

# Spin evolution of NSs

# Evolution of neutron stars

**Thermal**



**Magneto-rotational**

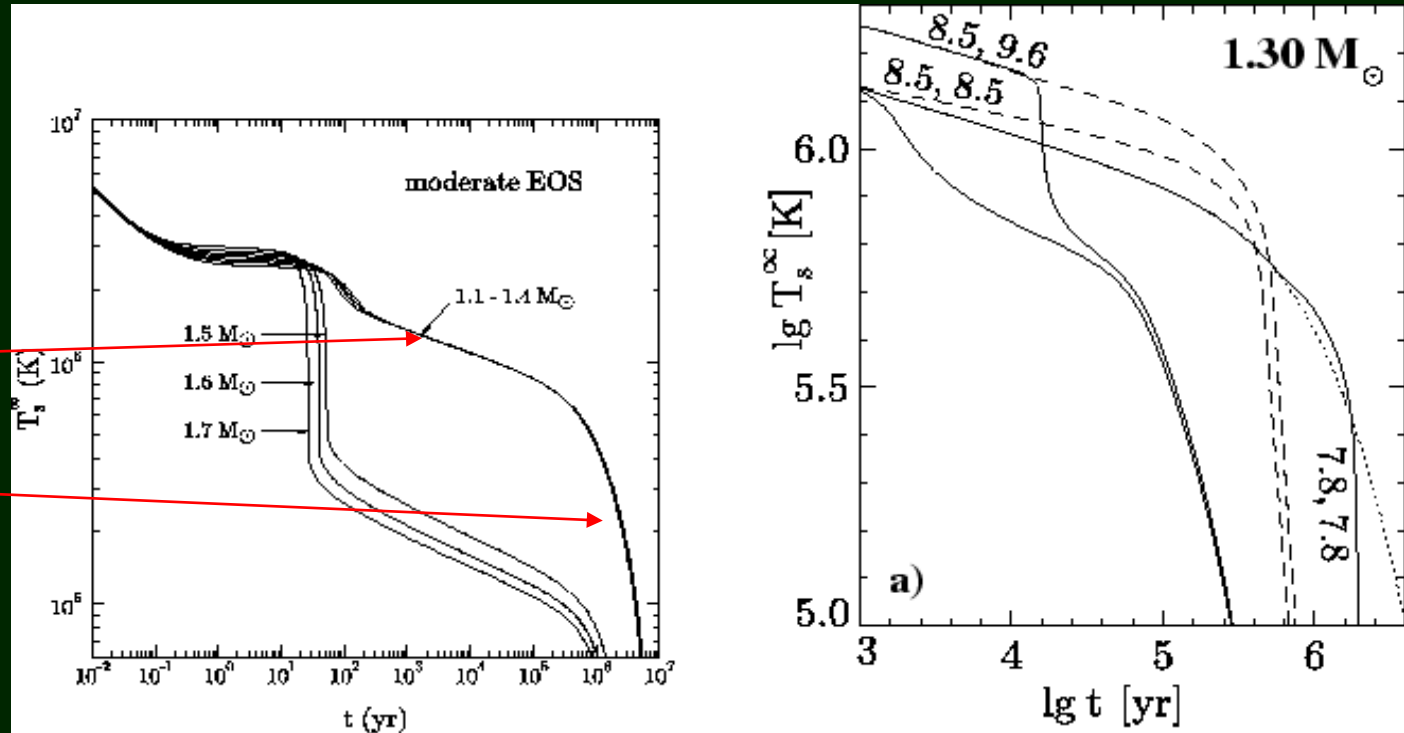
Observational appearance of a NS can depend on:

