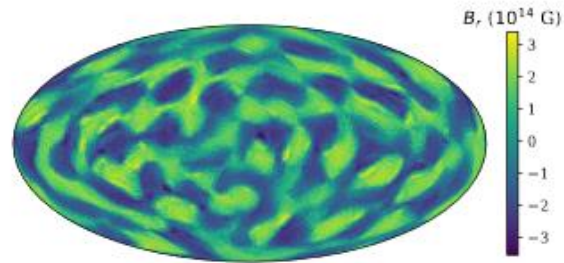


Tangled field - 2

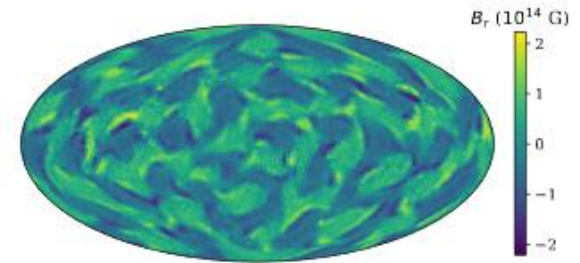
stochastic dynamo scenario



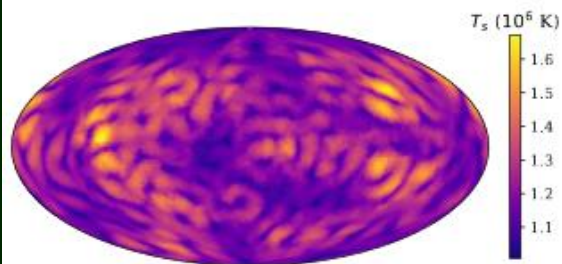
3.5 Kyr, (a)



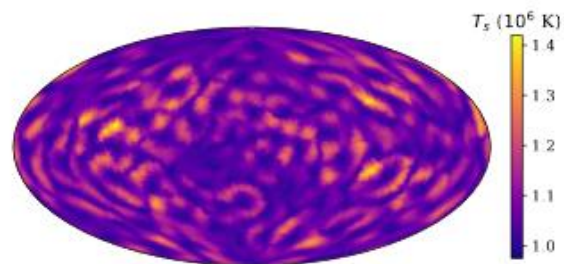
9.5 Kyr, (b)



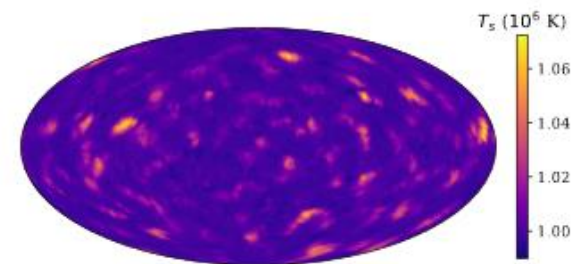
37.7 Kyr, (c)



(d)

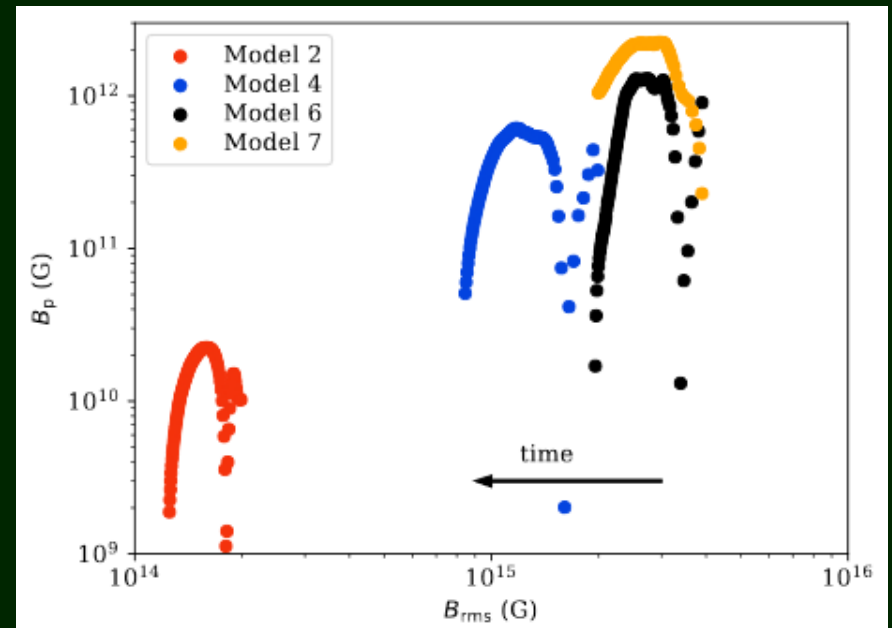
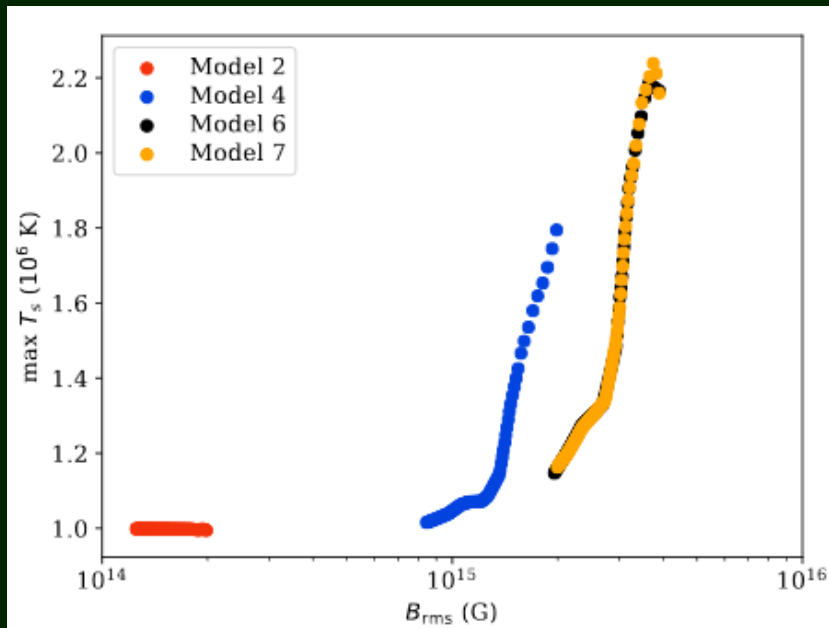
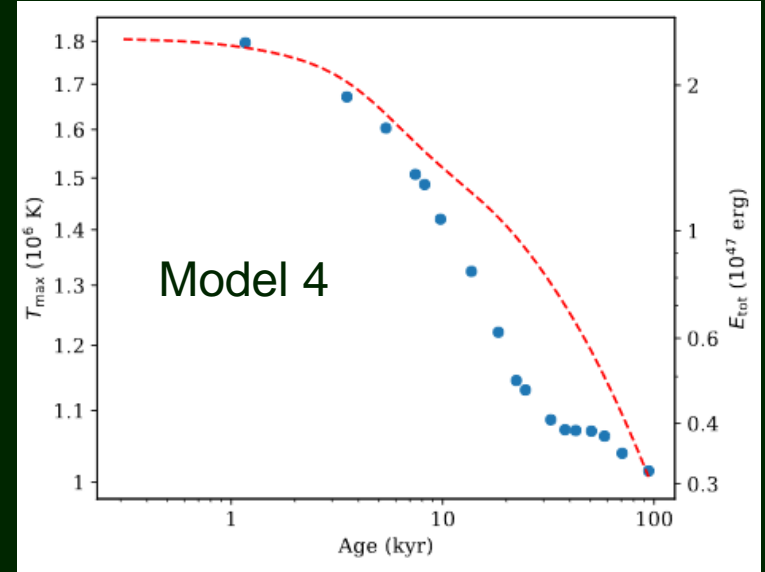
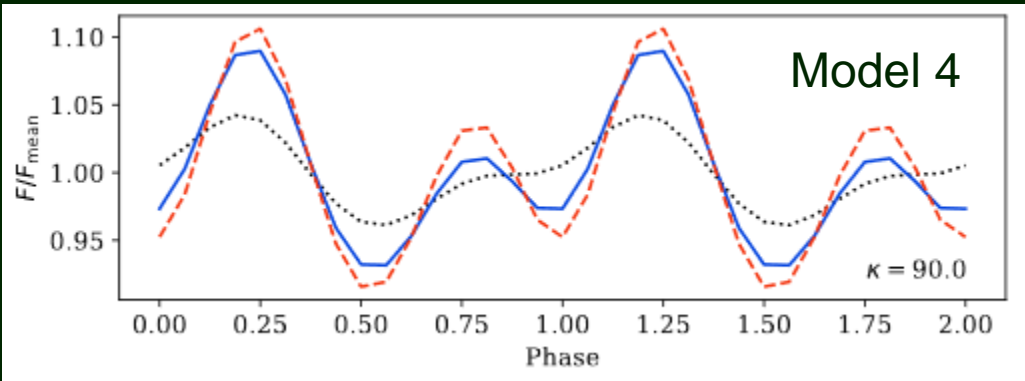


(e)



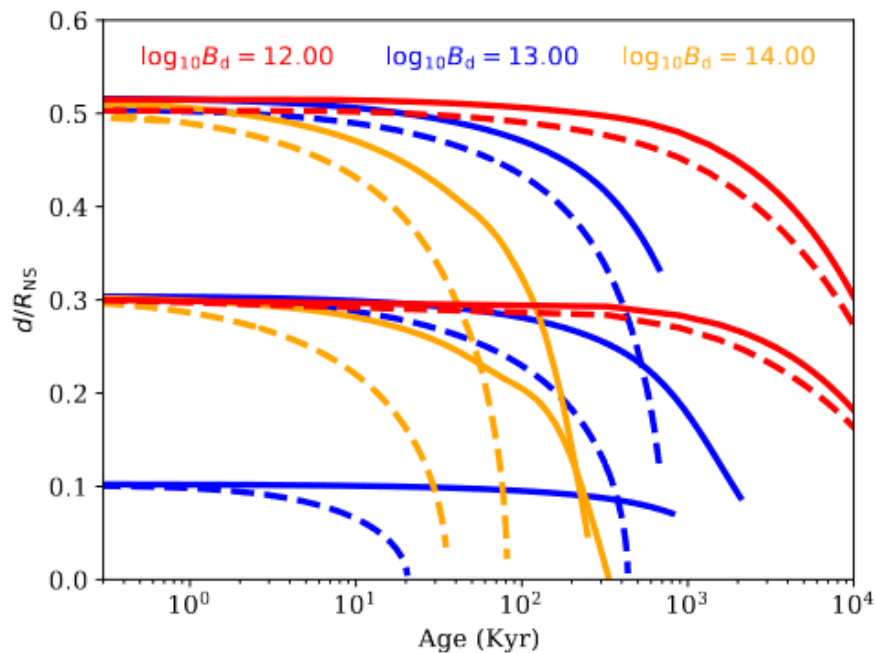
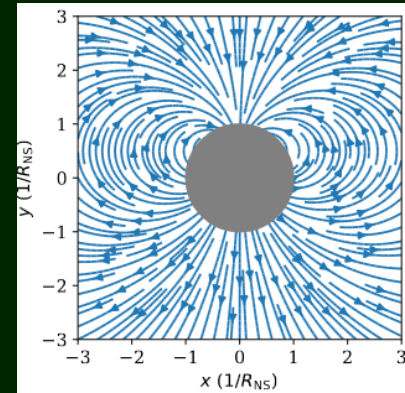
(f)

Tangled field - 2

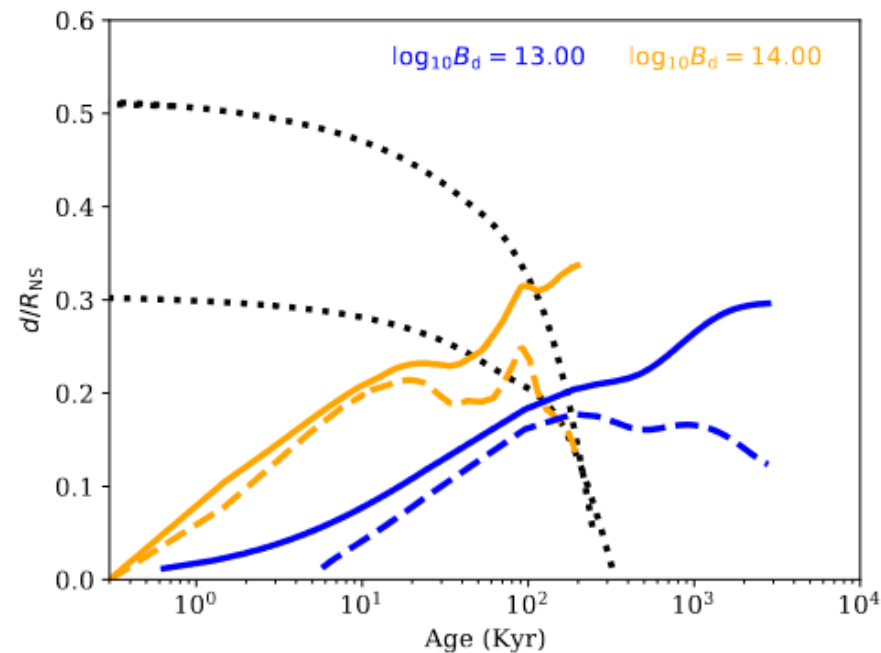


Off-centered dipoles

Numerical modeling off-centered dipoles is not an easy task. For the first time, the authors address this problem focusing only on the crustal field evolution.

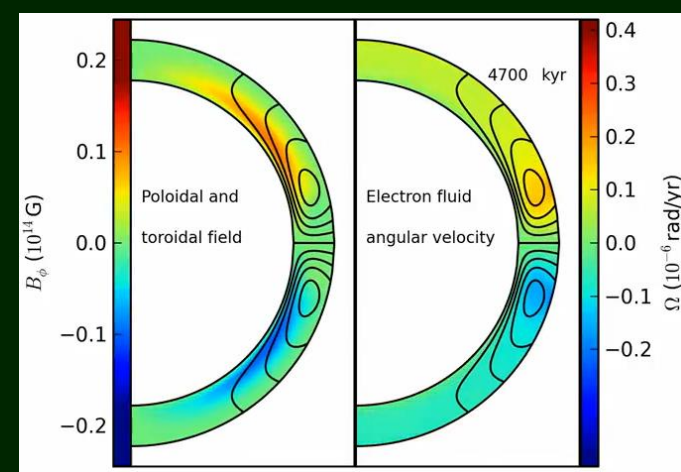
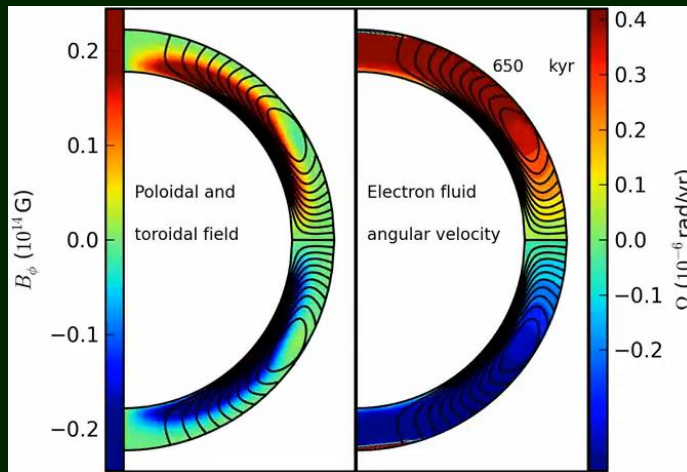
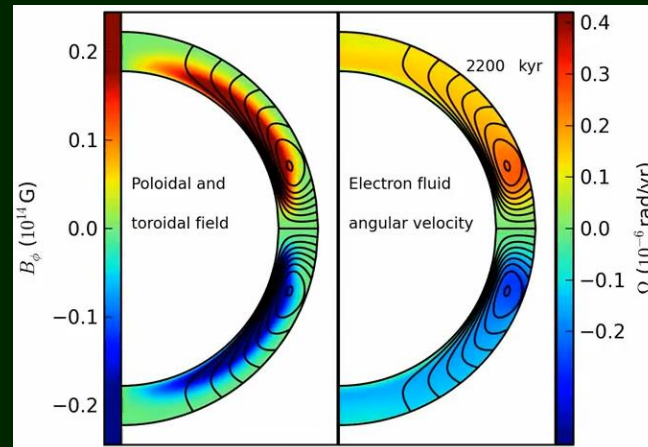
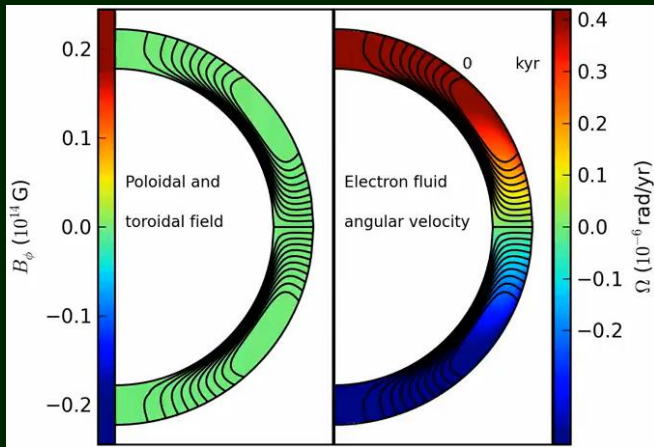


No toroidal field



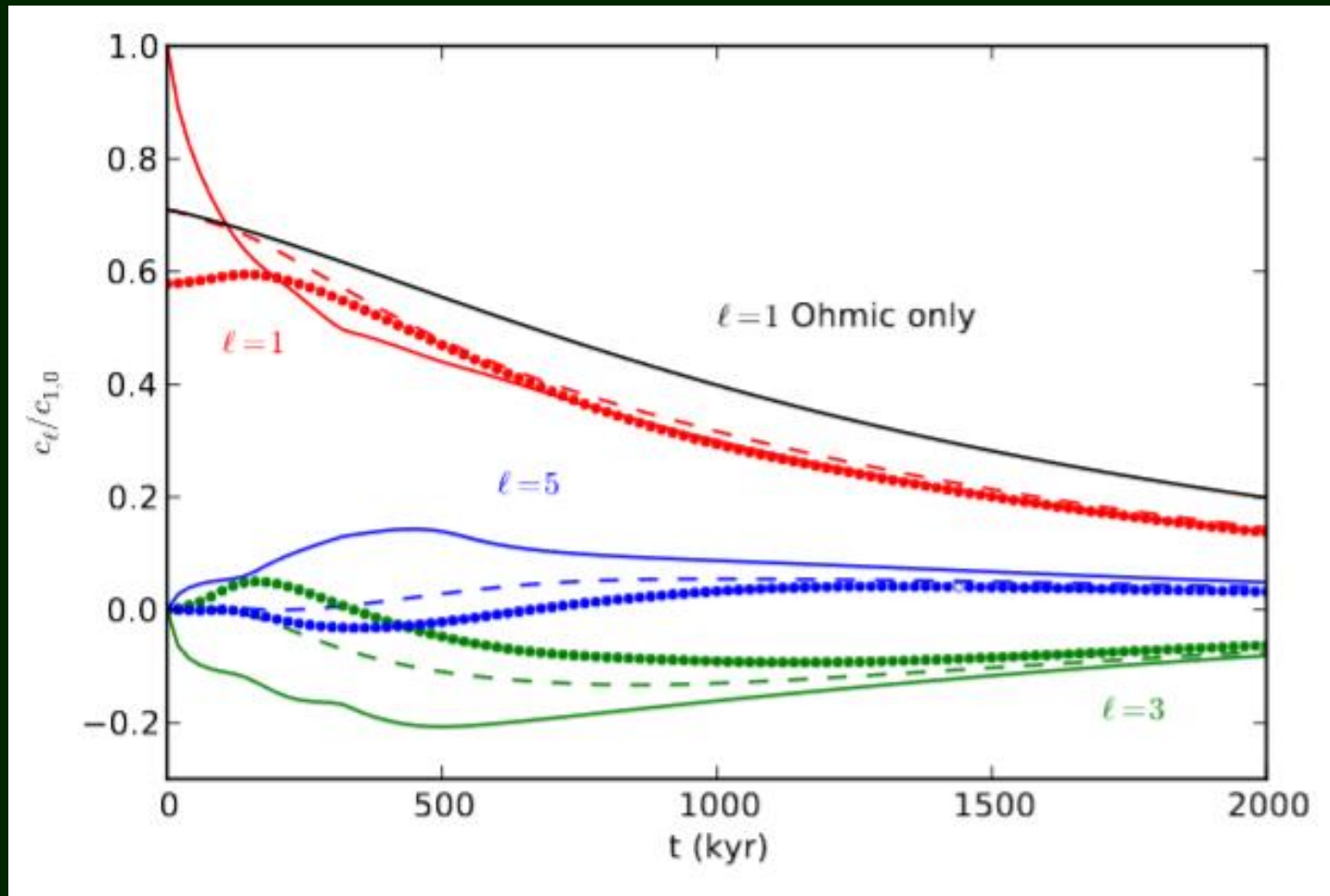
Initially centered dipole plus a toroidal field

Hall cascade and attractor



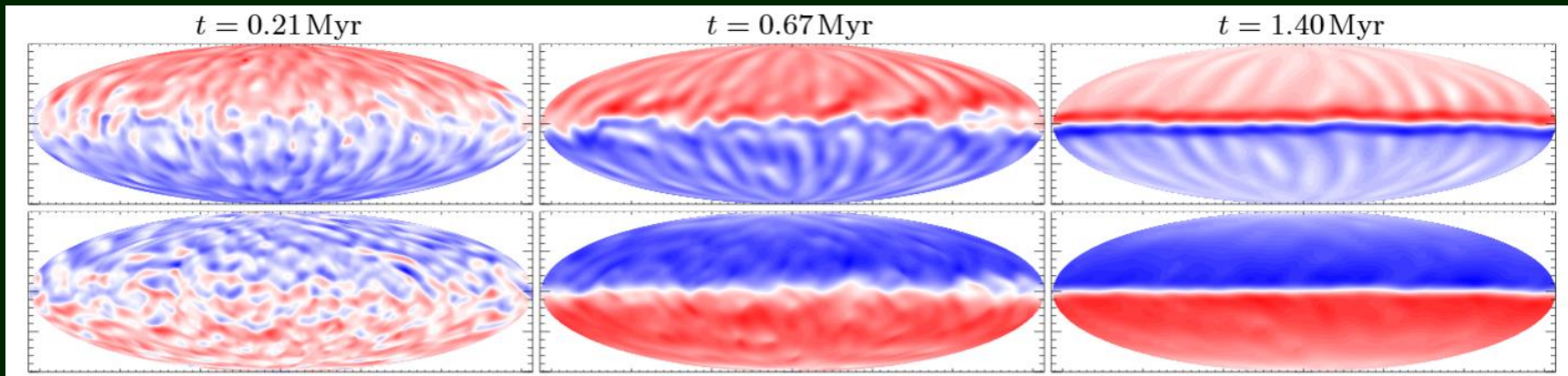
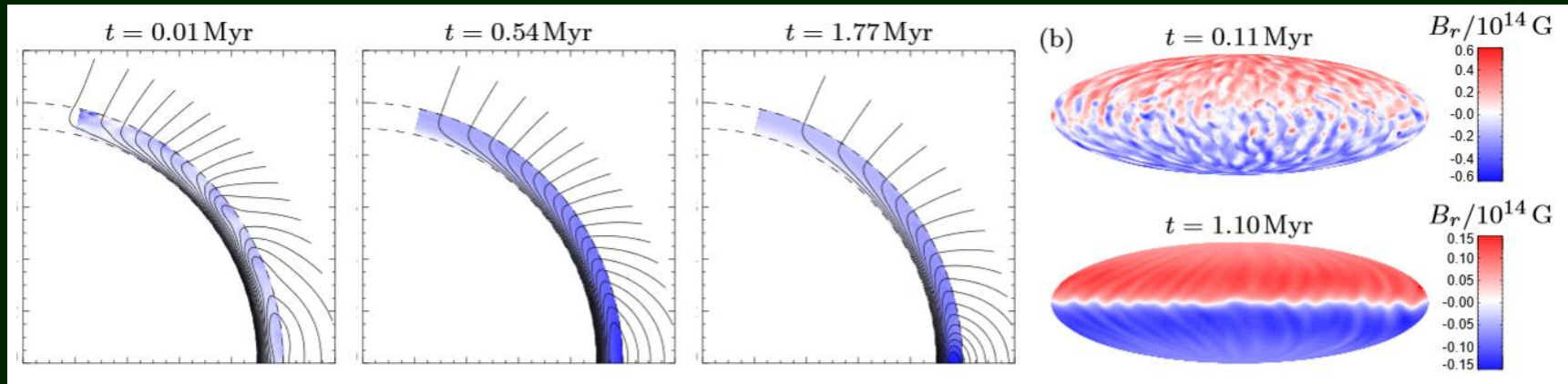
Hall cascade can reach the stage of so-called Hall attractor, where the field decay stalls for some time (Gourgouliatos, Cumming).

Evolution of different components



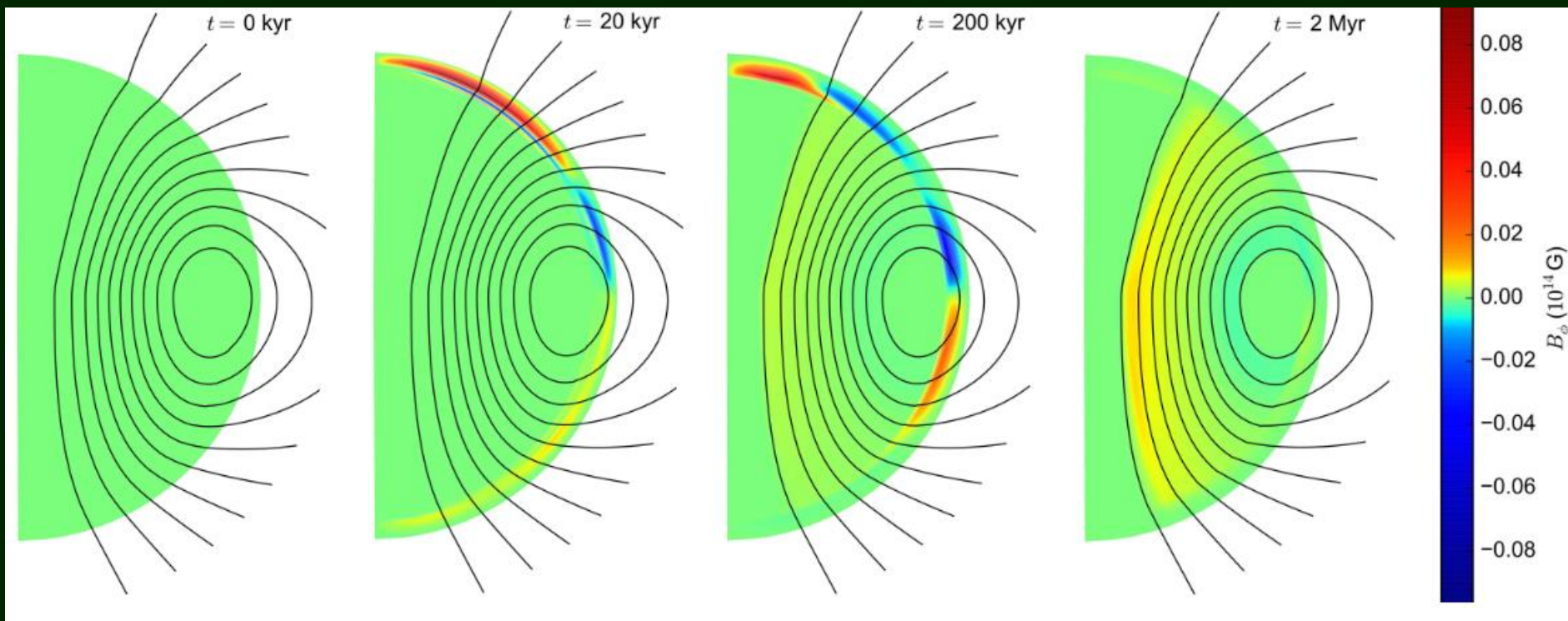
Hall attractor mainly consists of dipole and octupole (+15)

New studies of the hall cascade



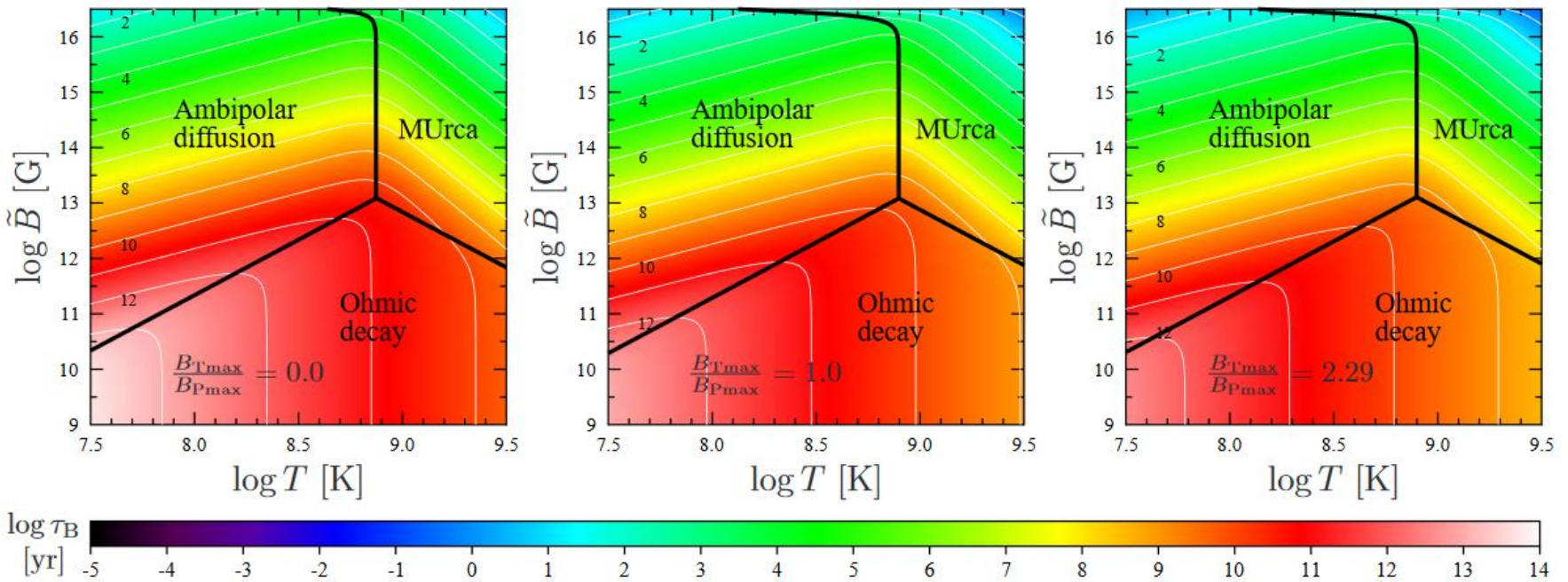
New calculations support the idea of a kind of stable configuration.

Core and crust field evolution



Hall attractor is confirmed.

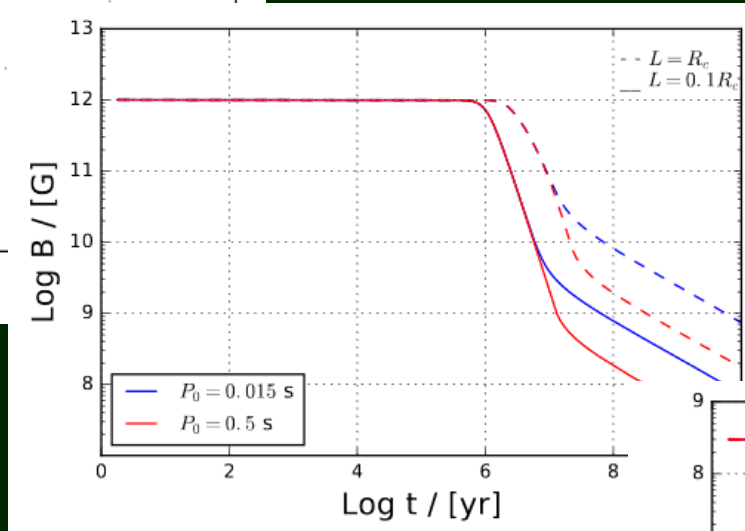
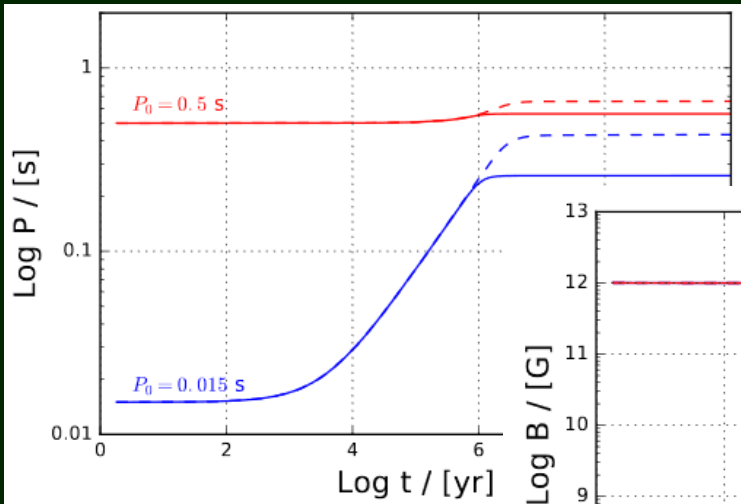
Core field evolution



Typical timescales for the magnetic field dissipation as functions of temperature and the magnetic field strength.

Field evolution due to ambipolar diffusion

Hypothesis: field decay in MSPs is caused by ambipolar diffusion in the NS core in the non-superfluid/superconductor regime.



