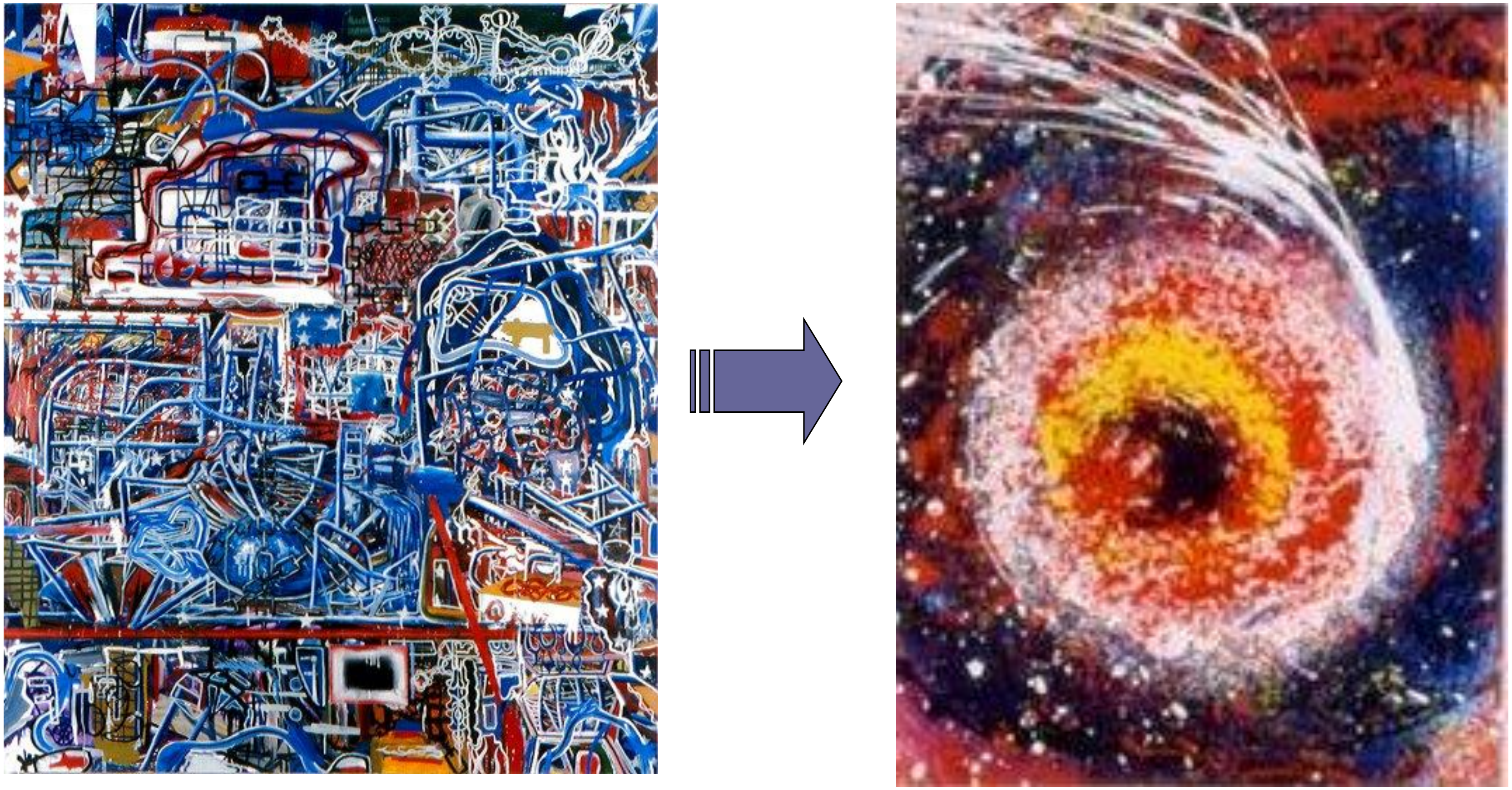
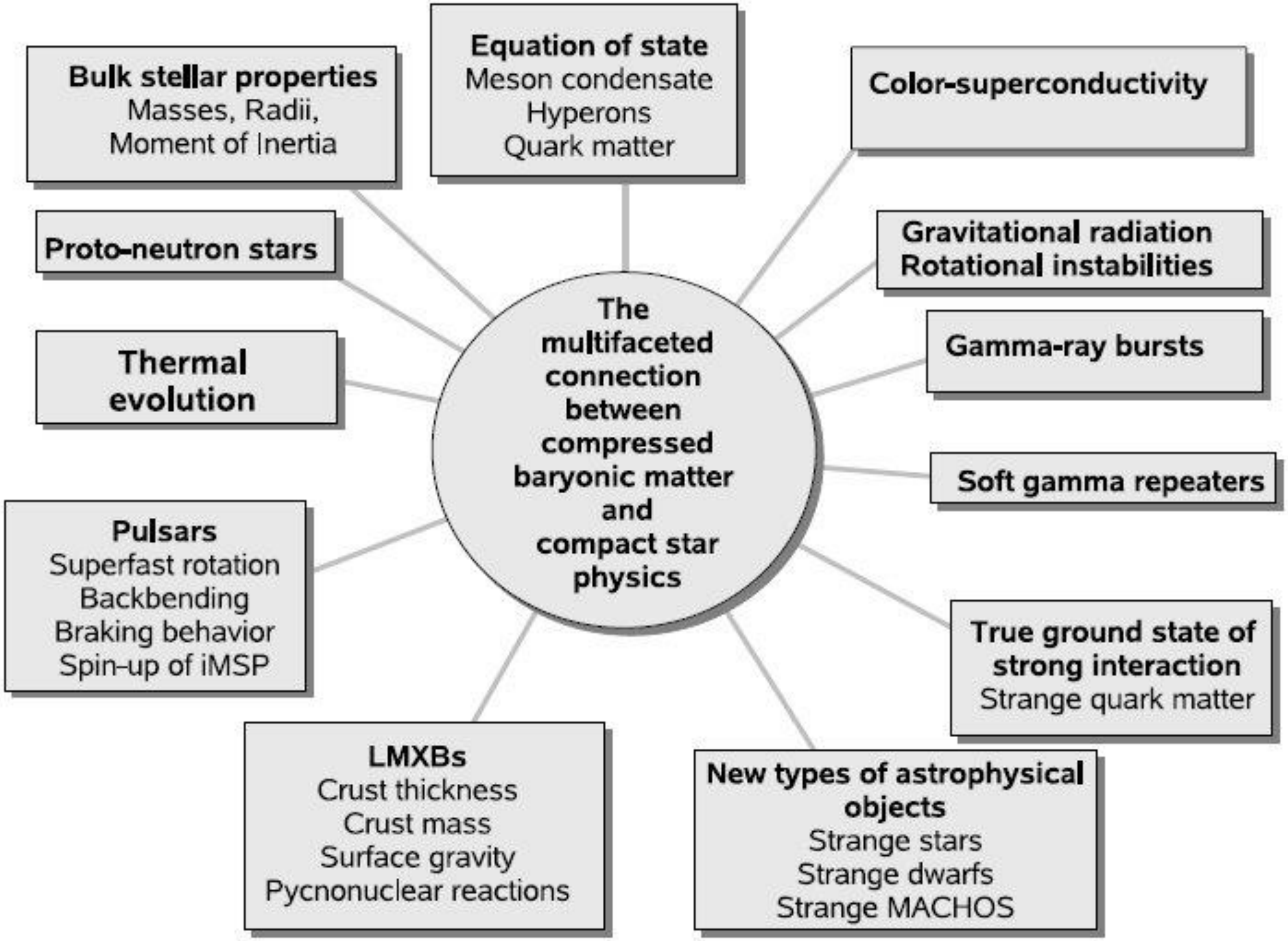
Internal structure of Neutron Stars

Artistic view



Astronomy meets QCD



The multifaceted connection between compressed baryonic matter and compact star physics

Bulk stellar properties
Masses, Radii,
Moment of Inertia

Equation of state
Meson condensate
Hyperons
Quark matter

Color-superconductivity

Proto-neutron stars

Gravitational radiation
Rotational instabilities

Thermal evolution

Gamma-ray bursts

Soft gamma repeaters

Pulsars
Superfast rotation
Backbending
Braking behavior
Spin-up of iMSP

True ground state of strong interaction
Strange quark matter

LMXBs
Crust thickness
Crust mass
Surface gravity
Pycnonuclear reactions

New types of astrophysical objects
Strange stars
Strange dwarfs
Strange MACHOS

arXiv: 0808.1279

Hydrostatic equilibrium for a star

$$\left\{\begin{array}{lll}
(1) & \dfrac{dP}{dr} = -\dfrac{Gm\rho}{r^2} & m = m(r) \\
(2) & \dfrac{dm}{dr} = 4\pi\rho\, r^2 & \\
(3) & \xcancel{\dfrac{dS}{dt} = Q} & \\
(4) & P = P(\rho) &
\end{array}\right.$$

For NSs we can take T=0
and neglect the third equation

For a NS effects of GR are also important.

$$r_g = \frac{2GM}{c^2} \approx 2.95 \; \frac{M}{M_{SUN}} \; \text{km}$$

$M/R \sim 0.15\,(M/M_\odot)(R/10\text{ km})^{-1}$
$J/M \sim 0.25\,(1\text{ ms}/P)\,(M/M_\odot)(R/10\text{km})^{2}$

Lane-Emden equation. Polytrops.

$$P = K\rho^{\gamma}, \quad K, \gamma = \text{const}, \quad \gamma = 1 + \frac{1}{n}$$

$$\frac{dP}{dr} = -\frac{Gm\rho}{r^2} = g\rho, \quad g = -\frac{Gm}{r^2} = -\frac{d\varphi}{dr}$$

$$\frac{dP}{dr} = -\rho\,\frac{d\varphi}{dr}, \quad \Delta\varphi = 4\pi \mathrm{G}\rho$$

$$\rho = \rho_c\,\Theta^n, \quad \Theta = 1 \ \text{при} \ r = 0$$

$$P = K\rho_c^{1+1/n}\,\Theta^{1+n}, \quad \frac{dP}{dr} = (n+1)K\rho_c^{1+1/n}\,\Theta^n\,\frac{d\Theta}{dr}$$

$$\frac{d\varphi}{dr} = -(n+1)K\rho_c^{1/n}\,\frac{d\Theta}{dr}$$

$$\Delta\Theta = -\frac{4\pi G\rho_c^{1-1/n}}{(n+1)K}\,\Theta^n$$

$$\xi = r/a, \quad a^2 = (n+1)K\rho_c^{1/n-1}/(4\pi G)$$

$$\boxed{\frac{1}{\xi^2}\frac{d}{d\xi}\xi^2\frac{d}{d\xi}\Theta = -\Theta^n}$$

$$\Theta = \Theta(\xi)$$

$$0 \le \xi \le \xi_1$$

$$\Theta(0) = 1, \quad \Theta'(0) = 0$$

$$\Theta(\xi_1) = 0$$

Properties of polytropic stars

Analytic solutions:

$$n=0 \qquad \Theta = 1-\frac{\xi^2}{6} \qquad \xi_1 = \sqrt{6}$$

$$n=1 \qquad \Theta = \frac{\sin\xi}{\xi} \qquad \xi_1 = \pi$$

$$n=5 \qquad \Theta = \frac{1}{\sqrt{1+\xi^2/3}} \qquad \xi_1 = \infty$$

$$M = 4\pi\int_0^R dr\, r^2\rho = 4\pi\rho_c a^3\xi_1^2\,|\Theta'(\xi_1)|$$

$$\frac{\rho_c}{\bar{\rho}} = \frac{4\pi R^3\rho_c}{3M} = \frac{\xi_1}{3|\Theta'(\xi_1)|}$$

↓ γ=5/3 ↓ γ=4/3

n	**0**	**1**	**1.5**	**2**	**3**
ξ_1	**2.449**	**3.142**	**3.654**	**4.353**	**6.897**
$\lvert\Theta'_1\rvert$	**0.7789**	**0.3183**	**0.2033**	**0.1272**	**0.04243**
$\rho_c/\bar{\rho}$	**1**	**3.290**	**5.991**	**11.41**	**54.04**

$$M \sim \rho_c^{(3-n)/(2n)}$$

$$R \sim \rho_c^{(1-n)/(2n)}$$

$$M \sim R^{(3-n)/(1-n)}$$

$$n=0 \qquad M \sim R^3$$

$$n=1 \qquad M \sim \rho_c \qquad R = \mathrm{const}$$

$$n=1.5 \qquad M \sim \sqrt{\rho_c} \sim R^{-3}$$

$$n=3 \qquad M = \mathrm{const} \qquad R \sim \rho_c^{-1/3}$$

Useful equations

White dwarfs

1. Non-relativistic electrons

   γ=5/3, K=$(3^{2/3}\ \pi^{4/3}/5)\ (\hbar^2/m_e m_u^{5/3}\mu_e^{5/3})$;

   μ_e-mean molecular weight per one electron

   K=$1.0036\ 10^{13}\ \mu_e^{-5/3}$ (CGS)

2. Relativistic electrons

   γ=4/3, K=$(3^{1/3}\ \pi^{2/3}/4)\ (\hbar c/m_u^{4/3}\mu_e^{4/3})$;

   K=$1.2435\ 10^{15}\ \mu_e^{-4/3}$ (CGS)

Neutron stars

1. Non-relativistic neutrons

   γ=5/3, K=$(3^{2/3}\ \pi^{4/3}/5)\ (\hbar^2/m_n^{8/3})$;

   K=$5.3802\ 10^9$ (CGS)