- Temperature
- Period
- Magnetic field
- Velocity

# Evolution of NSs: temperature

Neutrino  
cooling stage

Photon  
cooling stage

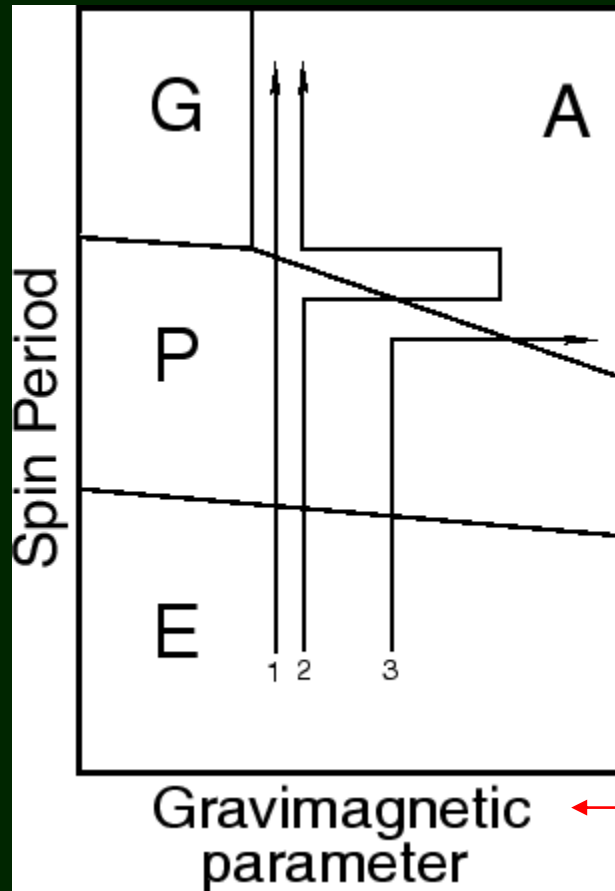


First papers on the thermal evolution appeared already in early 60s, i.e. before the discovery of radio pulsars.

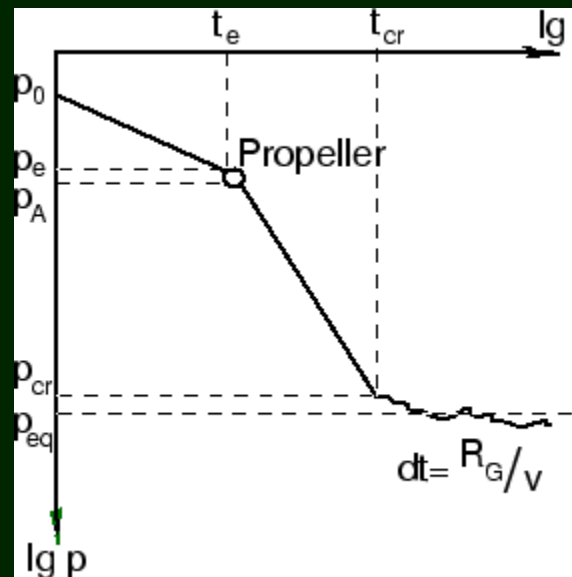
[Yakovlev et al. (1999) Physics Uspekhi]

# Evolution of neutron stars: rotation + magnetic field

Ejector → Propeller → Accretor → Georotator



- 1 – spin down
- 2 – passage through a molecular cloud
- 3 – magnetic field decay

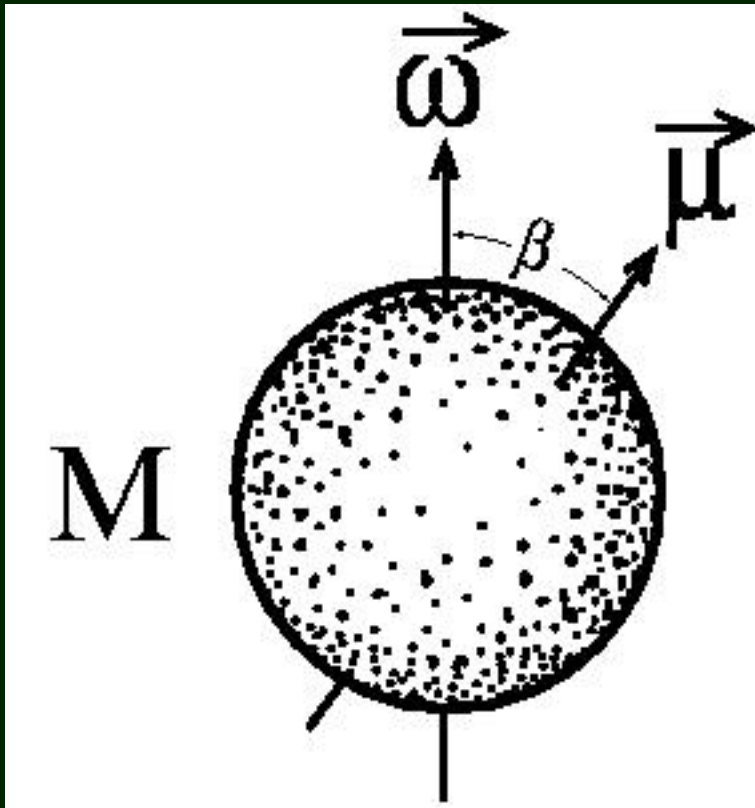


astro-ph/0101031

$\dot{M}/\mu^2$

See the book by Lipunov (1987, 1992)