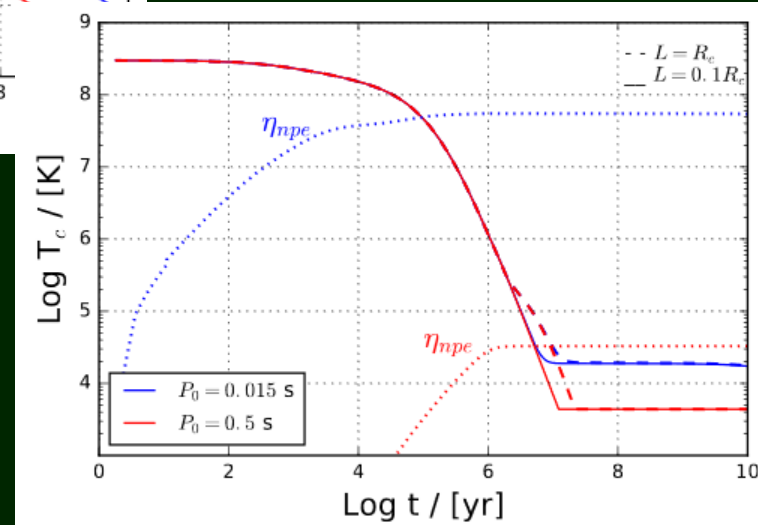
$$t_{Ohm} \ll t_{AD}$$

$$\dot{B} \approx -\frac{B}{t_{AD}}$$

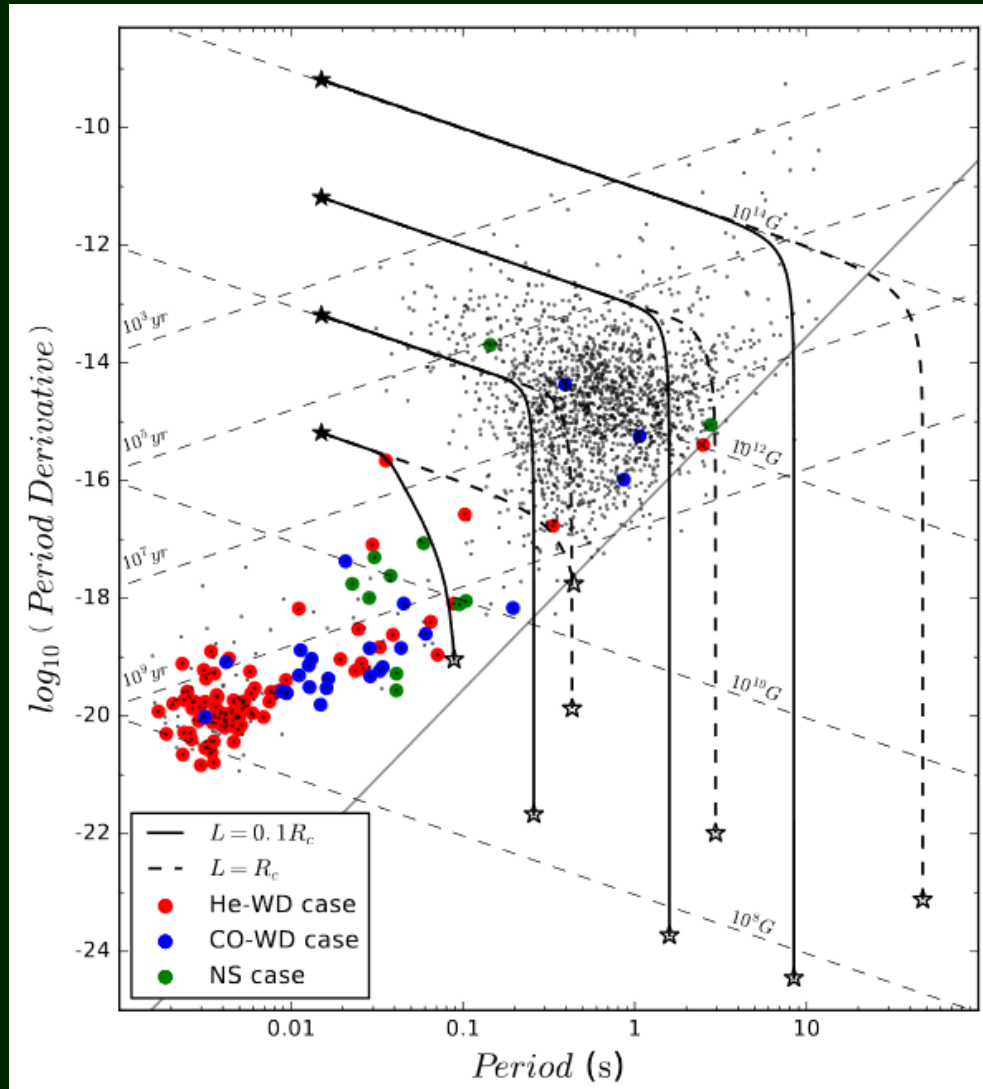
the magnetic field is transported by the charged particles at the ambipolar diffusion velocity

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{v}_{AD} \times \vec{B})$$

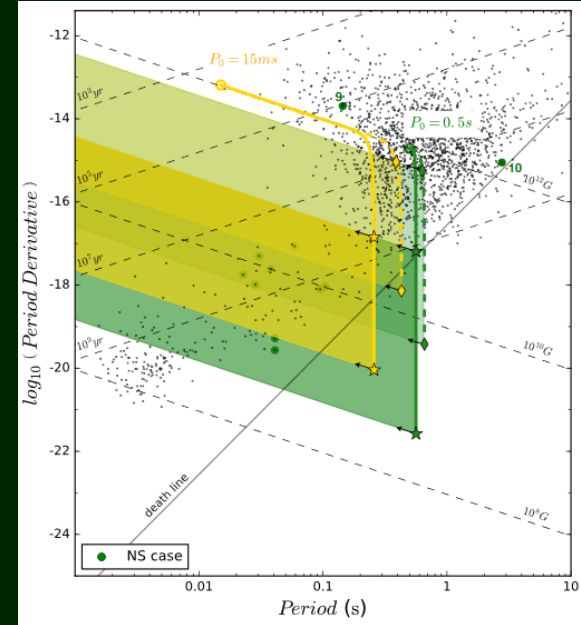
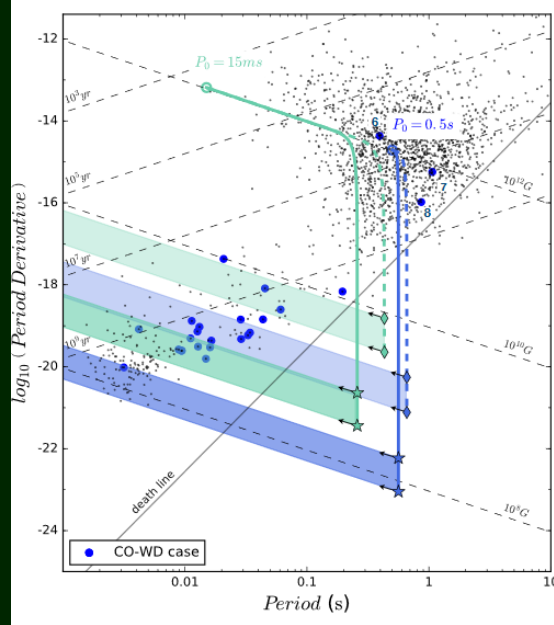
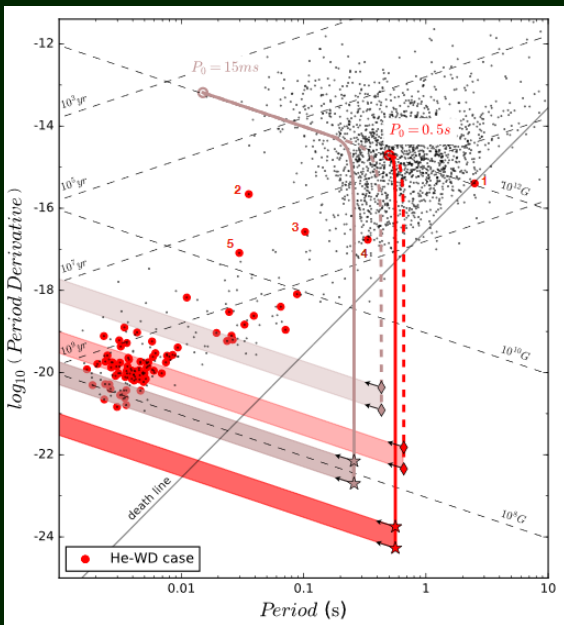
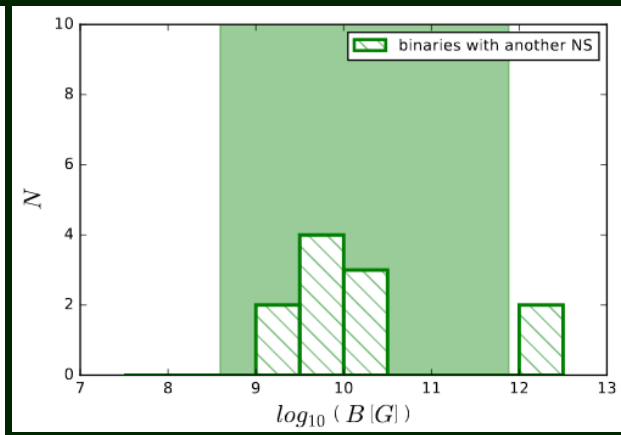
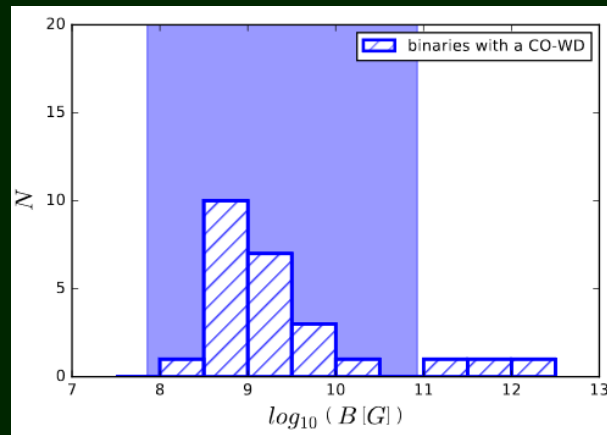
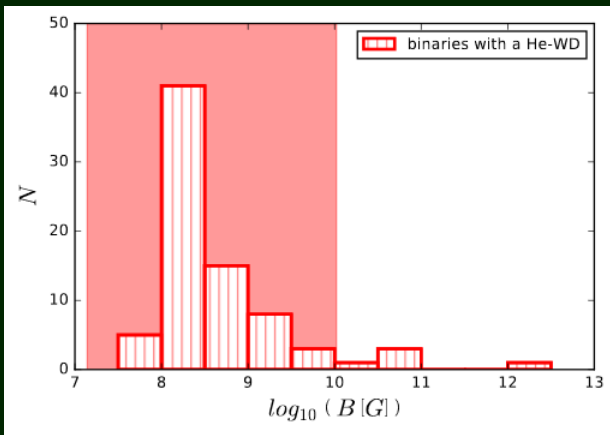
$$t_{AD} = 3 \times 10^9 \frac{T_4^2 L_5^2}{B_8^2} \text{ yr,}$$



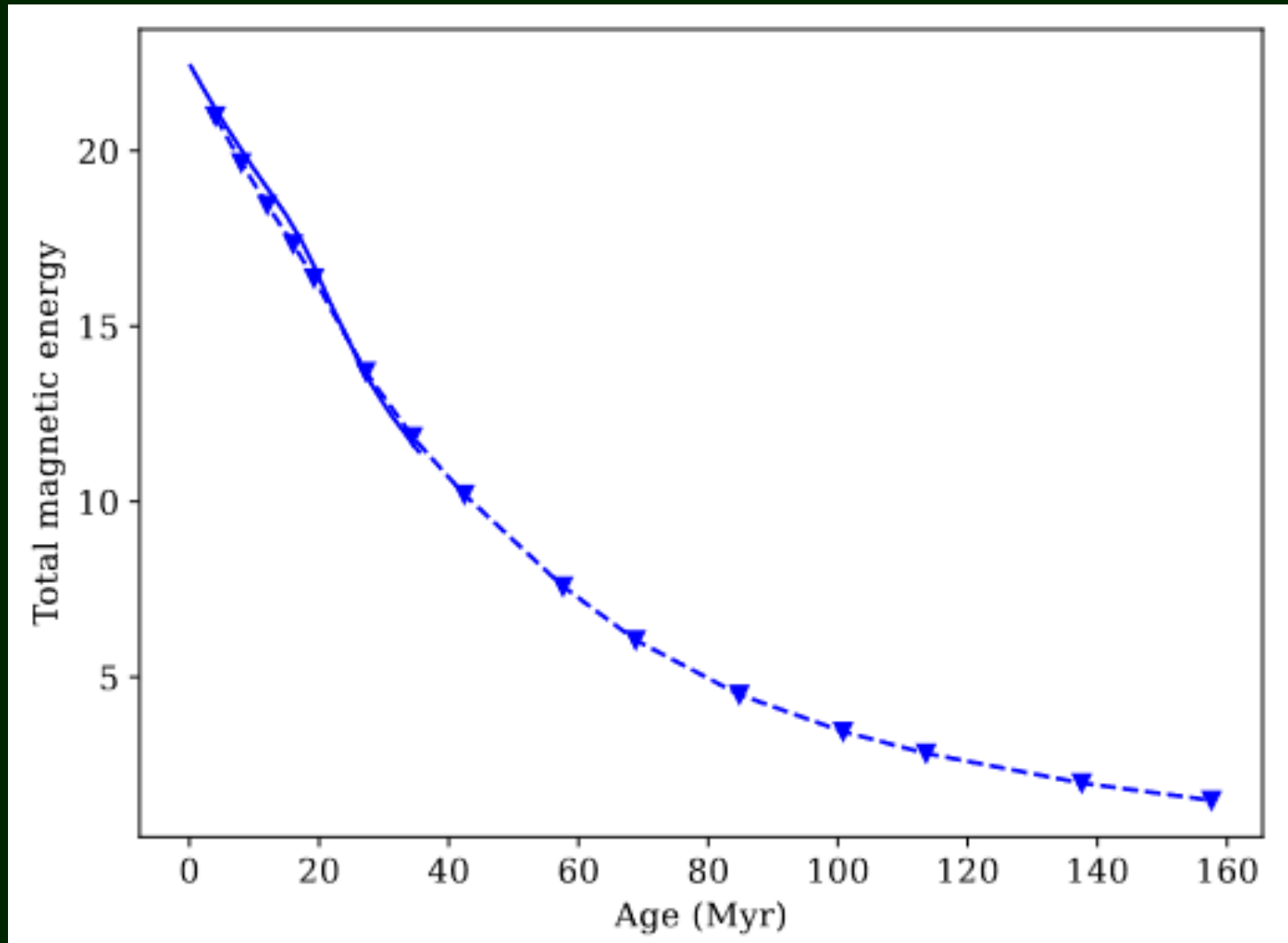
Evolution on the P-Pdot diagram



Different types of companions



Ambipolar diffusion in the core



Joint core + crust field evolution

Ambipolar diffusion +
Hall/Ohm effects

$$v_{amb} = \frac{x_n^2 t_{pn}}{m_p^*} \left[\frac{f_{mag}}{n_c} - \nabla(\Delta\mu) \right]$$

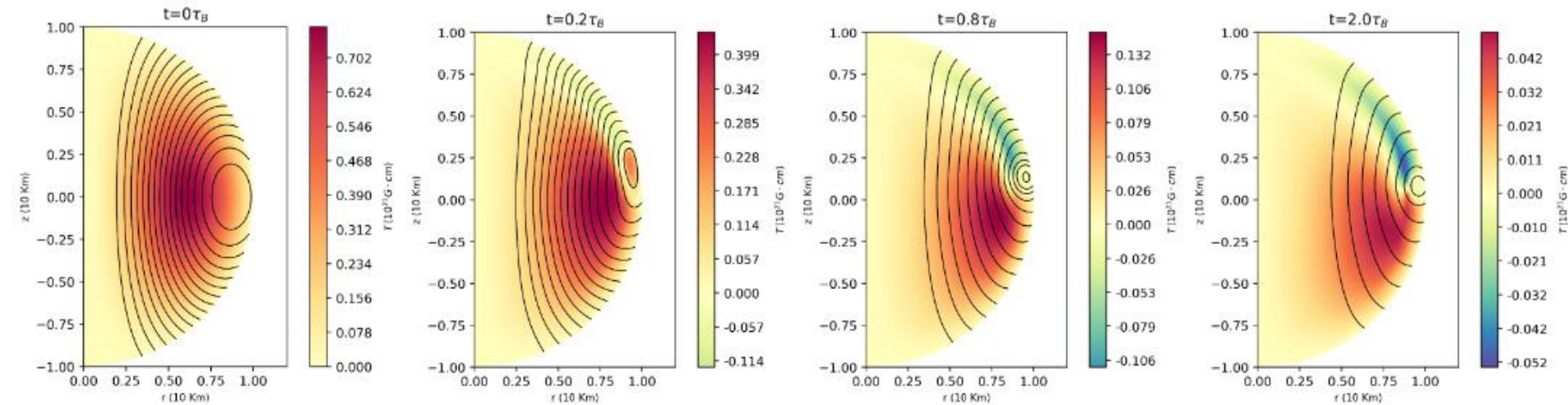
$$\frac{\partial B}{\partial t} = -\nabla \times \left(\frac{c}{4\pi e n_c} (\nabla \times B) \times B + \frac{c^2}{4\pi\sigma} \nabla \times B \right) + \nabla \times (v_p \times B)$$

$$\tau_{Ohmic} = \frac{4\pi\sigma L^2}{c^2},$$

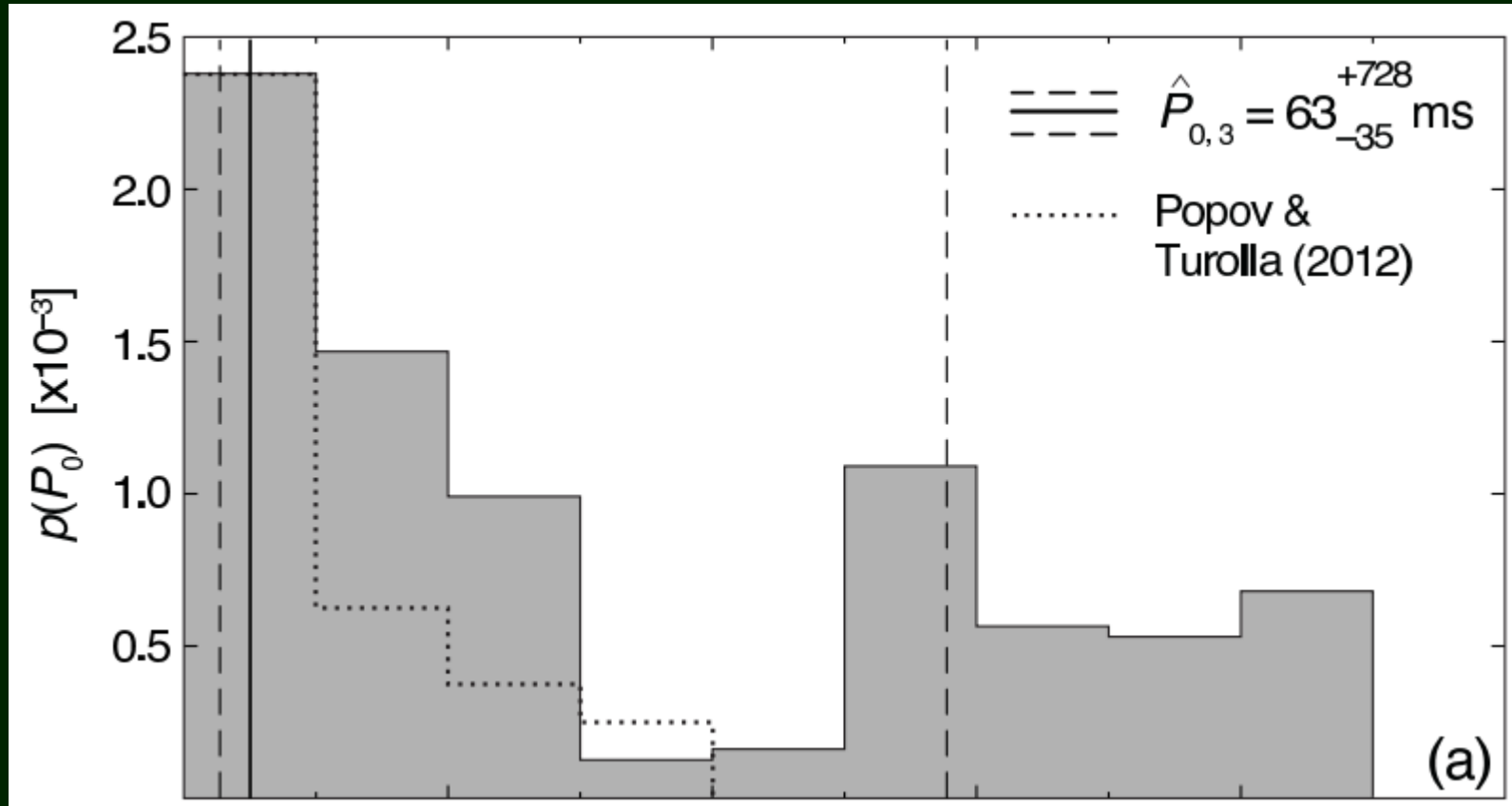
$$\tau_{Hall} = \frac{4\pi n_c e L^2}{c B_0}.$$

$$\tau_{AD} = \frac{4\pi n_c m_p^* L^2}{x_n^2 t_{pn} B^2}$$

Evolution of the field with a mixed (poloidal+toroidal) initial configuration



Wide initial spin period distribution



Based on kinematic ages. Mean age – few million years.
Note, that in Popov & Turolla (2012) only NSs in SNRs
were used, i.e. the sample is much younger!
Can it explain the difference?

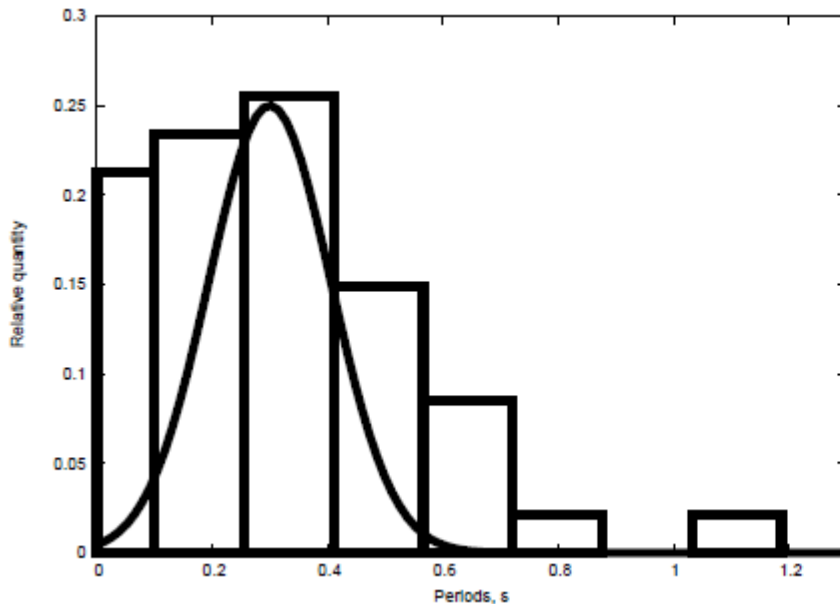
Magnetic field decay and P_0

One can suspect that magnetic field decay can influence the reconstruction of the initial spin period distribution.

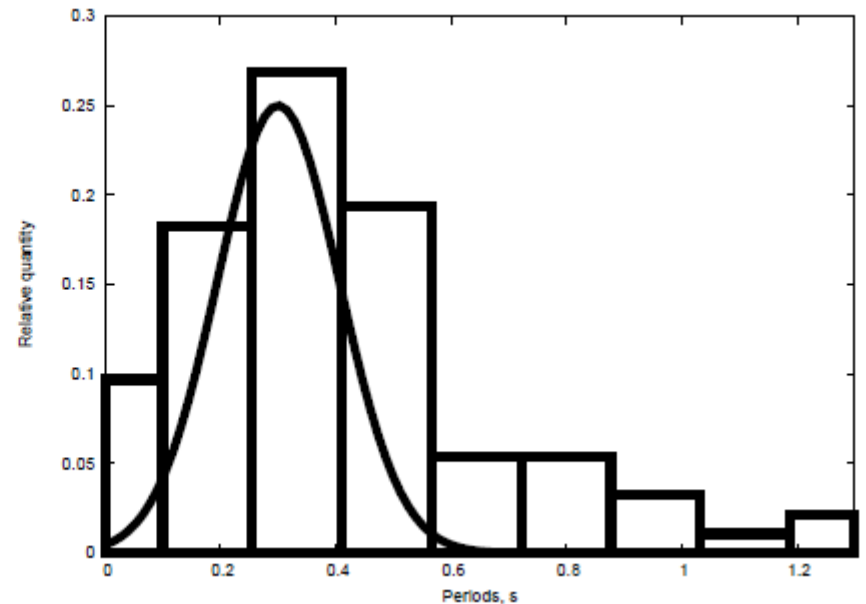
Exponential field decay with $\tau=5$ Myrs.

$\langle P_0 \rangle = 0.3$ s, $\sigma_P = 0.15$ s; $\langle \log B_0 / [G] \rangle = 12.65$, $\sigma_B = 0.55$

$$P_0 = P \sqrt{1 - \frac{t}{\tau}}$$

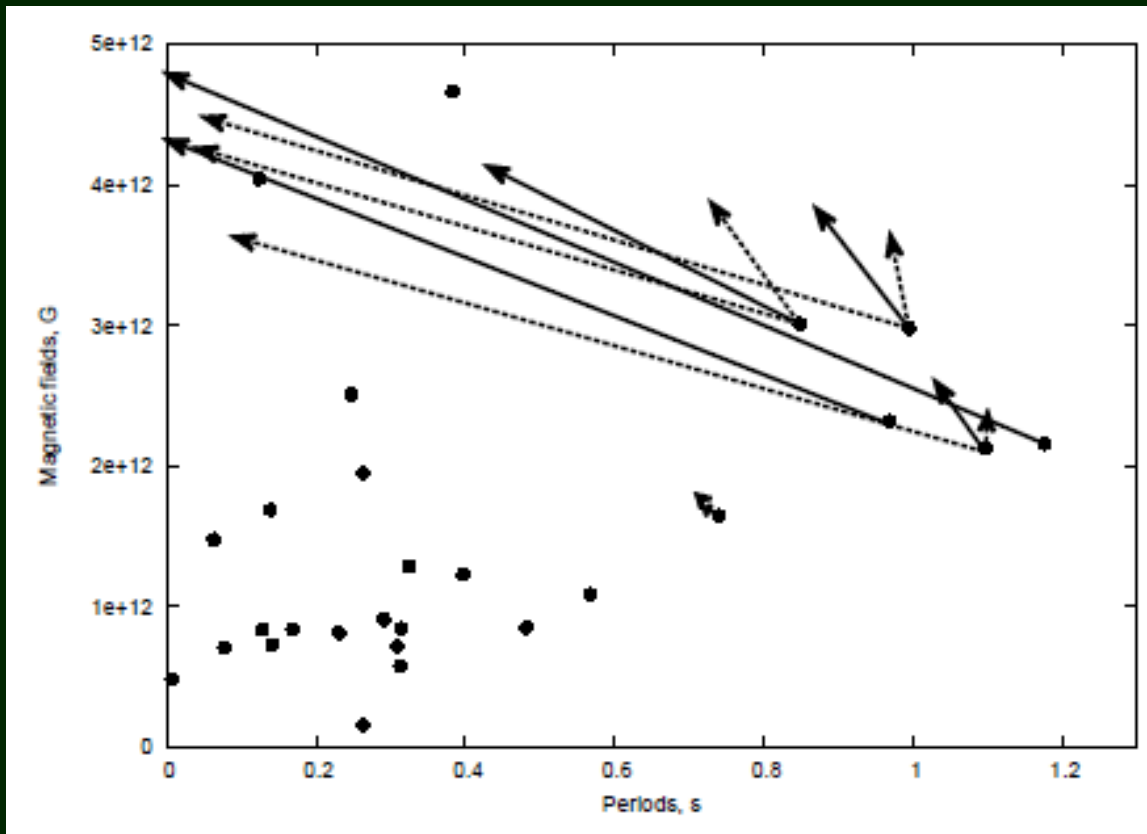


$\tau < 10^7$ yrs, $10^5 < t$



$10^5 < t < 10^7$ yrs

Real vs. reconstructed P_0

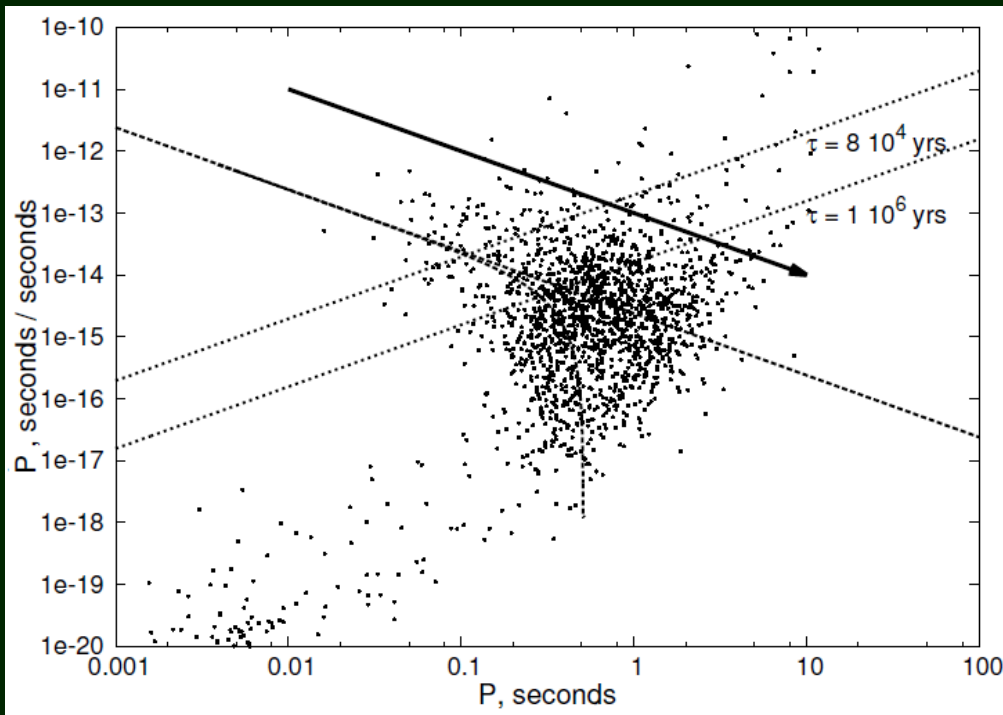


Arrows point to initial parameters of pulsars if the exponential magnetic field decay was operating.

How significantly the reconstructed initial periods changed due to not taking into account the exponential field decay

Modified pulsar current

We perform a modified pulsar current analysis. In our approach we analyse the flow not along the spin period axis, as it was done in previous studies, but study the flow along the axis of growing characteristic age.



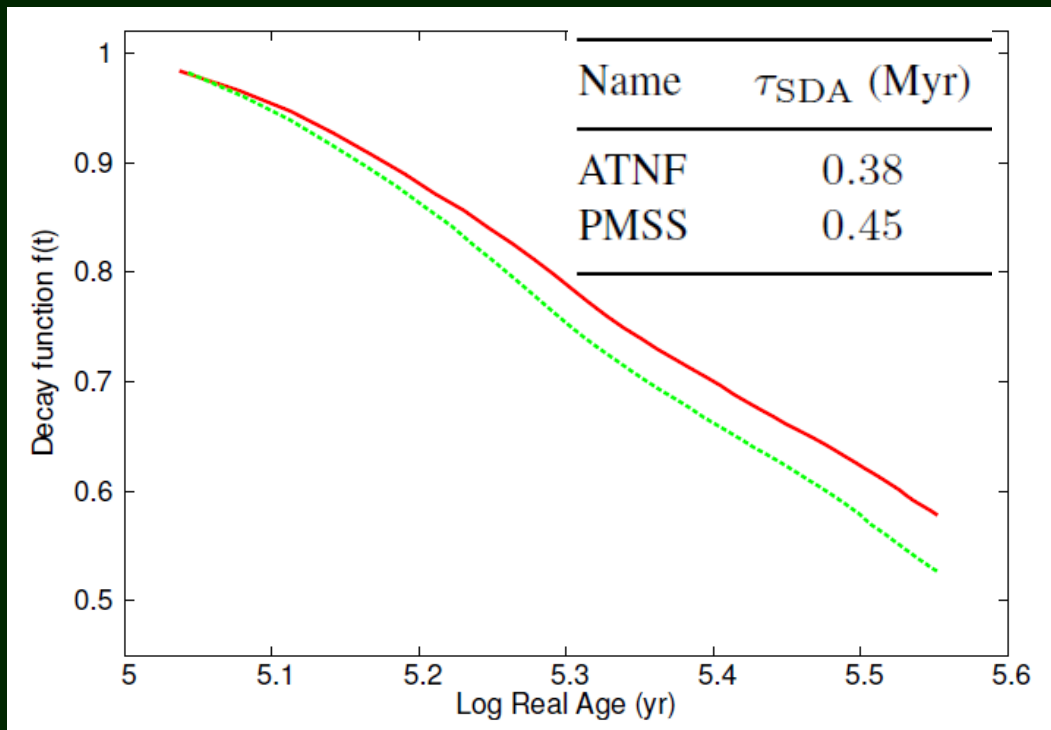
The idea is to probe magnetic field decay. Our method can be applied only in a limited range of ages.

We use distribution in characteristic ages to reconstruct the field evolution.

Application to real data

We apply our methods to large observed samples of radio pulsars to study field decay in these objects. As we need to have as large statistics as possible, and also we need uniform samples, in the first place we study sources from the ATNF catalogue (Manchester et al. 2005).

Then we apply our methods to the largest uniform subsample of the ATNF — to the PMSS (stands for the Parkes Multibeam and Swinburne surveys) (Manchester et al. 2001).



We reconstruct the magnetic field decay in the range of true (statistical) ages:

$$8 \cdot 10^4 < t < 3.5 \cdot 10^5 \text{ yrs}$$

which corresponds to characteristic ages

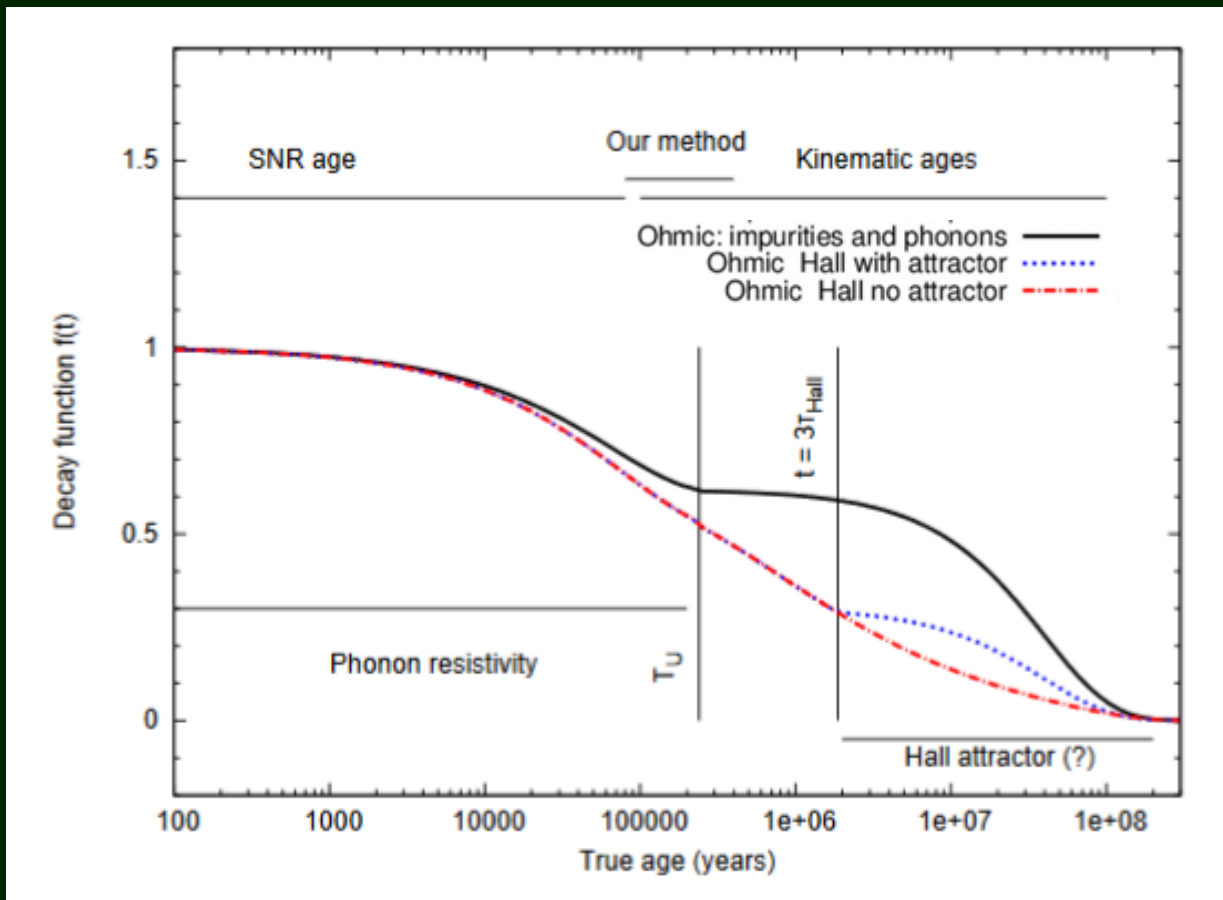
$$8 \cdot 10^4 < \tau < 10^6 \text{ yrs.}$$

In this range, the field decays roughly by a factor of two.