2. Relativistic neutrons

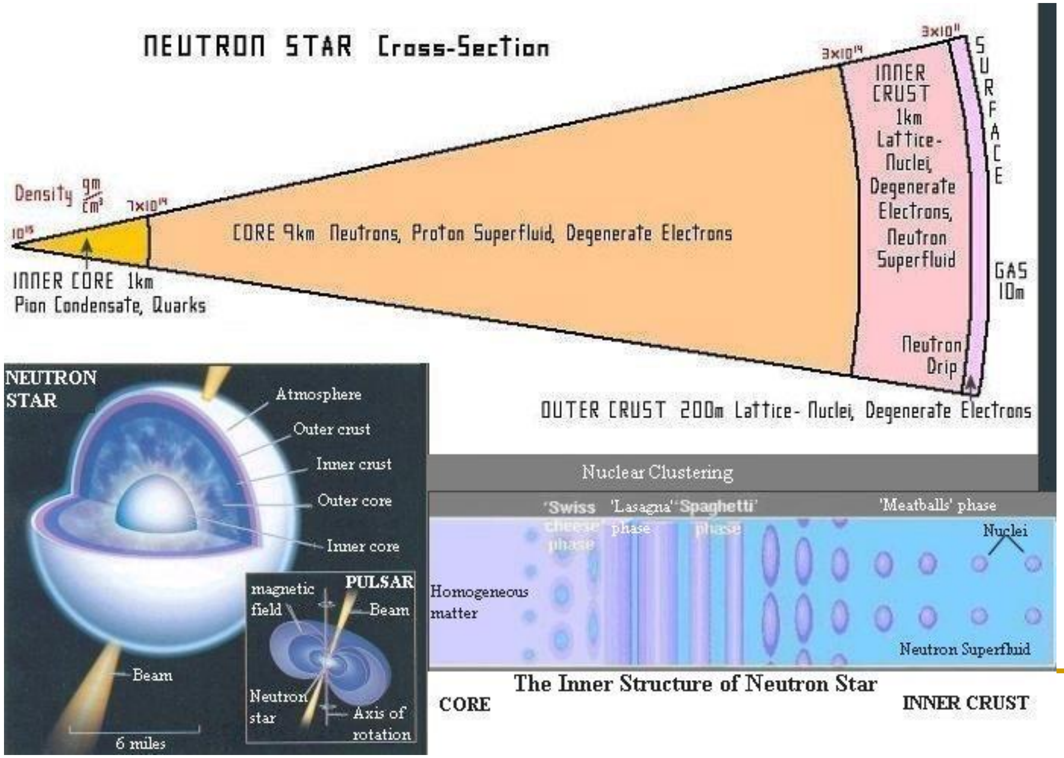
   γ=4/3, K=$(3^{1/3}\ \pi^{2/3}/4)\ (\hbar c/m_n^{4/3})$;

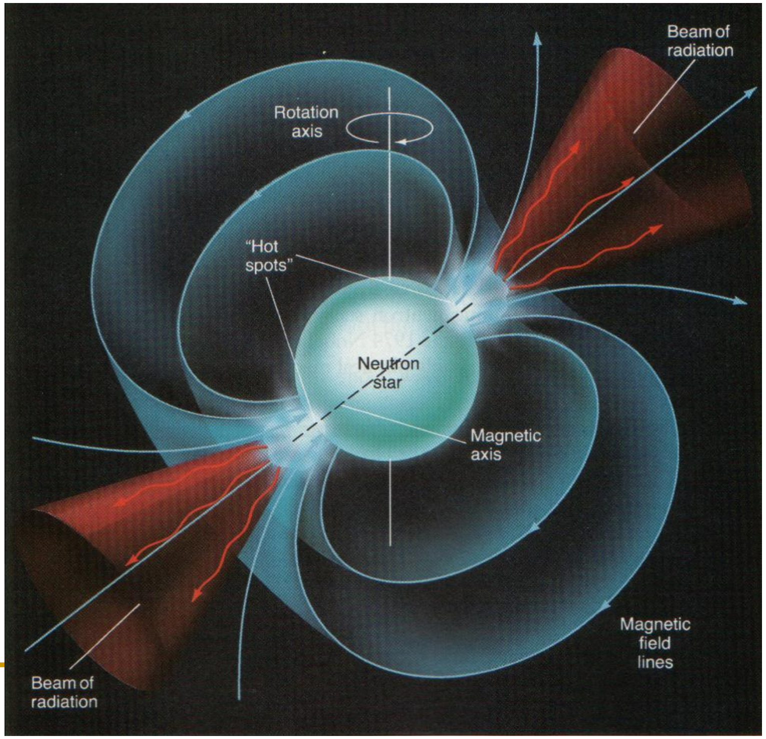
   K=$1.2293\ 10^{15}$ (CGS)

[Shapiro, Teukolsky]

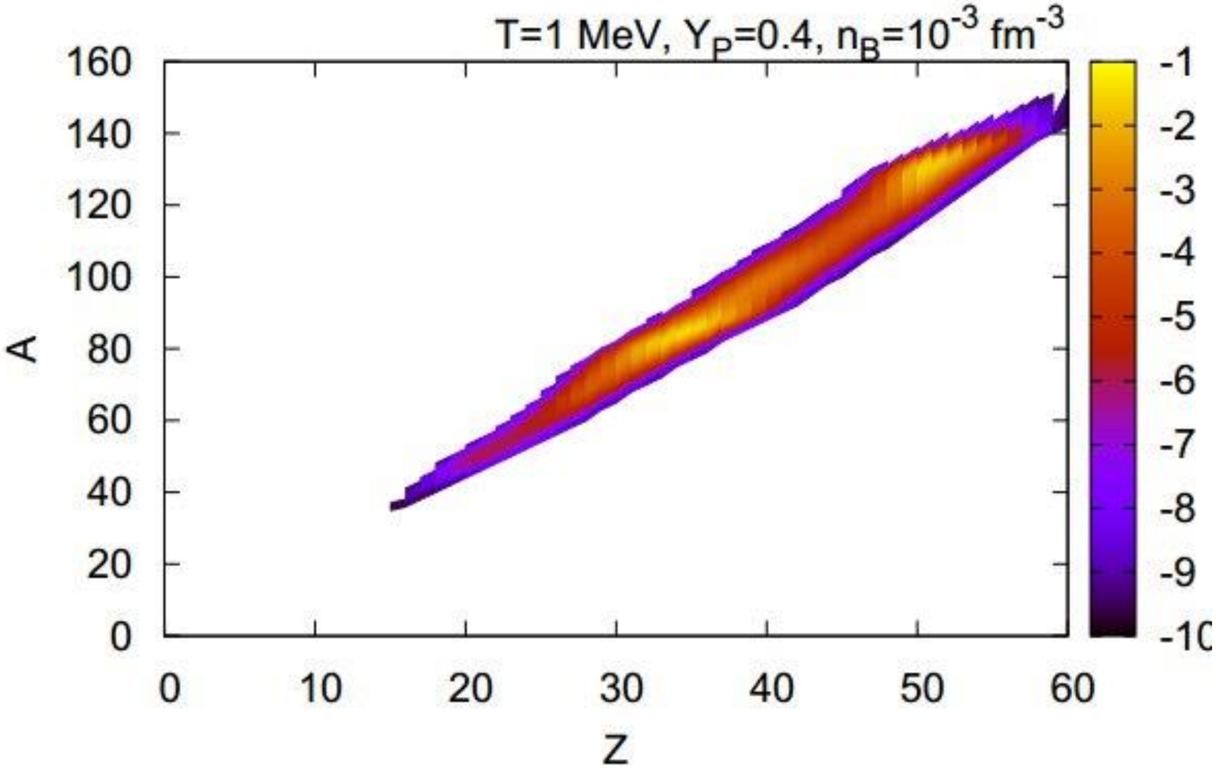
Neutron stars

Superdense matter and superstrong magnetic fields





Proto-neutron stars

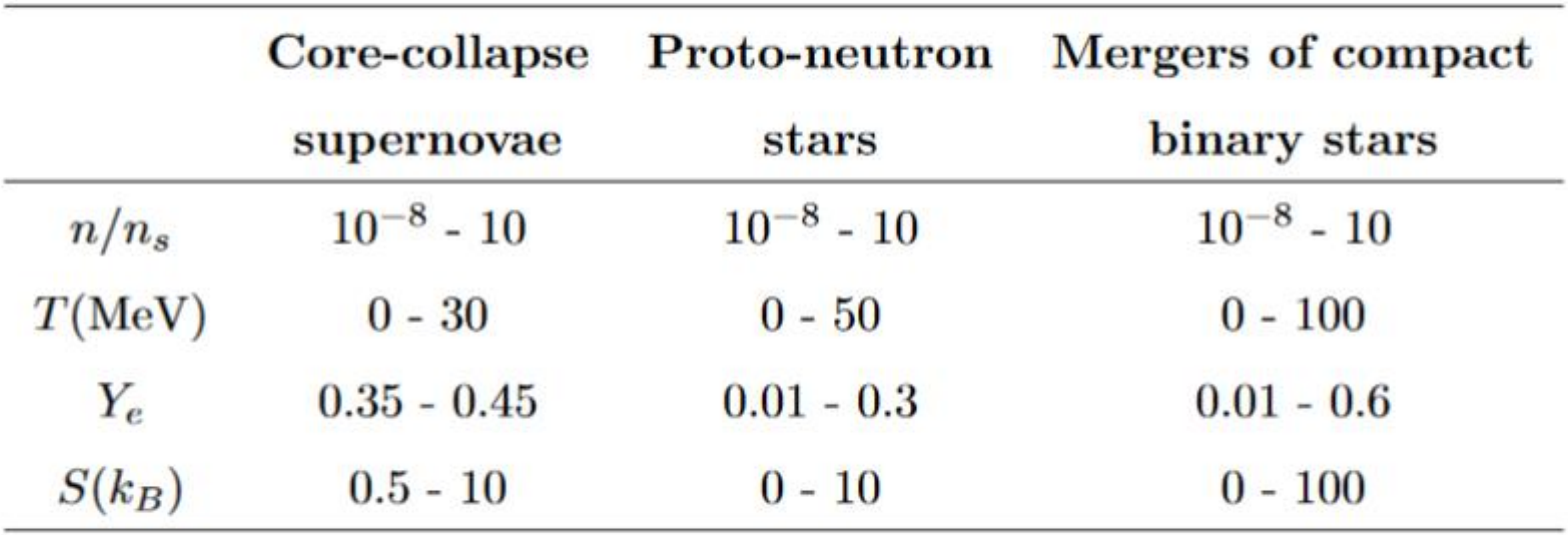


Mass fraction of nuclei in the nuclear chart for matter at T = 1 MeV, $n_B = 10^{-3}$ fm^{-3}, and $Y_P = 0.4$. Different colors indicate mass fraction in Log_{10} scale.

1202.5791

NS EoS are also important for SN explosion calculation, see 1207.2184

EoS for core-collapse, proto-NS and NS-NS mergers



	Core-collapse supernovae	Proto-neutron stars	Mergers of compact binary stars
n/n_s	10^{-8} - 10	10^{-8} - 10	10^{-8} - 10
T(MeV)	0 - 30	0 - 50	0 - 100
Y_e	0.35 - 0.45	0.01 - 0.3	0.01 - 0.6
$S(k_B)$	0.5 - 10	0 - 10	0 - 100

Wide ranges of parameters

1512.07820

Astrophysical point of view

Astrophysical appearence of NSs
is mainly determined by:

- **Spin**
- **Magnetic field**
- **Temperature**
- **Velocity**
- **Environment**

The first four are related to the NS structure!

Equator and radius

$$\mathbf{ds^2=c^2dt^2e^{2\Phi}-e^{2\lambda}dr^2-r^2[d\theta^2+\sin^2\theta d\varphi^2]}$$

In flat space $\Phi(r)$ and $\lambda(r)$ are equal to zero.

- $t=\mathrm{const},\ r=\mathrm{const},\ \theta=\pi/2,\ 0<\Phi<2\pi \Rightarrow l=2\pi r$
- $t=\mathrm{const},\ \theta=\mathrm{const},\ \varphi=\mathrm{const},\ 0<r<r_0 \Rightarrow dl=e^{\lambda}dr \Rightarrow l=\int_0^{r_0} e^{\lambda}dr\neq r_0$

Gravitational redshift

$$d\tau = dt\, \overset{<1}{e^{\Phi}}, \qquad \nu_r = \frac{dN}{d\tau} = e^{-\Phi}\frac{dN}{dt}$$

→ Frequency emitted at r

$$r \to \infty \qquad \Phi \to 0 \qquad \nu_\infty = \frac{dN}{dt}$$

→ Frequency detected by an observer at infinity

$$\nu_\infty = \nu_r\, e^{\Phi} \qquad \Rightarrow \qquad \Phi(r)$$

→ This function determines gravitational redshift

$$e^{2\lambda} \equiv \frac{1}{1 - \dfrac{2Gm}{c^2 r}}$$

It is useful to use m(r) – gravitational mass inside r – instead of λ(r)

Outside of the star

$$r > R \Rightarrow m(r) = M = \text{const}$$

$$\mathrm{e}^{2\Phi} = 1 - \frac{2GM}{c^2 r} = 1 - \frac{r_g}{r}, \qquad r_g = \frac{2GM}{c^2}$$