# Magnetic rotator



Observational appearances of NSs (if we are not speaking about cooling) are mainly determined by  $P$ ,  $\dot{P}$ ,  $V$ ,  $B$ , (and, probably, by the inclination angle  $\chi$ ), and properties of the surrounding medium.  $B$  is not evolving significantly in most cases, so it is important to discuss spin evolution.

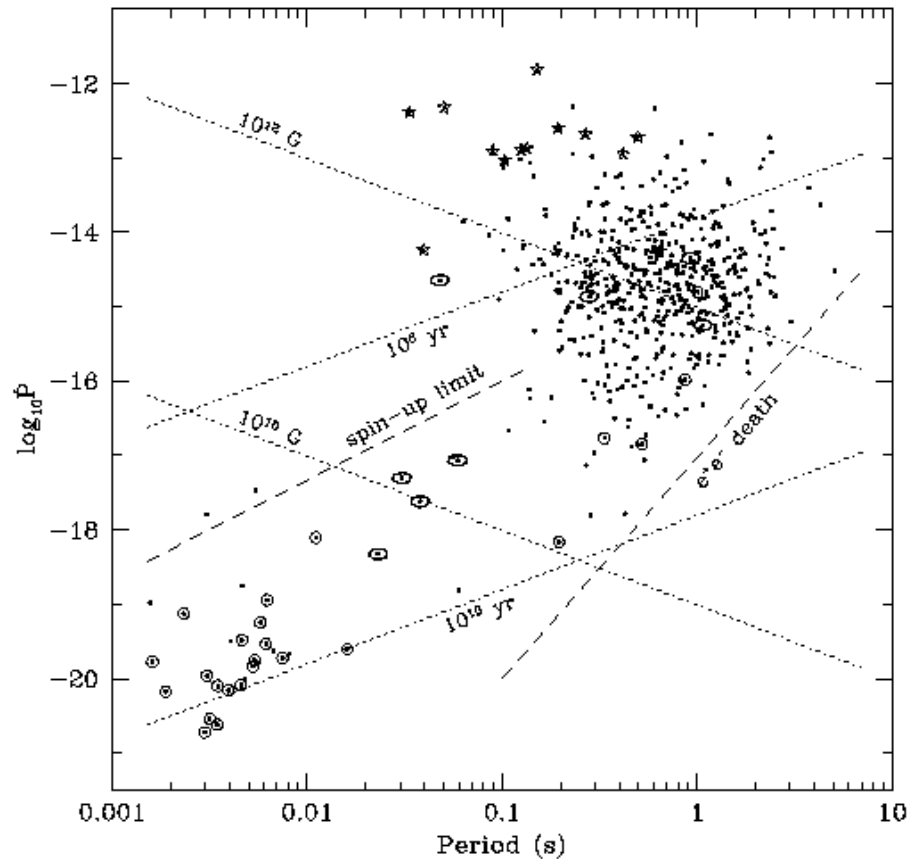
Together with changes in  $B$  (and  $\chi$ )  
one can speak about

## **magneto-rotational evolution**

We are going to discuss the main stages of this evolution, namely:

*Ejector, Propeller, Accretor, and Georotator* following the classification by Lipunov

# Magneto-rotational evolution of radio pulsars



For radio pulsar magneto-rotational evolution is usually illustrated in the P-Pdot diagram.

However, we are interested also in the evolution after this stage.

$$L_m = \frac{2}{3} \frac{\mu^2 \omega^4}{c^3} \sin^2 \beta = \kappa_t \frac{\mu^2}{R_l^3} \omega,$$

$$B \sim 3.2 \times 10^{19} (PdP/dt)^{1/2} \text{ G}.$$

Spin-down.

Rotational energy is released.  
The exact mechanism is still unknown.