With an exponential fit this corresponds to the decay time scale $\sim 4 \cdot 10^5$ yrs.

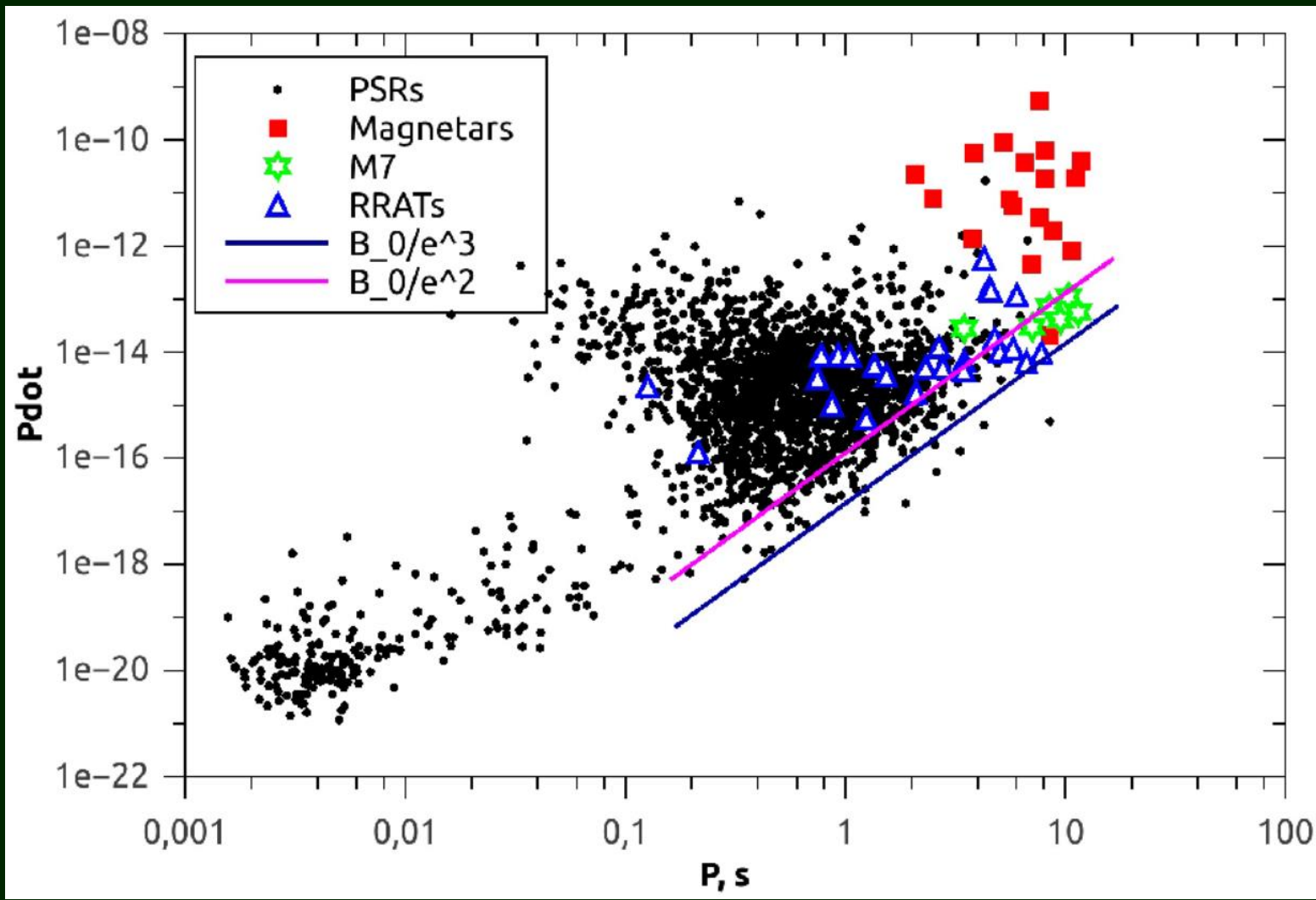
Note, this decay is limited in time.

Comparison of different options



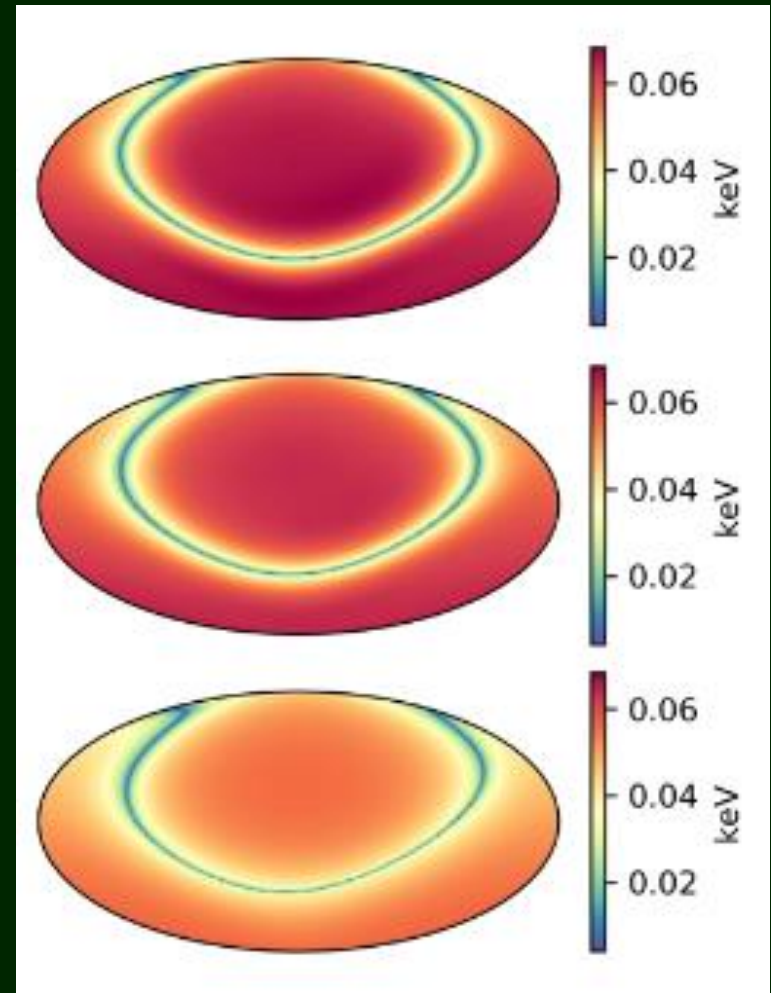
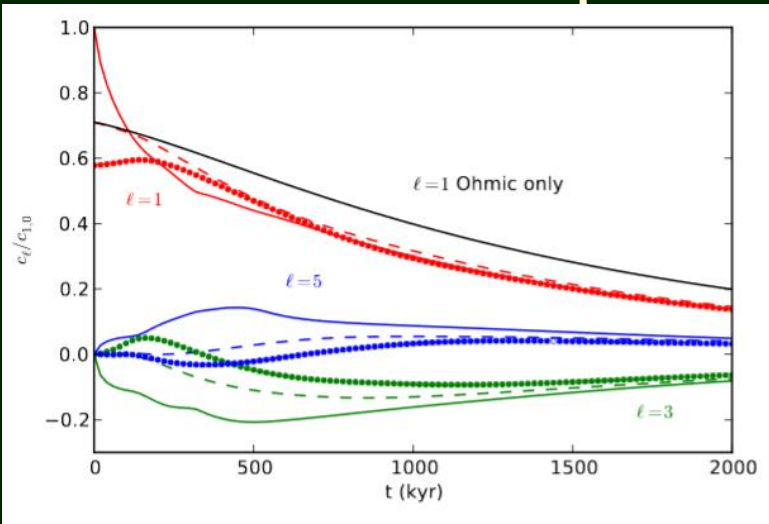
We think that at the ages $\sim 10^5$ yrs and below for normal pulsars we see mostly Ohmic decay, which then disappears as NSs cool down below the critical T.

Getting close to the attractor

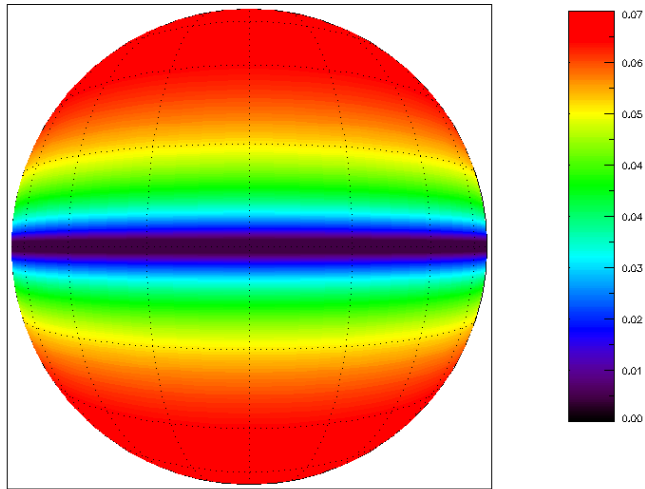


Thermal maps and Hall attractor

1311.7004



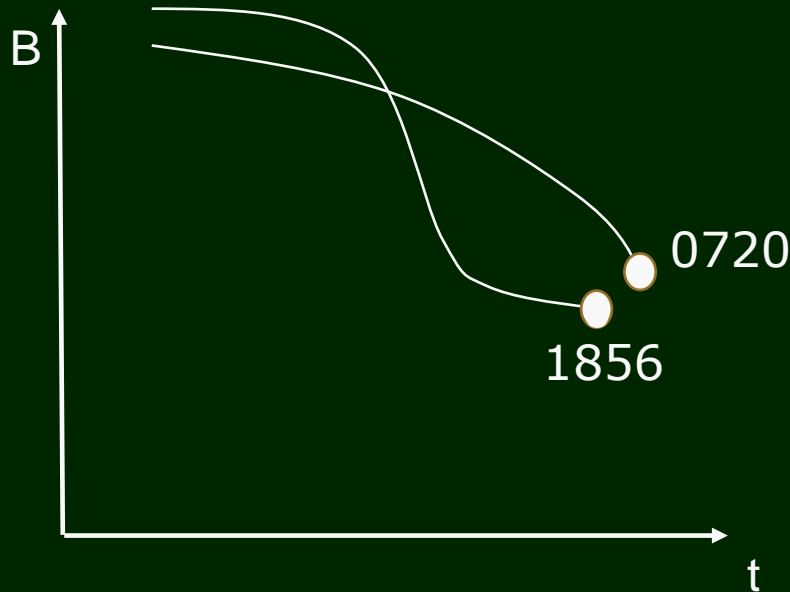
Dipole+octupole+l5



1610.05050

2105.00684

Tracks on the P-dot diagram



Kinematic age is larger for 0720, but characteristic age – for 1856.

It seems that 1856 is now on a more relaxed stage of the magneto-rotational evolution.

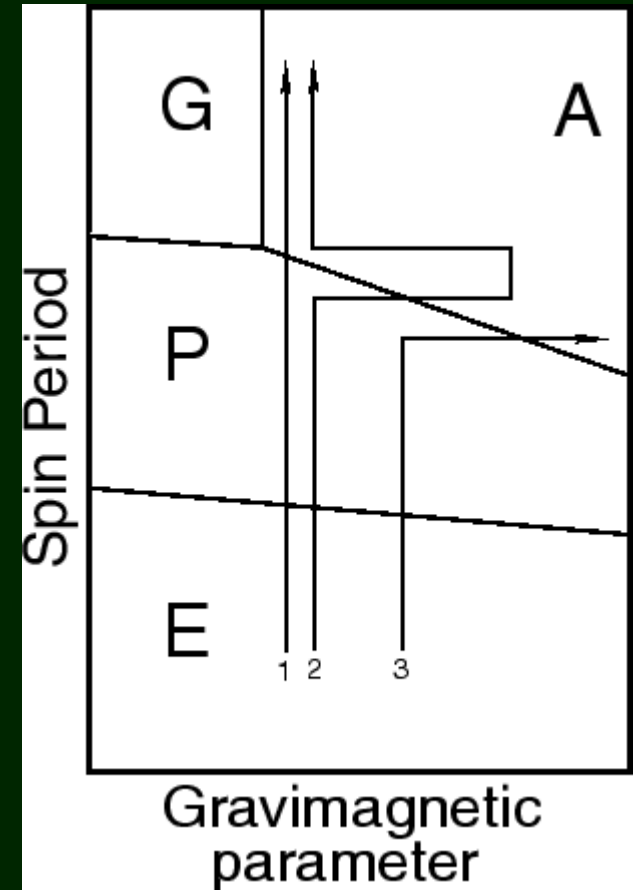
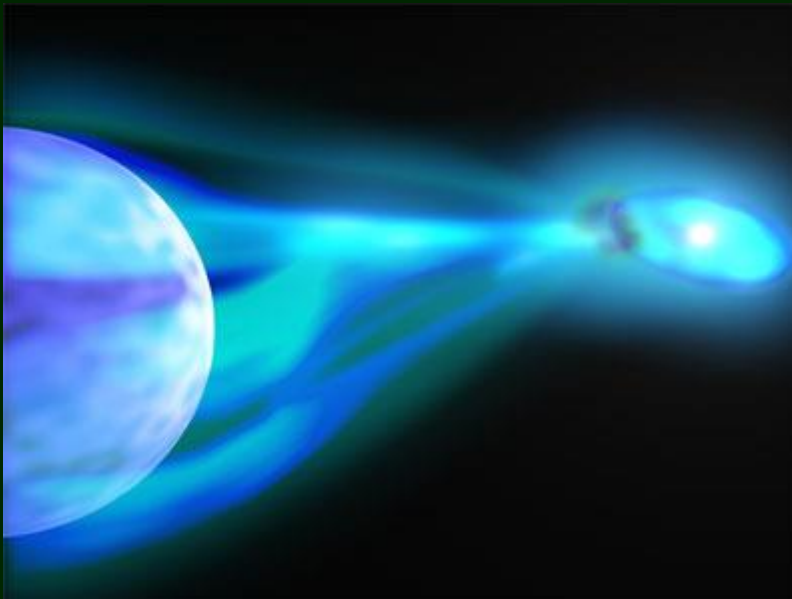
RX J0720 shows several types of activity, but RX J1856 is a very quiet source.

Non-monotonic evolution?

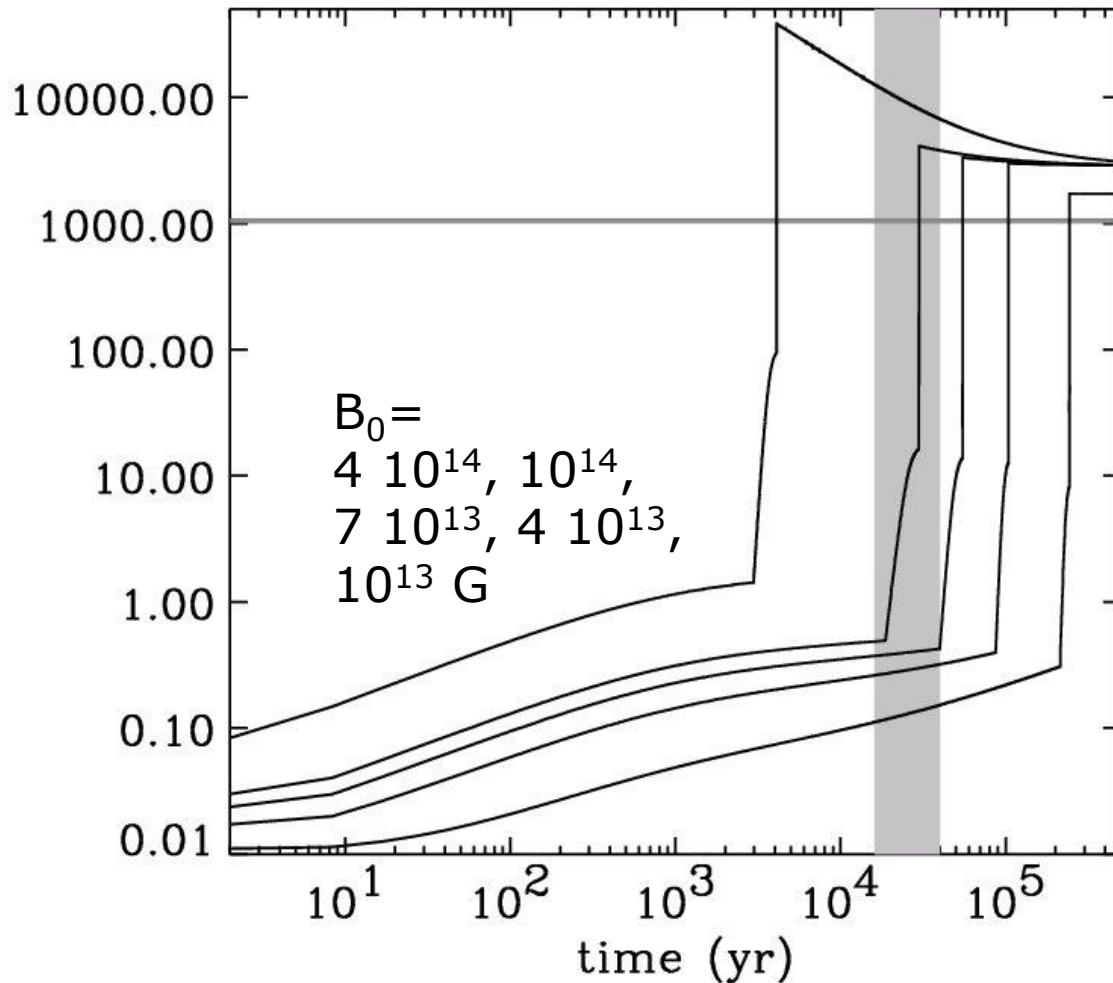
SXP 1062

A peculiar source was discovered in SMC.
Be/Xray binary, $P=1062$ sec.
A SNR is found. Age $\sim 10^4$ yrs.
(1110.6404; 1112.0491)

Typically, it can take ~ 1 Myr for a NS
with $B \sim 10^{12}$ G to start accretion.



Evolution of SXP 1062



A model of a NS with initial field $\sim 10^{14}$ G which decayed down to $\sim 10^{13}$ G can explain the data on SXP 1062.

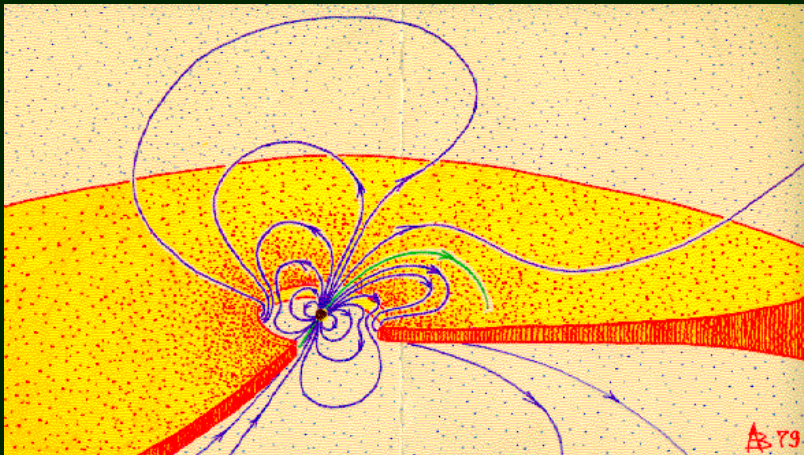
1112.2507

Some new data in
1304.6022

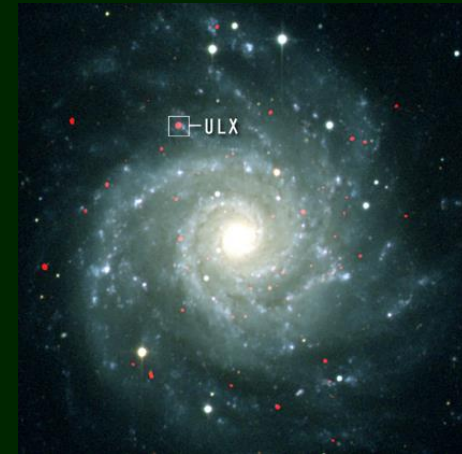
Many other scenarios have been proposed.
We need new observational data.

Accreting magnetars

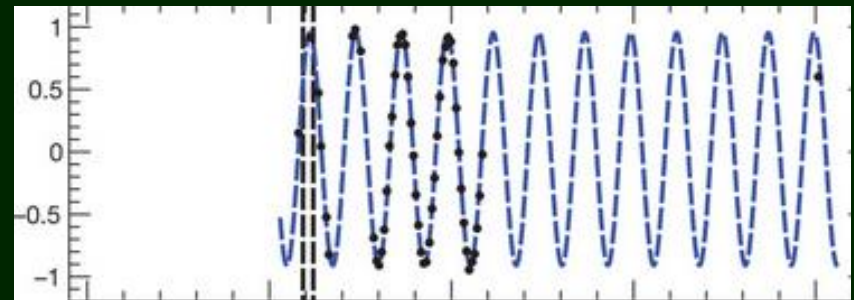
Typically magnetic fields of neutron stars in accreting X-ray binaries are estimated with indirect methods.



- Spin-up
- Spin-down
- Equilibrium period
- Accretion model
-

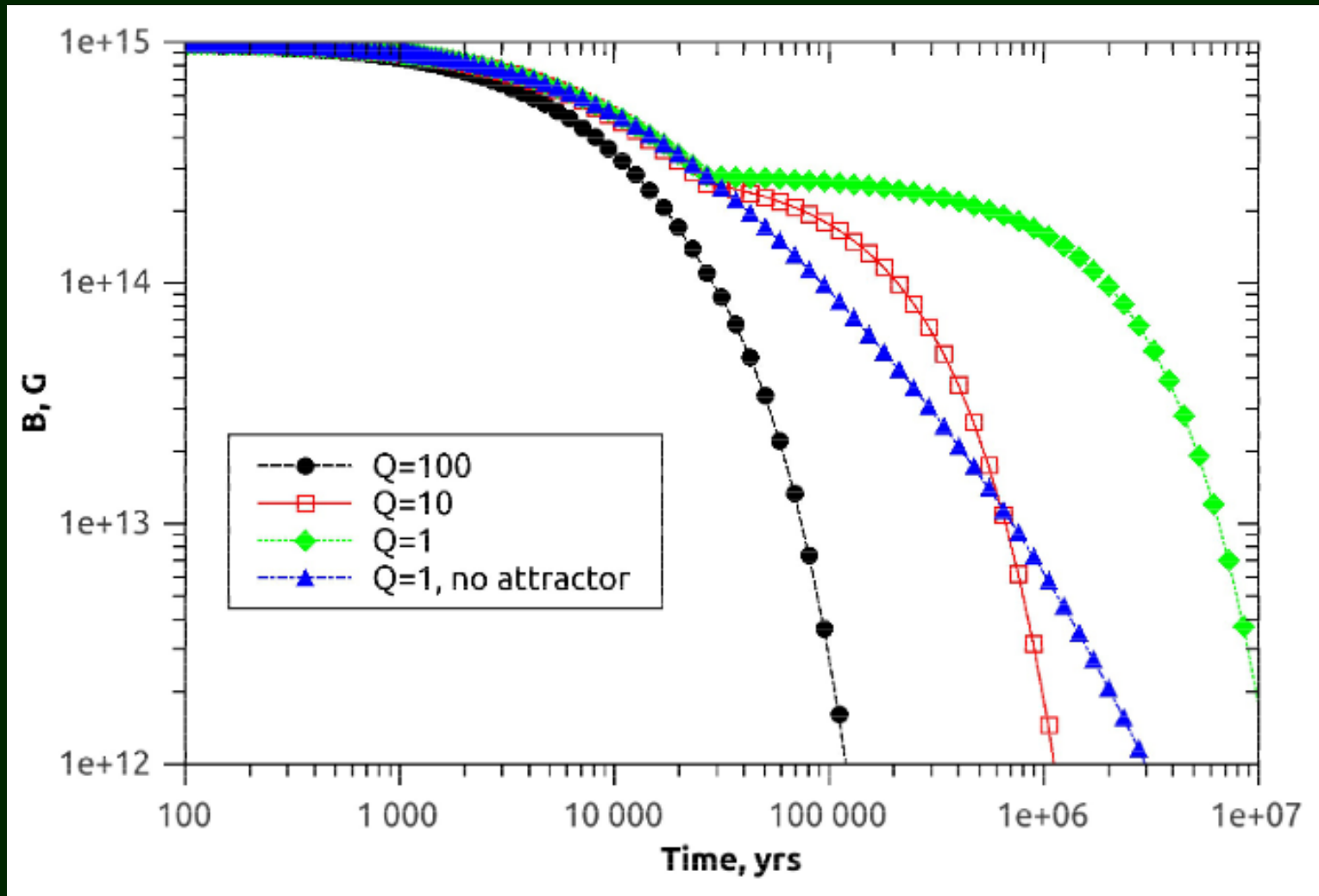


- ULX. NuSTAR J095551+6940.8 (M82 X-2). [Ekşi et al. \(2015\)](#).
- ULX. NGC 5907. [Israel et al. \(2017a\)](#)
- ULX. NGC 7793 P13. [Israel et al. \(2017b\)](#).
- 4U0114+65. [Sanjurjo et al. \(2017\)](#).
- 4U 2206+54. [Ikhsanov & Beskrovnaya \(2010\)](#).
- SXP1062. [Fu & Li \(2012\)](#)
- Swift J045106.8-694803. [Klus et al. \(2013\)](#).

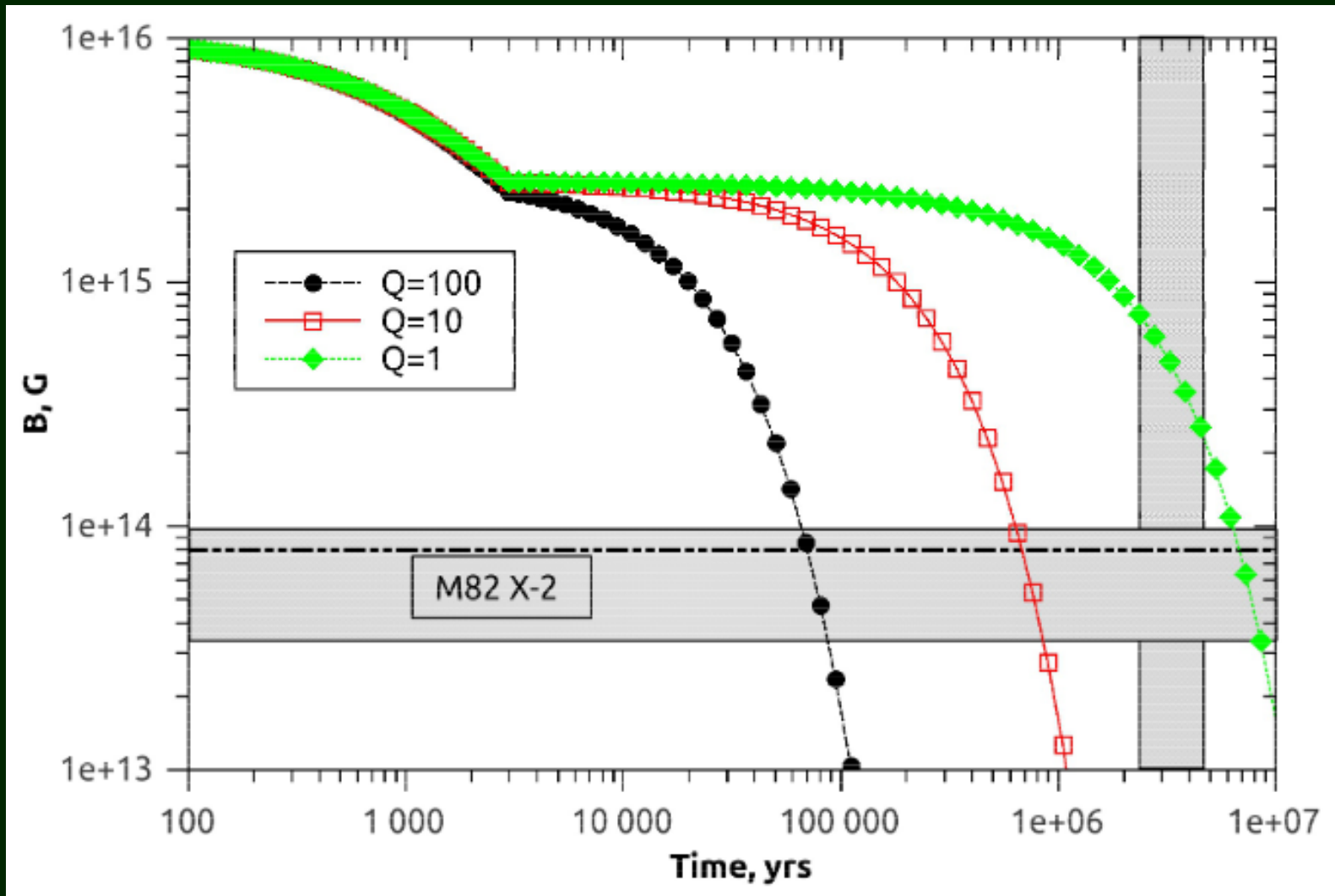


1709.10385, see also 2104.09076 on field determination in PULX and 2201.07507 for a brief review.

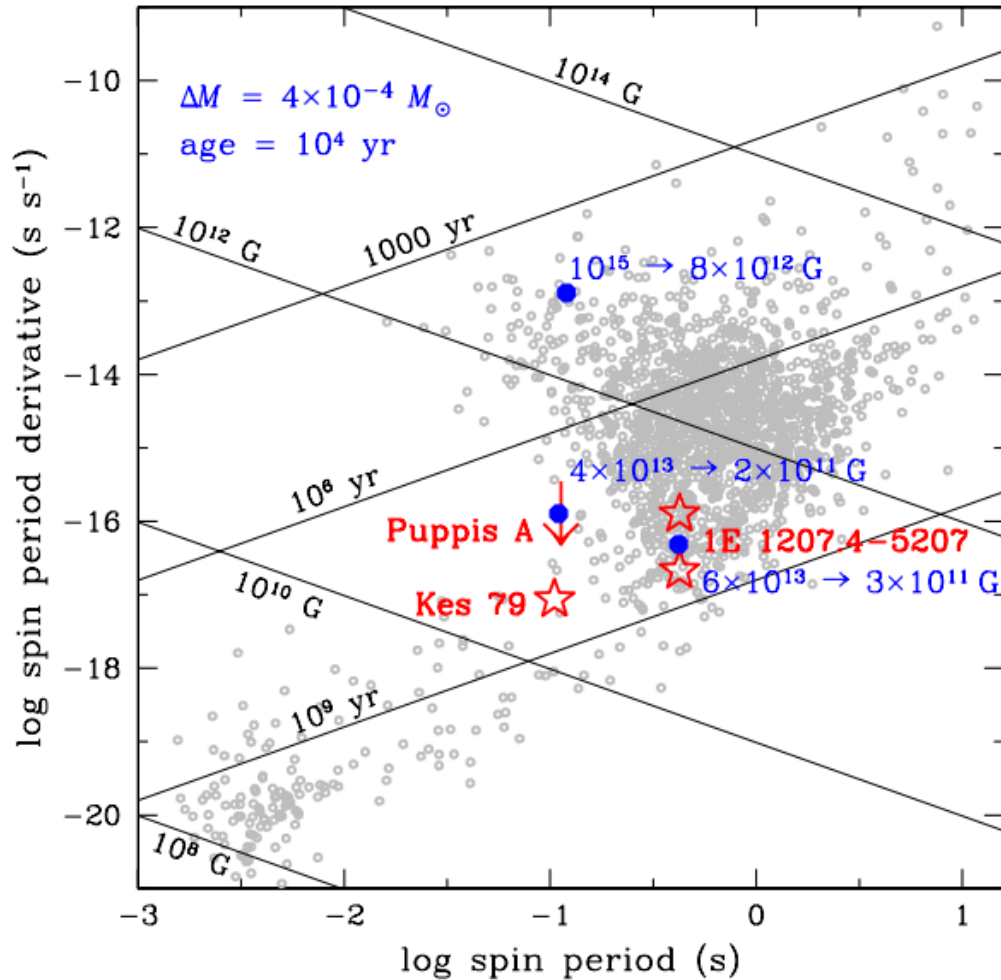
Field evolution in a magnetar



Parameters of ULX M82 X-2



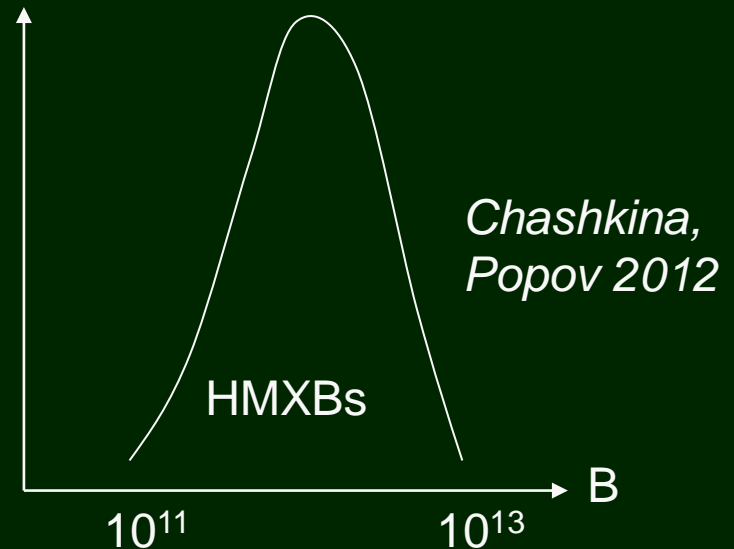
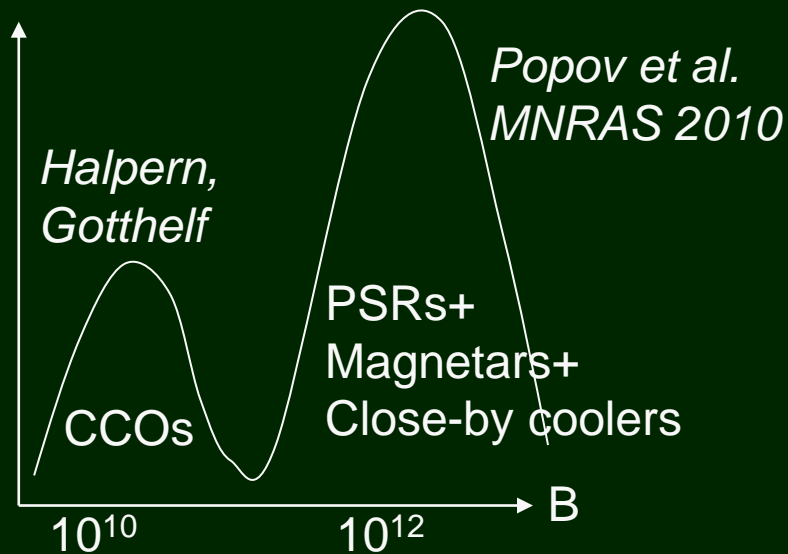
Anti-magnetars



Note, that there is no room for antimagnetars from the point of view of birth rate in many studies of different NS populations.

New results 1301.2717
Spins and derivative are measured for
PSR J0821-4300 and
PSR J1210-5226

Evolution of CCOs



Among young isolated NSs about 1/3 can be related to CCOs. If they are anti-magnetars, then we can expect that 1/3 of NSs in HMXBs are also low-magnetized objects.