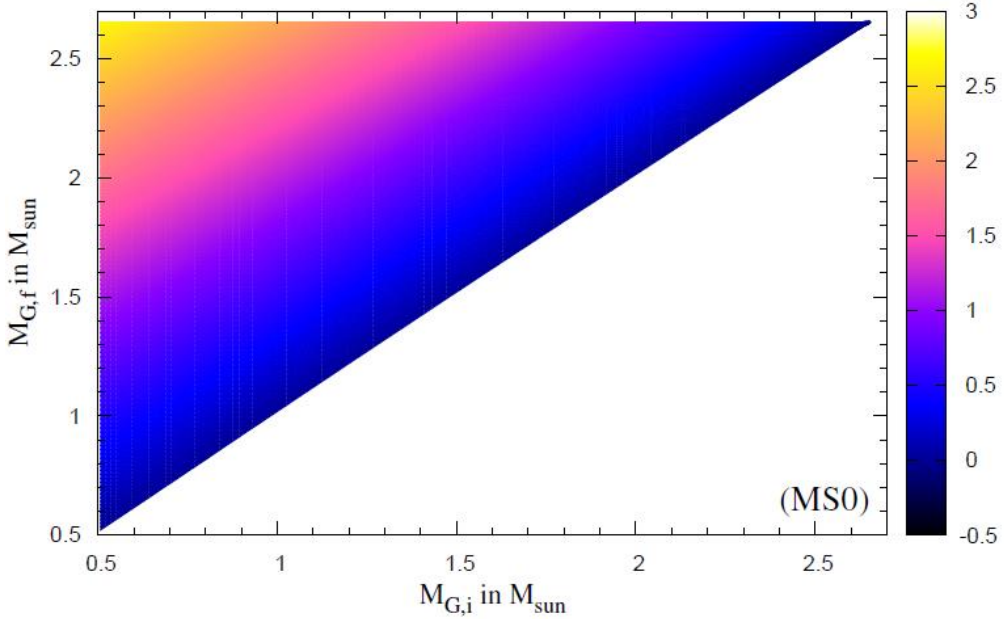
$$ds^2 = \left(1 - \frac{r_g}{r}\right) c^2\ dt^2 - \left(1 - \frac{r_g}{r}\right)^{-1} dr^2 - r^2\ d\Omega^2$$

$$\nu_\infty = \nu_r \sqrt{1 - \frac{r_g}{r}}$$ → redshift

Bounding energy → $\Delta M = M_b - M \sim 0.2\ M_{\text{sun}}$

Apparent radius → $R_\infty = R / \sqrt{1 - r_g / R}$

Bounding energy



If you drop a kilo on a NS, then you increase its mass for < kilo

M_{acc} is shown with color

$M_{G,i}$ ($M_{\odot}$)	ΔM_G ($M_{\odot}$)	$M_{B,i}$ ($M_{\odot}$)		M_{acc} (ΔM_B) ($M_{\odot}$)	
		APR	MS0	APR	MS0
1.4	0.57	1.554	1.525	0.768	0.712
1.5	0.47	1.681	1.647	0.641	0.591
1.6	0.37	1.811	1.767	0.511	0.470
1.7	0.27	1.943	1.892	0.379	0.345
1.8	0.17	2.080	2.018	0.242	0.219
1.9	0.07	2.221	2.146	0.101	0.091

$M_{acc}=\Delta M_G+\Delta BE/c^2=\Delta M_B$

BE- binding energy

$BE=(M_B-M_G)c^2$

1102.2912

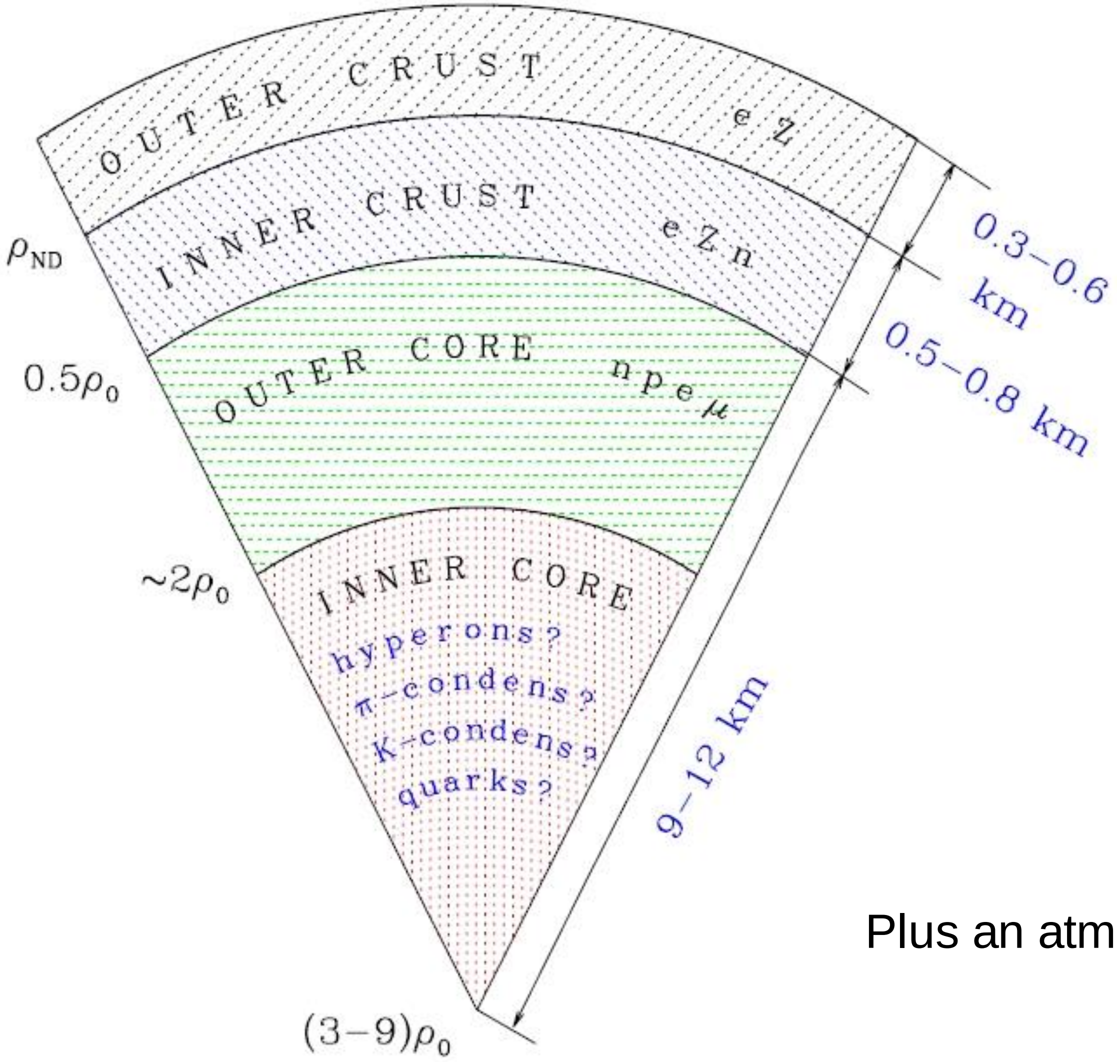
TOV equation

$$\boxed{R_{ik} - \frac{1}{2} g_{ik} R = \frac{8\pi G}{c^4} T_{ik}}$$

$$\left\{ \begin{array}{ll} (1) & \boxed{\frac{dP}{dr} = -\frac{G\rho m}{r^2}\left(1+\frac{P}{\rho c^2}\right)\left(1+\frac{4\pi r^3 P}{mc^2}\right)\left(1-\frac{2Gm}{rc^2}\right)^{-1}} \\ (2) & \frac{dm}{dr} = 4\pi r^2 \rho \\ (3) & \frac{d\Phi}{dr} = -\frac{1}{\rho c^2}\frac{dP}{dr}\left(1+\frac{P}{\rho c^2}\right)^{-1} \\ (4) & P = P(\rho) \end{array} \right.$$

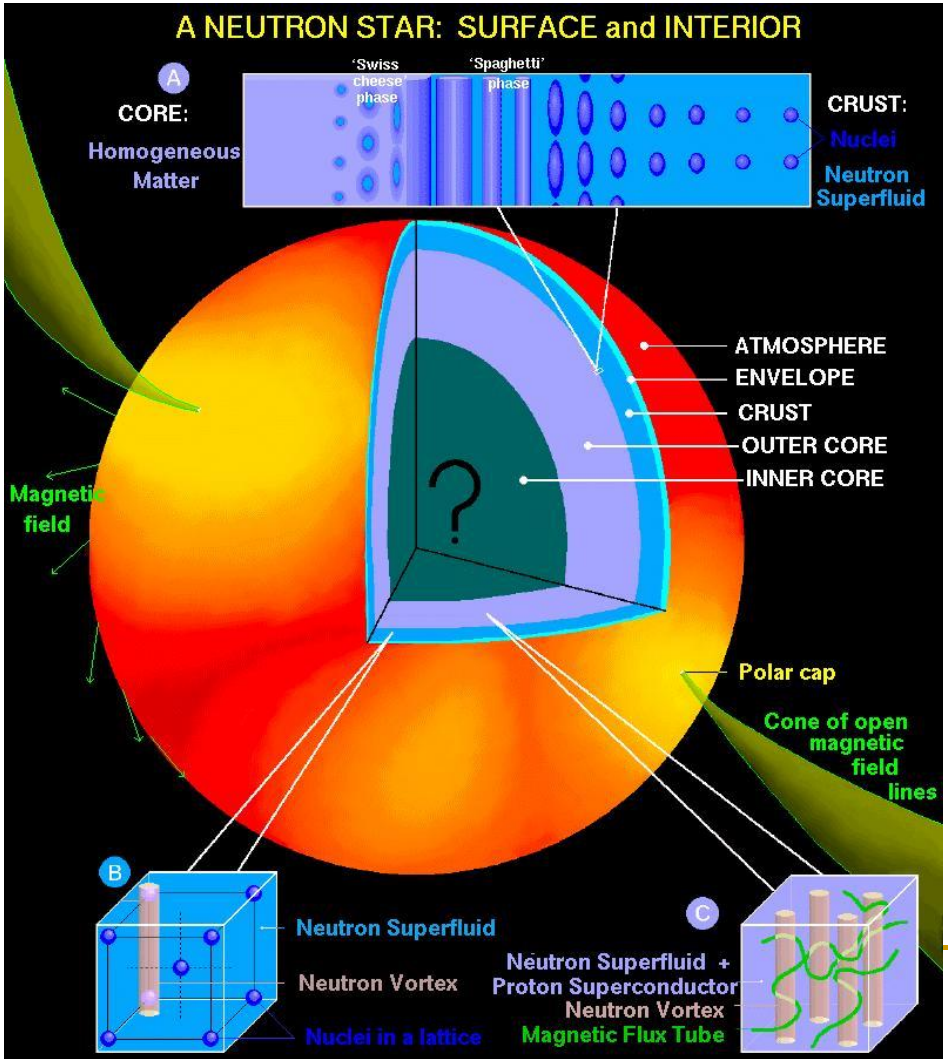
Tolman (1939)
Oppenheimer-Volkoff (1939)

Structure and layers

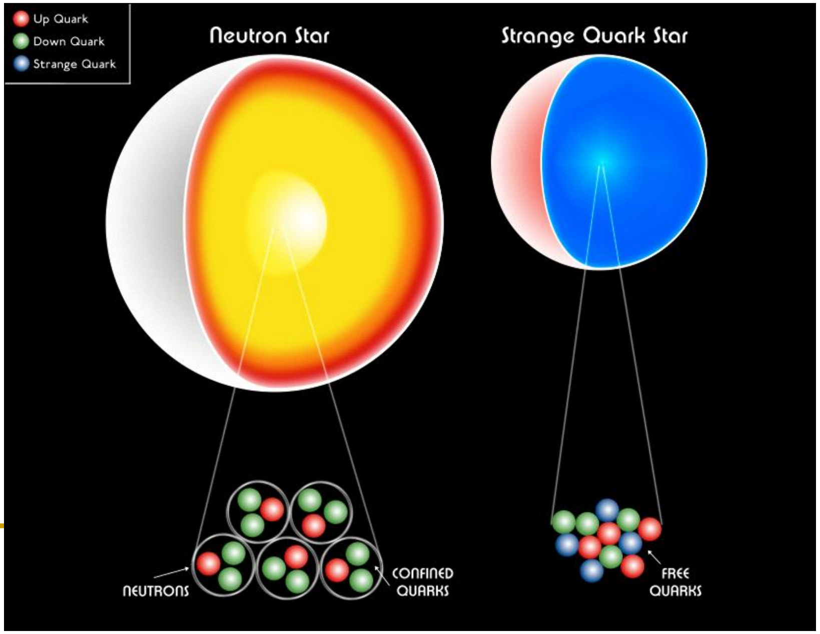


Plus an atmosphere...