# Radio pulsar braking: current losses

The model of pulsar emission is not known, and also the model for spin-down is not known, too. Well-known magneto-dipole formula is just a kind of approximation.

One of models is the **longitudinal current losses** model (Beskin et al. see astro-ph/0701261)

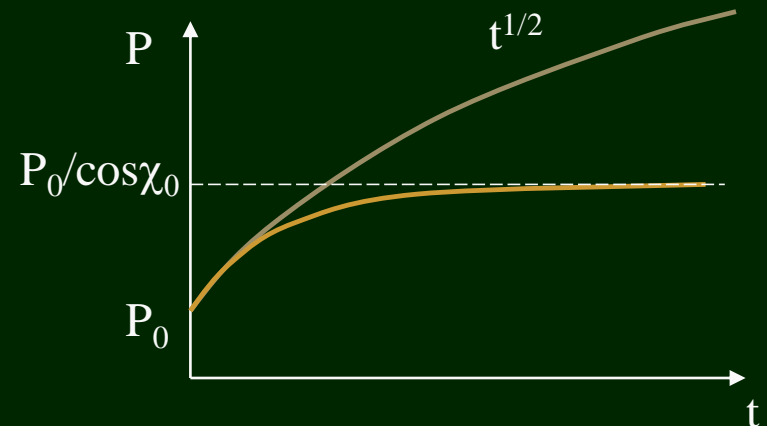
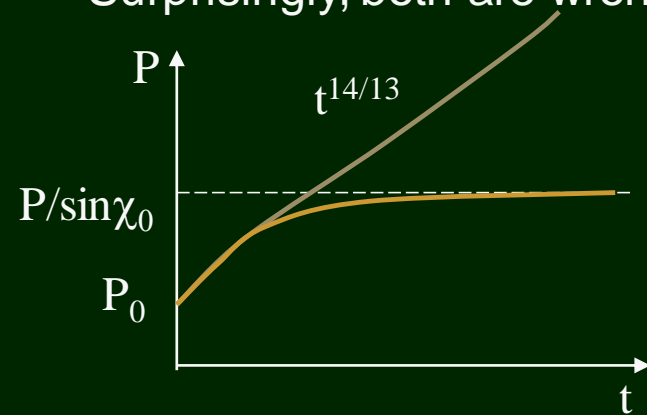
$$\dot{P} = 10^{-15} B_{12}^{10/7} P^{1/14} \cos^{3/2} \chi.$$

Longitudinal current losses

$$\dot{P} = 0.24 \times 10^{-15} B_{12}^2 P^{-1} \sin^2 \chi.$$

Magneto-dipole

Both models predict evolution of the angle between spin and magnetic axis. Surprisingly, both are wrong!



# Radio pulsar braking: braking index

$$n_{\text{br}} = \frac{\Omega \ddot{\Omega}}{\dot{\Omega}^2}.$$

Braking index (definition)

$$n_{\text{br}} = 3 + 2 \cot^2 \chi.$$

Magneto-dipole formula

$$n_{\text{br}} = 1.93 + 1.5 \tan^2 \chi.$$

Longitudinal current losses

For well-measured braking indices  $n < 3$ .