They are expected to have short spin periods < 1 sec.

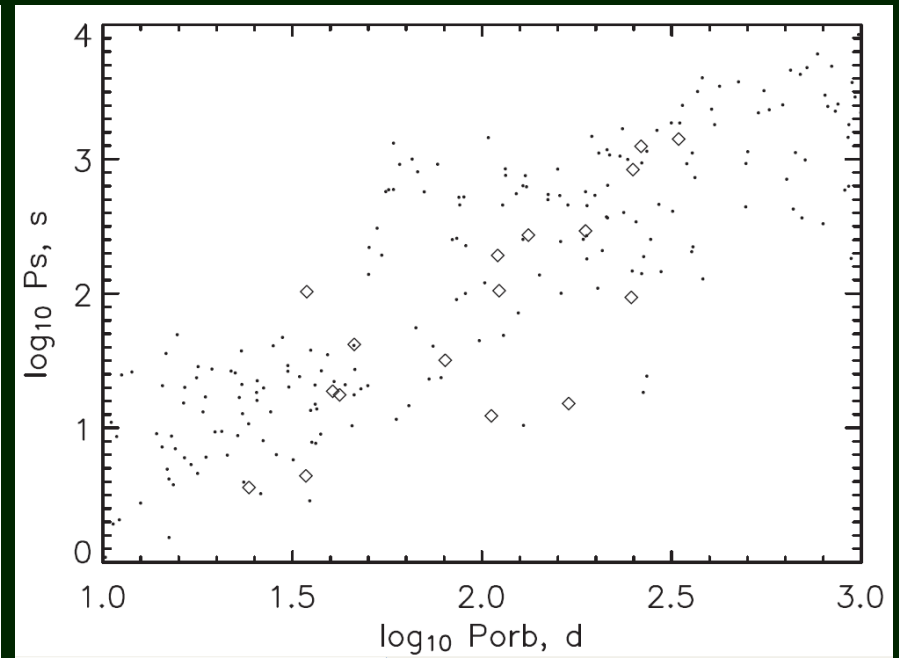
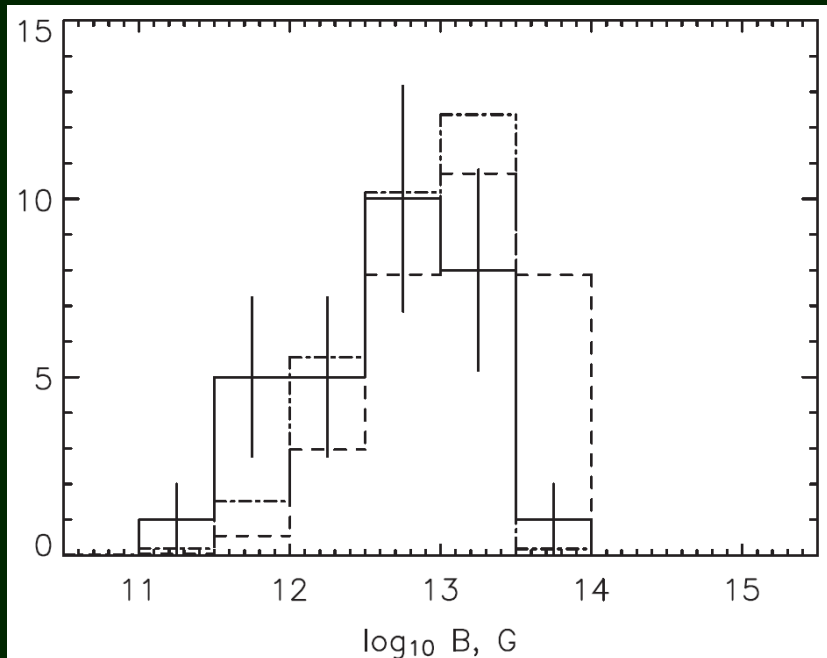
However, there are no many sources with such properties.

The only good example - SAX J0635+0533. An old CCO?

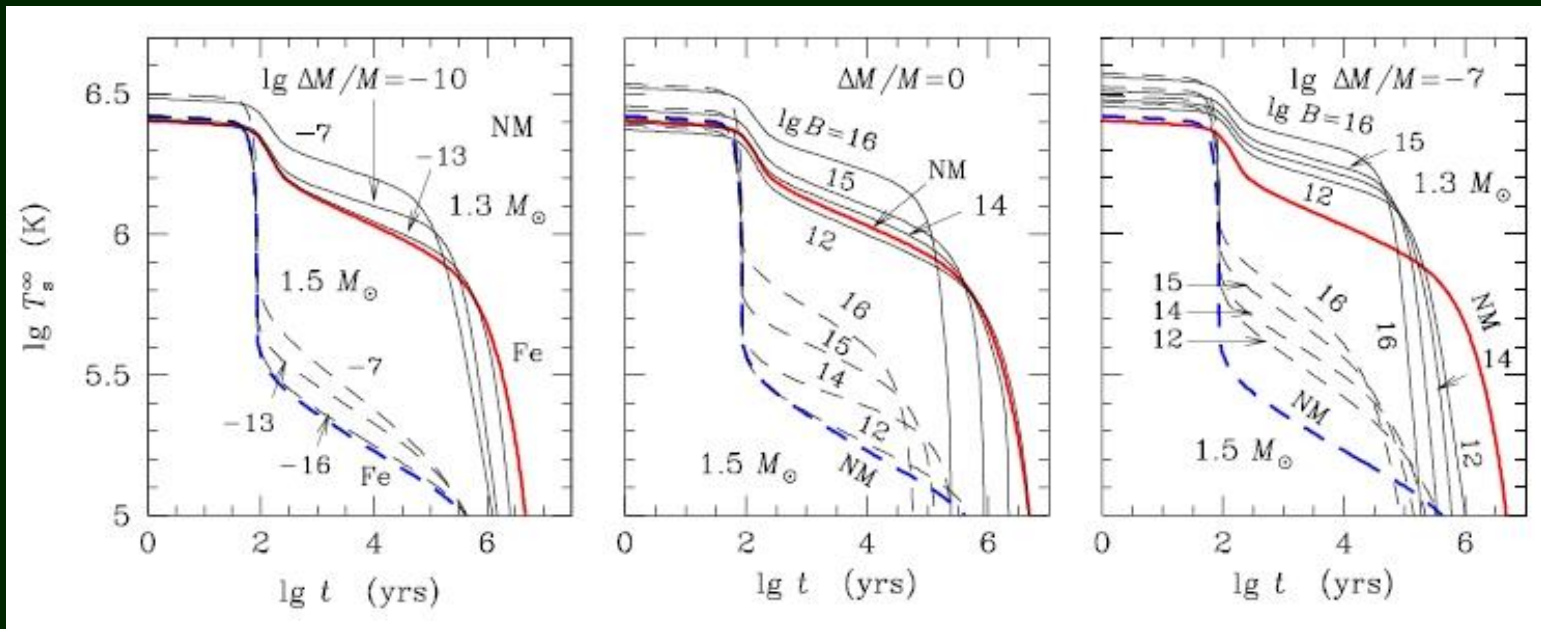
Possible solution: emergence of magnetic field (see physics in Ho 2011, Viganò, Pons 2012).

Observations vs. theory

We use observations of Be/X-ray binaries in SMC to derive magnetic field estimates, and compare them with prediction of the Pons et al. model.



Where are old CCOs?



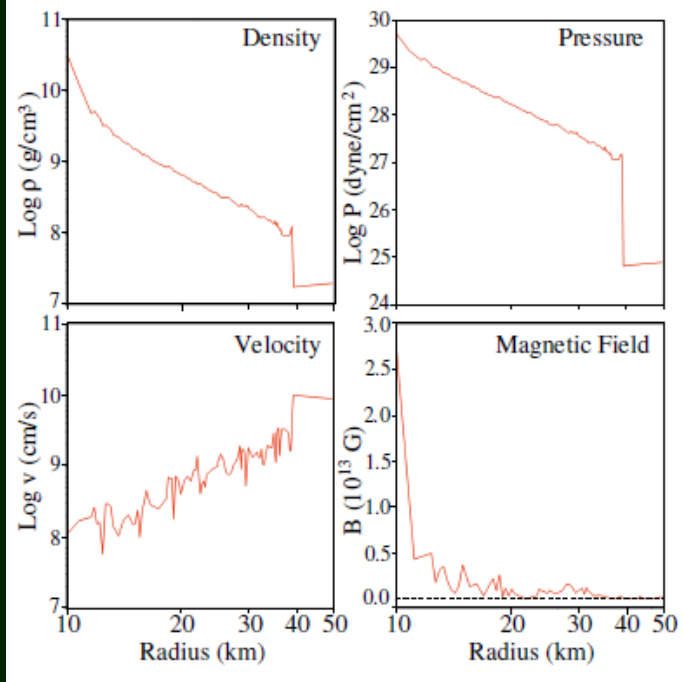
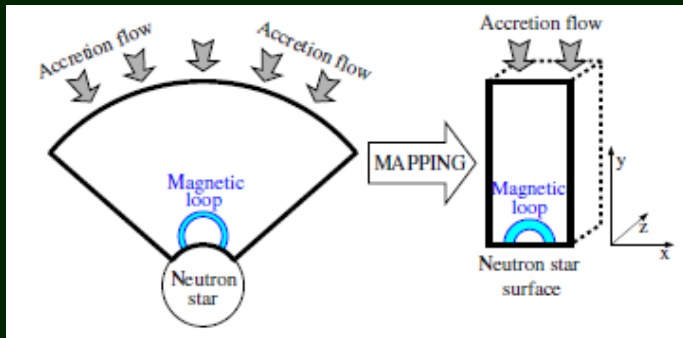
Yakovlev, Pethick 2004

According to cooling studies they have to be bright till at least 10^5 years. But only one candidate (2XMM J104608.7-594306 Pires et al.) to be a low-B cooling NS is known (Calvera is also a possible candidate).

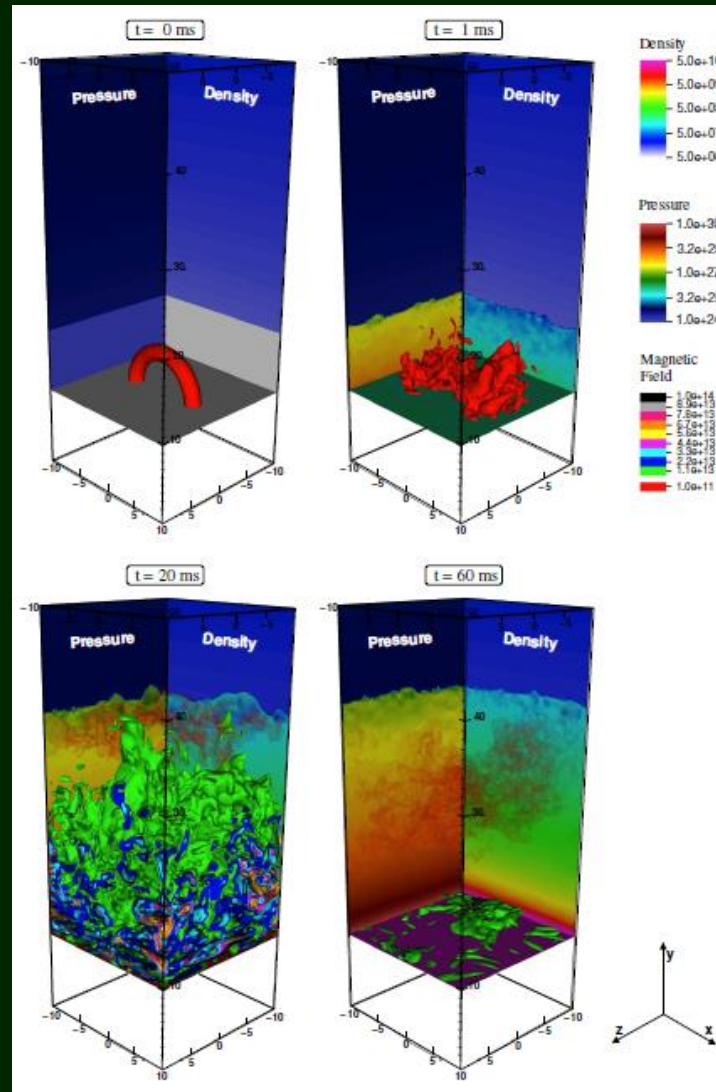
We propose that a large set of data on HMXBs and cooling NSs is in favour of field emergence on the time scale $10^4 \leq \tau \leq 10^5$ years (arXiv:1206.2819).

Some PSRs with thermal emission for which additional heating was proposed can be descendants of CCOs with emerged field.

How the field is buried



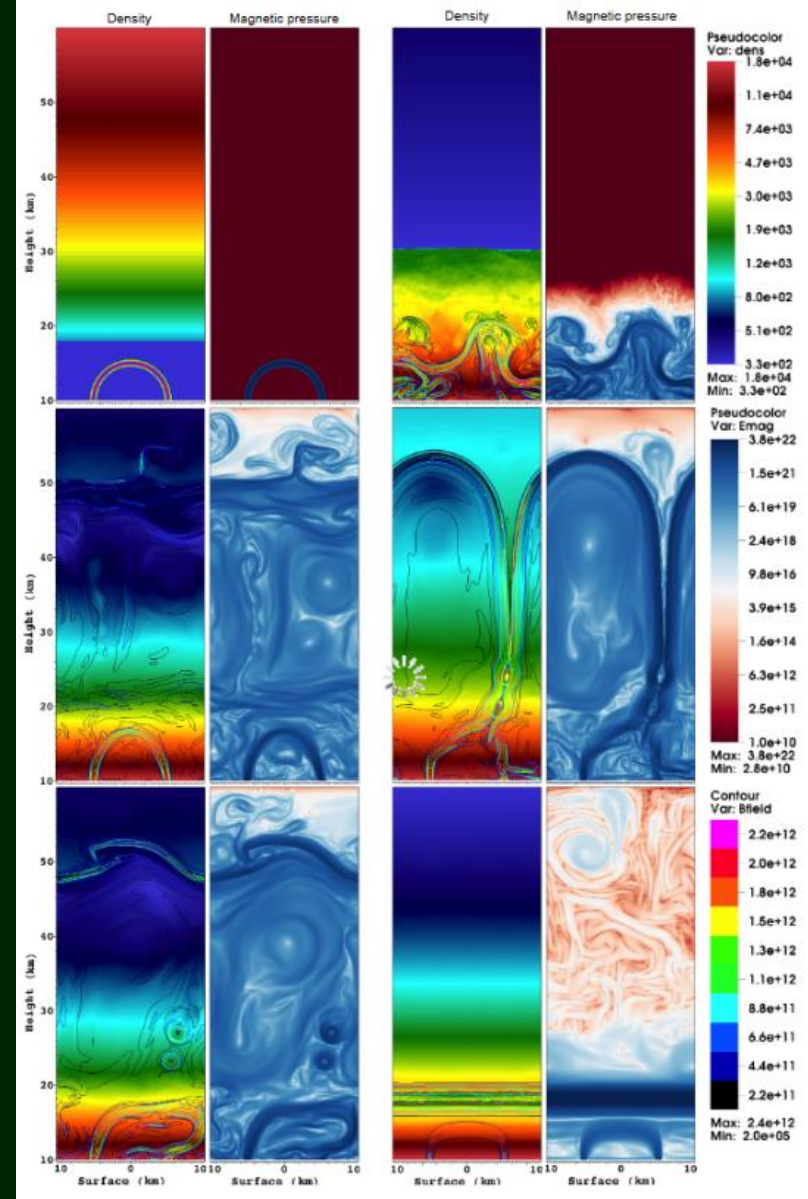
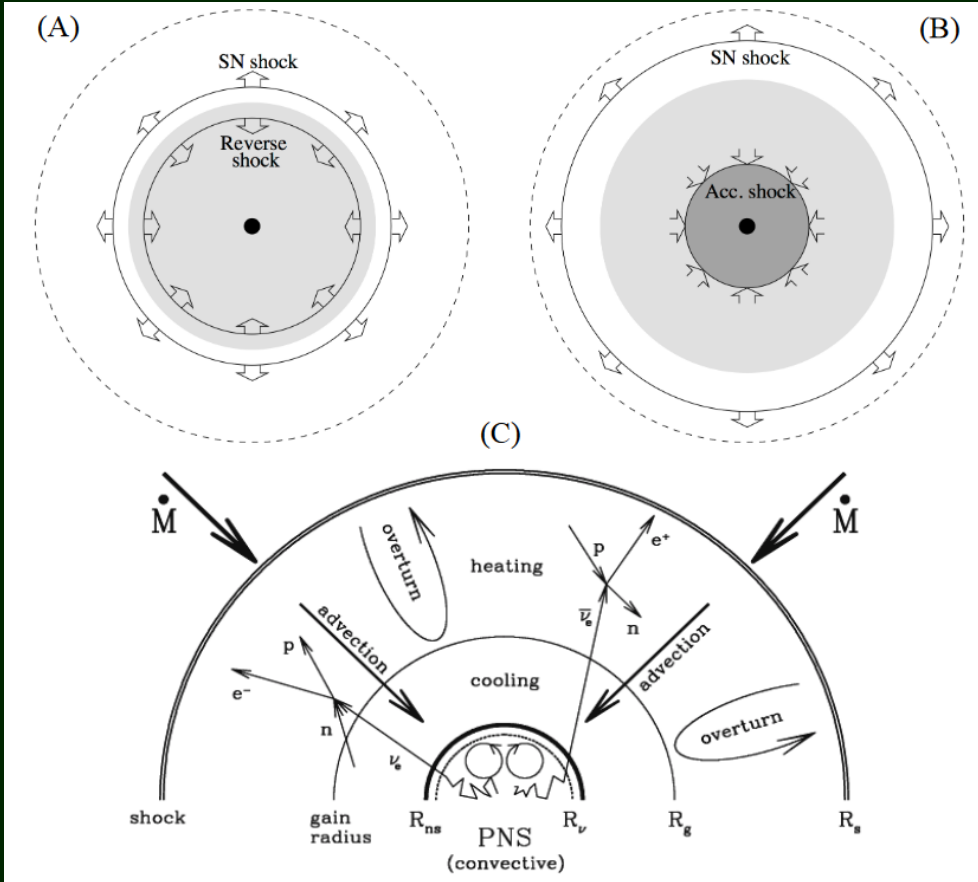
For $t=60$ msec



1212.0464

See 1210.7112 for a review of CCOs magnetic fields

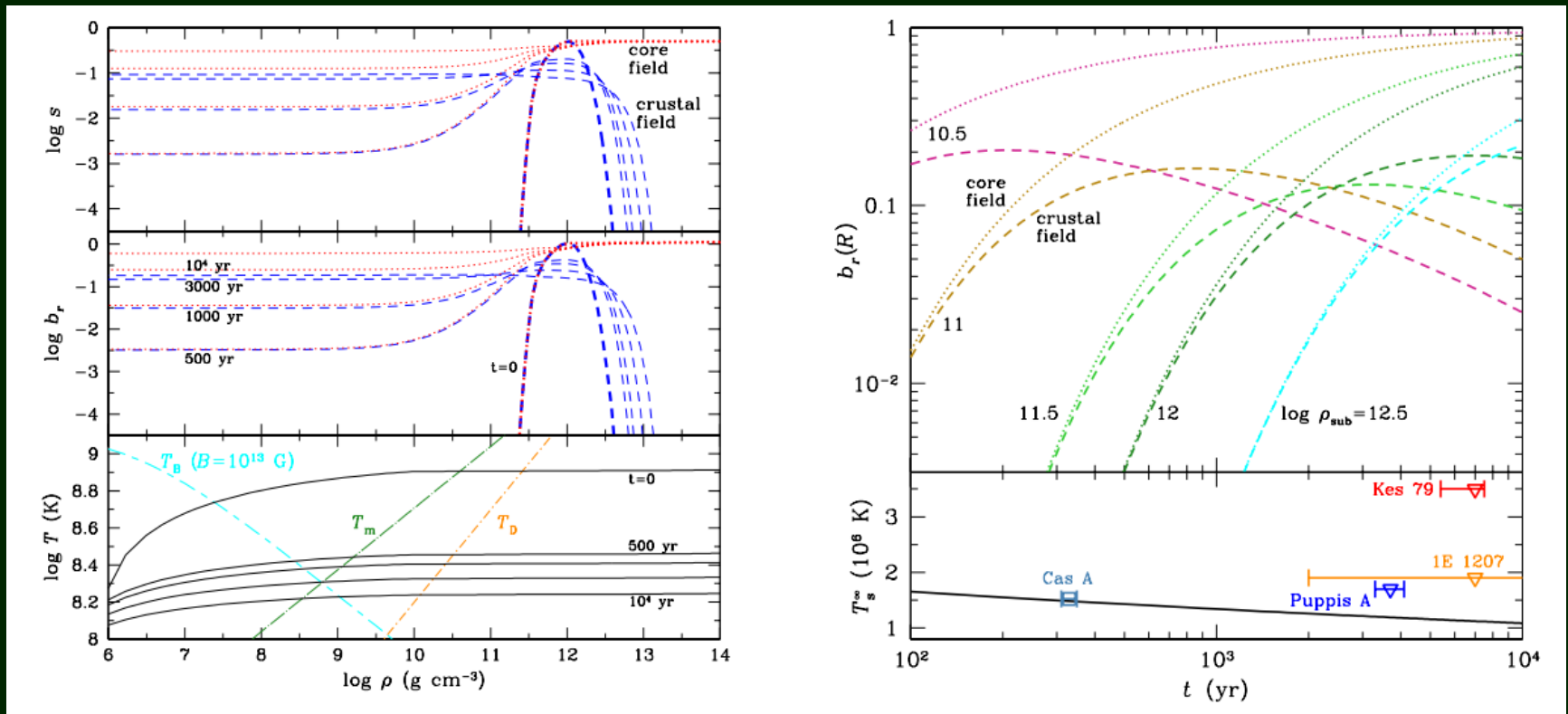
More advanced model



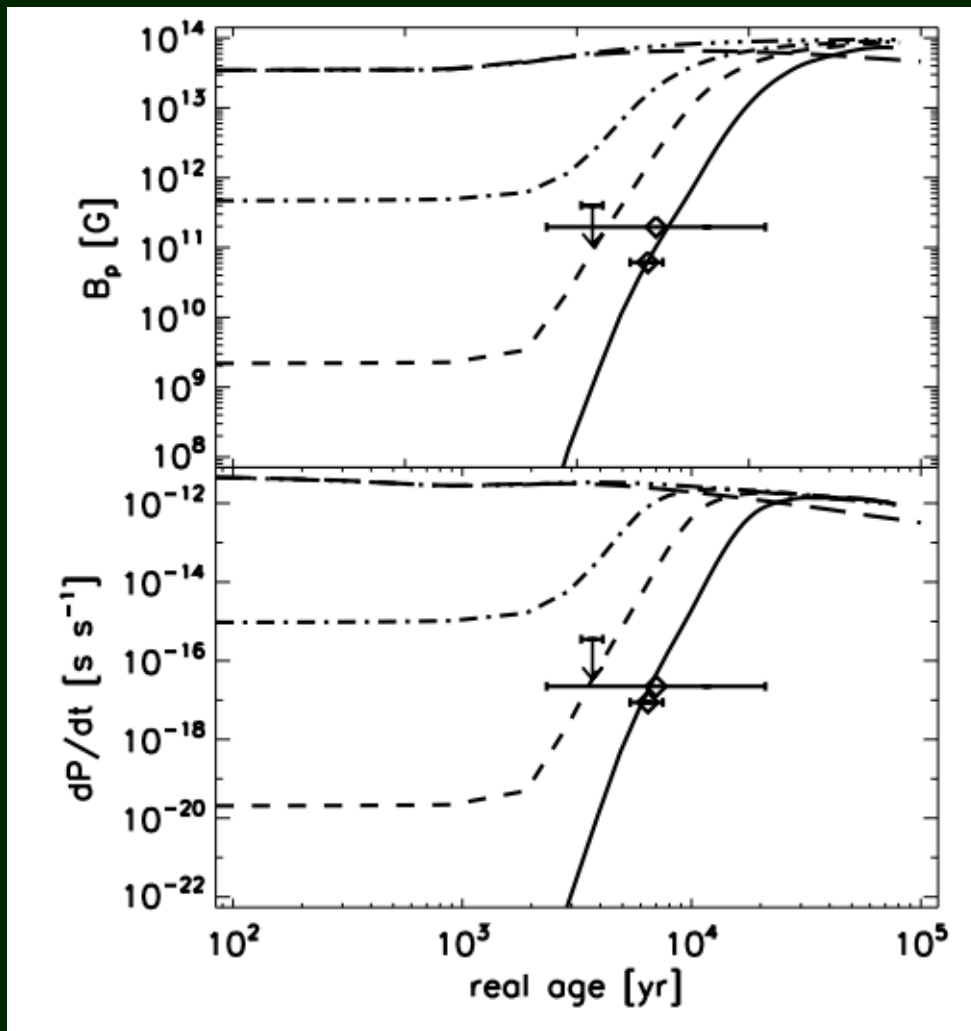
Emerging field: modeling

1D model of field emergence

Dashed – crustal, dotted – core field



Another model



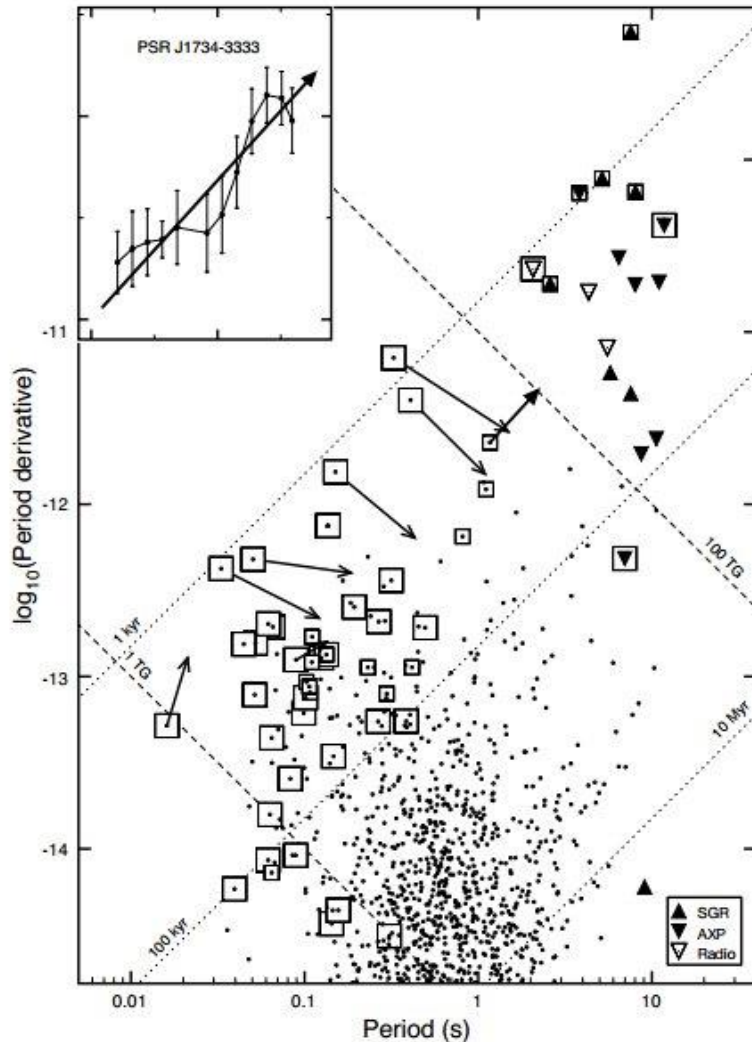
2D model with field decay

Ohmic diffusion dominates in field emergence, but Hall term also can be important.

Calculations confirm that emergence on the time scale 10^3 - 10^5 years is possible.

$$B_{0p} = 10^{14} \text{ G}$$

Emerged pulsars in the P-Pdot diagram



Emerged pulsars are expected to have
 $P \sim 0.1-0.5$ sec
 $B \sim 10^{11}-10^{12}$ G
Negative braking indices or at least $n < 2$.
About 20-40 of such objects are known.

Parameters of emerged PSRs:
similar to “injected” PSRs
(Vivekanand, Narayan, Ostriker).

The existence of significant fraction
of “injected” pulsars formally
do not contradict recent pulsar current studies
(Vranesevic, Melrose 2011).

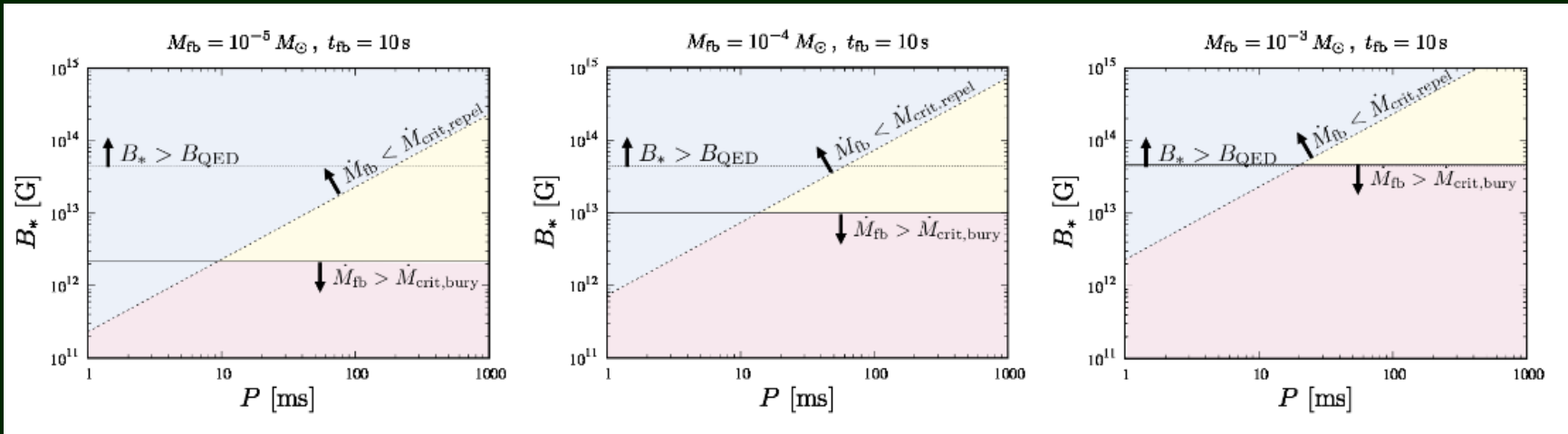
Part of PSRs supposed to be born with
long (0.1-0.5 s) spin periods can be
matured CCOs.