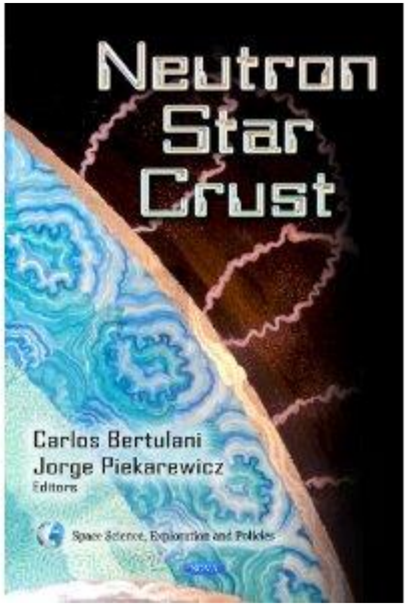
Neutron star interiors

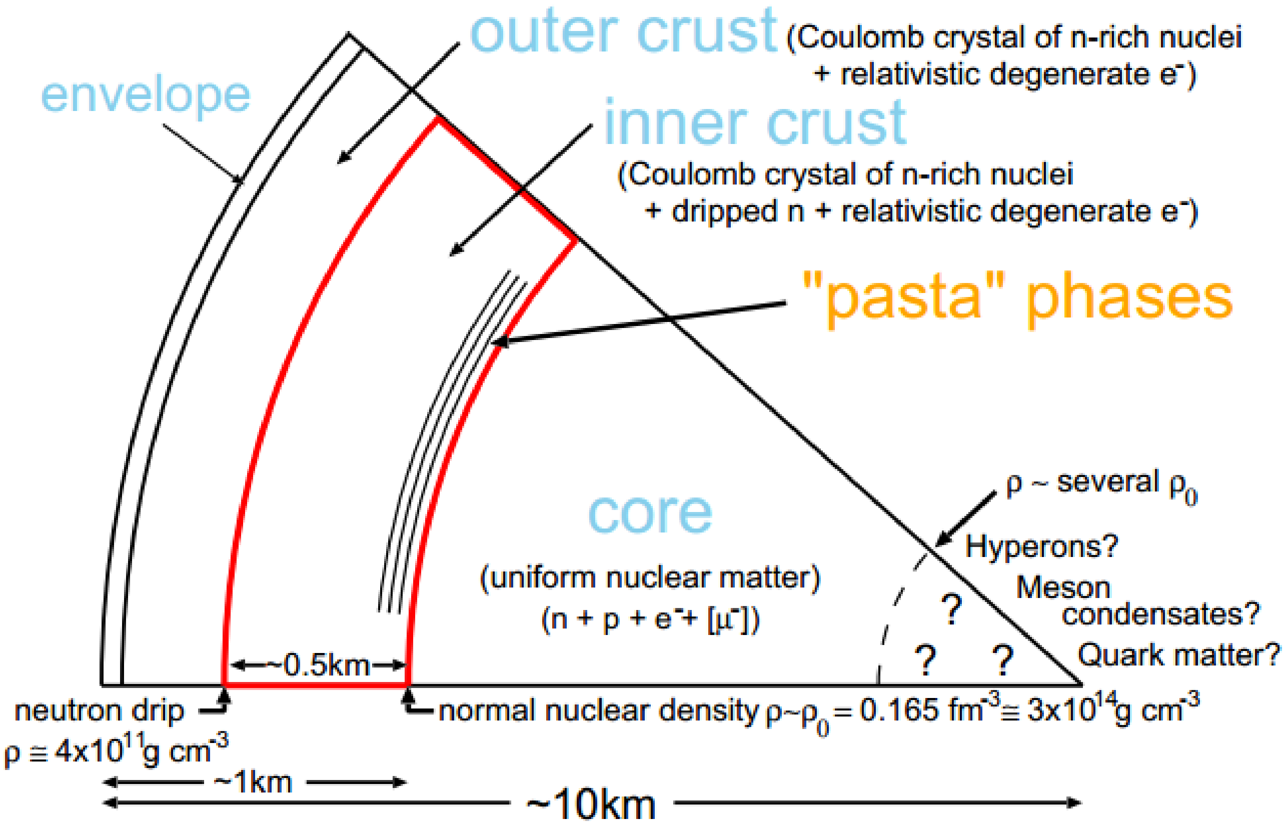


Radius: 10 km
Mass: 1-2 solar
Density: above the nuclear
Strong magnetic fields



Neutron star crust



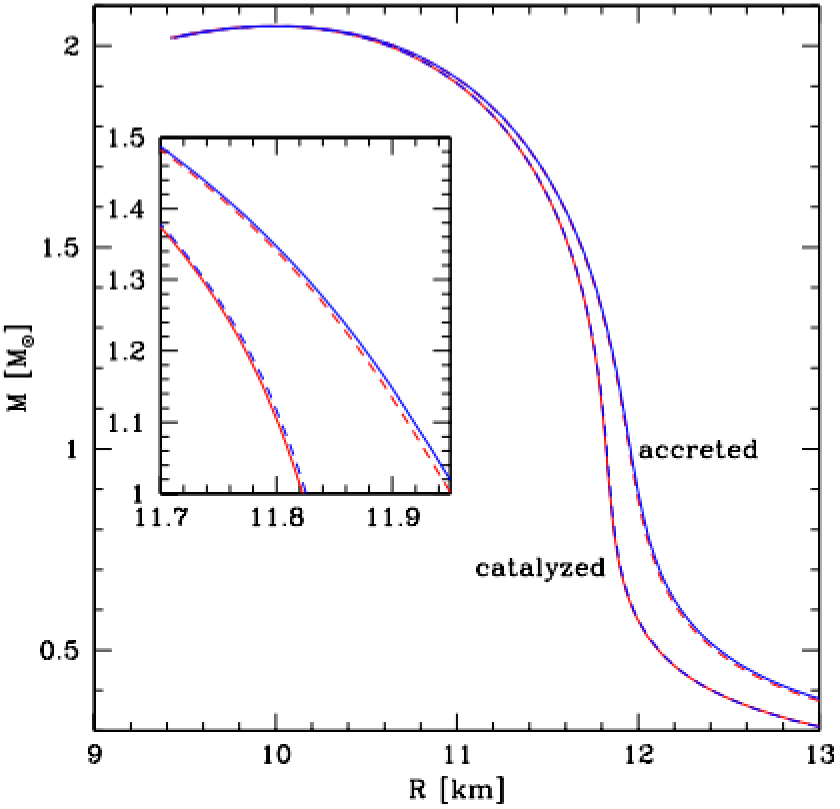


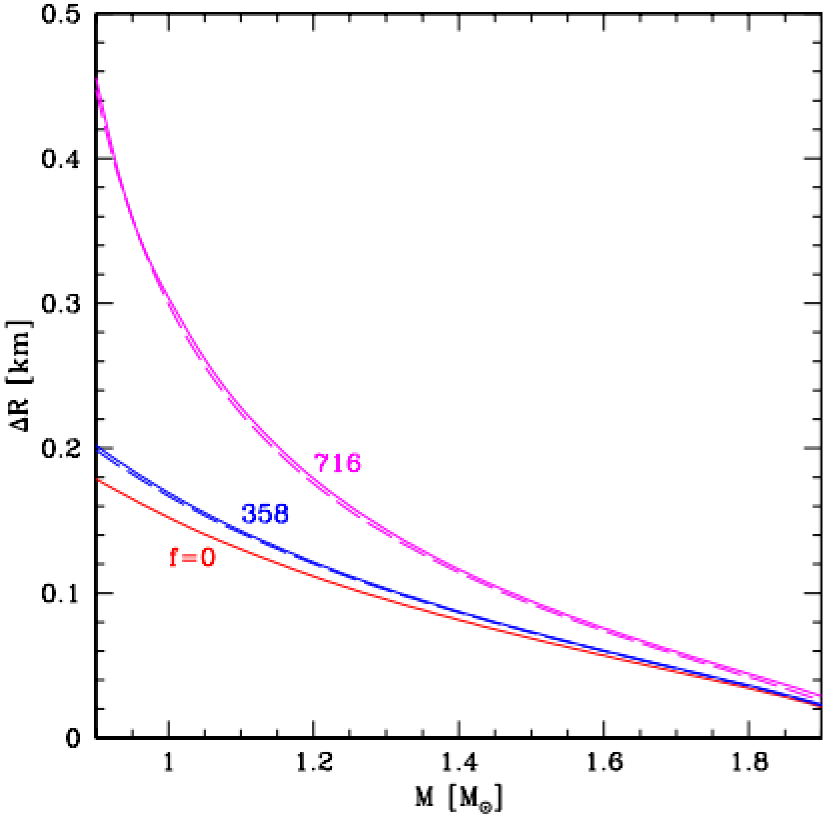
Many contributions to the book are available in the arXiv.

Mechanical properties of crusts are continuosly discussed, see 1208.3258

Accreted crust

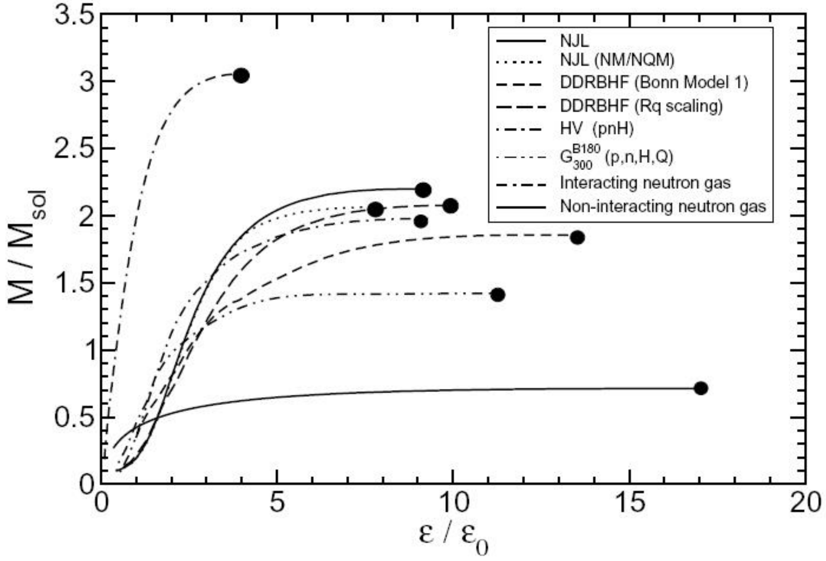
It is interesting that the crust formed by accreted matter differs from the crust formed from catalyzed matter. The former is thicker.





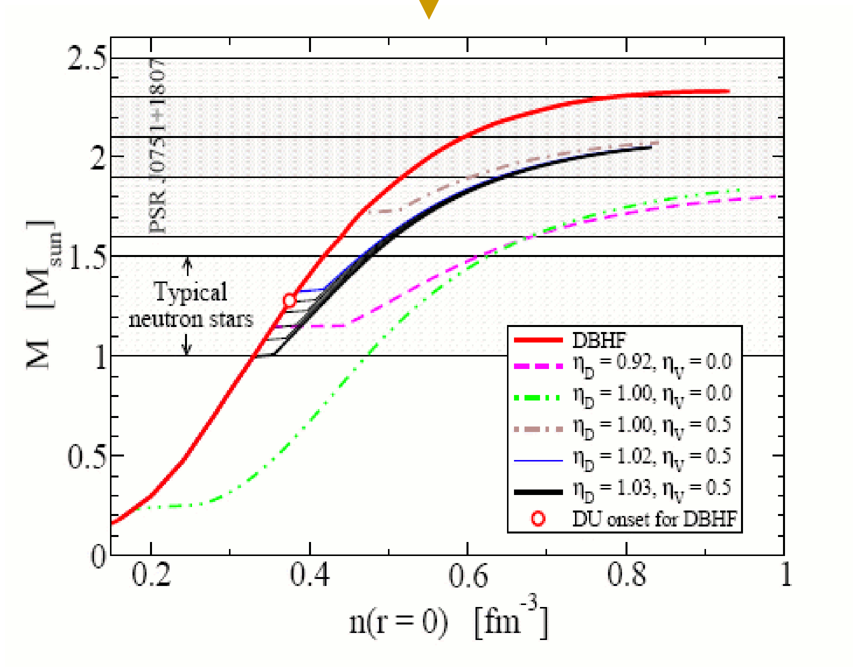
1104.0385

Configurations



NS mass vs. central density (Weber et al. arXiv: 0705.2708)

Stable configurations for neutron stars and hybrid stars (astro-ph/0611595).

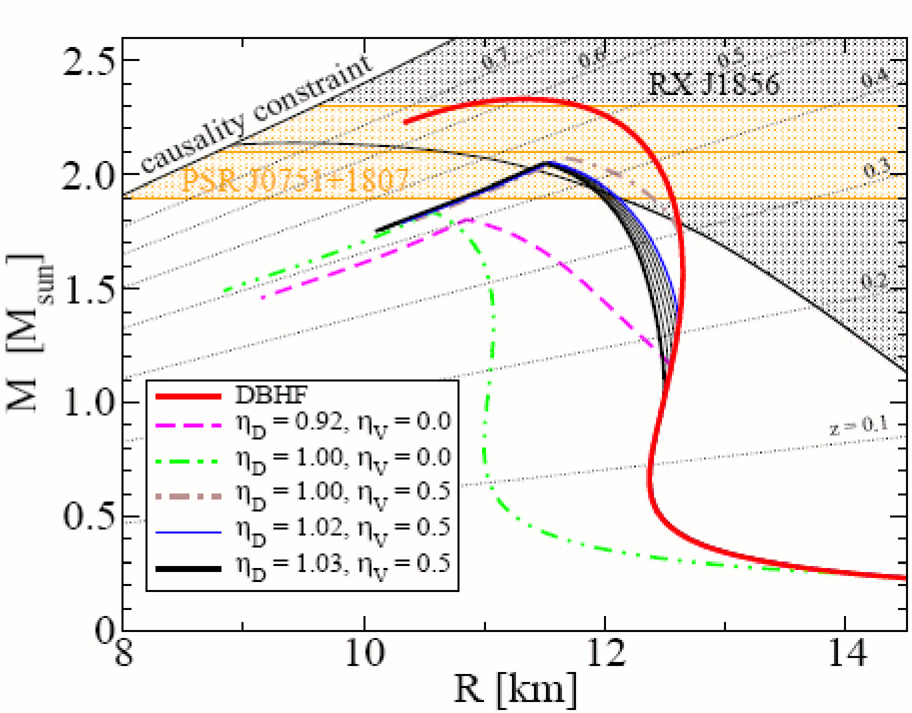


A RNS code is developed
and made available to the public
by Sterligioulas and Friedman
ApJ 444, 306 (1995)
http://www.gravity.phys.uwm.edu/rns/

Mass-radius

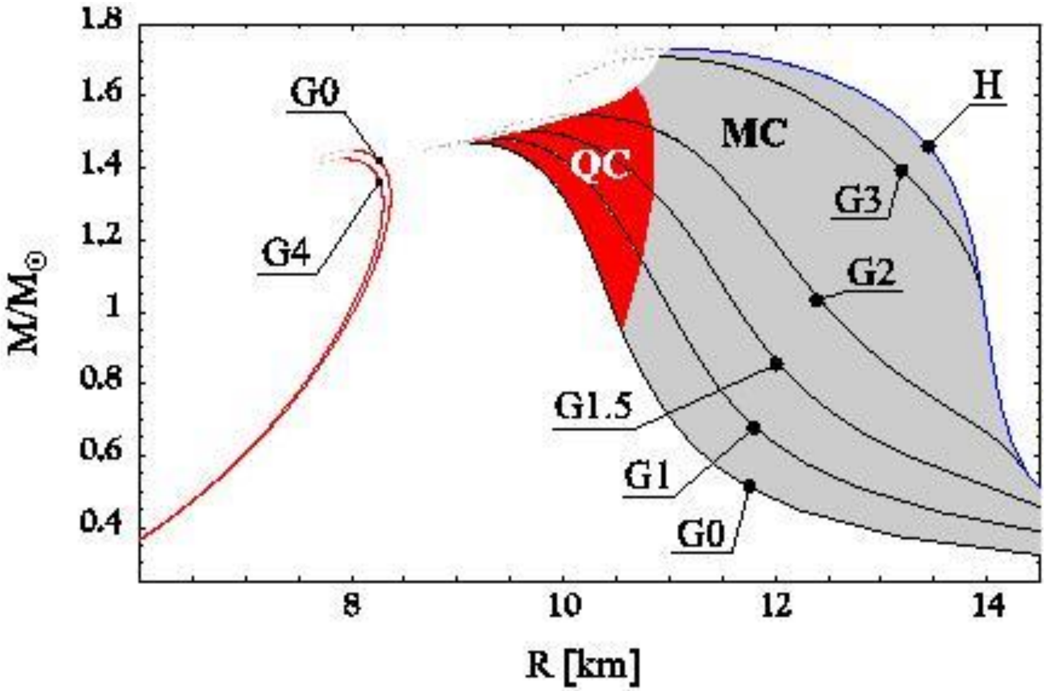
Mass-radius relations for CSs with possible phase transition to deconfined quark matter.