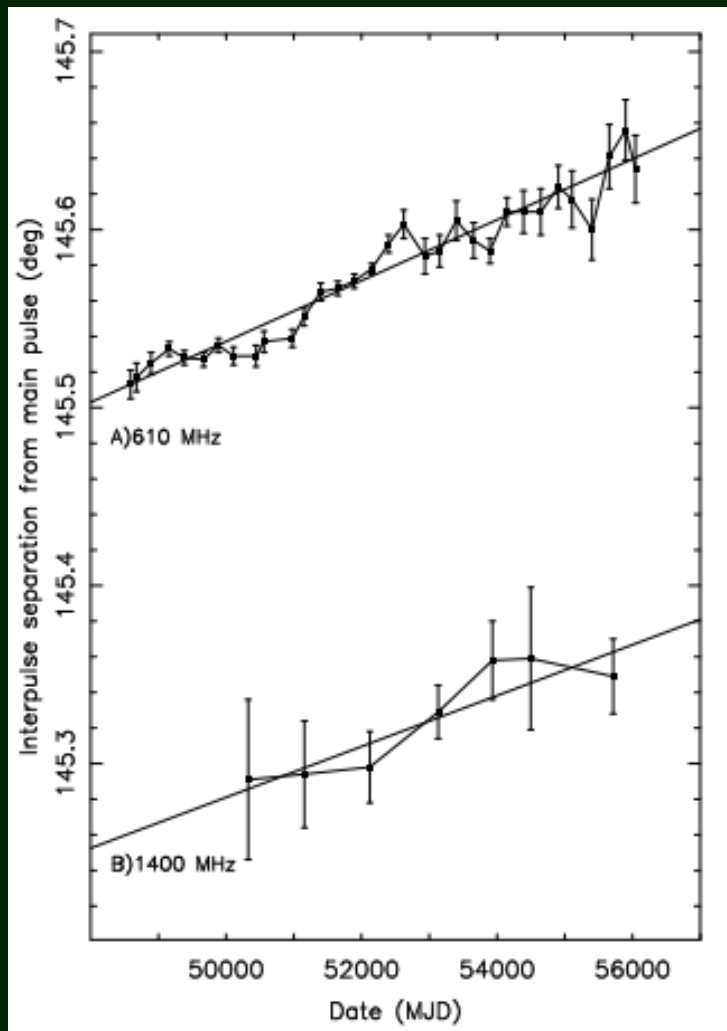
However, for many pulsars they are very large.

This can be simply an observational effect (microglitches, noise, etc.), but it can also be something real.

For example, related to the magnetic field evolution.



# Crab pulsar and angle evolution

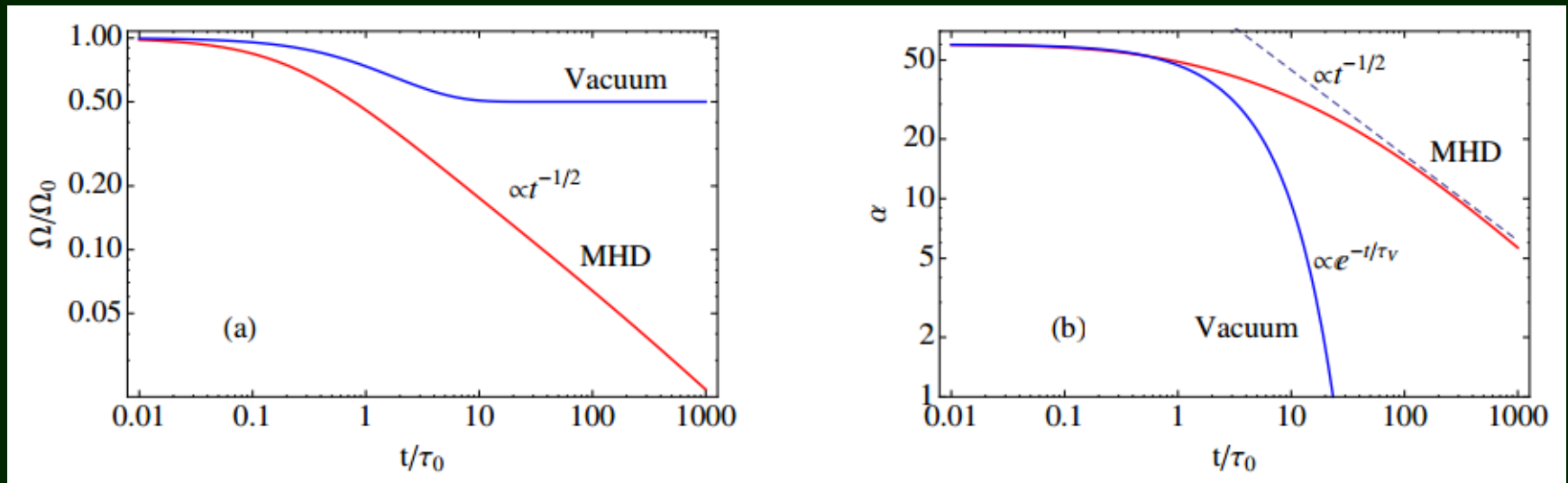


It seems that the angle is changing with the rate 0.6 degrees per century. It is visible as the separation between the main pulse and interpulse is changing.

The axis of the dipolar magnetic field is moving towards the equator.