Field, rotation, fallback

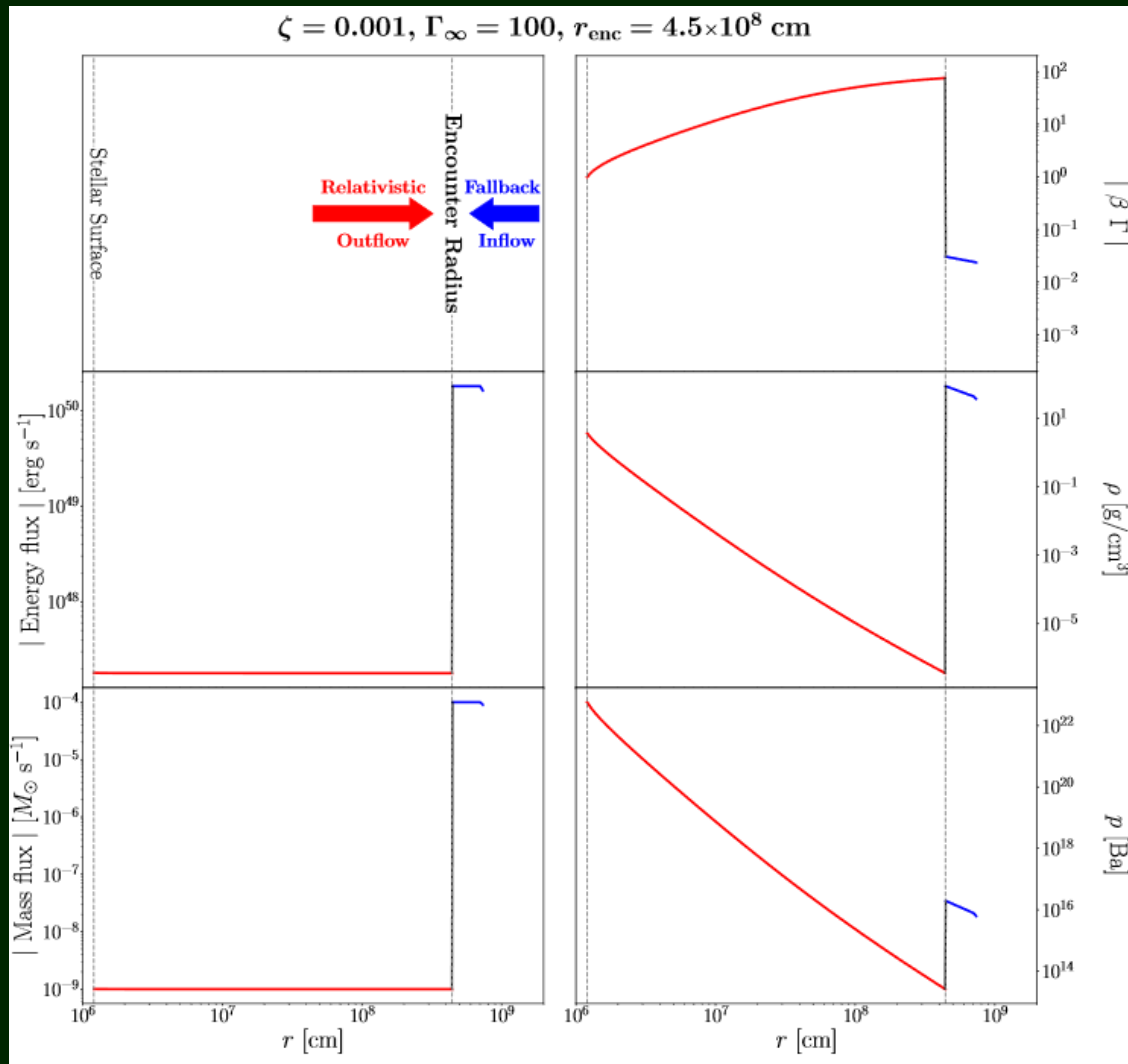
$$\dot{M}_{\text{crit,repul}} \sim 3 \times 10^{-5} M_{\odot} \text{s}^{-1} \frac{\xi_{\text{ss,crit}}}{0.2} \frac{(4\pi D_{\text{fb}} \sqrt{\xi_{\text{ss}}})_{\text{crit}}}{5.3} \left(\frac{B_*}{10^{13} \text{ G}} \right)^2 \left(\frac{P}{10 \text{ ms}} \right)^{-2} \left(\frac{t_{\text{fb}}}{20 \text{ s}} \right)^{2/3}$$

$$\frac{B_*^2}{8\pi} \lesssim \rho v^2 \sim \frac{\dot{M}}{4\pi R_*^2} \sqrt{\frac{GM_c}{R_*}}$$

$$\dot{M}_{\text{crit,bury}} \sim 3 \times 10^{-6} M_{\odot} \text{s}^{-1} \left(\frac{B_*}{10^{13} \text{ G}} \right)^2$$

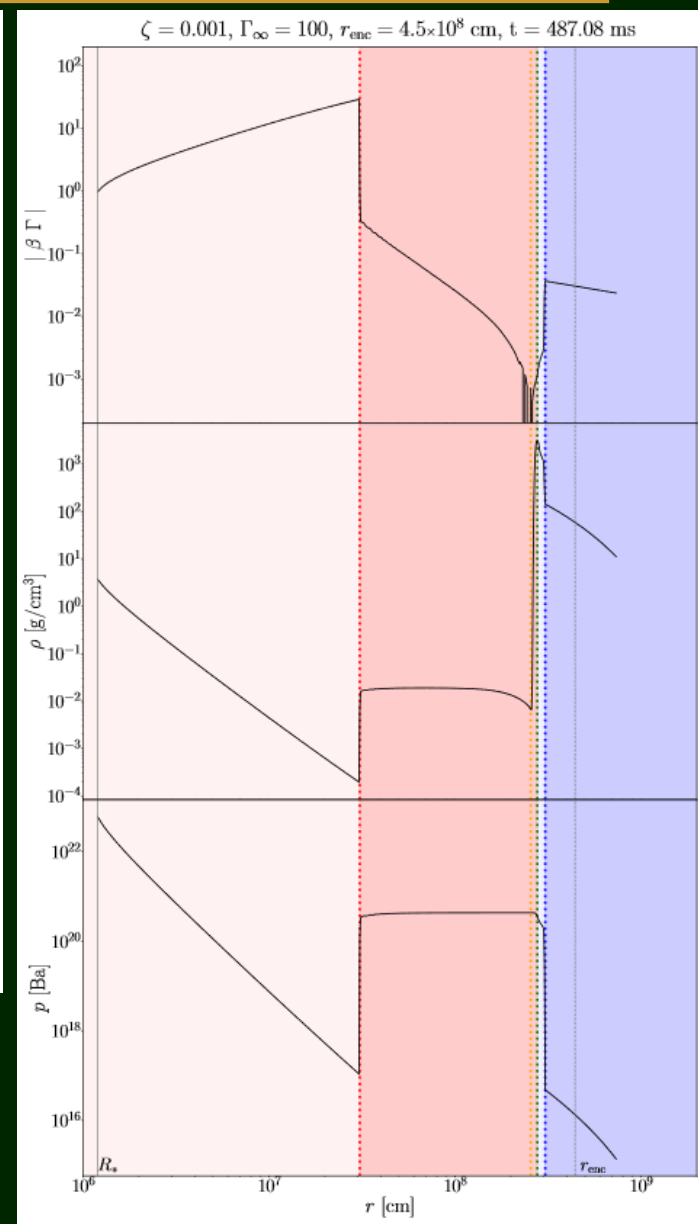
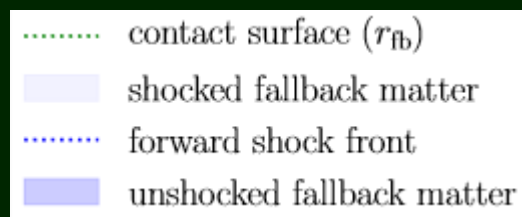
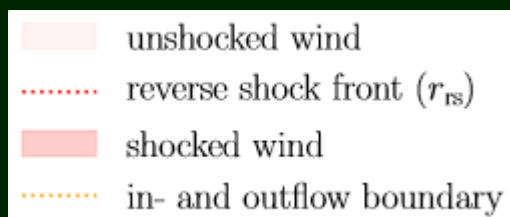
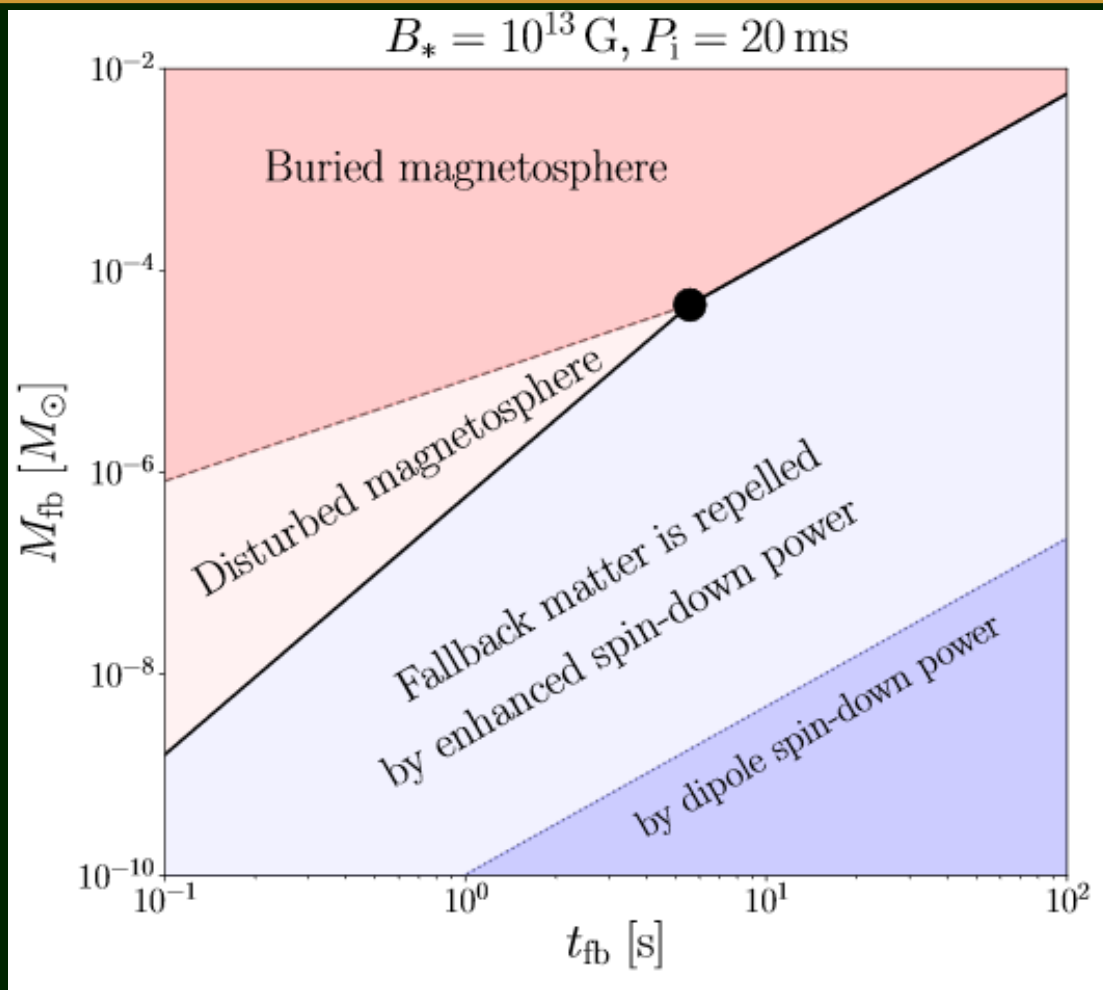


Fallback matter interacting with a NS

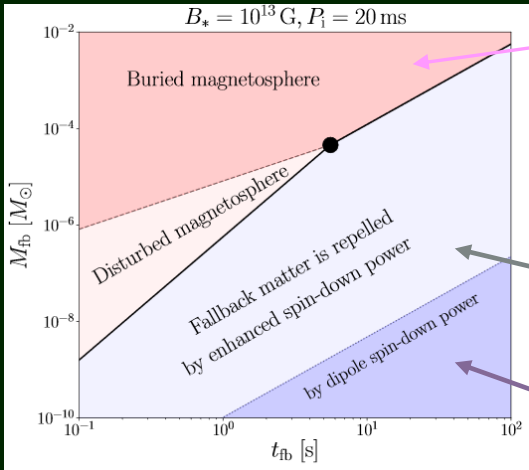


Fallback matter interacts with relativistic wind, magnetosphere, and finally – with the NS surface.

Depending on parameters (ΔM , field, spin, etc.) different regimes can appear. Thus, NSs can appear at different stages and can be observed as sources with different properties.



Variants of young NS properties



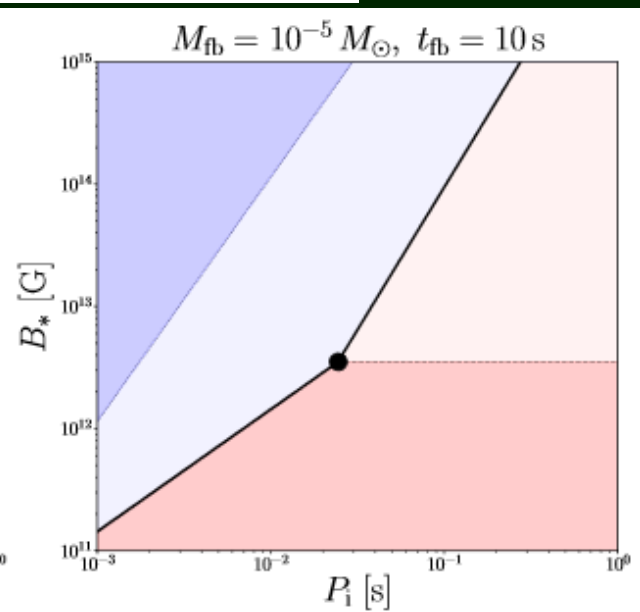
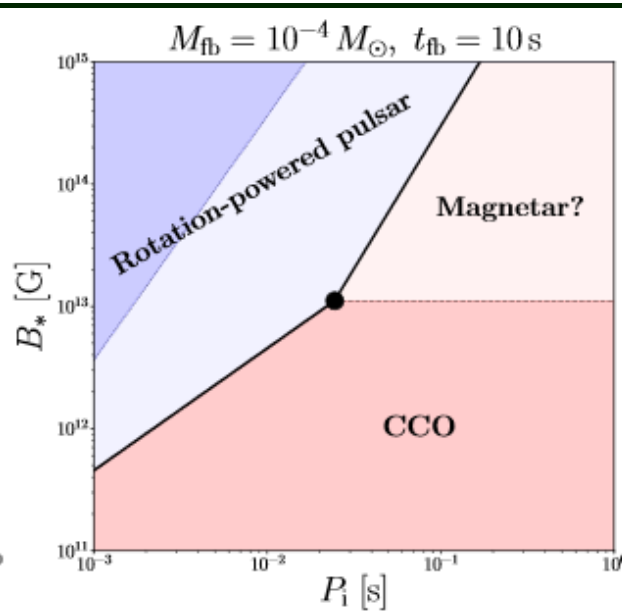
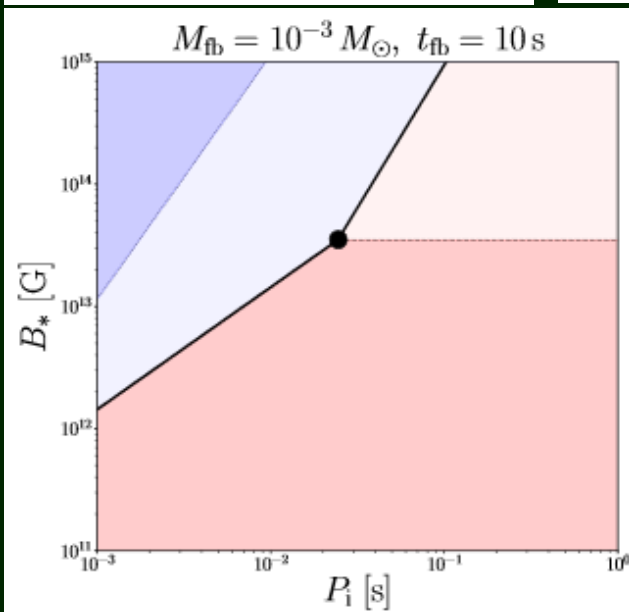
$$M_{\text{fb}} > 8.2 \times 10^{-5} M_{\odot} B_{*,13}^2 t_{\text{fb},1},$$

$$\dot{M}_{\text{fb}} = \dot{M}_{\text{fb,ini}} \times \begin{cases} 1 & t \leq t_{\text{fb}} \\ (t/t_{\text{fb}})^{-l} & t > t_{\text{fb}} \end{cases},$$

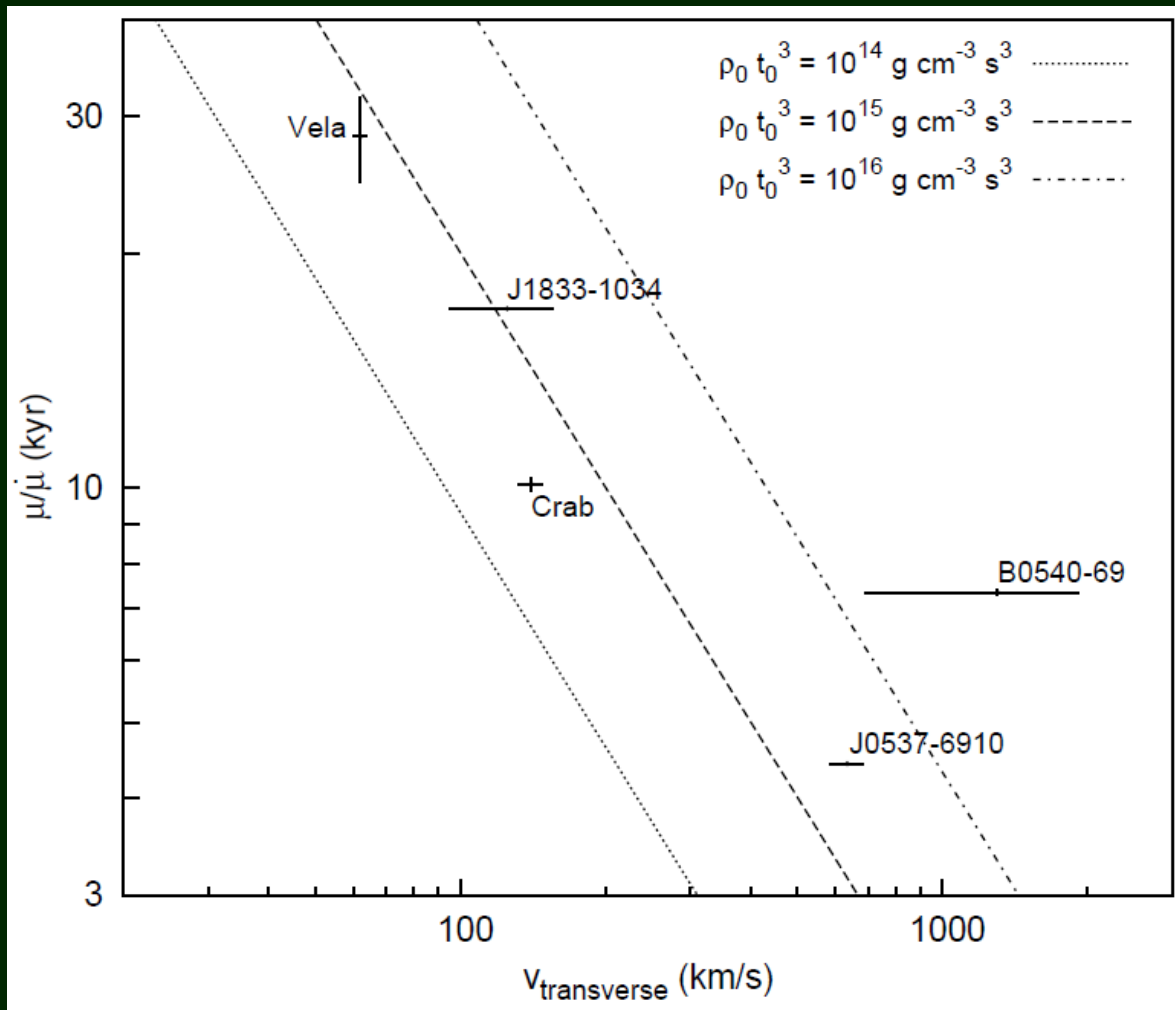
$$\dot{M}_{\text{fb,ini}} = \frac{l-1}{l} \frac{M_{\text{fb}}}{t_{\text{fb}}} \sim 1 \times 10^{-5} M_{\odot} \text{ s}^{-1} \left(\frac{l-1}{l} \right) M_{\text{fb},-4} t_{\text{fb},1}^{-1},$$

$$M_{\text{fb,crit}} \approx \begin{cases} 5.2 \times 10^{-3} M_{\odot} B_{*,13}^2 P_{i,-2}^{-14/3} t_{\text{fb},1}^{23/9} & r_{\Lambda} > R_* \\ 4.8 \times 10^{-4} M_{\odot} B_{*,13}^2 P_{i,-2}^{-2} t_{\text{fb},1}^{5/3} & r_{\Lambda} \leq R_* \end{cases}.$$

$$M_{\text{fb,crit}} \approx 7.7 \times 10^{-8} M_{\odot} (1 + \sin^2 \chi) B_{*,13}^2 P_{i,-2}^{-4} t_{\text{fb},1}^{5/3} \text{ (dipole).}$$



Growing field and kick velocities?

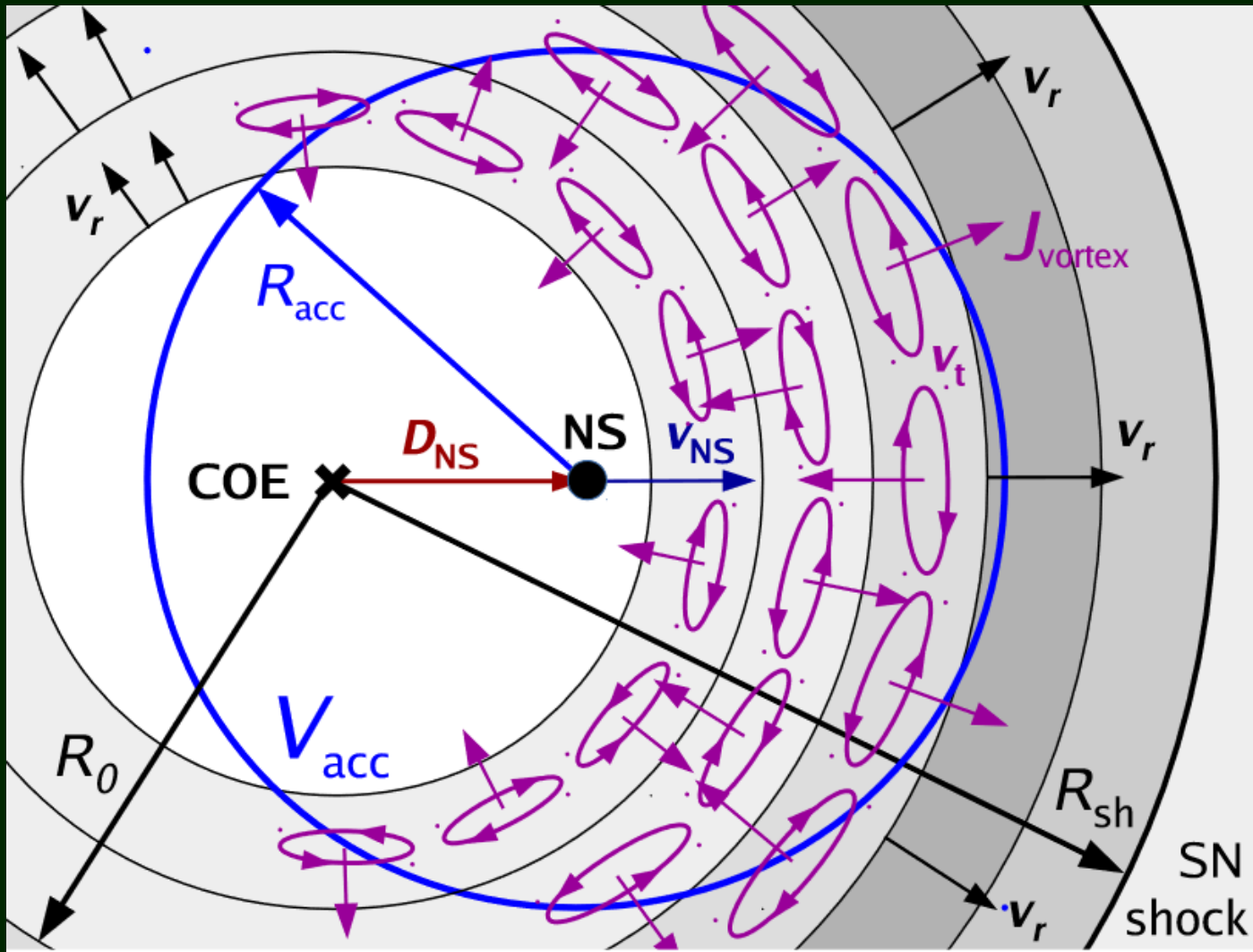


The idea is that $n < 3$ are explained as due to growing field. Then it is possible to estimate the timescale for growing and plot it vs. velocity.

Larger kick –
- smaller fallback –
- faster field growing

NSs with large kick velocities will accrete less amount of fallback material leading to shallower submergence of their fields and shorter time-scales for the growth of their fields.

Spin and kick from fallback

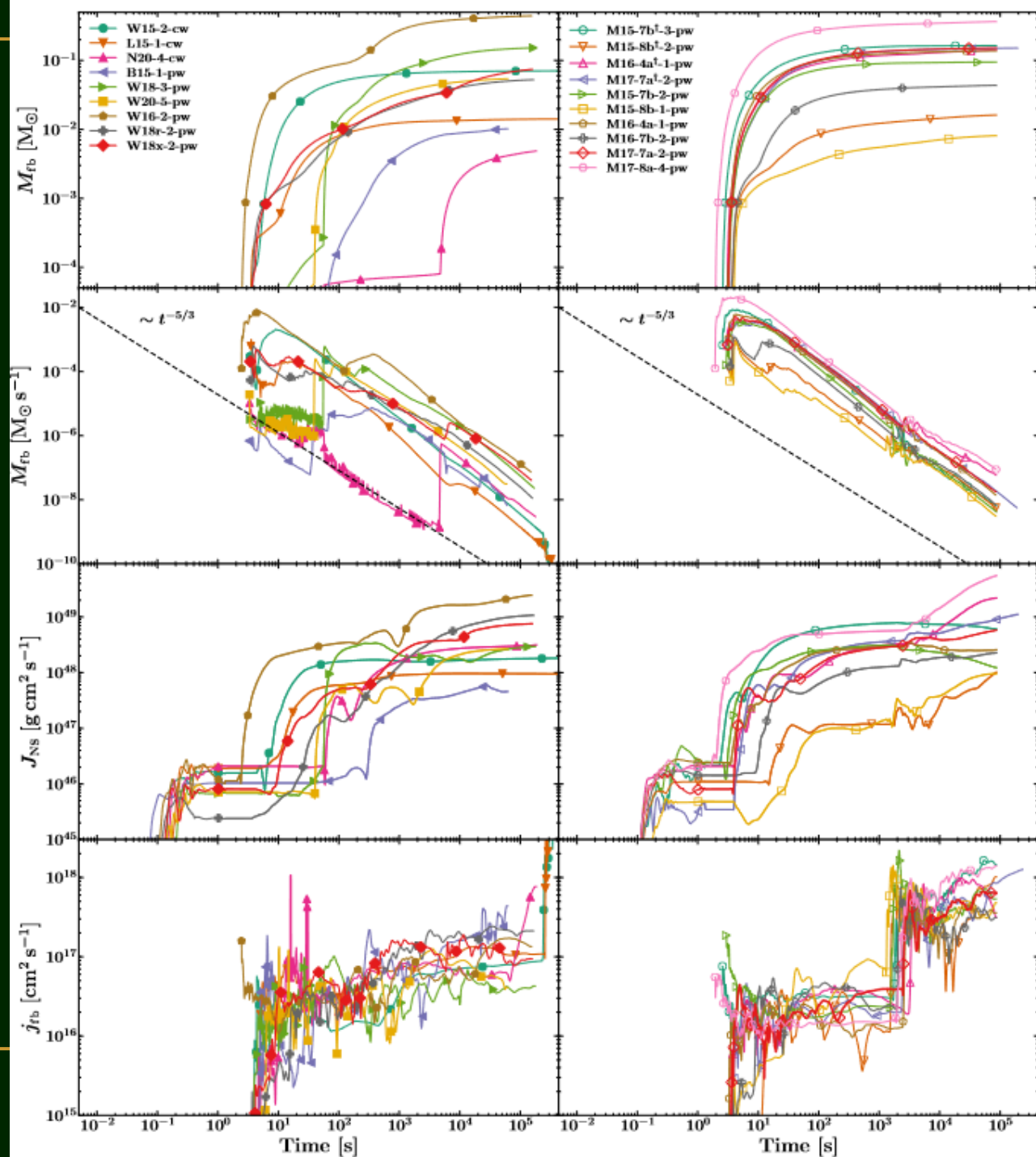


As the NS accretes
the matter expelled
in explosion,
it's mass grows,
and it spins-up.

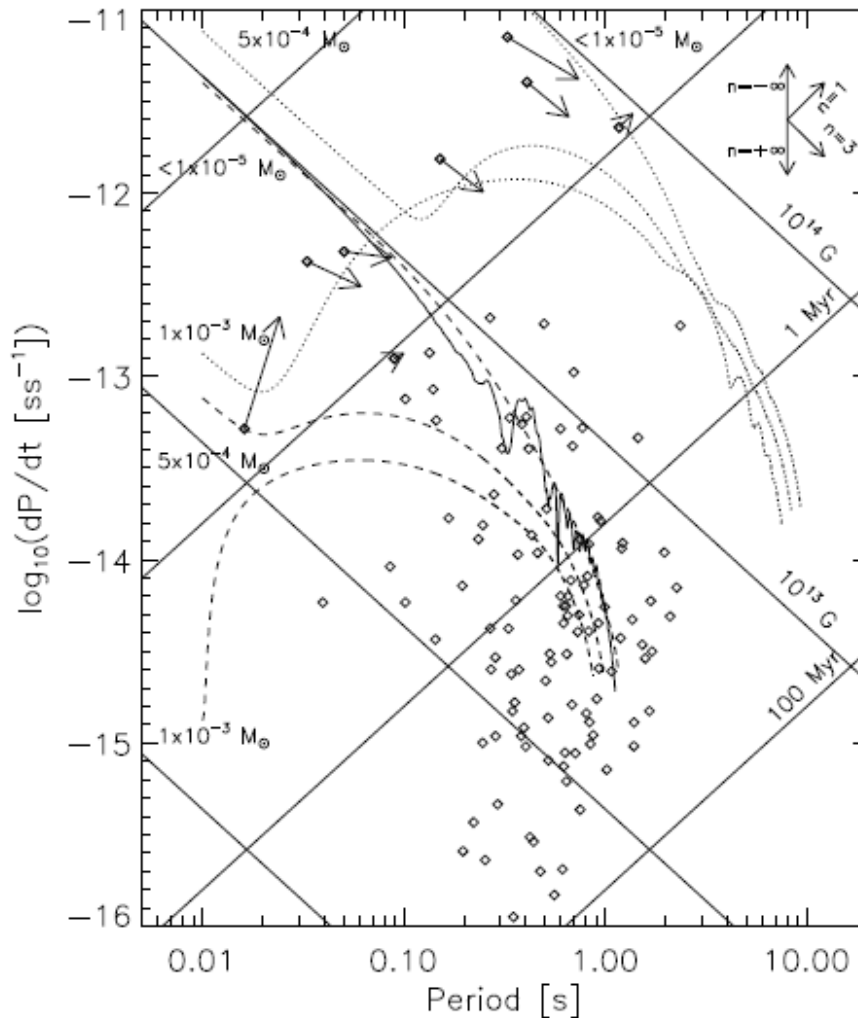
The fallback rate
approaches the
standard $\sim t^{-5/3}$ form.

$$T_{\text{spin}} = \frac{2\pi I_{\text{NS}}}{J_{\text{NS}}}$$

$$\approx 1.09 \text{ [s]} \left(\frac{M_{\text{NS}}}{1.5 M_{\odot}} \right) \left(\frac{R_{\text{NS}}}{12 \text{ km}} \right)^2 \left(\frac{J_{\text{NS}}}{10^{46} \text{ erg s}} \right)^{-1}$$



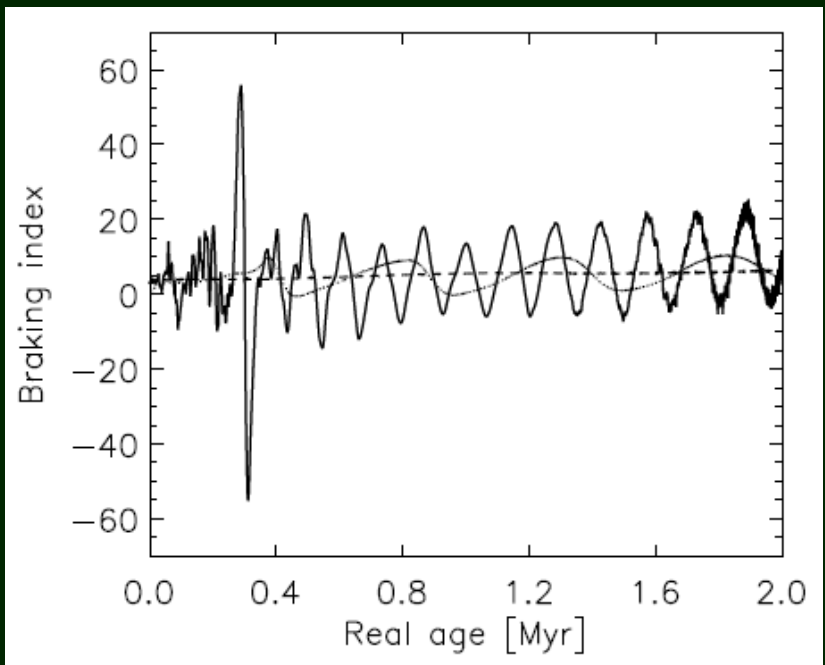
Evolution of PSRs with evolving field



Three stages:

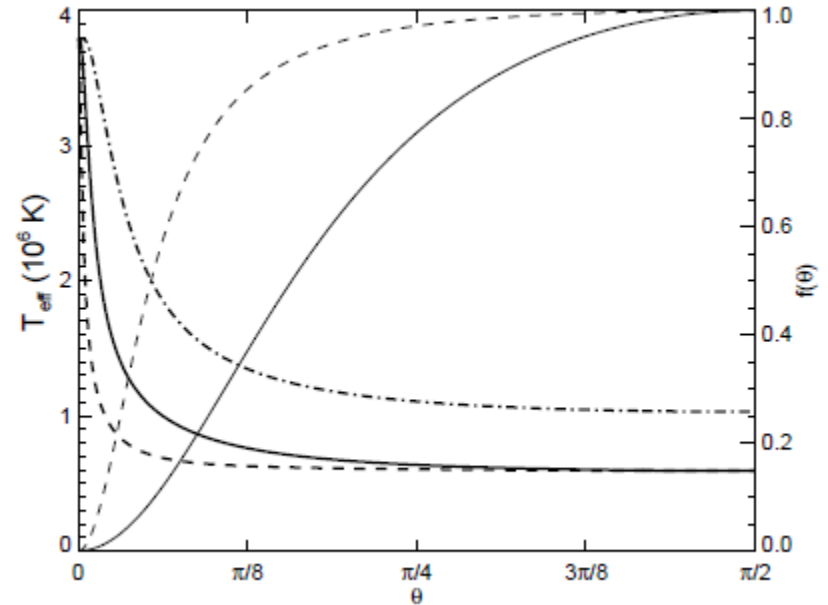
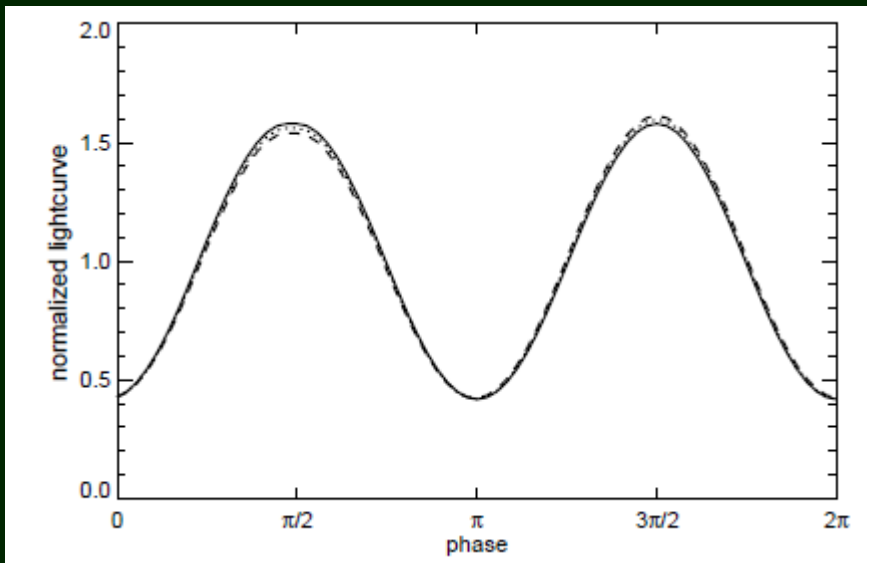
1. $n \leq 3$ Standard + emerging field
2. $n > 3$ Ohmic field decay
3. oscillating and large n – Hall drift

$$n = 3 - 4 \frac{\dot{B}_0}{B_0} \tau_c \equiv 3 - 4 \frac{\tau_c}{\tau_B},$$



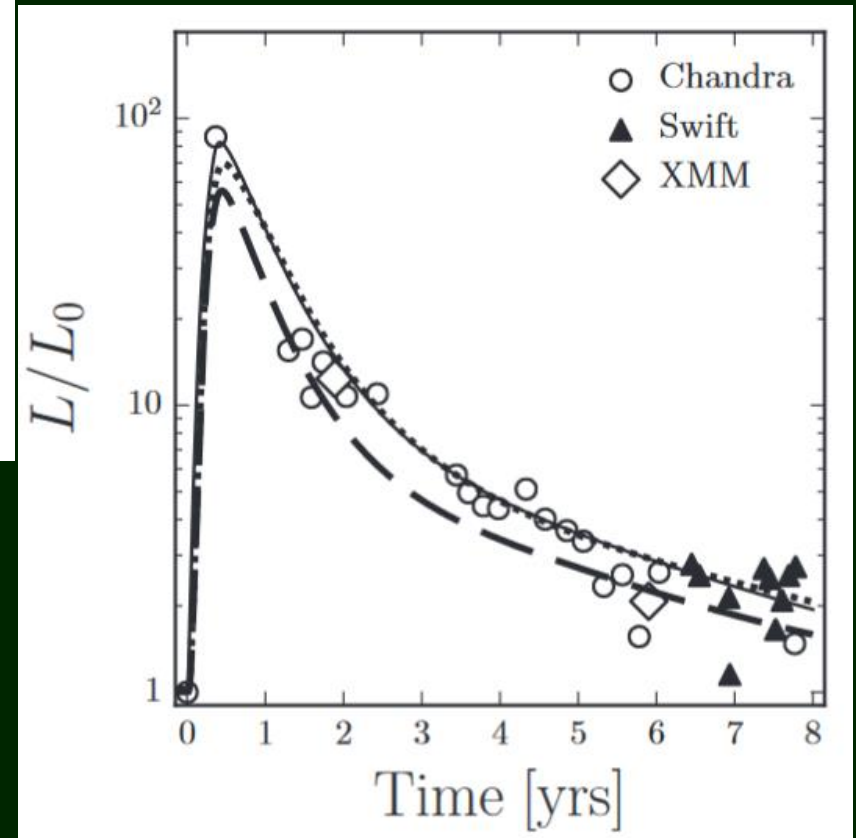
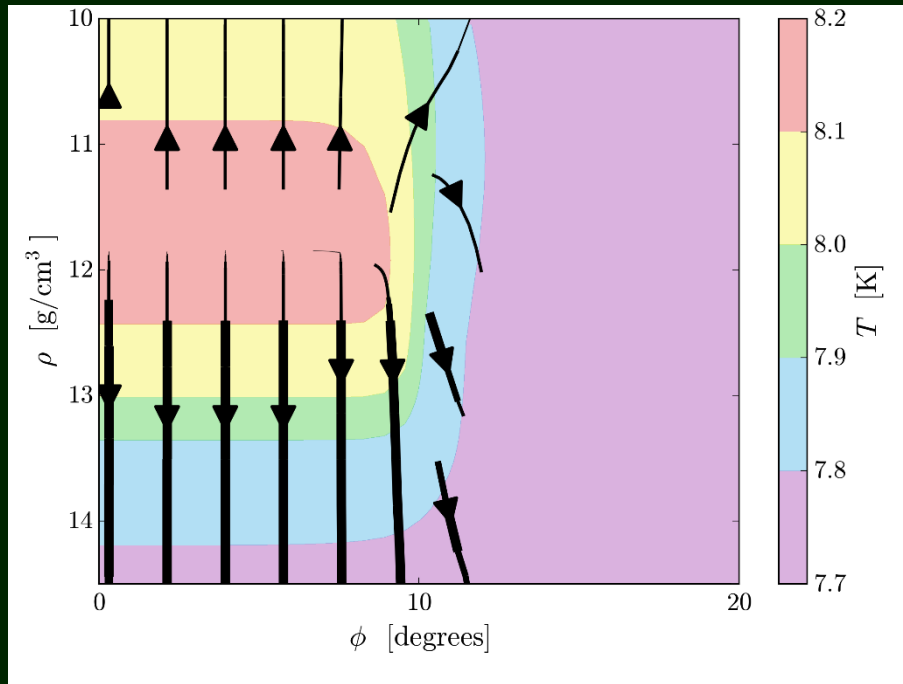
Buried field in Kes79?