About hyperon stars see a review in 1002.1658.
About strange stars and some other exotic options – 1002.1793



(astro-ph/0611595)

Mass-radius relation

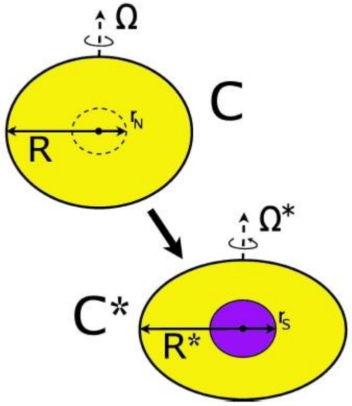


Main features

- Max. mass
- Diff. branches
  (quark and normal)
- Stiff and soft EoS
- Small differences for
  realistic parameters
- Softening of an EoS
  with growing mass

Rotation is neglected here.
Obviously, rotation results in:
- larger max. mass
- larger equatorial radius

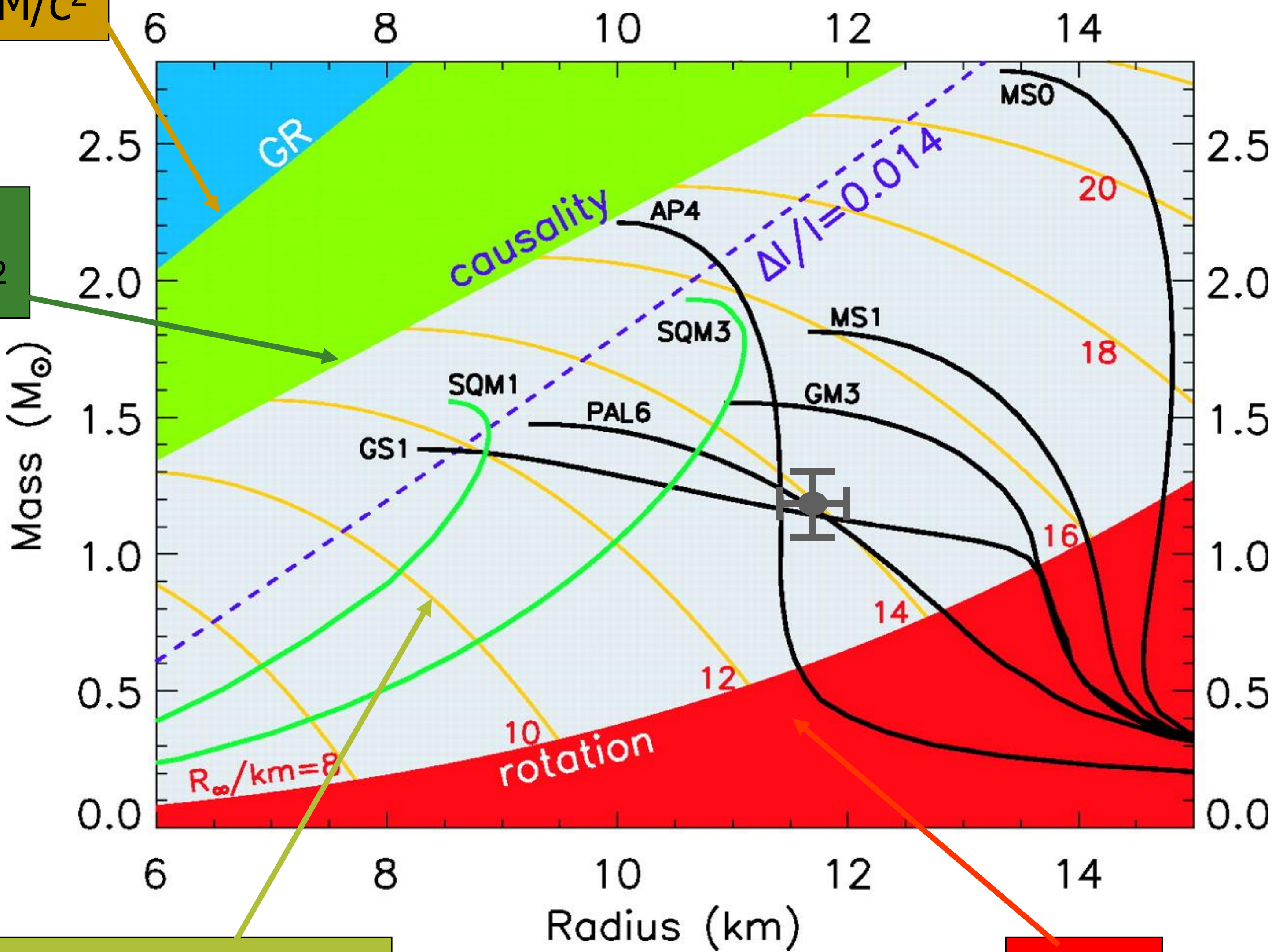
Spin-down can result in phase transition,
as well as spin-up (due to accreted mass),
see 1109.1179



Haensel, Zdunik
astro-ph/0610549

$R=2GM/c^2$

$P=\rho$
$R\sim3GM/c^2$

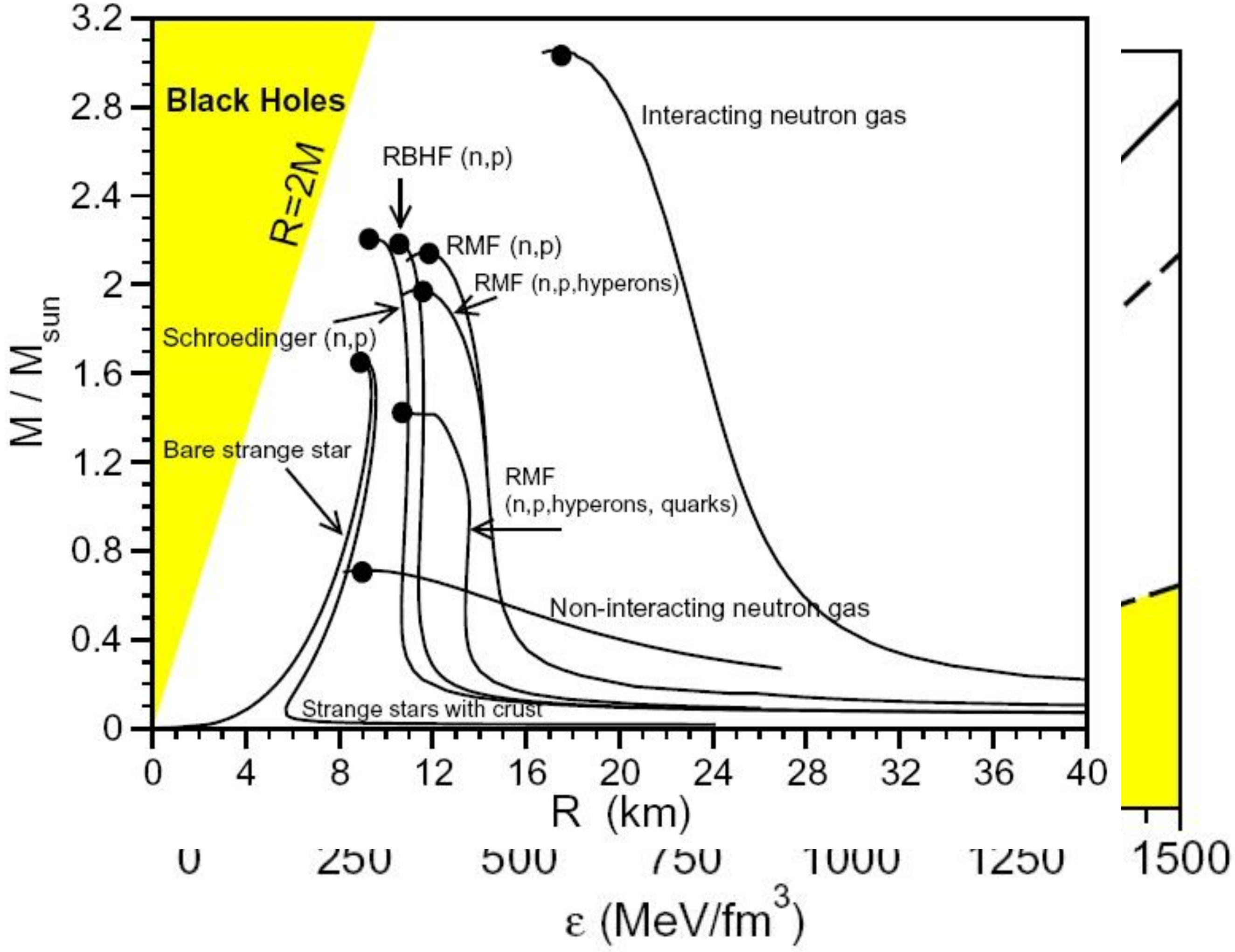


$R_\infty=R(1-2GM/Rc^2)^{-1/2}$

$\omega=\omega_K$

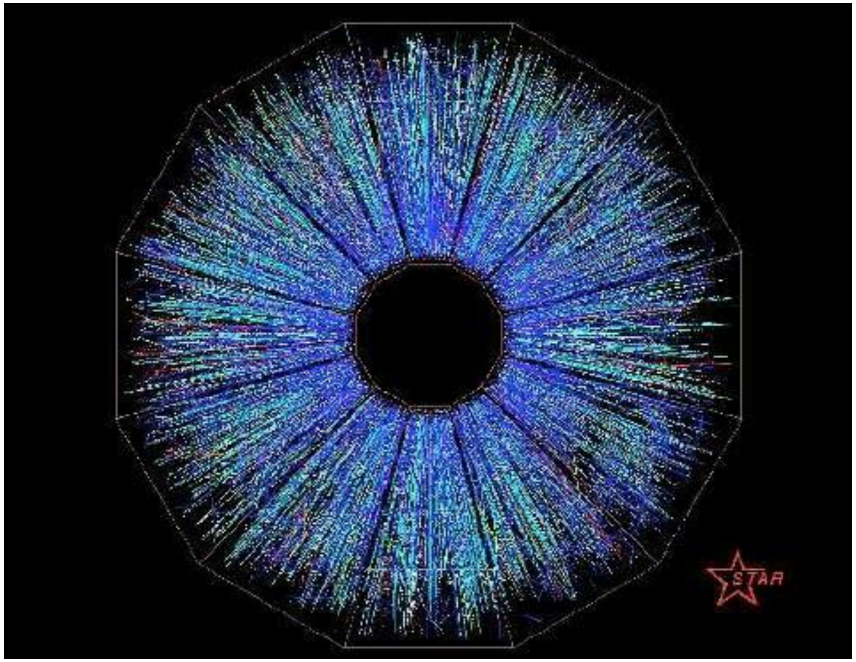
Lattimer & Prakash (2004)

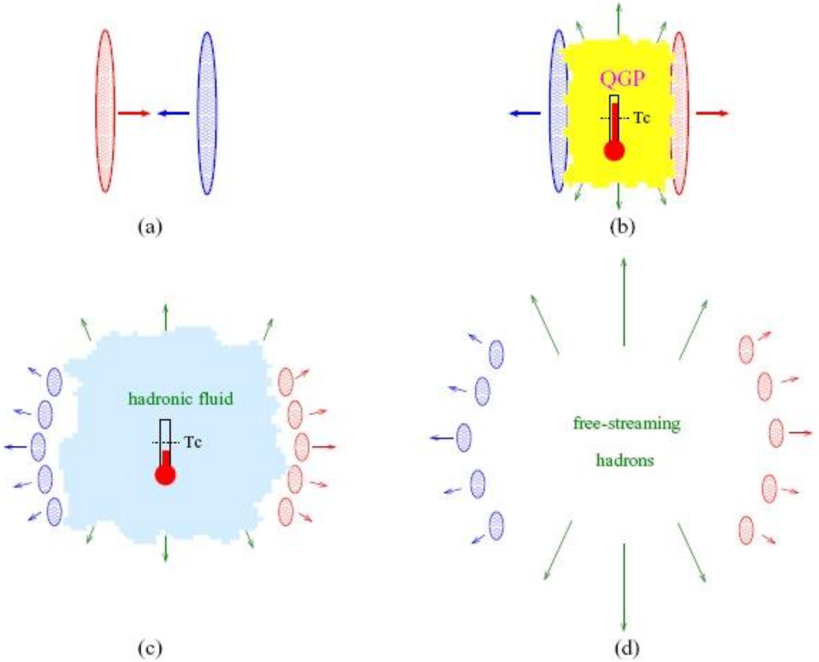
EoS



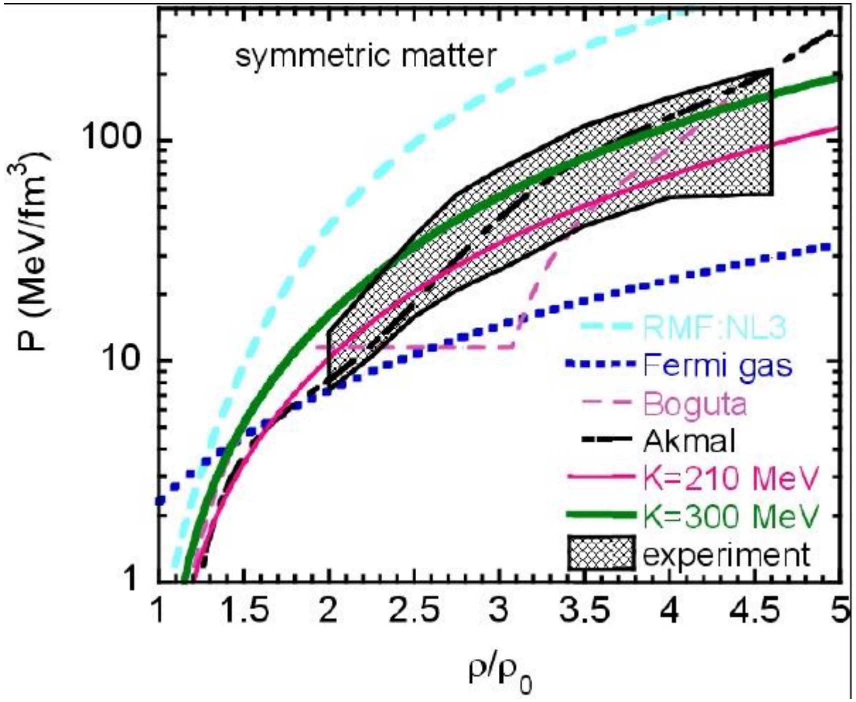
(Weber et al. ArXiv: 0705.2708)