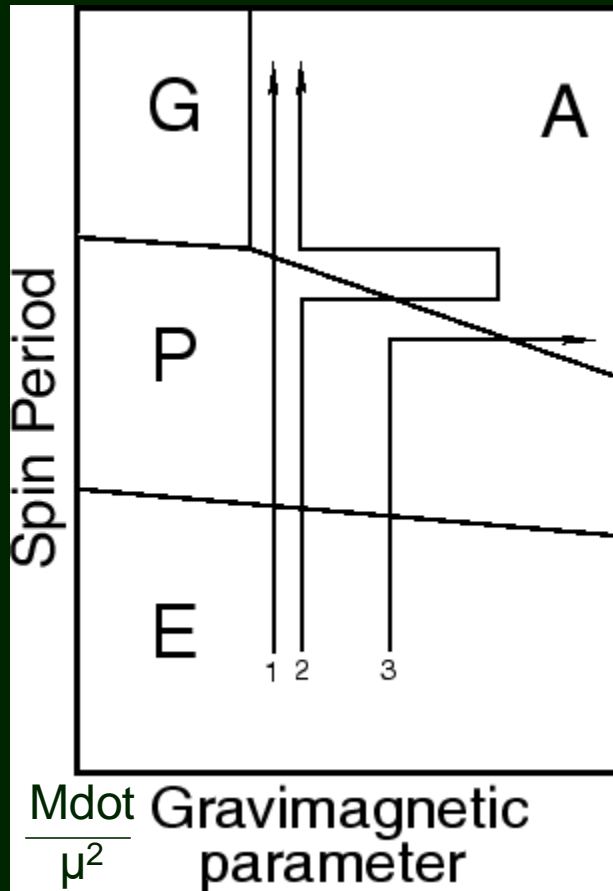
# Theoretical studies of the angle evolution

The authors studied the case of plasma filled magnetosphere.  
The angle should evolve towards zero.

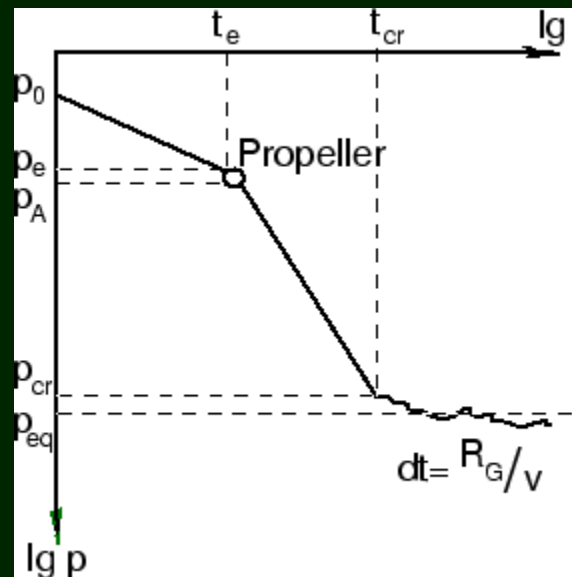


# Magneto-rotational evolution of NSs

Ejector → Propeller → Accretor → Georotator



- 1 – spin down
- 2 – passage through a molecular cloud
- 3 – magnetic field decay



astro-ph/0101031

See the book by Lipunov (1987, 1992)

# Critical radii -I

Transitions between different evolutionary stages can be treated in terms of critical radii

- Ejector stage. Radius of the light cylinder.  $R_l=c/\omega$ .  
Shvartsman radius.  $R_{sh}$ .
- Propeller stage. Corotation radius.  $R_{co}$
- Accretor stage. Magnetospheric (Alfven) radius.  $R_A$
- Georotator stage. Magnetospheric (Alfven) radius.  $R_A$

As observational appearance is related to interaction with the surrounding medium the radius of *gravitational capture* is always important.  $R_G=2GM/V^2$ .