The idea is to reconstruct surface temperature distribution, and then calculate which field configuration can produce it.

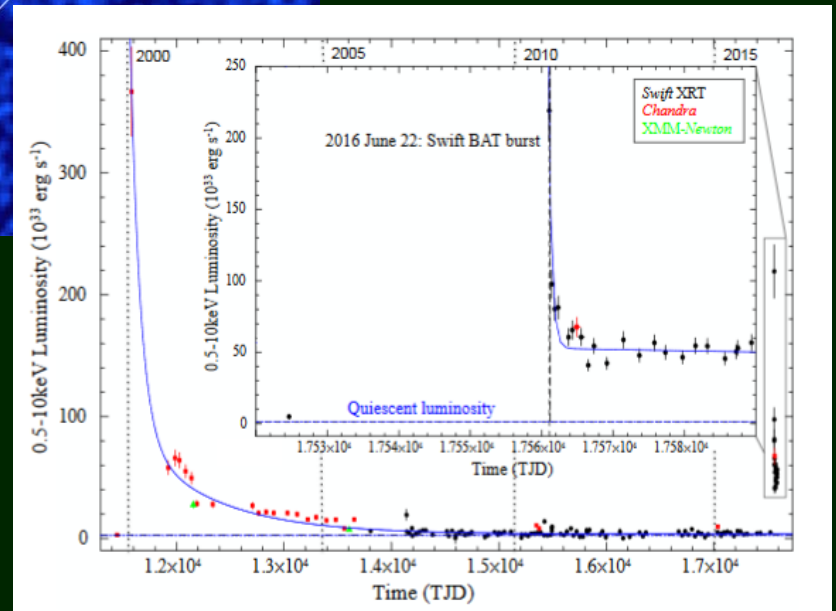
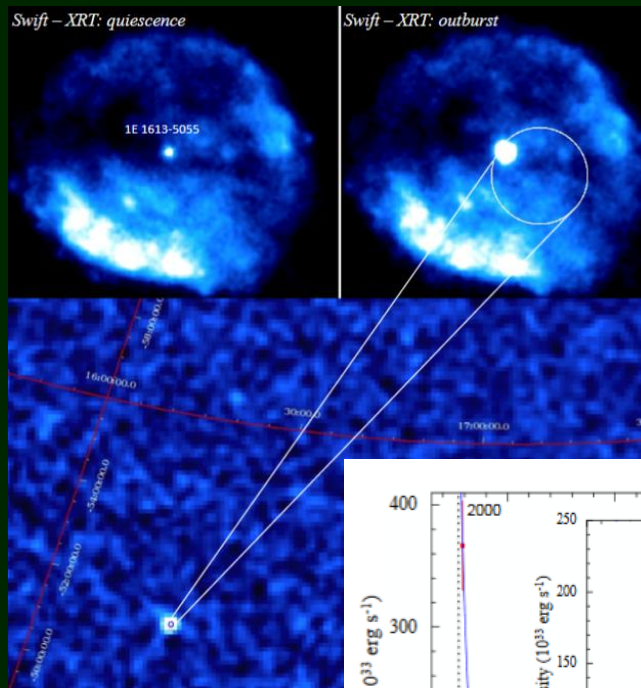
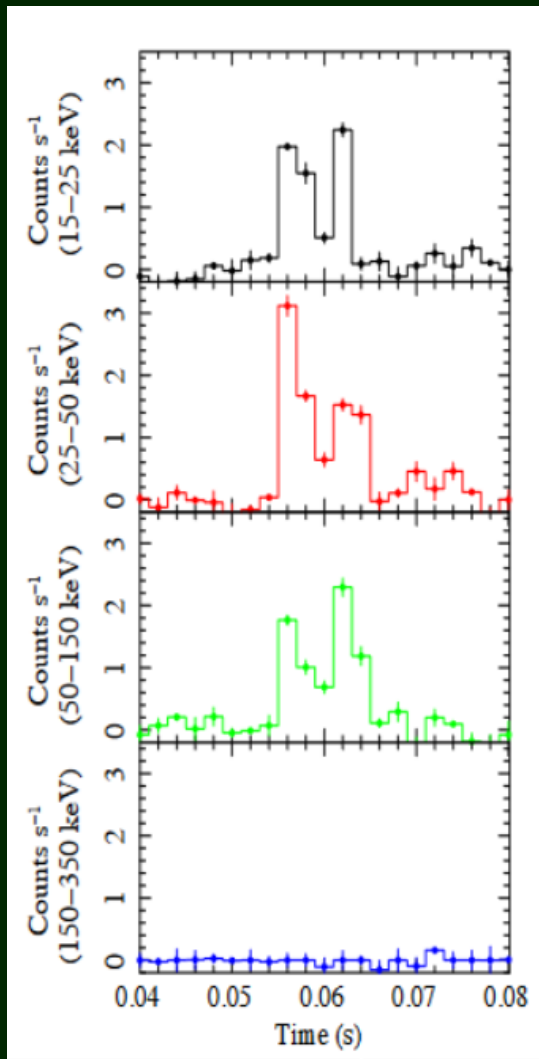


Very large pulse fraction (64%) in the anti-magnetar Kes 79. Large sub-surface magnetic field can explain the existence of compact hot spots. Then the field must have been buried in a fall-back episode.

Hidden magnetar in RCW103



Not so hidden!



Fallback discs

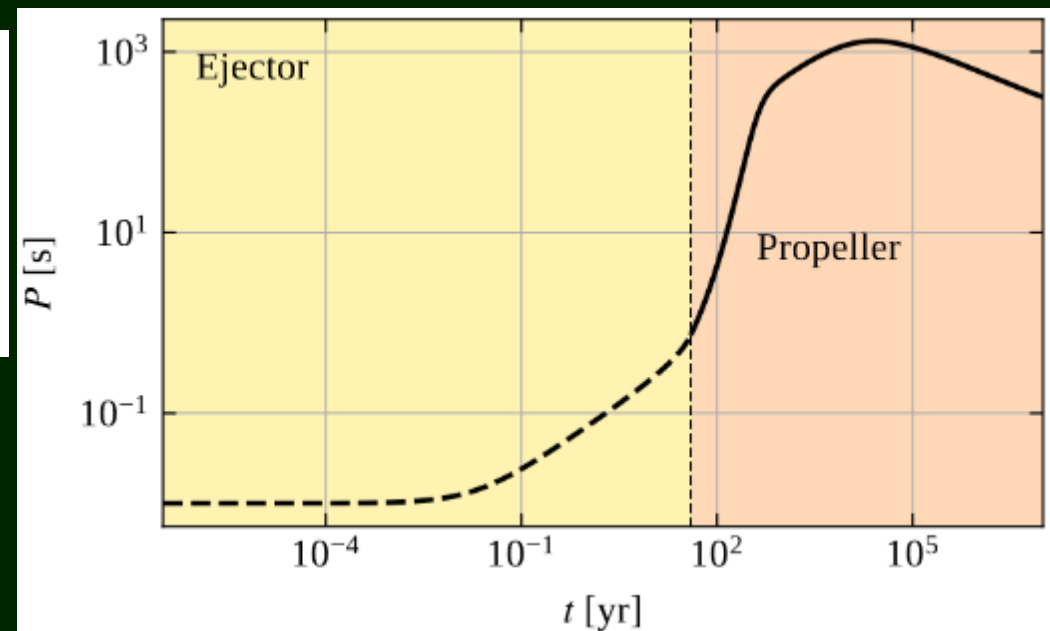
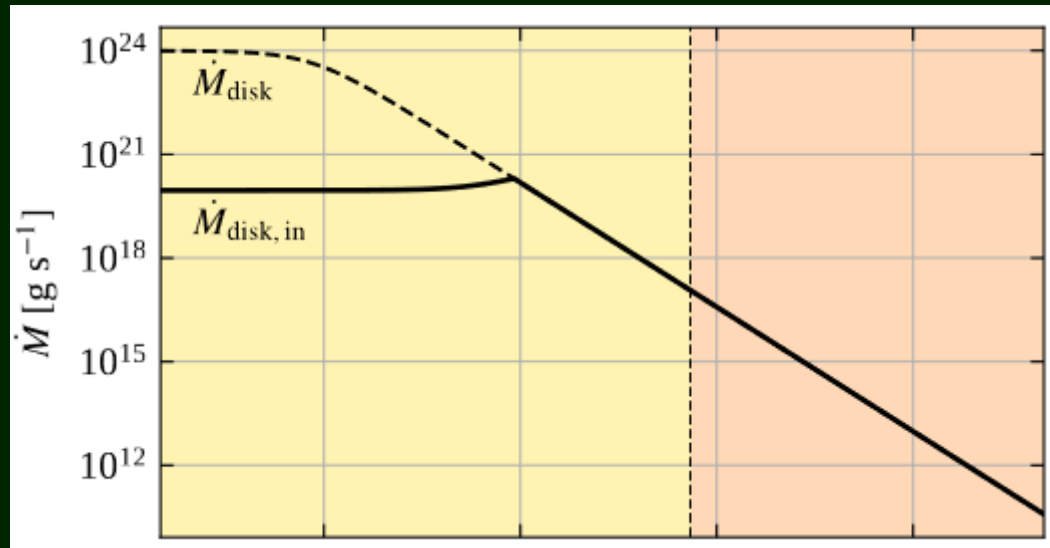
$$\dot{M}_d(t) = \dot{M}_{d,0} \left(1 + \frac{t}{t_v}\right)^{-\alpha}$$

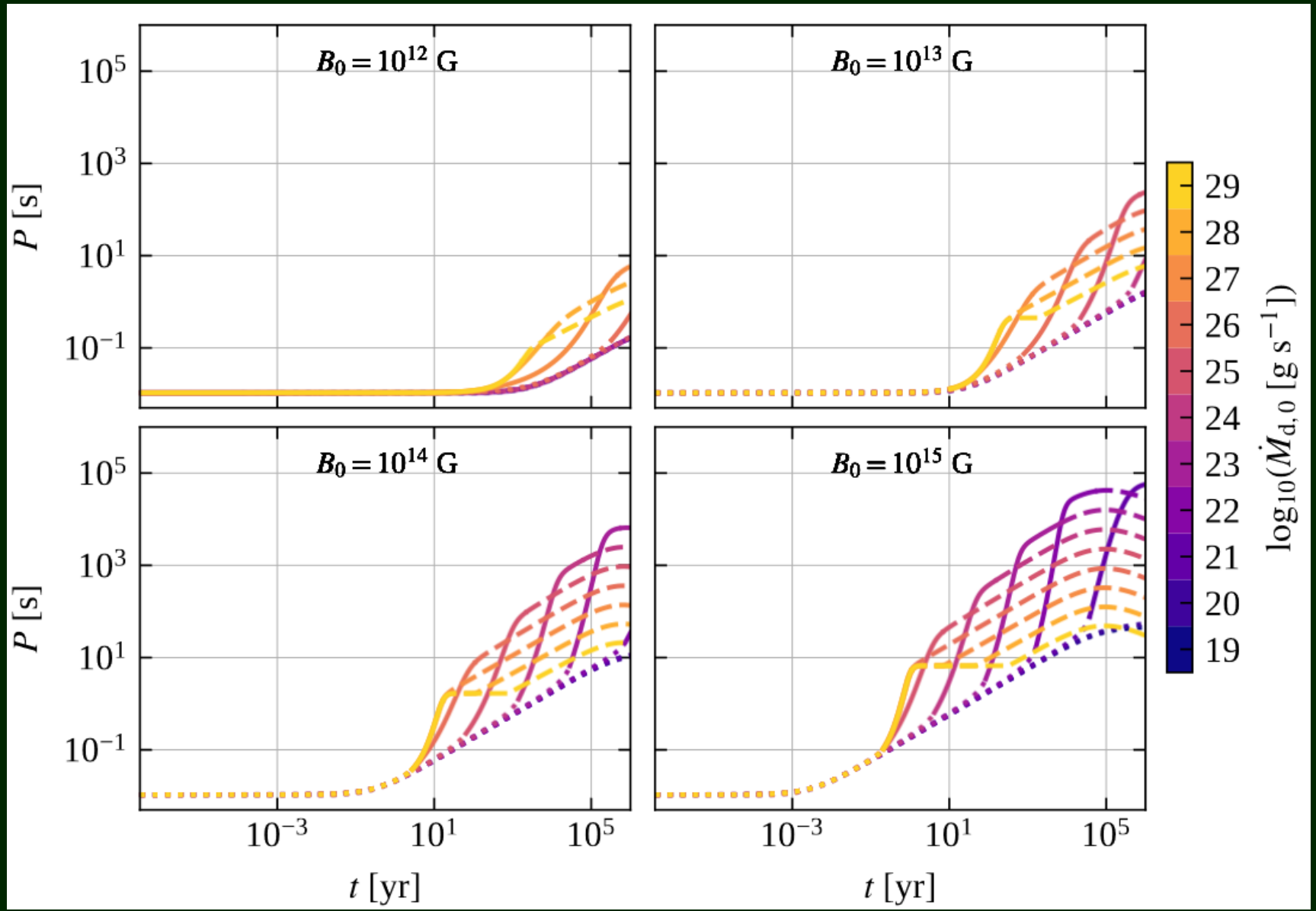
$$I_{\text{NS}}\dot{\omega} = N_{\text{tot}} = N_{\text{acc}} + N_{\text{dip}}$$

$$B(t) = B_0 \frac{e^{-t/\tau_{\text{Ohm}}}}{1 + \frac{\tau_{\text{Ohm}}}{\tau_{\text{Hall},0}} [1 - e^{-t/\tau_{\text{Ohm}}}]}$$

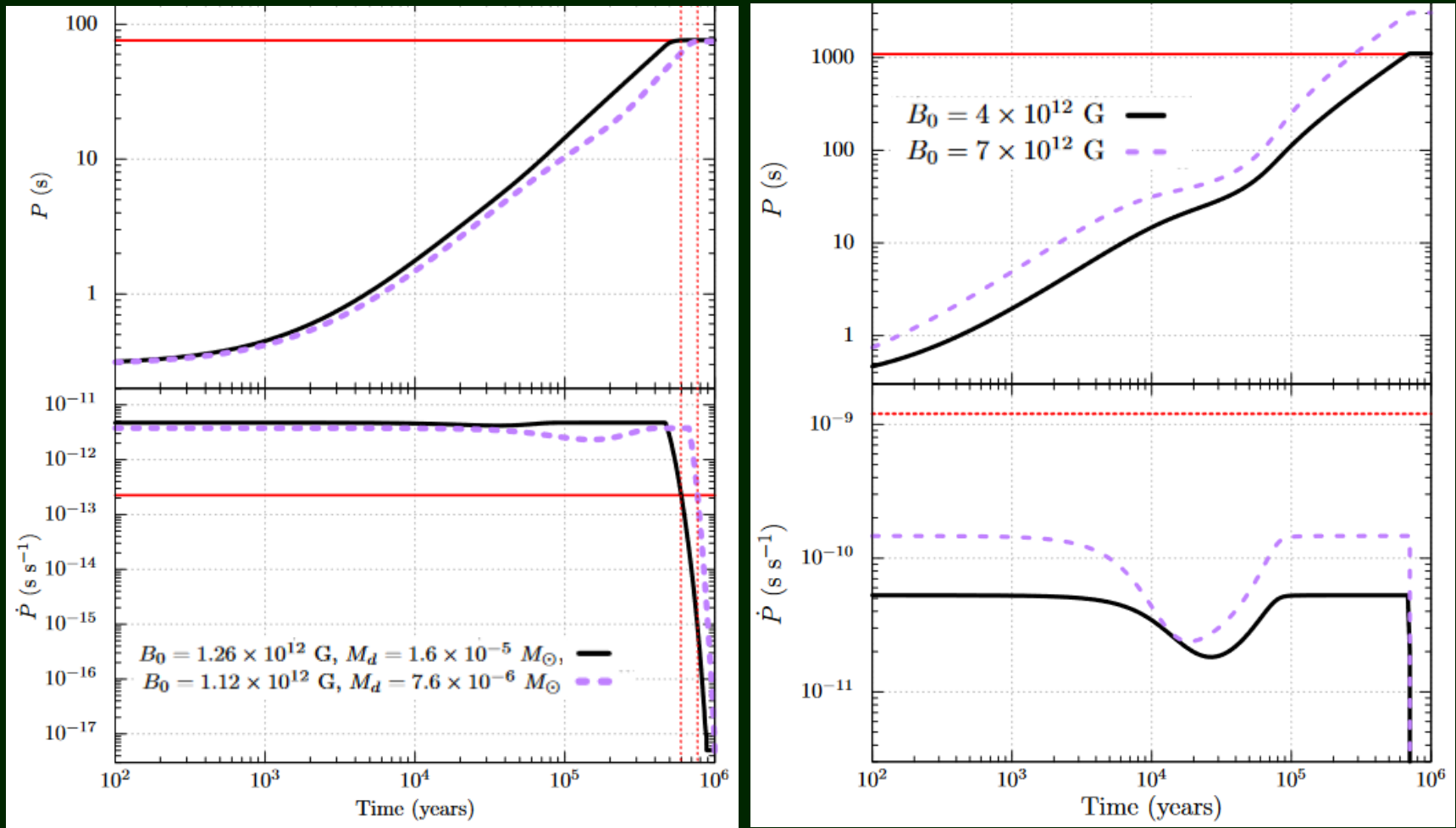
$$N_{\text{tot}} = \begin{cases} \dot{M}_{d,\text{in}} r_{\text{lc}}^2 [\Omega_{\text{K}}(r_{\text{lc}}) - \omega] - I_{\text{NS}} \beta B^2 \omega^3 & \text{if } r_m > r_{\text{lc}}, \\ \dot{M}_{d,\text{in}} r_m^2 [\Omega_{\text{K}}(r_m) - \omega] - I_{\text{NS}} \left(\frac{r_{\text{lc}}}{r_m}\right)^2 \beta B^2 \omega^3 & \text{if } r_m \leq r_{\text{lc}}. \end{cases}$$

It is possible to explain long spin periods (~ 1000 sec) of recently discovered radio transients.





Fallback discs and long spin period of radio transients



2212.10501

2202.06852

It is necessary to measure \dot{P} to probe the scenario

GRBs and fallback onto magnetars

Giant X-ray flares in GRB happen after $\sim 30-10^5$ s.

Rotational energy $\sim 2 \cdot 10^{52}$ erg P_{ms}^{-2}

$$\dot{M}_{\text{D}}(t) = \dot{M}_{\text{fb}} - \dot{M}_{\text{acc}} - \dot{M}_{\text{prop}},$$

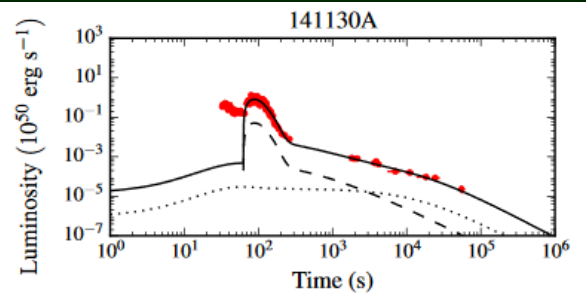
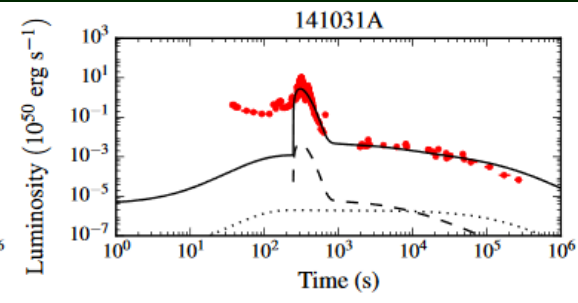
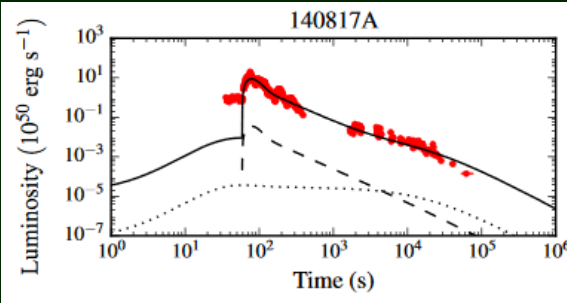
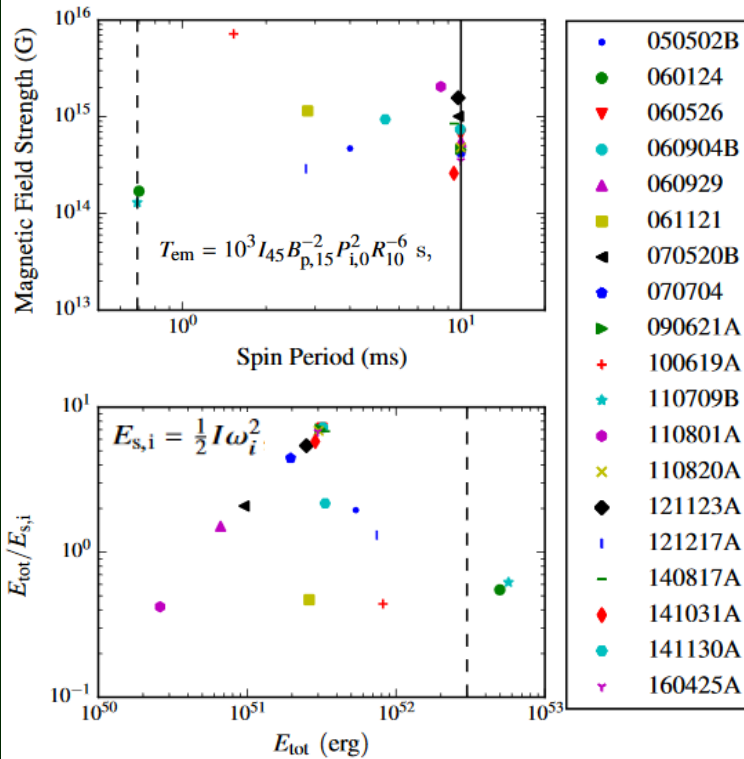
$$\dot{M}_{\text{fb}}(t) = \frac{M_{\text{fb}}}{t_{\text{fb}}} \left(\frac{t + t_{\text{fb}}}{t_{\text{fb}}} \right)^{-\frac{5}{3}}$$

$$L_{\text{dip}} = -\tau_{\text{dip}}\omega$$

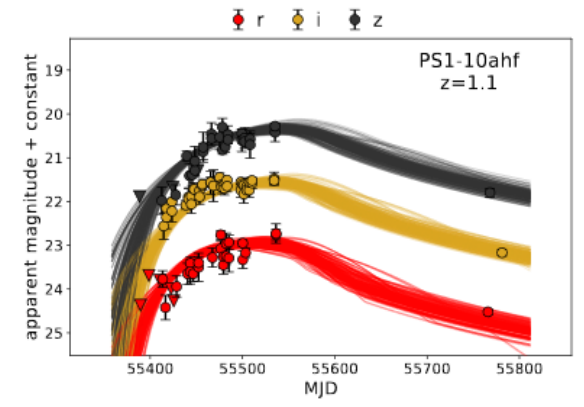
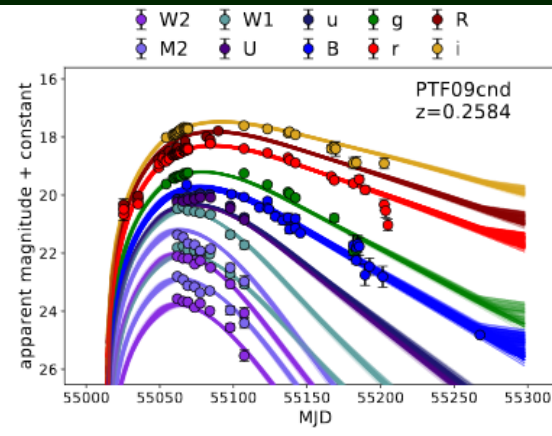
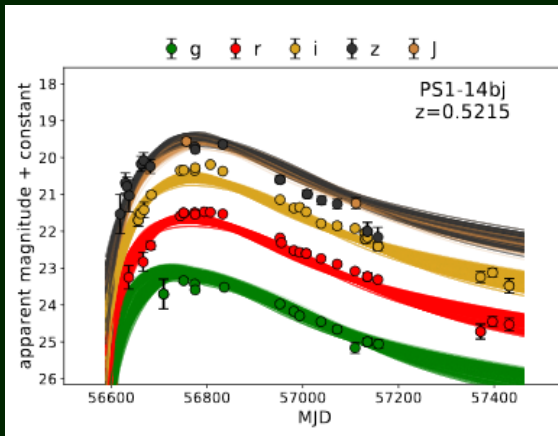
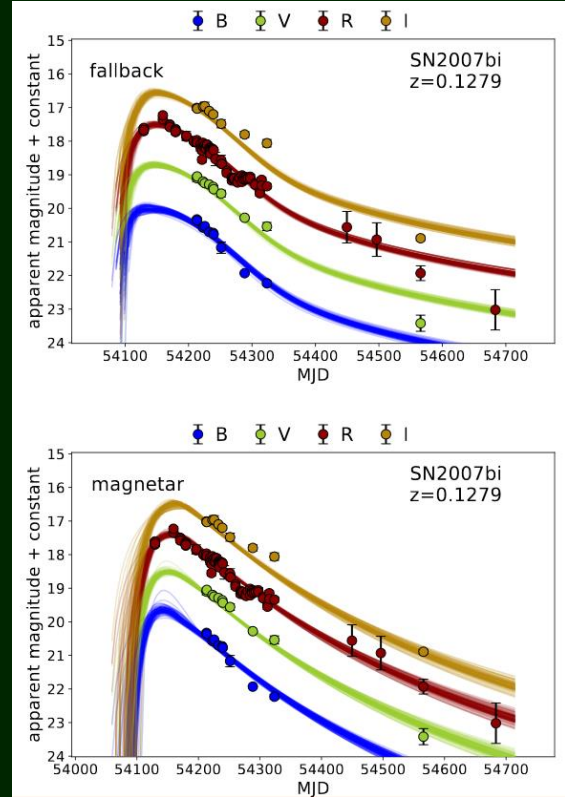
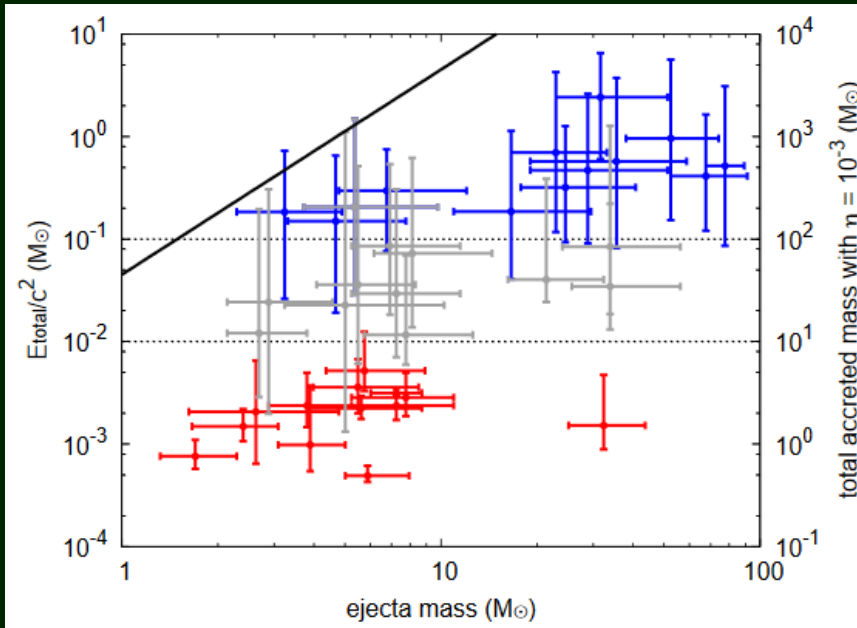
$$L_{\text{prop}} = -\tau_{\text{acc}}\omega$$

τ_{acc} and τ_{dip} are the accretion and dipole torques

$$L_{\text{tot}} = \frac{1}{f_{\text{B}}} \left(\eta_{\text{prop}} L_{\text{prop}} + \eta_{\text{dip}} L_{\text{dip}} \right)$$



Fallback to power SLSN

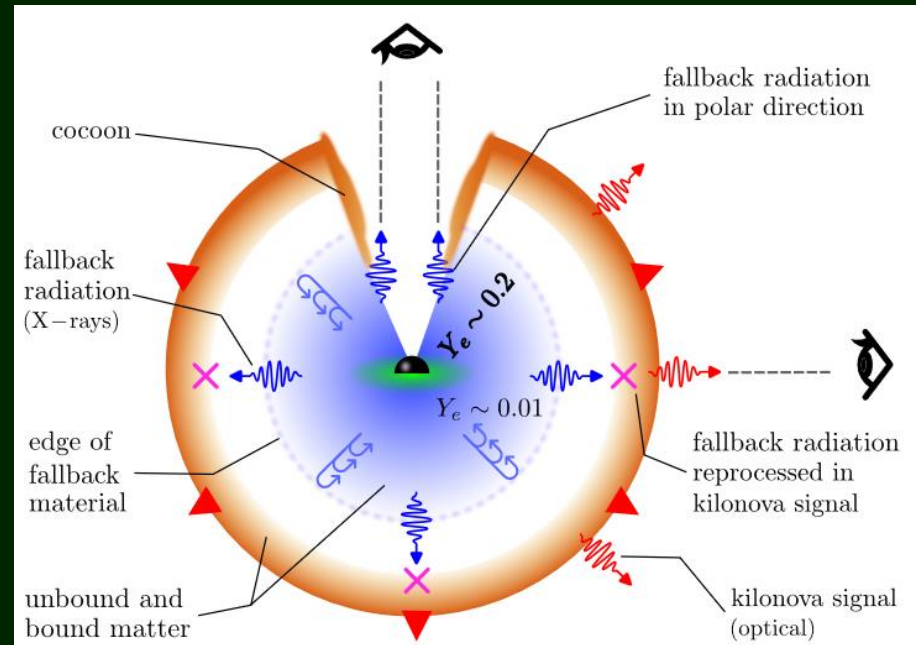
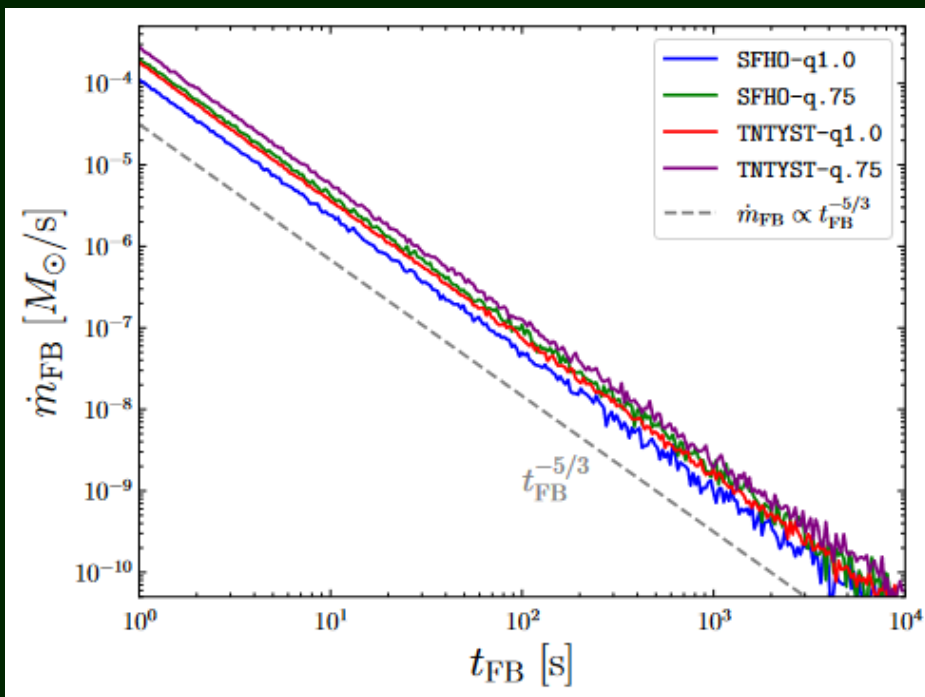


1806.00090

$$L_{\text{fallback}}(t) = \begin{cases} L_1 \left(\frac{t_{\text{tr}}}{1 \text{ sec}}\right)^{-\frac{5}{3}} & \equiv L_{\text{flat}} \quad (t < t_{\text{tr}}) \\ L_1 \left(\frac{t}{1 \text{ sec}}\right)^{-\frac{5}{3}} & \quad (t \geq t_{\text{tr}}) \end{cases}$$

Fallback after NS-NS coalescence

Accreted mass always $> \sim 0.001 M_{\odot}$
 Accretion rate $\sim t^{-5/3}$



The luminosity can be $> 10^{48}$ erg/s for hundreds of seconds and thus, can explain the extended emission in short GRBs

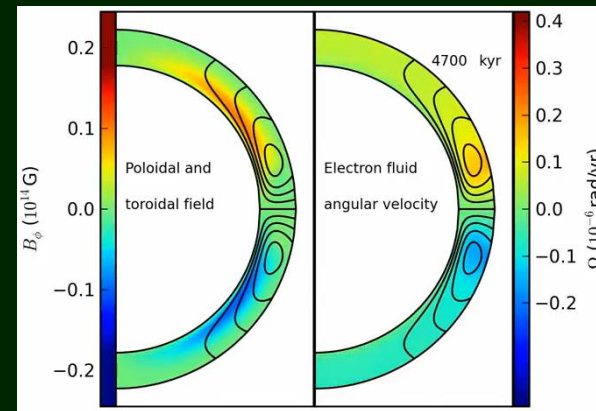
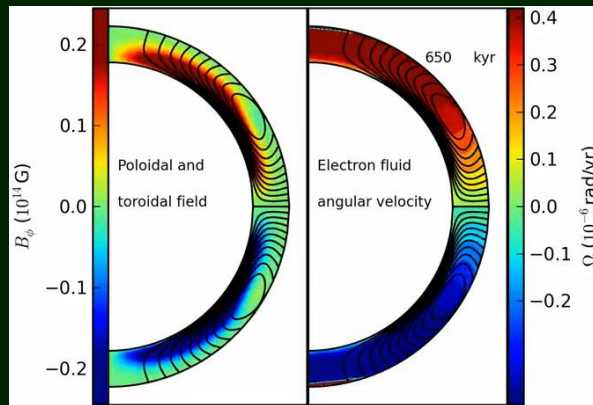
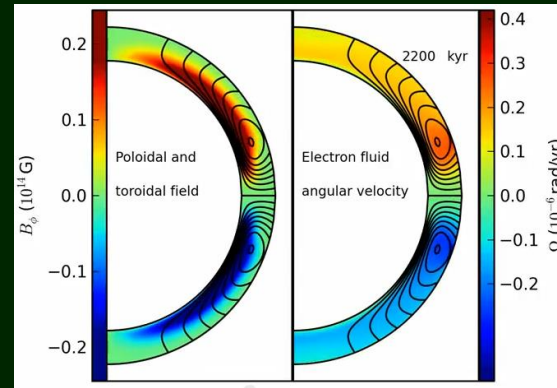
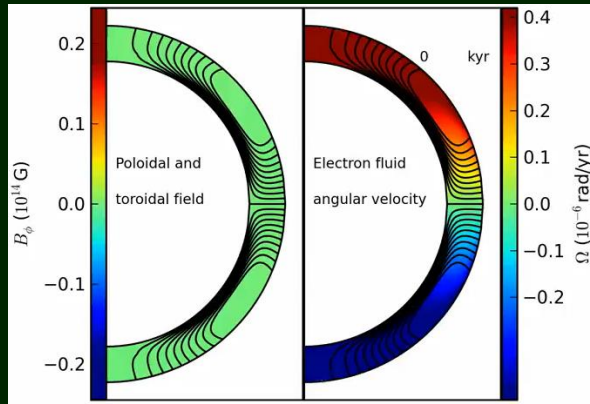
Conclusions

- Decaying magnetic field results in additional heating of a NS and decreasing its spin-down rate
- Field decay can be more important for large initial fields, for “standard” fields ($\sim 10^{12}$ G) it is not important
- It is possible to describe different types of young NSs (PSRs, magnetars, M7 etc.) in the model with decaying magnetic field
- Re-merging magnetic field can be an important ingredient
- With re-emerging field we can add to the general picture also CCOs.
- Recent studies indicate that in the life of normal radio pulsars there is a period when their magnetic field decay
- Hall cascade (and attractor) can be an important ingredient of the field evolution.
- At the moment we cannot state that we see the Hall attractor in the population of normal radio pulsars
- Also, we do not see that any of the M7 NSs are at the attractor stage, as its properties are predicted by GC2013
- Probably, the attractor stage is reached later, or its properties are different from the predicted ones.