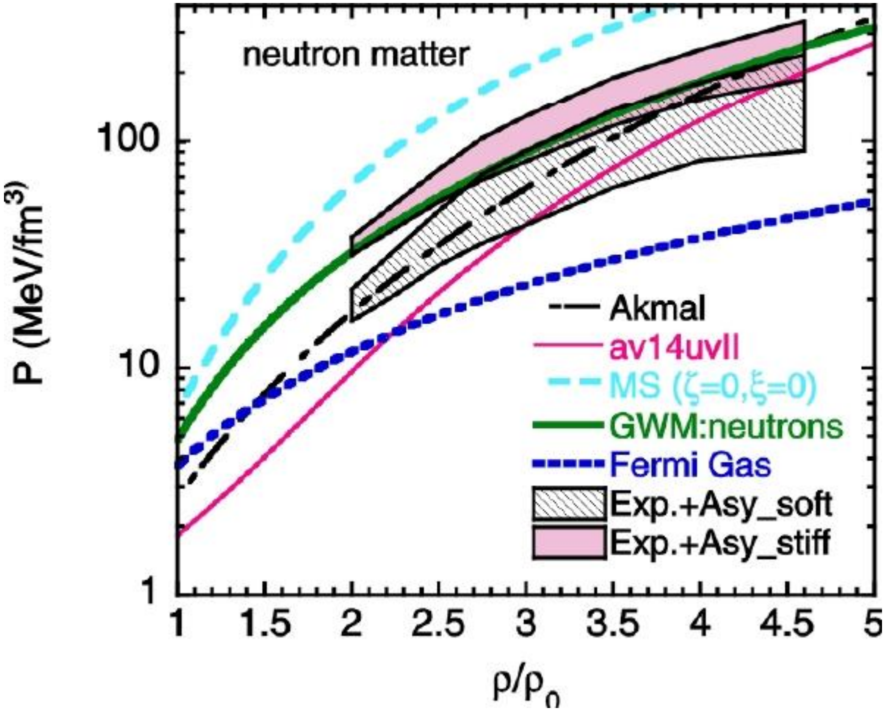
Au-Au collisions





Experimental results and comparison





1 Mev/fm^3 = 1.6 10^{32} Pa

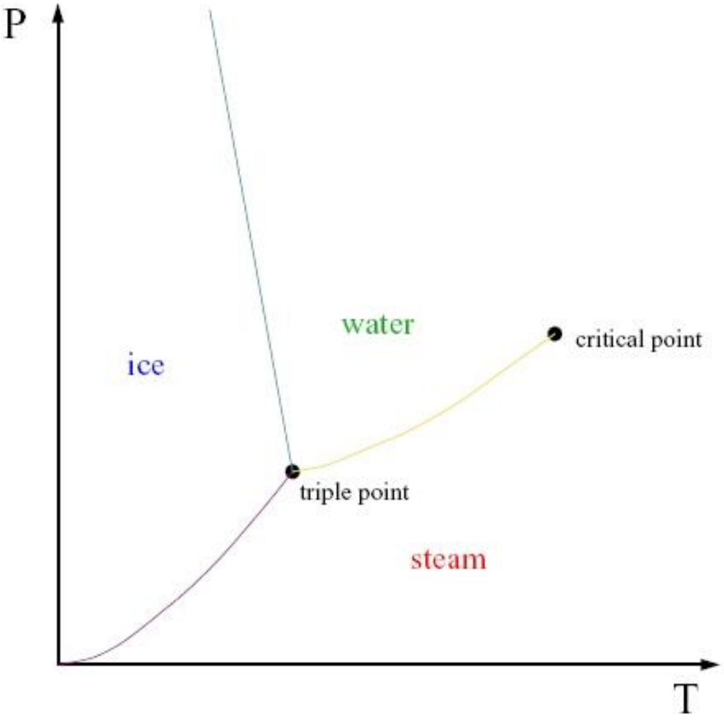
GSI-SIS and AGS data

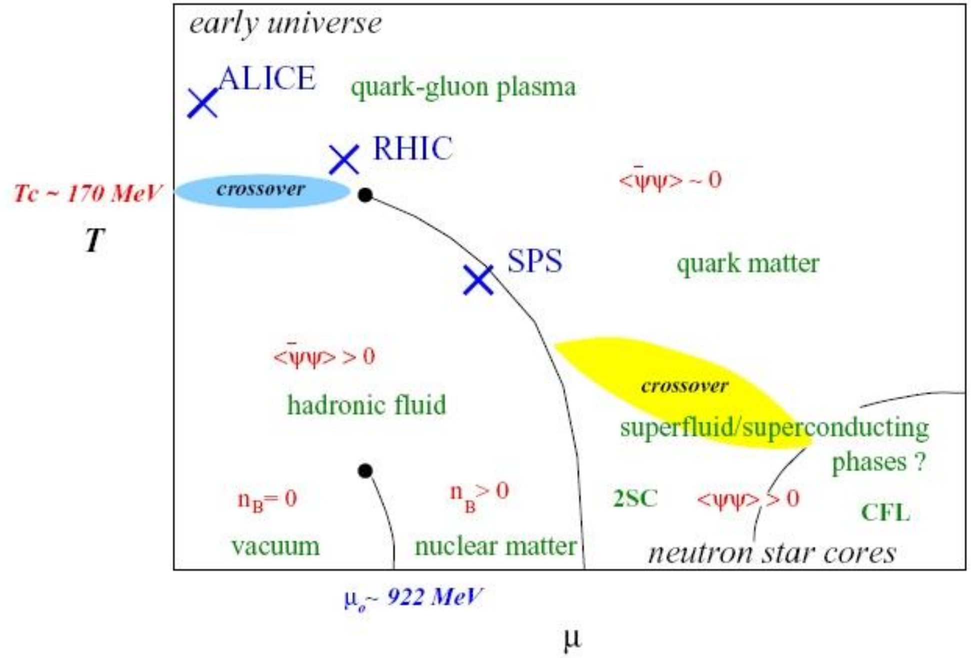
Danielewicz et al. nucl-th/0208016

New heavy-ion data and discussion: 1211.0427

Also laboratory measurements of lead nuclei radius can be important, see 1202.5701

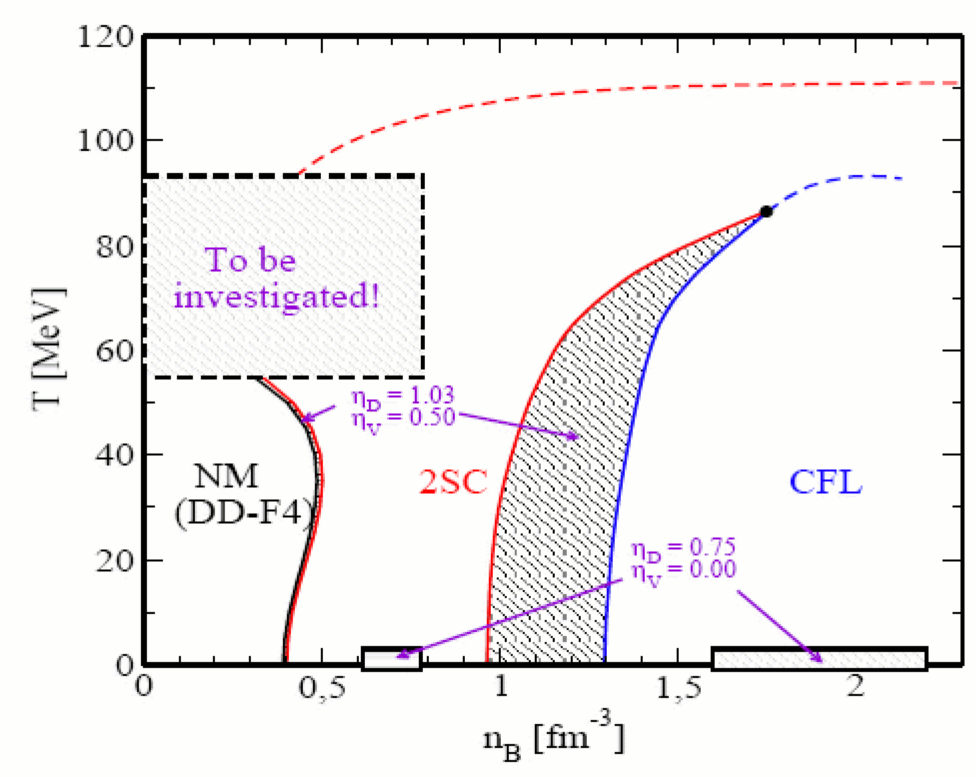
Phase diagram





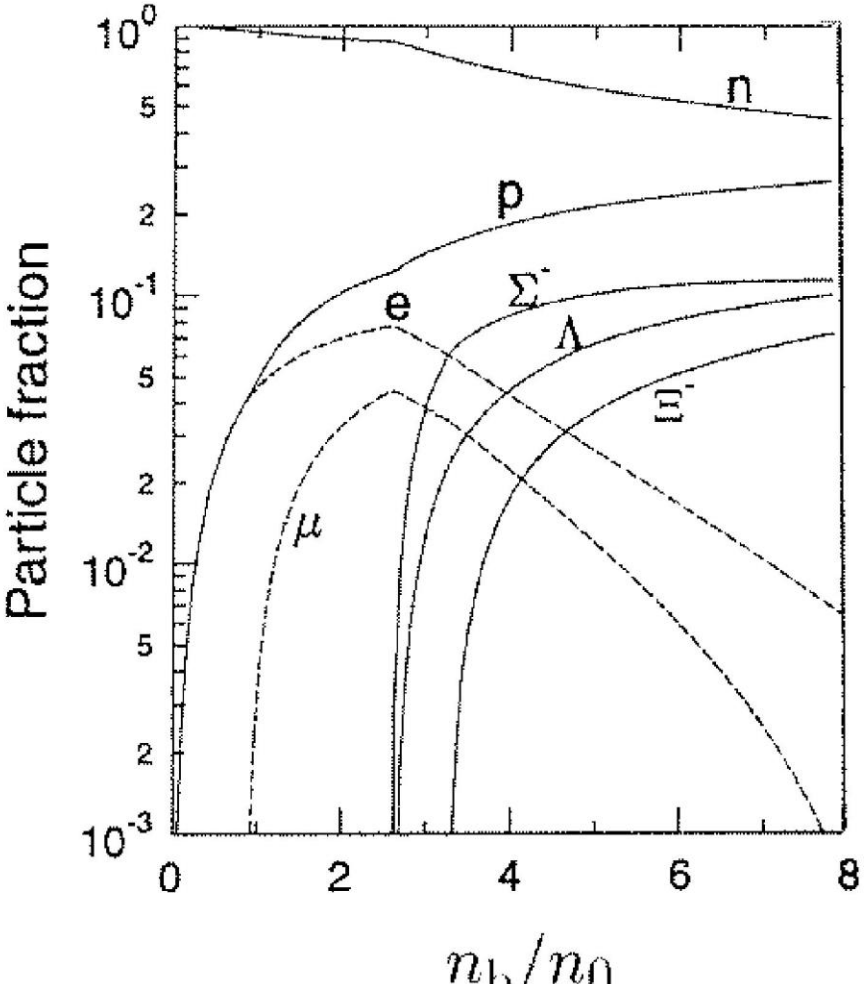
Phase diagram

Phase diagram for isospin symmetry using the most favorable hybrid EoS studied in astro-ph/0611595.

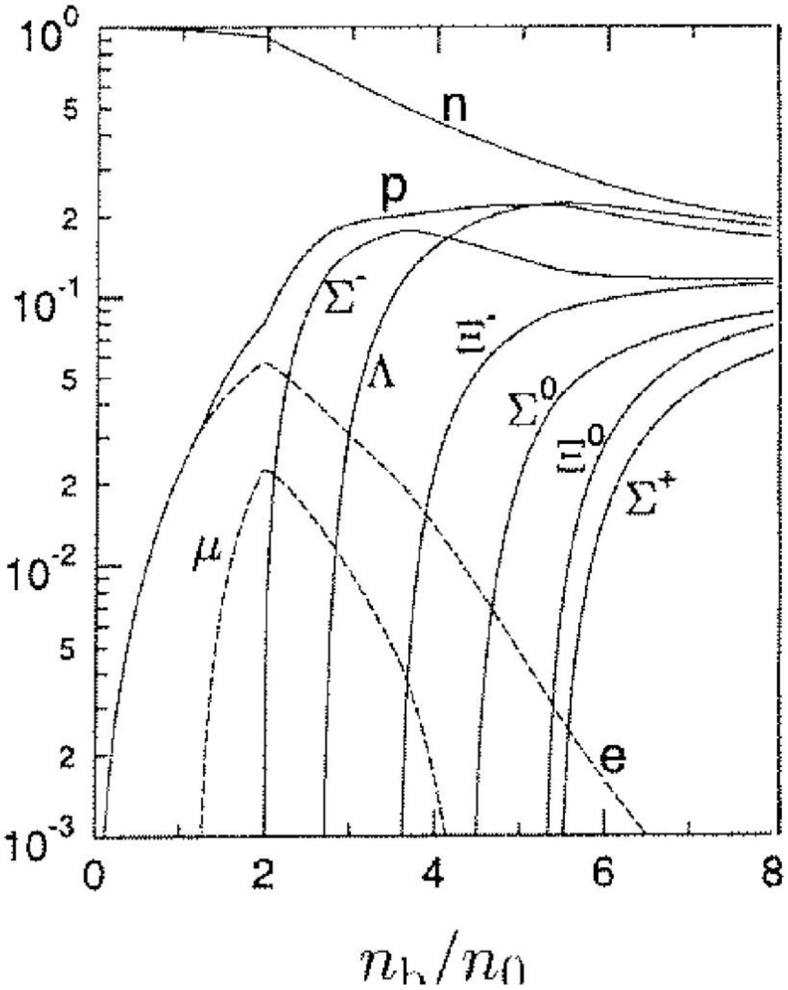


(astro-ph/0611595)

Particle fractions

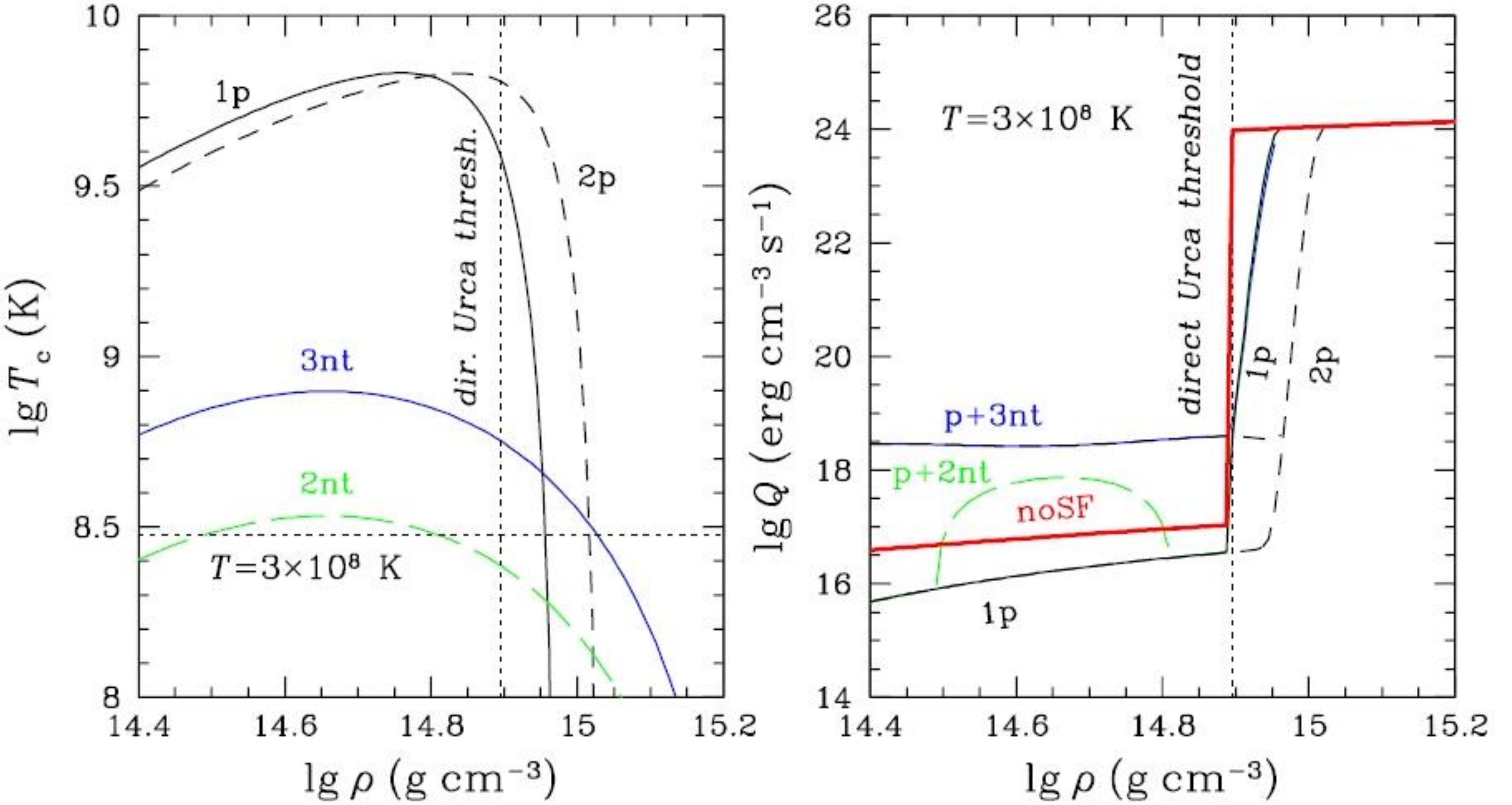


Effective chiral model of Hanauske et al. (2000)



Relativistic mean-field model TM1 of Sugahara & Toki (1971)

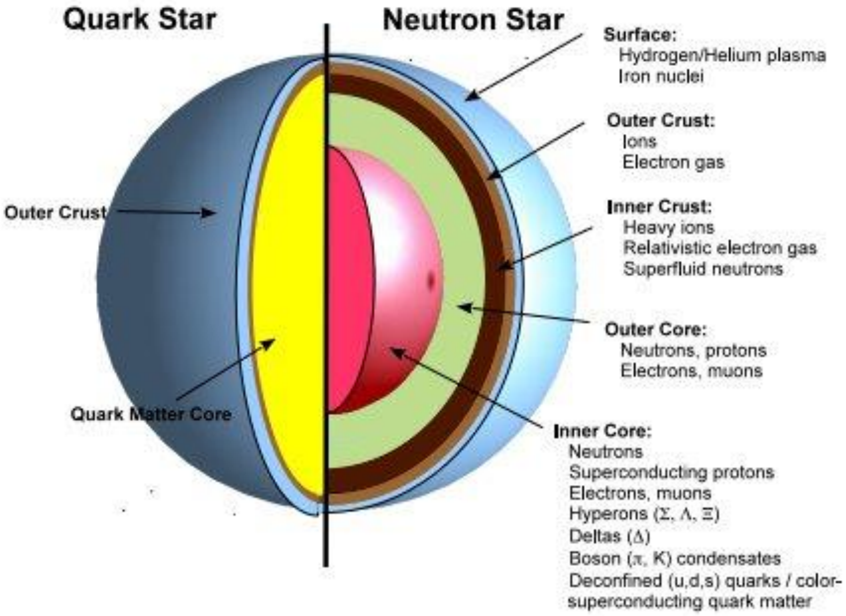
Superfluidity in NSs

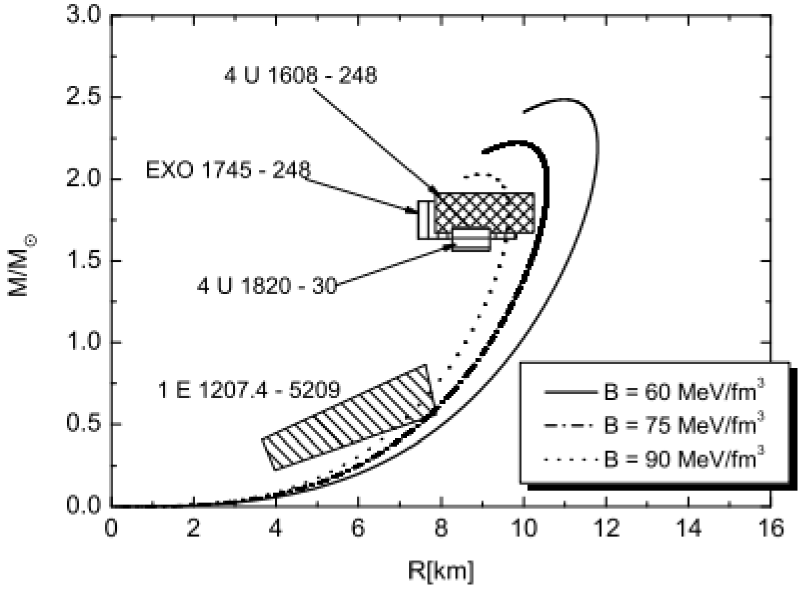


(Yakovlev)