Papers to read

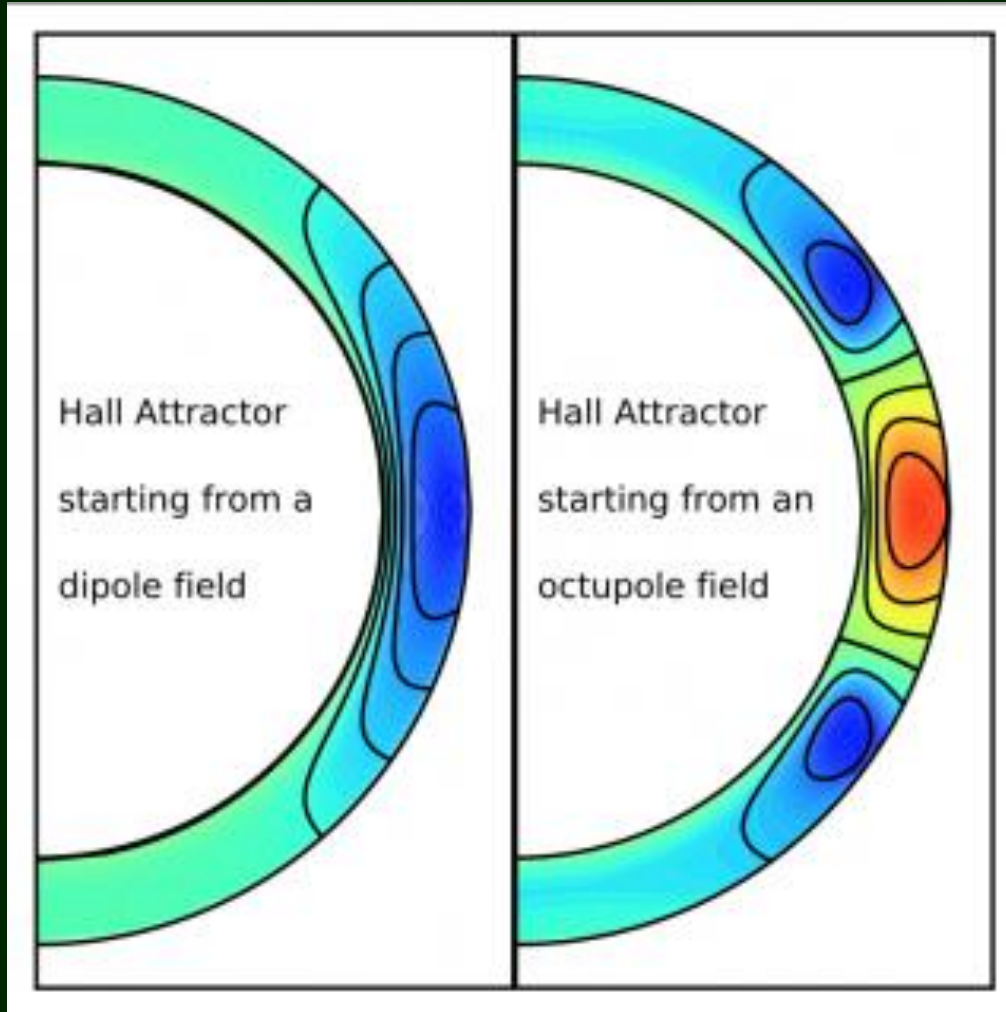
- Pons, Geppert “Magnetic field dissipation in neutron star crusts: from magnetars to isolated neutron stars ” astro-ph/0703267
- Popov et al. “Population synthesis studies of isolated neutron stars with magnetic field decay” MNRAS (2009) arXiv: 0910.2190
- Ho “Evolution of a buried magnetic field in the central compact object neutron stars ” arXiv:1102.4870
- Cumming et al. “MAGNETIC FIELD EVOLUTION IN NEUTRON STAR CRUSTS DUE TO THE HALL EFFECT AND OHMIC DECAY” astro-ph/0402392
- Igoshev et al. “Magnetic Field Evolution in Neutron Star Crusts: Beyond the Hall Effect” arXiv:2201.08345
- ➔ • Igoshev et al. “Evolution of neutron star magnetic fields” arXiv: [2109.05584](https://arxiv.org/abs/2109.05584)
- ➔ • Abolmasov et al. “Spin evolution of neutron stars ” arXiv: 2402.04331, Sec. 2

Hall cascade and attractor



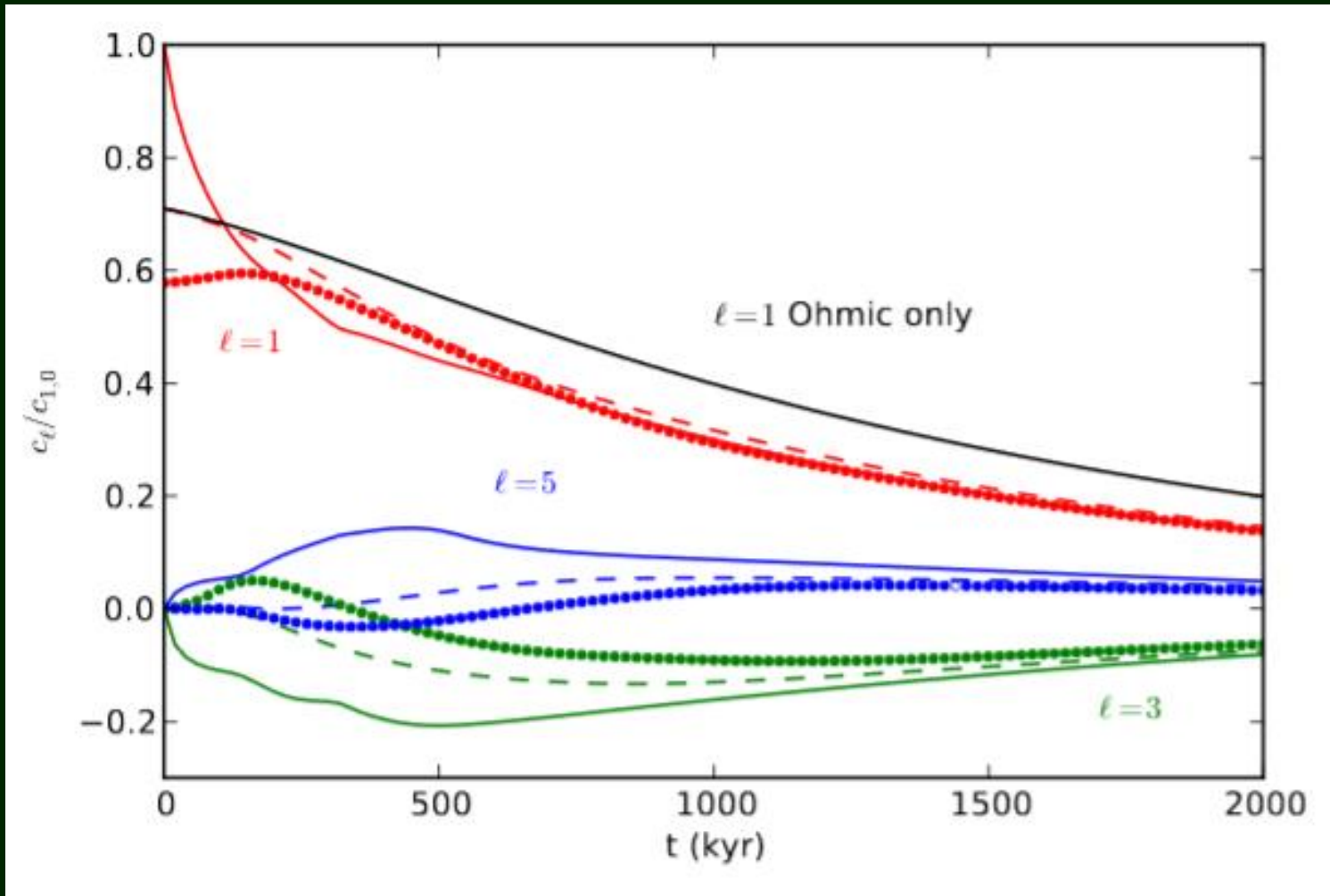
Hall cascade can reach the stage of so-called Hall attractor, where the field decay stalls for some time (Gourgouliatos, Cumming).

Hall attractor



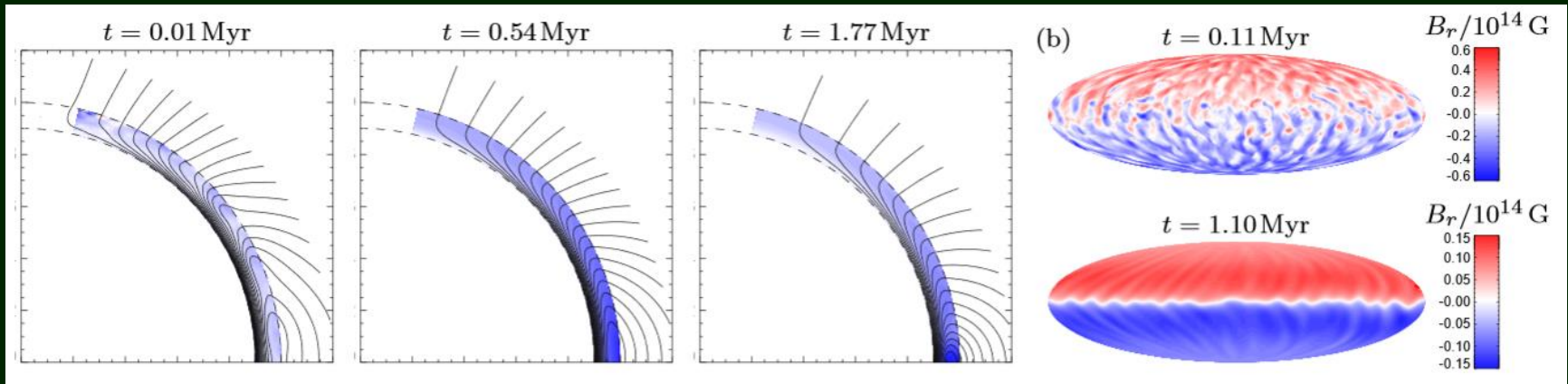
After some time the Hall cascade decays as the field finds a new| stable configuration.

Evolution of different components

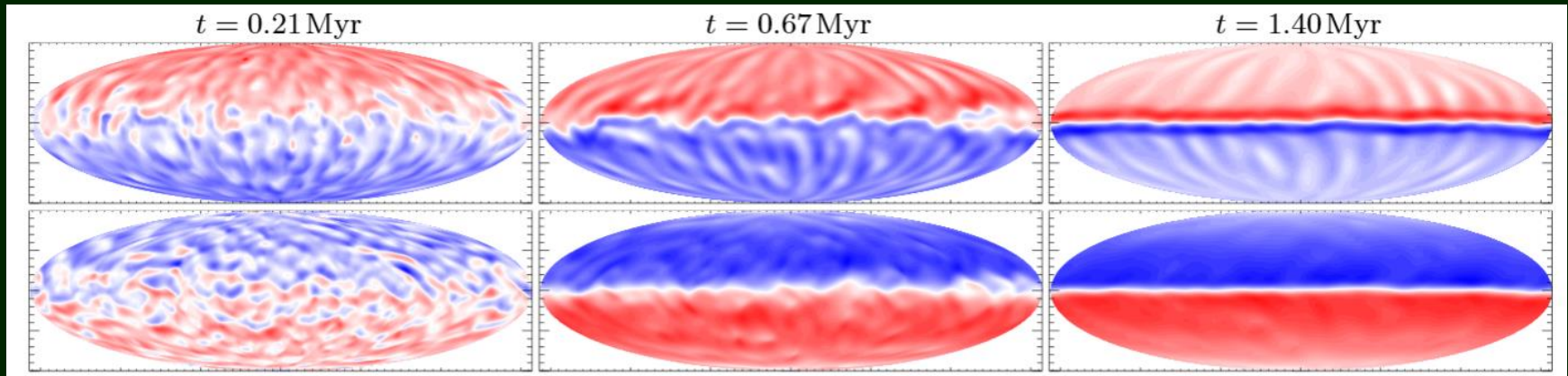


Hall attractor mainly consists of dipole and octupole

New studies of the hall cascade



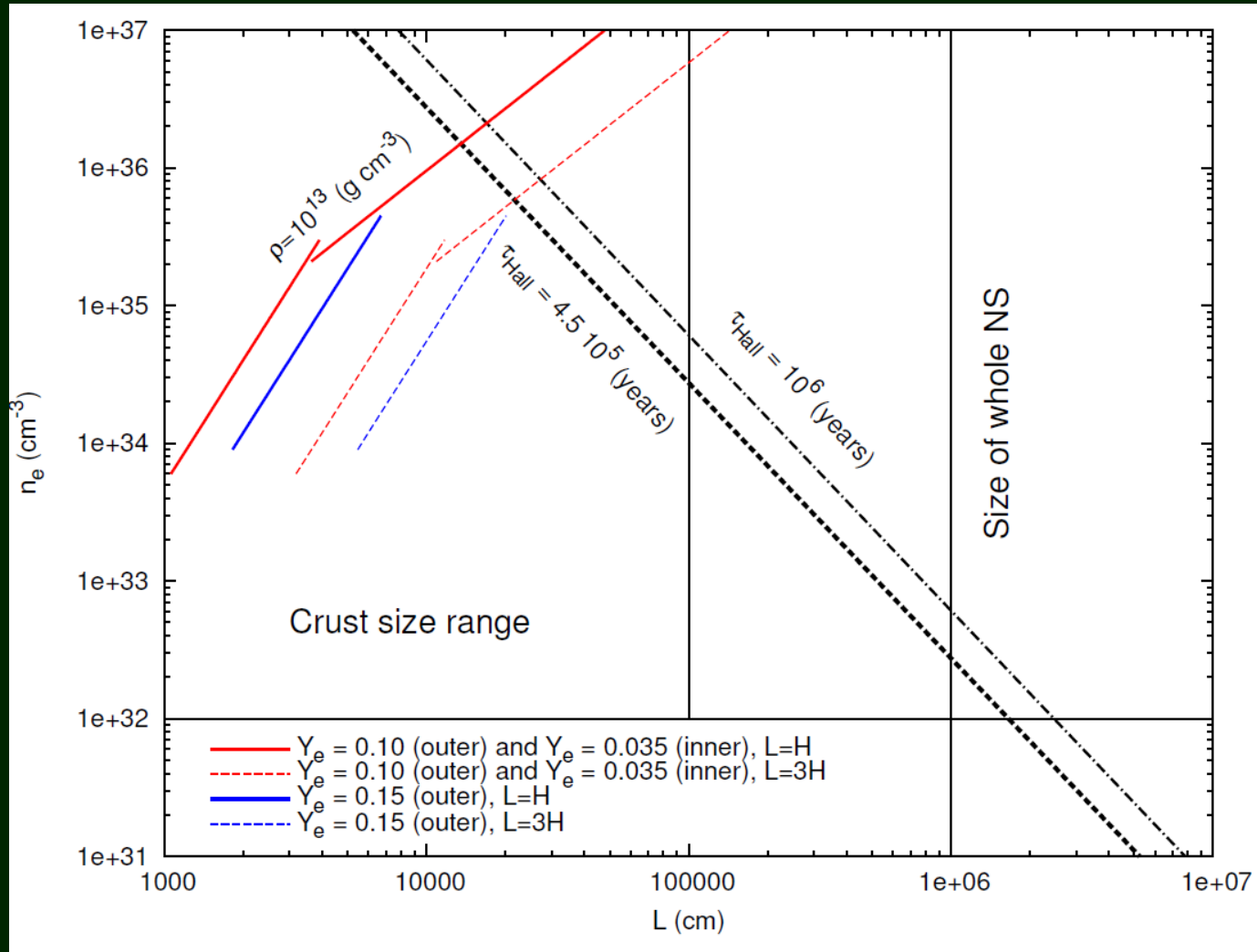
Toroidal
Poloidal



New calculations support the idea of a kind of stable configuration.

1501.05149

Where the currents are located?



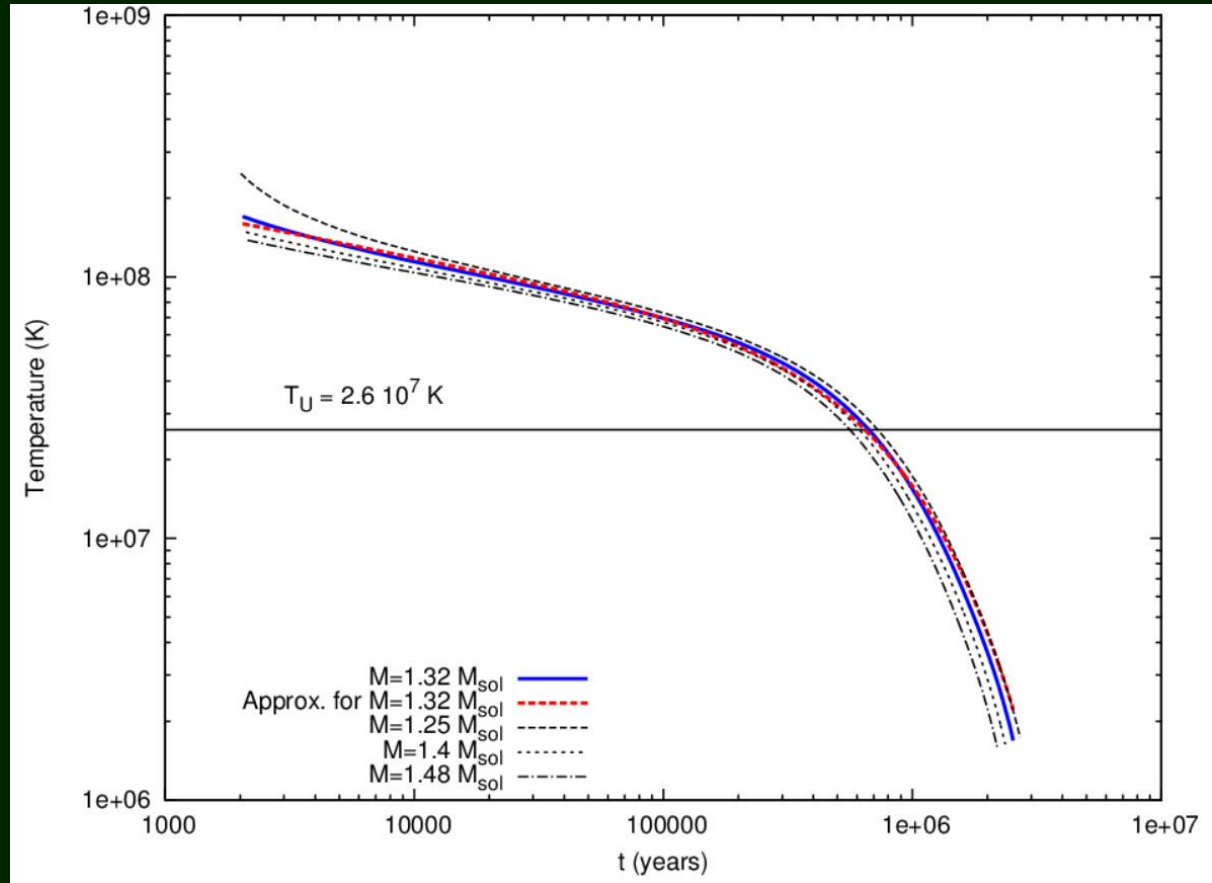
$$\tau_{\text{Hall}} \approx \frac{4\pi e L^2 n_e}{cB}$$

$$L \approx H = P(\rho) / (\rho g)$$

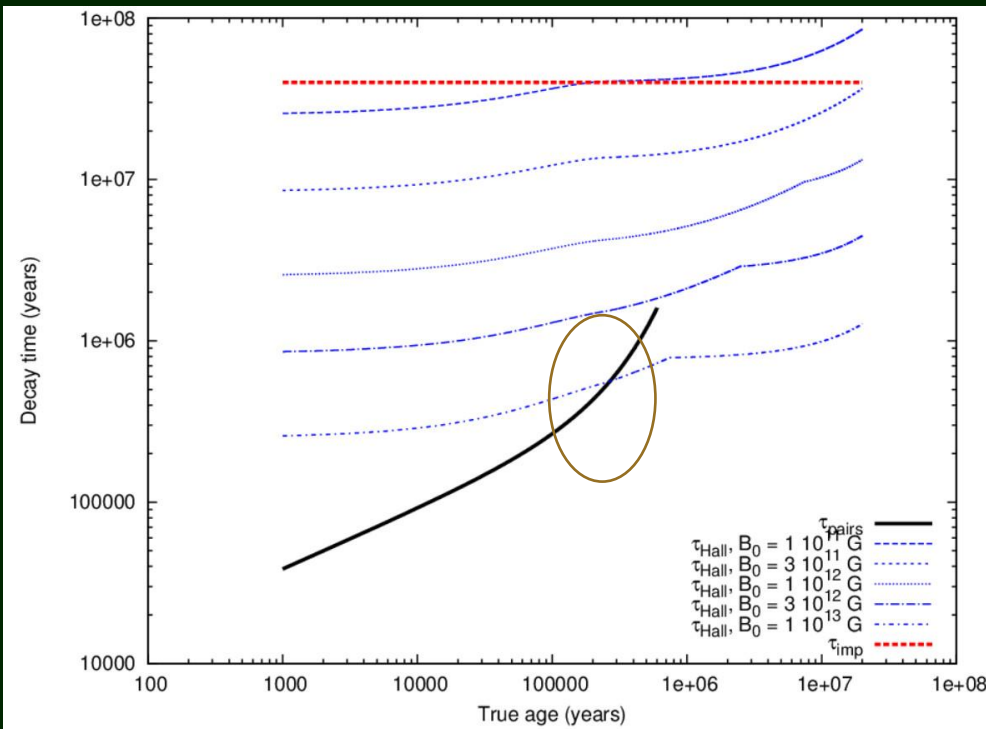
Thermal evolution

Calculations are made by Shternin et al.

We fit the numerical results to perform a population synthesis of radio pulsars with decaying field.



Different decay time scales



In the range of ages interesting for us the Hall rate is about the same value as the rate of Ohmic dissipation due to phonons.

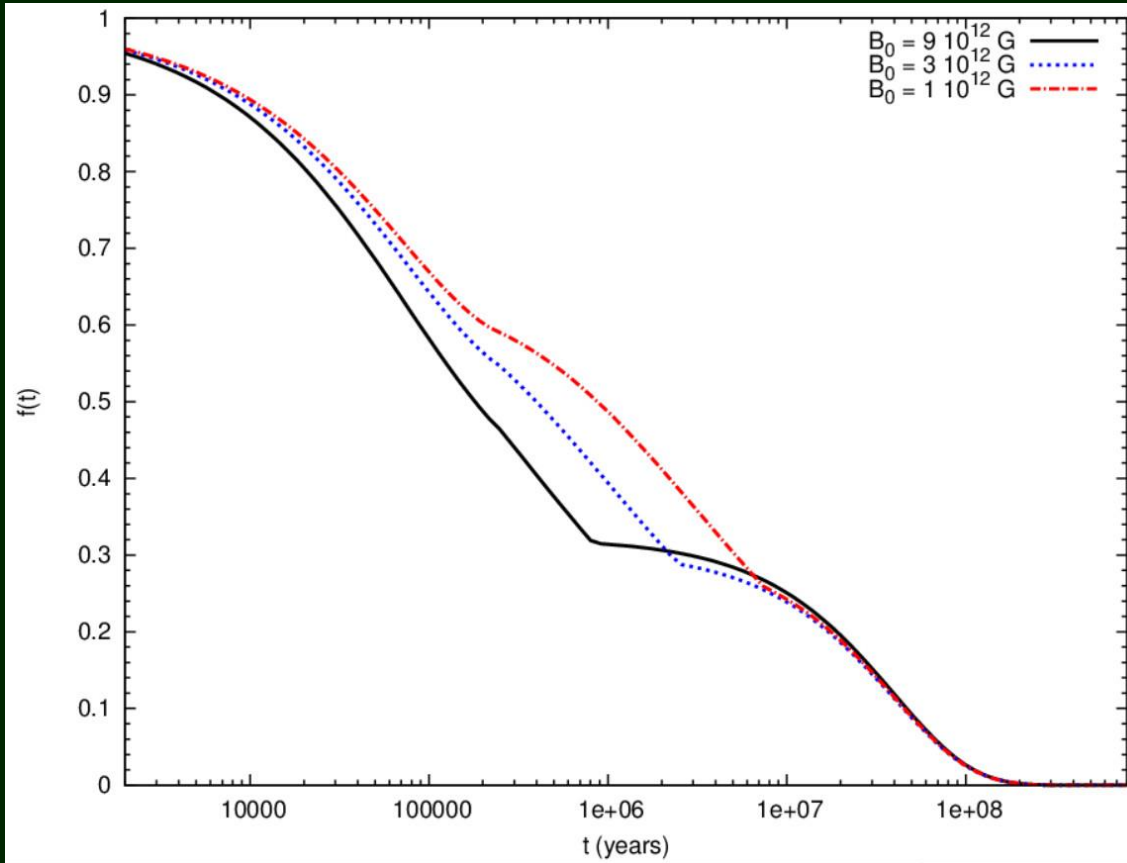
$$B = B_0 \frac{\exp(-t/\tau_{\text{Ohm}})}{1 + \frac{\tau_{\text{Ohm}}}{\tau_{\text{Hall}}}(1 - \exp(-t/\tau_{\text{Ohm}}))}$$

$$\tau_{\text{imp}} = 5.7 \frac{\rho_{14}^{5/3}}{Q} \left(\frac{Z}{30}\right) \left(\frac{Y_e}{0.05}\right)^{1/3} \left(\frac{Y_n}{0.8}\right)^{10/3} \times \left(\frac{f}{0.5}\right)^2 \left(\frac{g_{14}}{2.45}\right) \text{ Myrs,}$$

$$\tau_{\text{phonon}} = 2.2 \frac{\rho_{14}^{15/6}}{T_8^2} \left(\frac{Y_e}{0.05}\right)^{5/3} \left(\frac{Y_n}{0.8}\right)^{10/3} \times \left(\frac{f}{0.5}\right)^2 \left(\frac{g_{14}}{2.45}\right)^{-2} \text{ Myrs,}$$

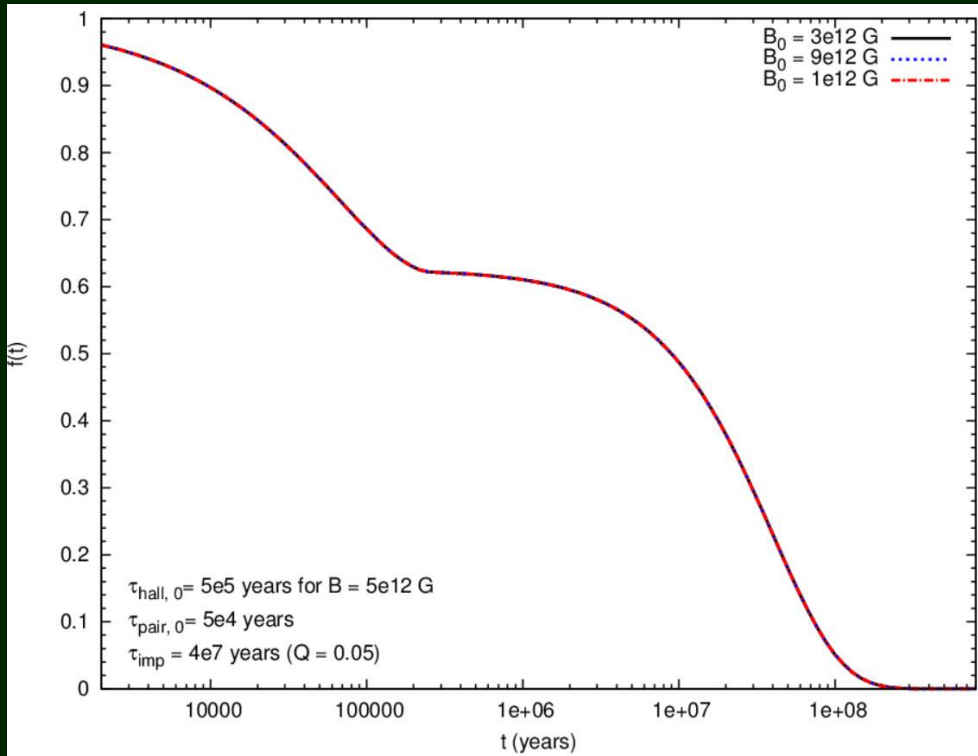
Magnetic field evolution

$$B = B_0 \frac{\exp(-t/\tau_{\text{Ohm}})}{1 + \frac{\tau_{\text{Ohm}}}{\tau_{\text{Hall}}}(1 - \exp(-t/\tau_{\text{Ohm}}))}$$

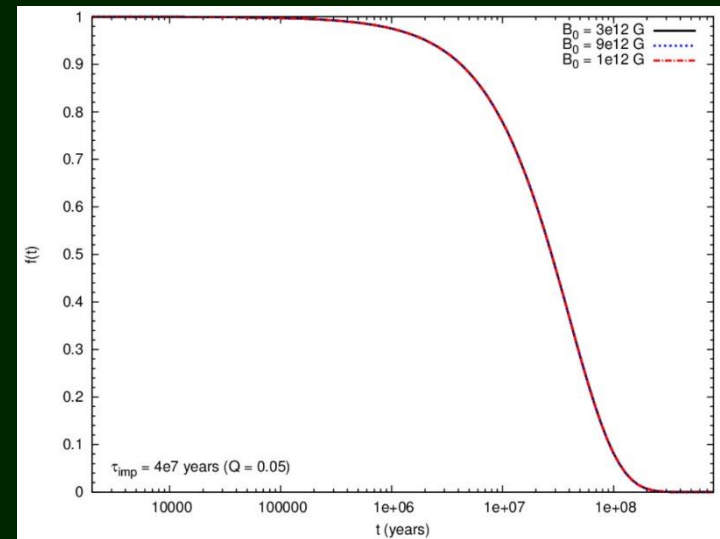


All inclusive:
- Hall
- Phonons
- Impurities

Only Ohmic decay

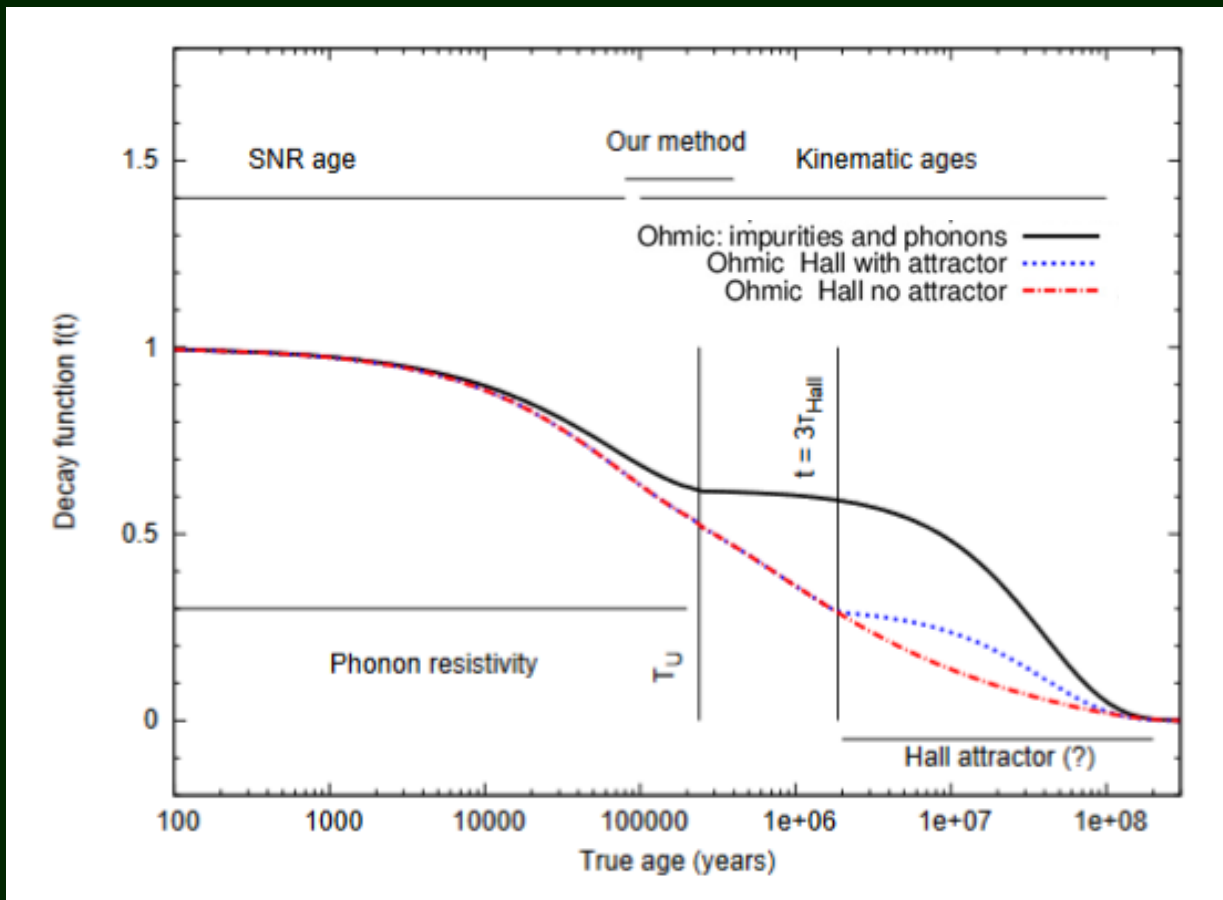


Here the Hall cascade is switched off



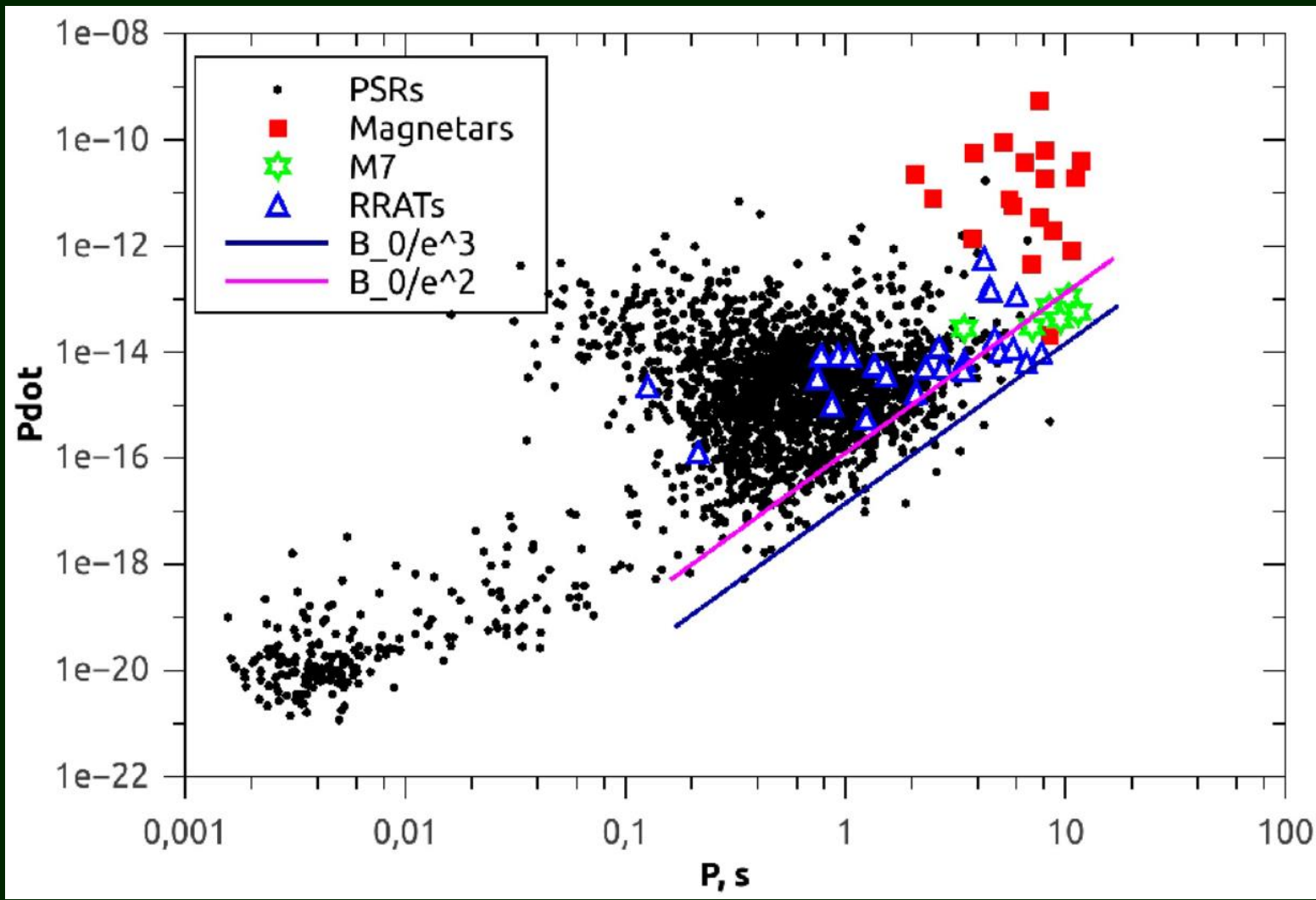
In one figure we have Ohmic decay only due to impurities, on another one – phonons are added.

Comparison of different options



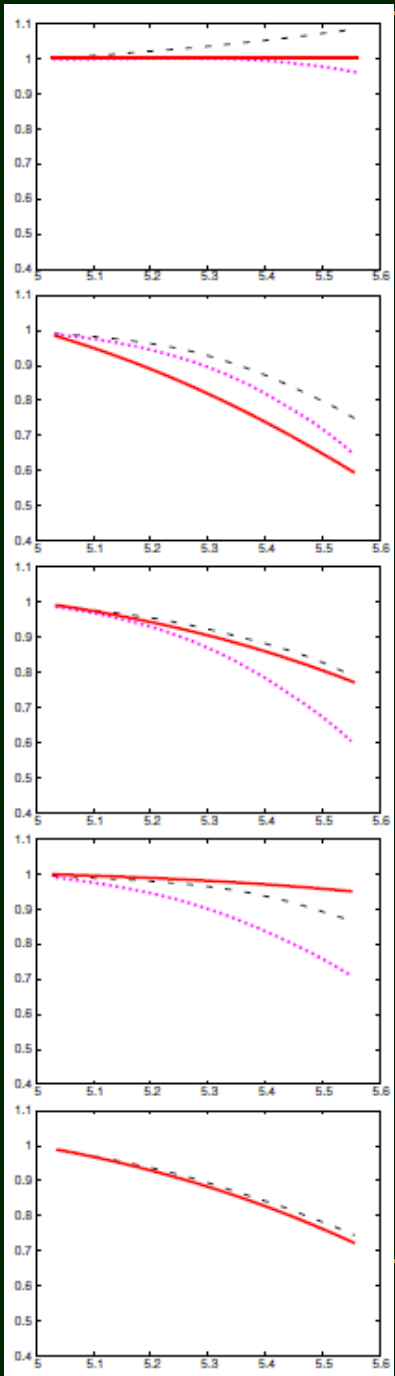
We think that at the ages $\sim 10^5$ yrs and below for normal pulsars we see mostly Ohmic decay, which then disappears as NSs cool down below the critical T.

Getting close to the attractor



Tests

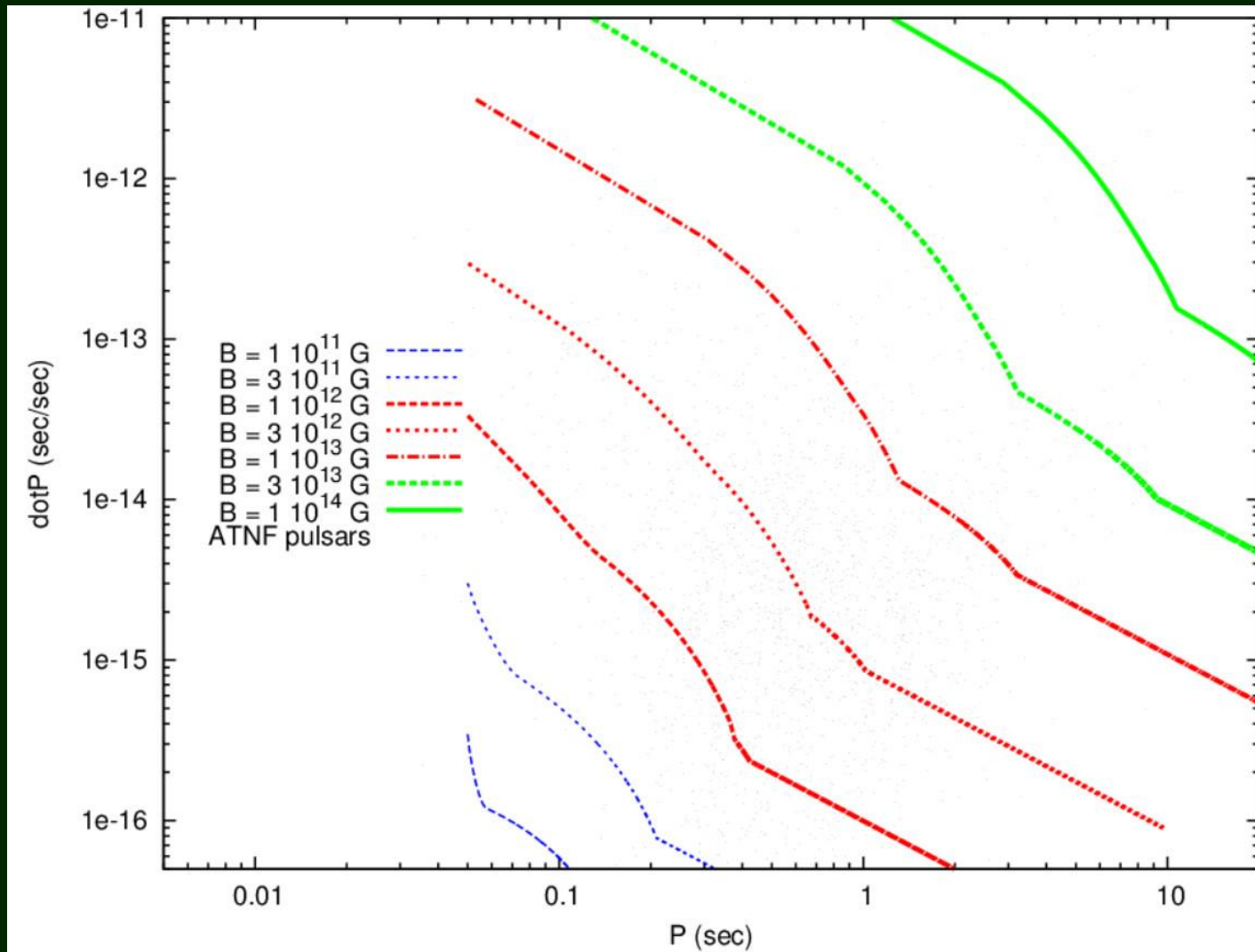
We make extensive tests of the method and obtain that in most of the cases it is able to uncover non-negligible magnetic field decay (more than a few tens of per cent during the studied range of ages) in normal radio pulsars for realistic initial properties of neutron stars.



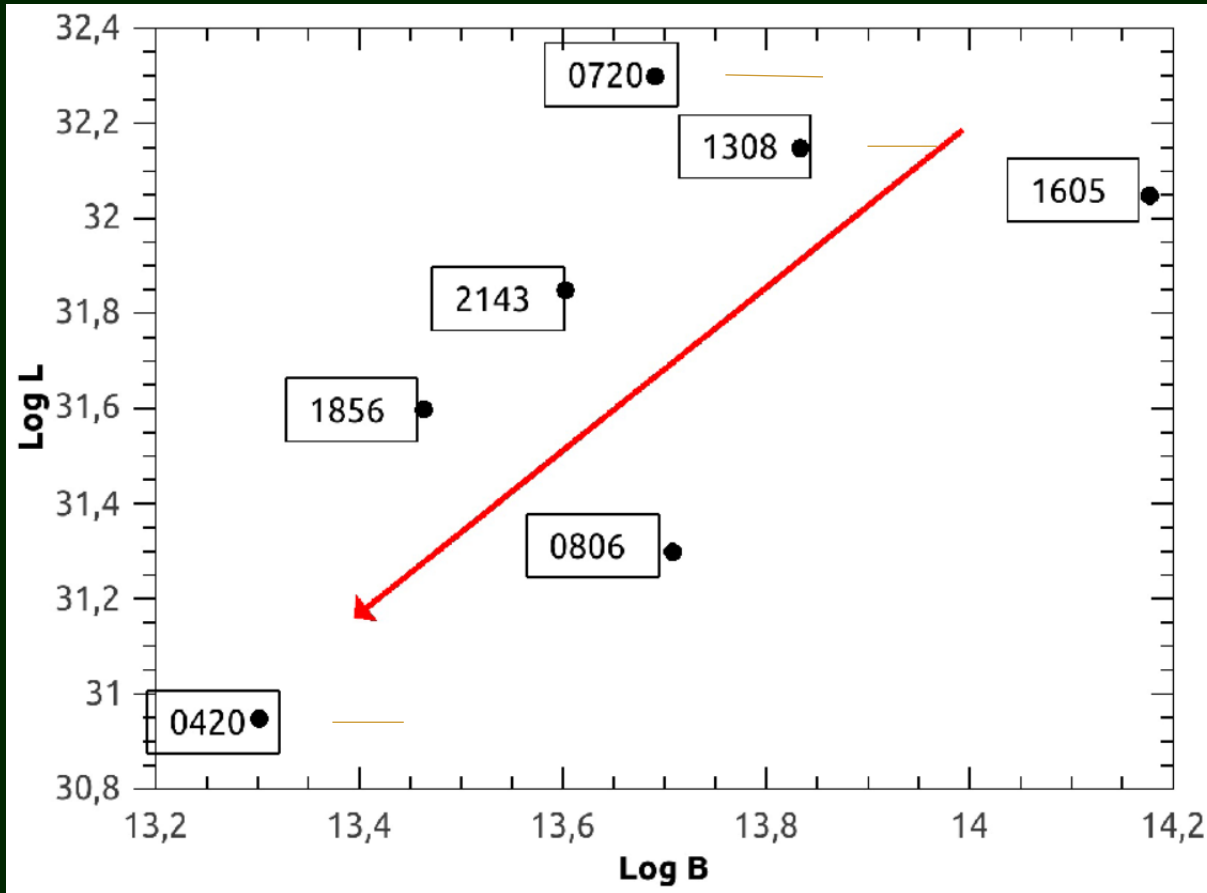
Name	$\log \mu_{B_0}$ [G]	$\log \sigma_{B_0}$ [G]	μ_{P_0} [s]	σ_{P_0} [s]	α	τ_D [Myr]	τ_{SDA} [Myr]
A1	12.60	0.47	0.33	0.23	0.50	∞	∞
A2	12.95	0.55	0.30	0.15	0.50	∞	10
B1	12.60	0.47	0.33	0.23	0.50	0.5	1.00
B2	12.95	0.55	0.30	0.15	0.50	0.5	0.690
C1	12.60	0.47	0.33	0.23	0.50	1	1.15
C2	12.95	0.55	0.30	0.15	0.50	1	0.560
D1	12.60	0.47	0.33	0.23	0.50	5	2.00
D2	12.95	0.55	0.30	0.15	0.50	5	0.80
E	13.04	0.55	0.22	0.32	0.44	~ 0.8	0.880

(Synthetic samples are calculated by Gullon, Pons, Miralles)

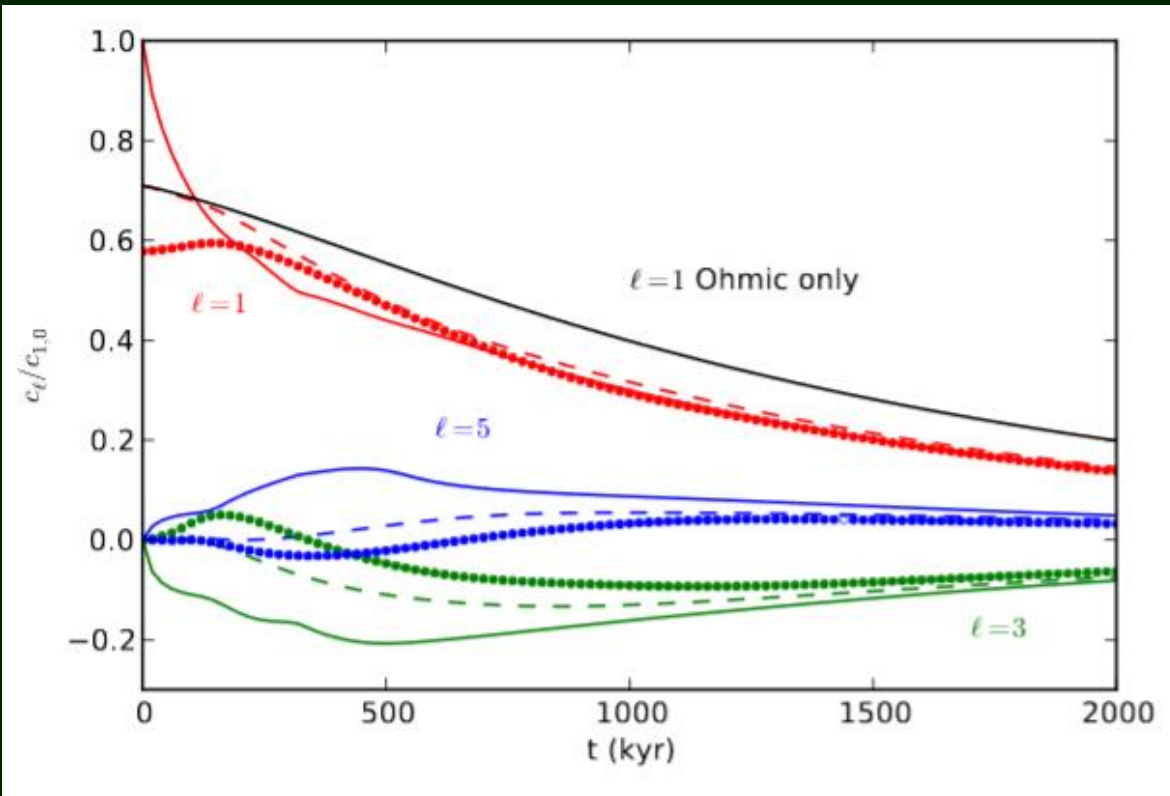
Evolution with field decay



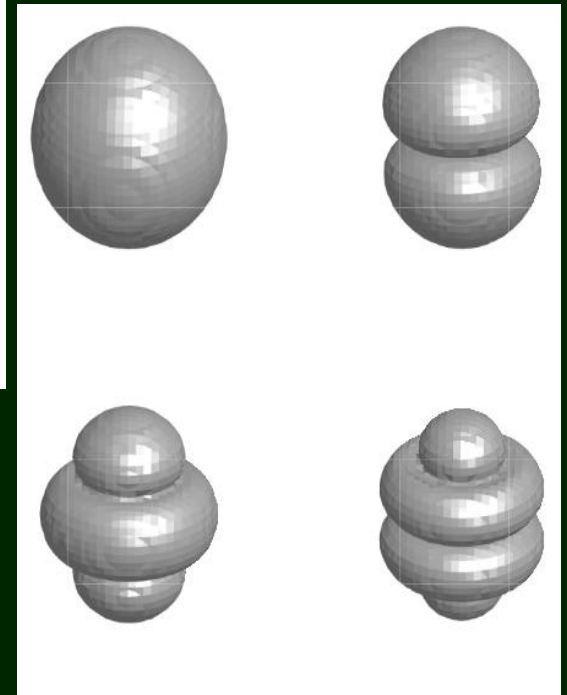
Who is closer to the attractor stage?



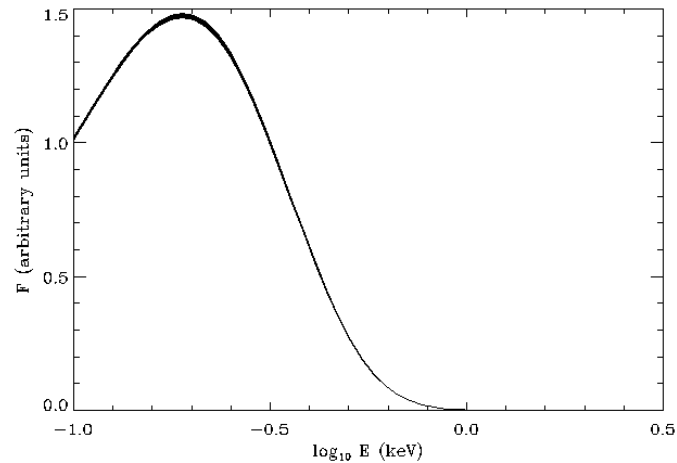
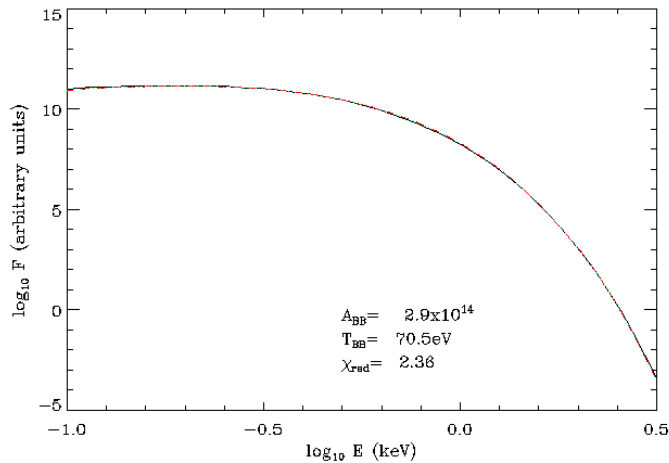
Evolution of different components



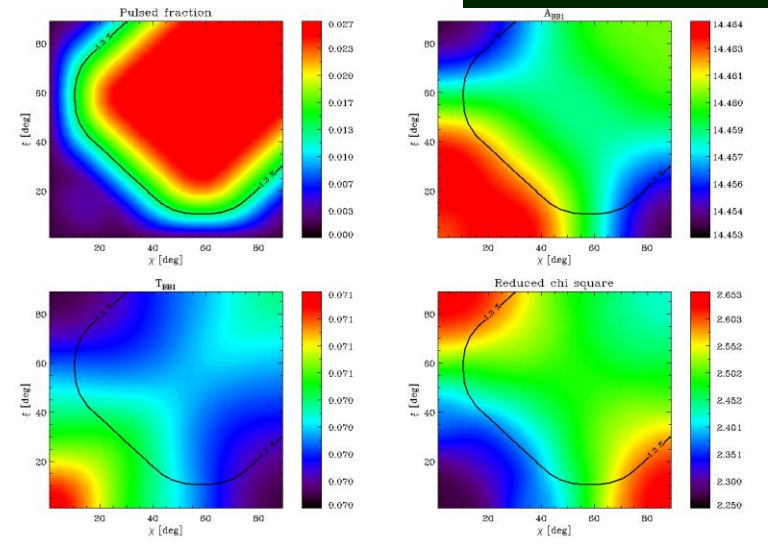
Hall attractor mainly consists of dipole and octupole



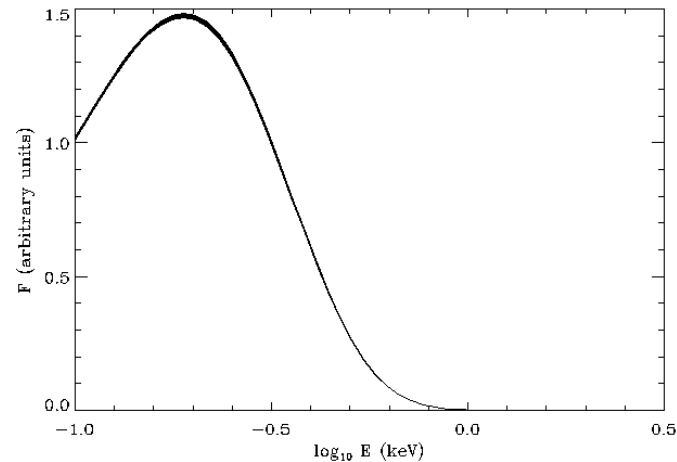
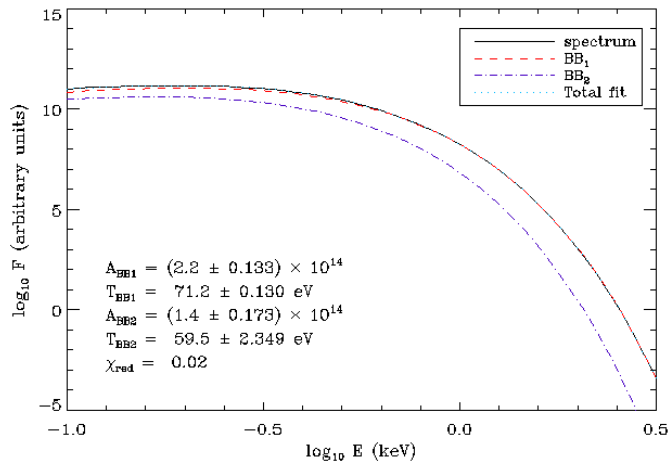
Spectral fits: single blackbody



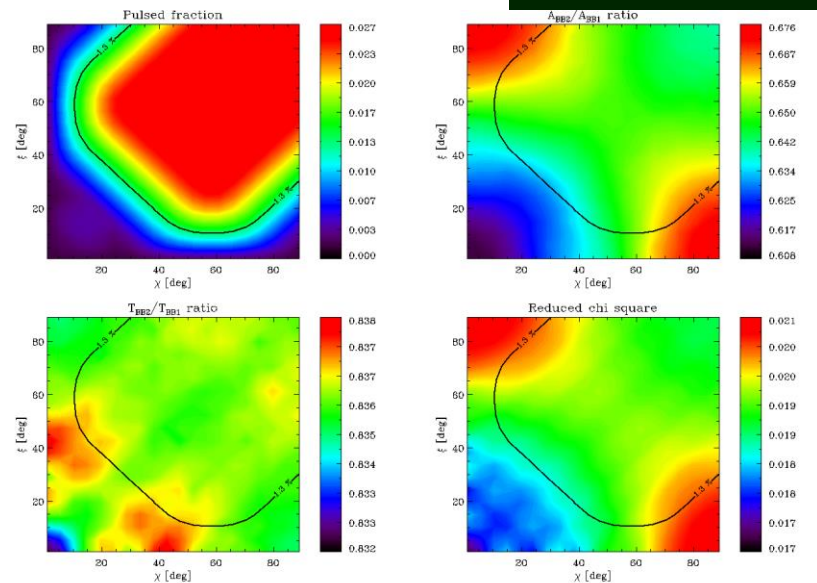
Single black body does not provide a good fit, even using, in addition, a line, or condensed surface.



Spectral fits: two black bodies



Formally, two black bodies is the best fit for 1856.
And for dipole+octupole we can obtain a very good fit.
But



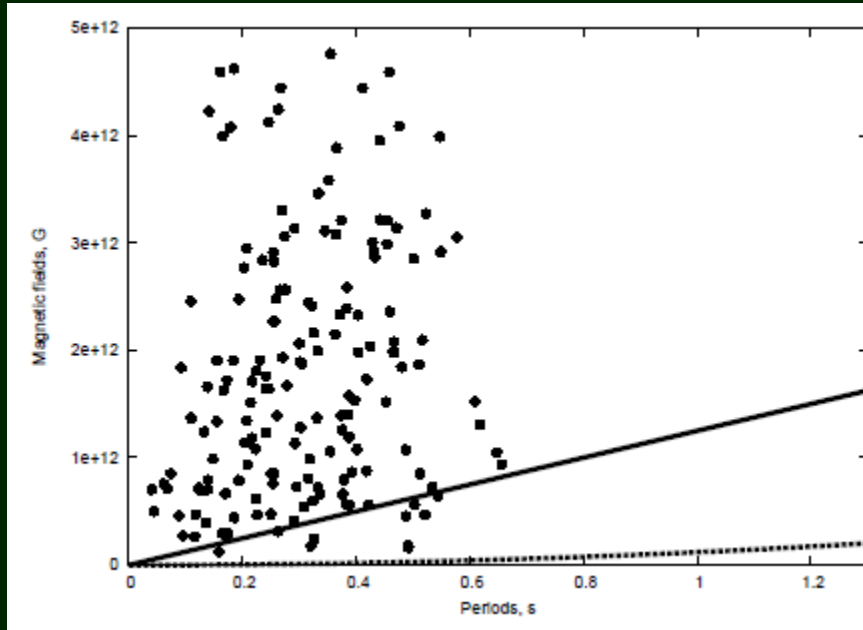
Observational data

Parameter	Single BB	Two BB
N_H [10^{19} cm^{-2}]	$4.8^{+0.2}_{-0.2}$	$12.9^{+2.2}_{-2.3}$
kT_h^∞ [eV]	$61.5^{+0.1}_{-0.1}$	$62.4^{+0.6}_{-0.4}$
R_h^∞ [km]	$5.0^{+0.1}_{-0.1}$	$4.7^{+0.2}_{-0.3}$
kT_s^∞ [eV]	-	$38.9^{+4.9}_{-2.9}$
R_s^∞ [km]	-	$11.8^{+5.0}_{-0.4}$
σ_{sys}	1.5%	0.6%
χ^2_ν	1.12	1.11

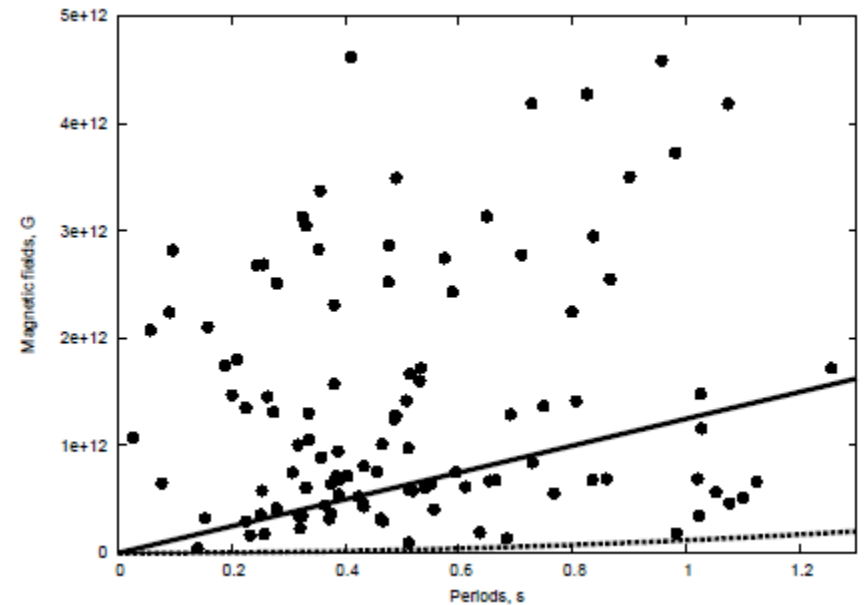
Two black bodies is the best fit.
The colder component corresponds to larger surface area.
This is in contrast with our results for the Hall attractor proposed by GC2013 (dipole + octupole).

Synthetic populations

Constant field

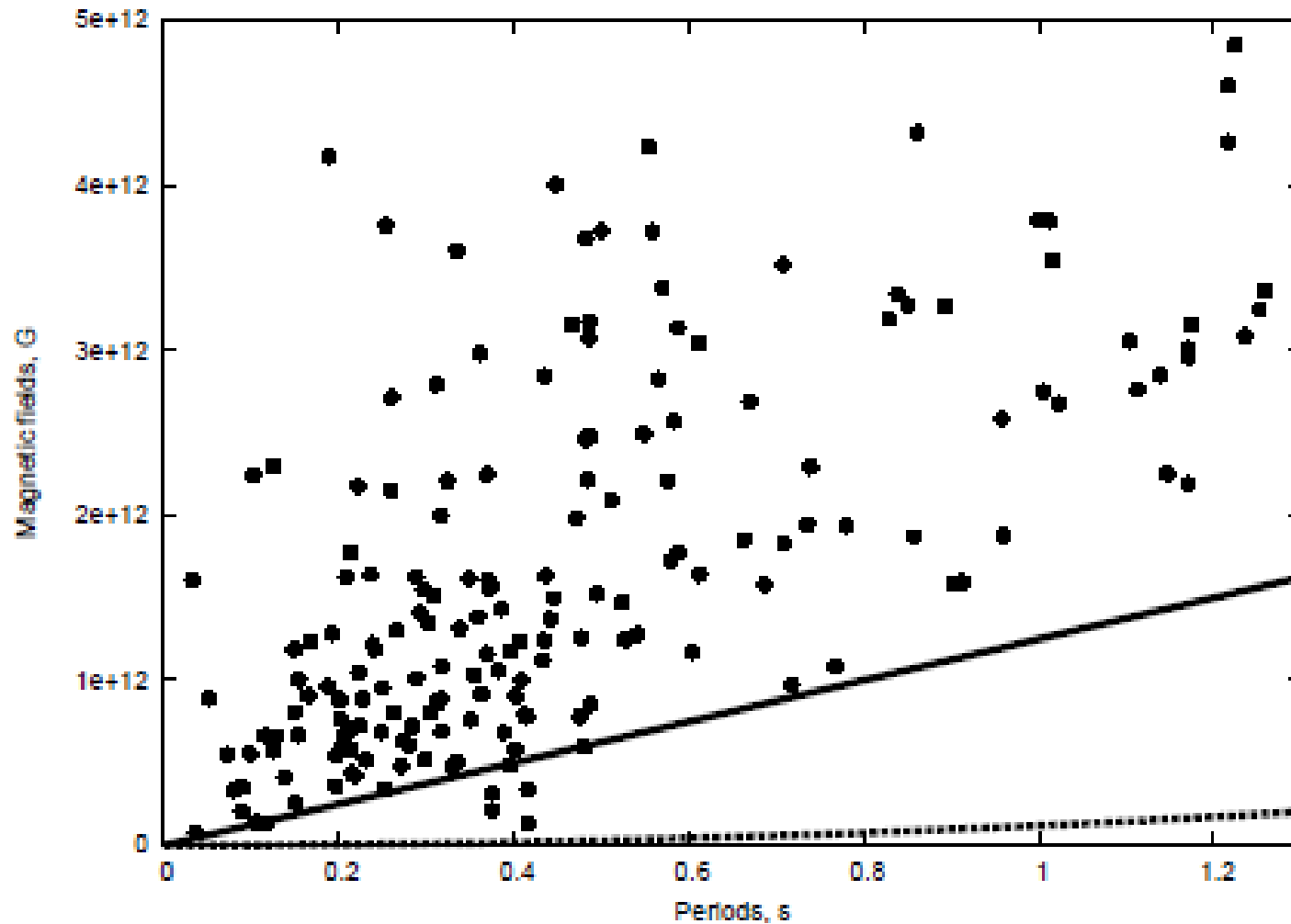


Exponential decay

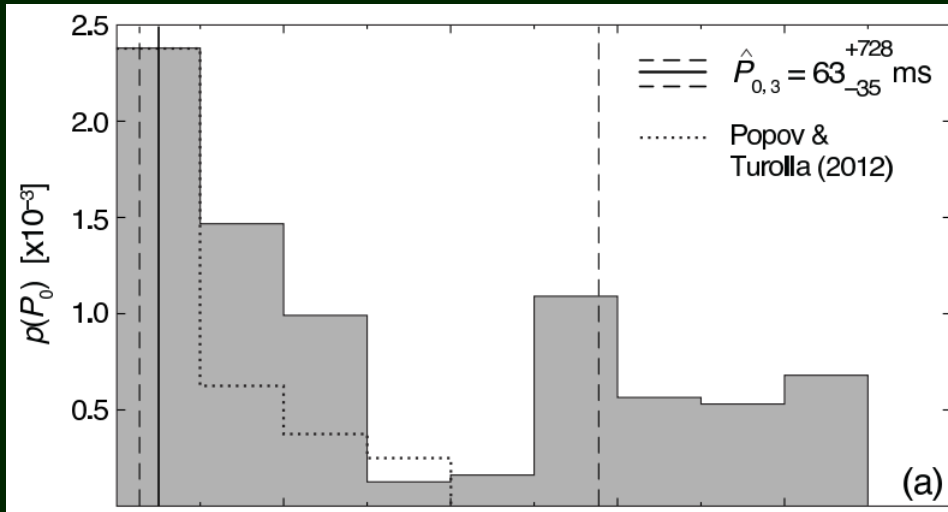


$$P_0 = P \sqrt{1 - \frac{t_{\text{true}}}{\tau}}$$

Fitting the field decay



Another option: emerging field



The problem is just with few (6) most long-period NSs. Is it possible to hide them when they are young, and make them visible at the age ~few million years?

Yes! Emerging magnetic field!!!

Then we need correlations between different parameters.