Яковлев и др. УФН 1999

Quark stars

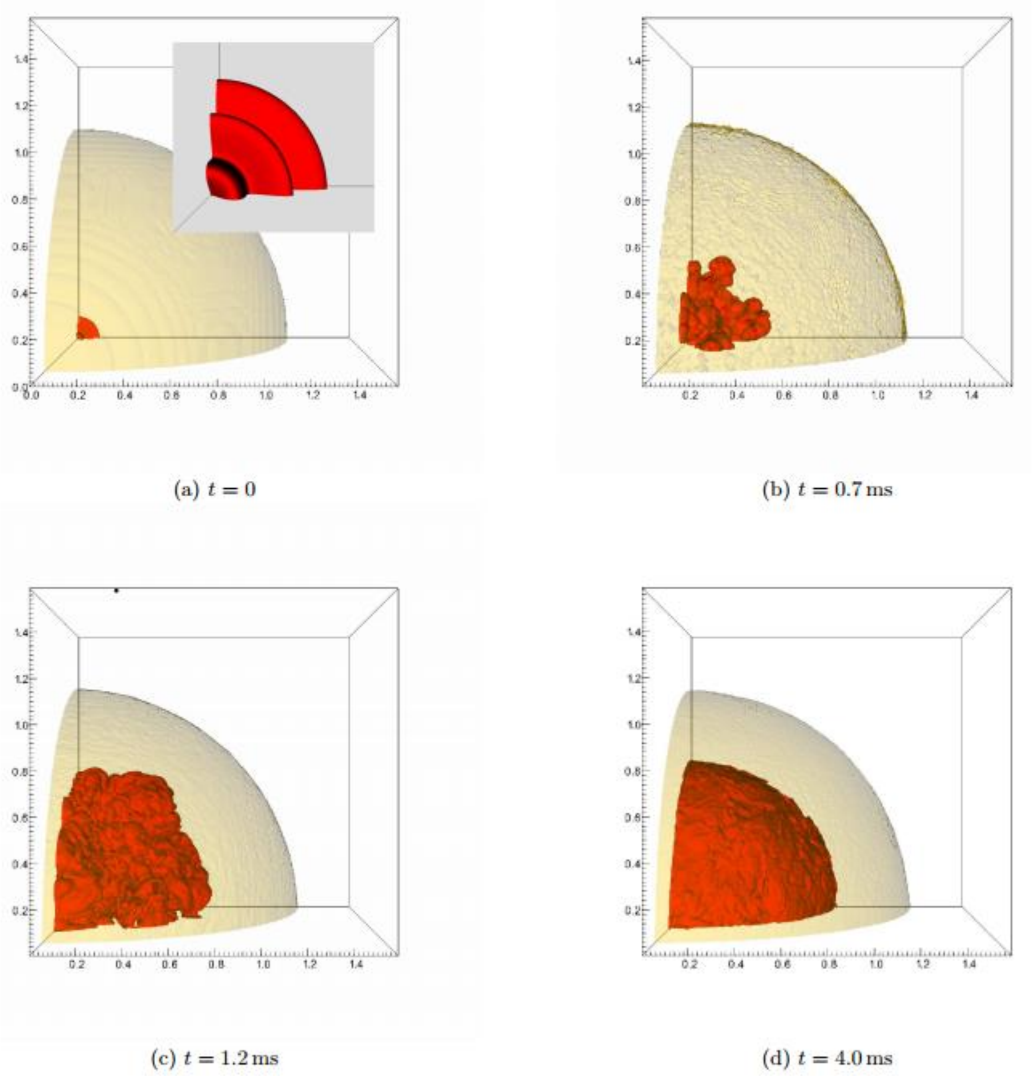




1210.1910

See also 1112.6430

Formation of quark stars



(a) $t = 0$ (b) $t = 0.7\,\mathrm{ms}$

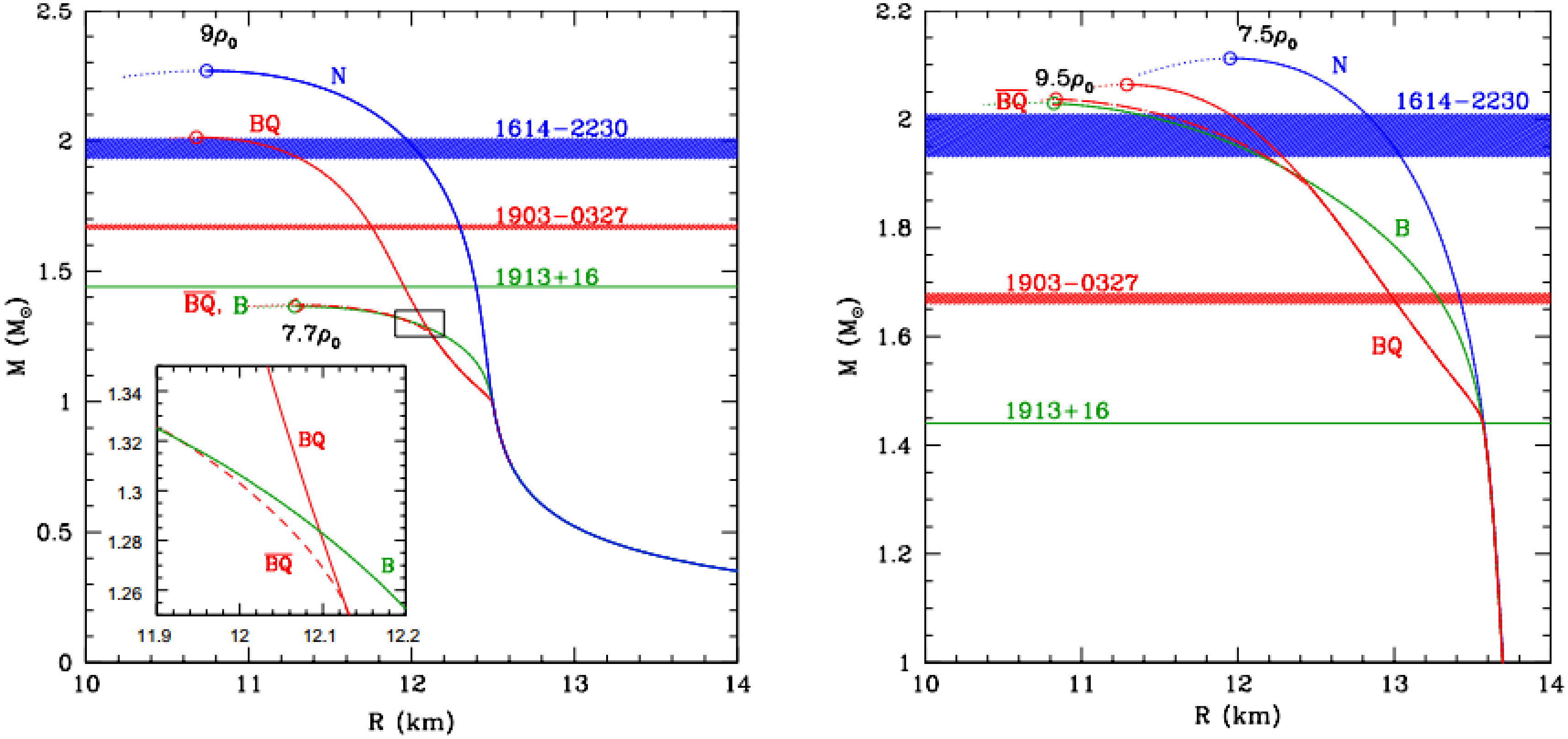
(c) $t = 1.2\,\mathrm{ms}$ (d) $t = 4.0\,\mathrm{ms}$

Turbulent deflagration,
as in SNIa.

Neutrino signal due to
conversion of a NS into
a quark star was calculated
in 1304.6884

1109.0539

Hybrid stars

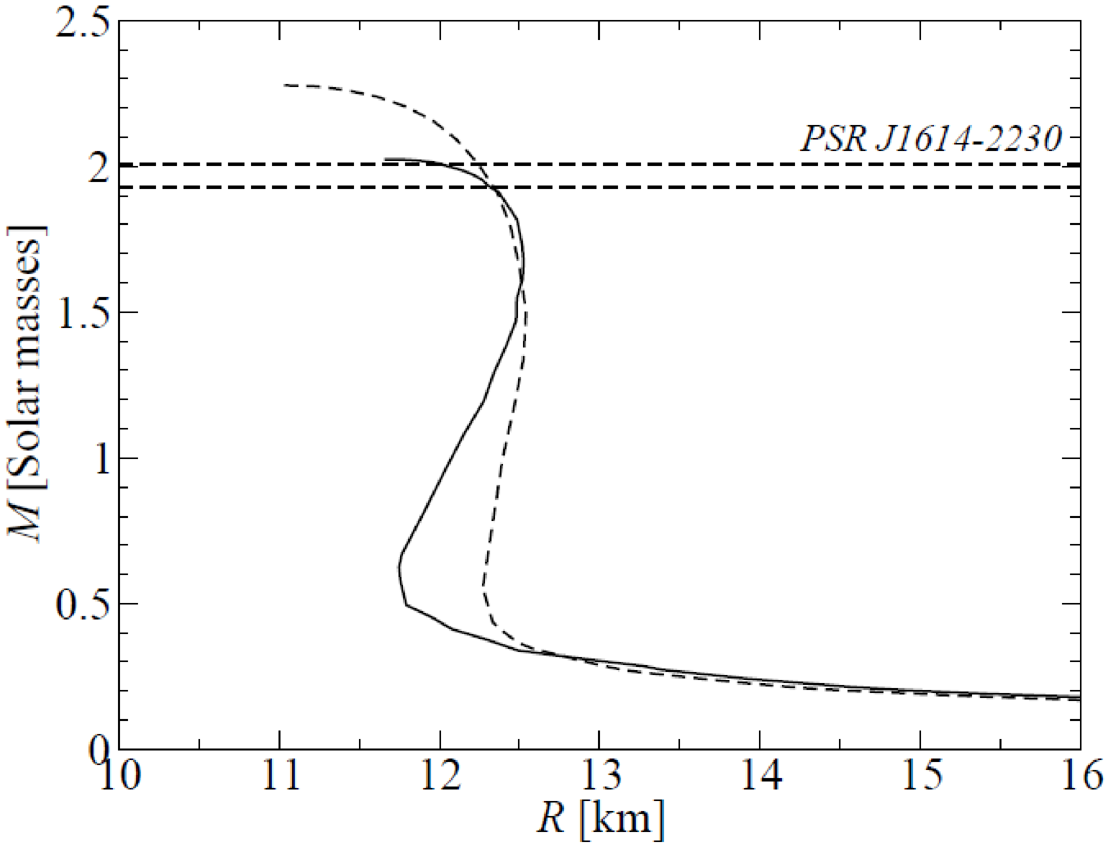


1211.1231

See also **1302.4732**

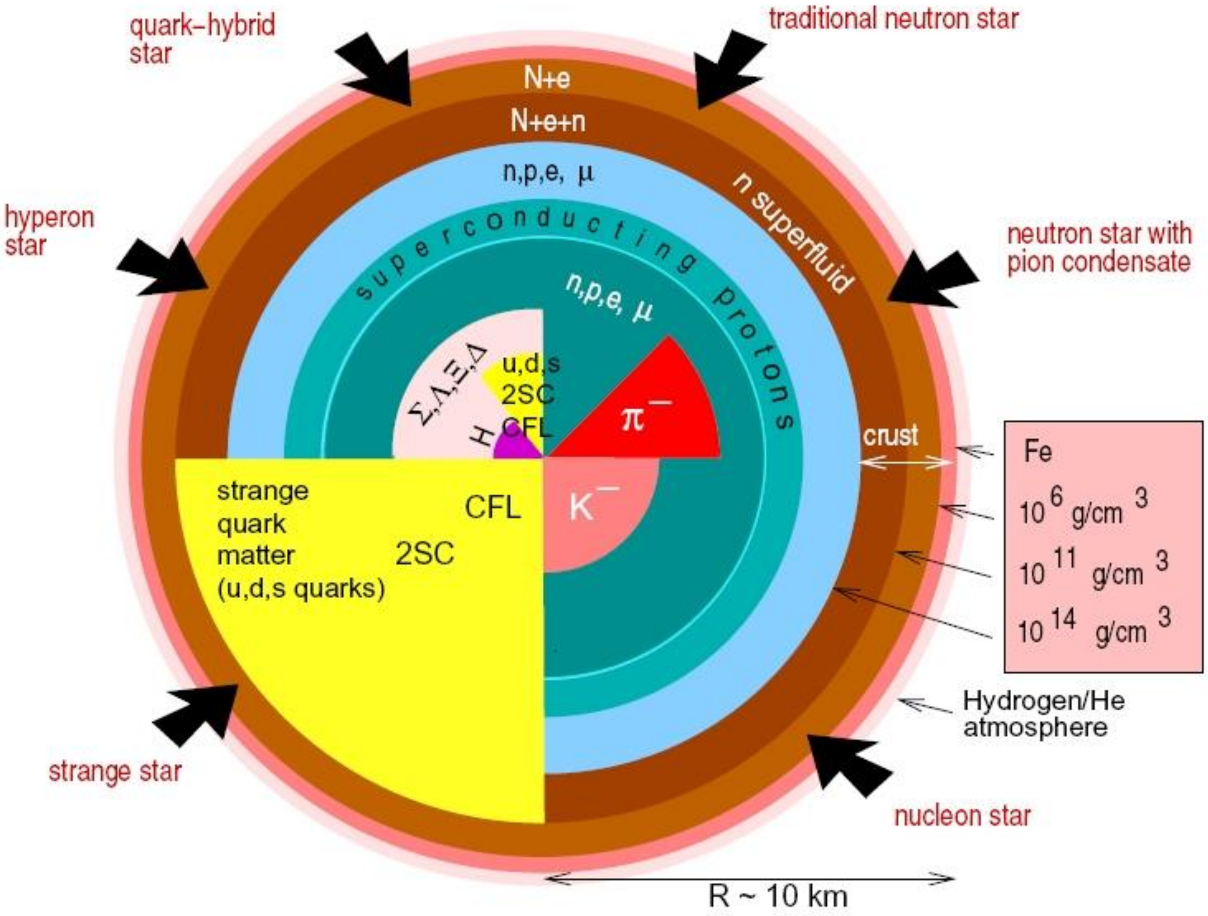
Massive hybrid stars

Stars with quark cores can be massive, and so this hypothesis is compatible with existence of pulsars with M>2 Msolar



1304.6907

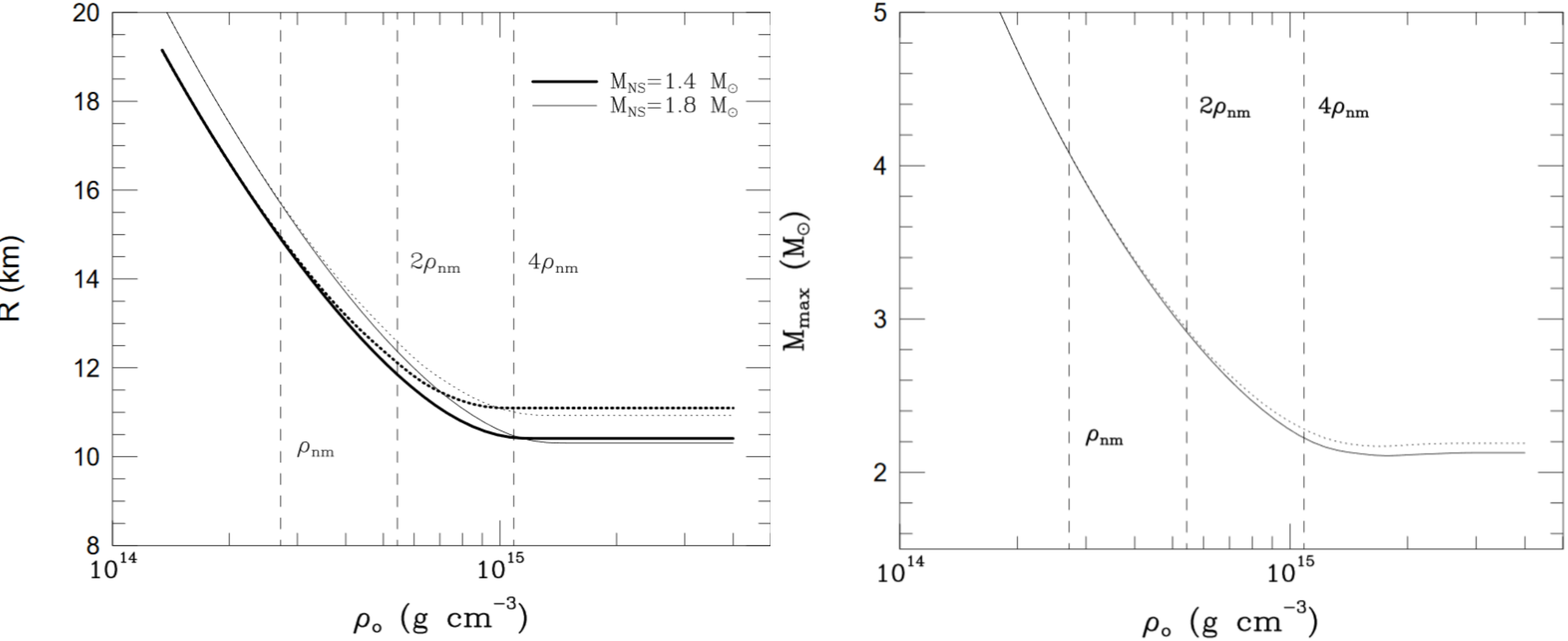
NS interiors: resume



(Weber et al. ArXiv: 0705.2708)

Maximum mass

Maximum mass of NSs depends on the EoS, however, it is possible to make calculations on the base of some fundamental assumptions.



astro-ph/9608059

Seminal paper: Rhoades, Ruffini 1974
http://prl.aps.org/abstract/PRL/v32/i6/p324_1

Papers to read

1. astro-ph/0405262 Lattimer, Prakash "Physics of neutron stars"

2. 0705.2708 Weber et al. "Neutron stars interiors and equation of state …"

3. physics/0503245 Baym, Lamb "Neutron stars"

4. 0901.4475 Piekarewicz "Nuclear physics of neutron stars" (first part)

5. 0904.0435 Paerels et al. "The Behavior of Matter Under Extreme Conditions"

6. 1512.07820 Lattimer, Prakash "The EoS of hot dense matter …."

7. 1001.3294 Schmitt "Dense matter in compact stars - A pedagogical introduction "

8. 1303.4662 Hebeler et al. "Equation of state and neutron star properties constrained by nuclear physics and observation "

9. 1210.1910 Weber et al. Structure of quark star

10. 1302.1928 Stone "High density matter "

+ the book by Haensel, Yakovlev, Potekhin

Lectures on the Web

Lectures can be found at my homepage:

http://xray.sai.msu.ru/~polar/html/presentations.html