# Critical radii-II

## 1. Shvartsman radius

It is determined by  
relativistic particles wind

$$R_{\text{sh}} = \left( \frac{8\kappa_t \mu^2 G^2 M^2 \omega^4}{\dot{M}_c v_\infty^5 c^4} \right)^{1/2}, \quad R_{\text{sh}} > R_G$$

## 2. Corotation radius

$$\omega R_{\text{st}} < \sqrt{GM_x / R_{\text{st}}}$$

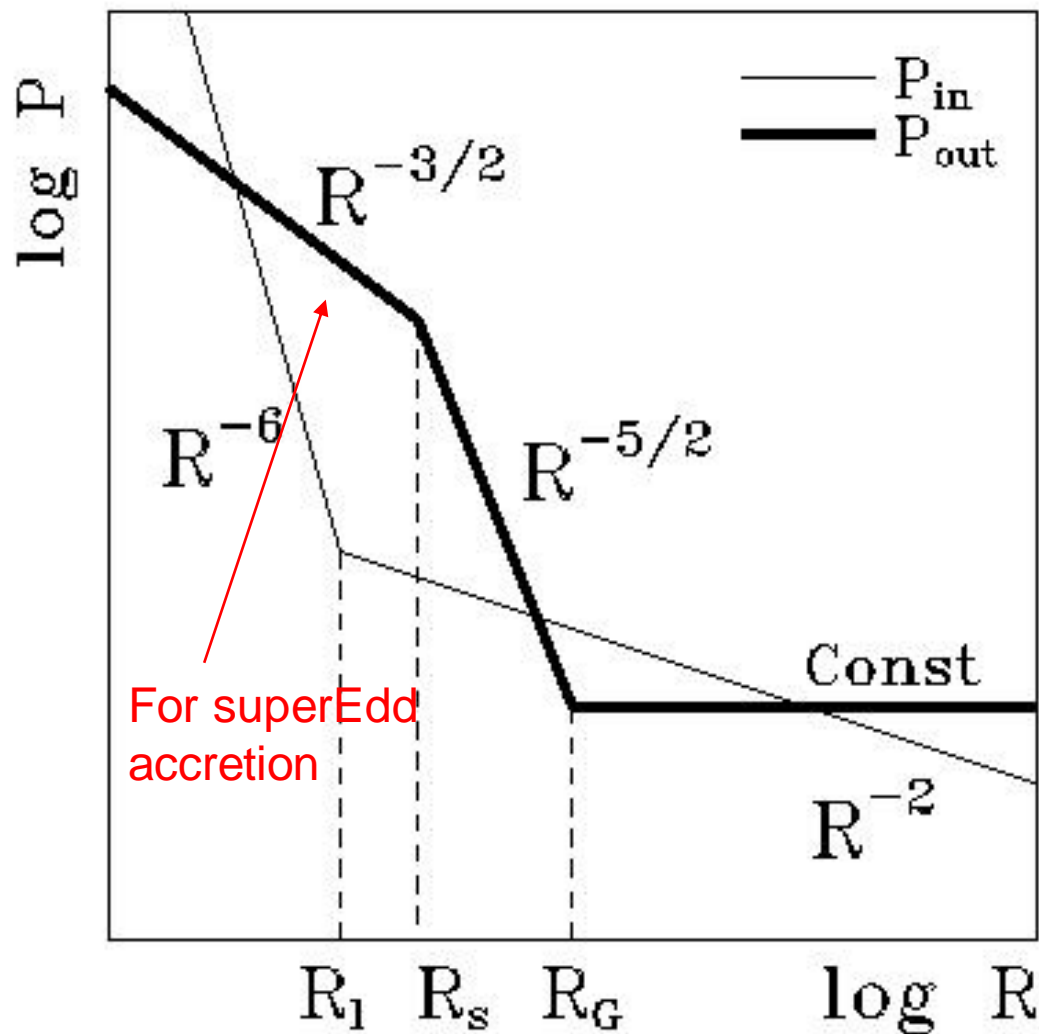
$$R_c = (GM_x / \omega^2)^{1/3} \sim 2.8 \times 10^8 m_x^{1/3} (P/1 \text{ s})^{2/3} \text{ cm}$$

## 3. Alfven radius

$$P_m(R_{\text{st}}) = P_a(R_{\text{st}})$$

$$R_A = \begin{cases} \left( \frac{2\mu^2 G^2 M^2}{\dot{M}_c v_\infty^5} \right)^{1/6}, & R_A > R_G \\ \left( \frac{\mu^2}{2\dot{M}_c \sqrt{2GM}} \right)^{2/7}, & R_A \leq R_G \end{cases}$$

# Pressure



$$P_m = \begin{cases} \frac{\mu^2}{8\pi R^6}, & R \leq R_t \\ \frac{L_m}{4\pi R^2 c}, & R > R_t \end{cases}$$

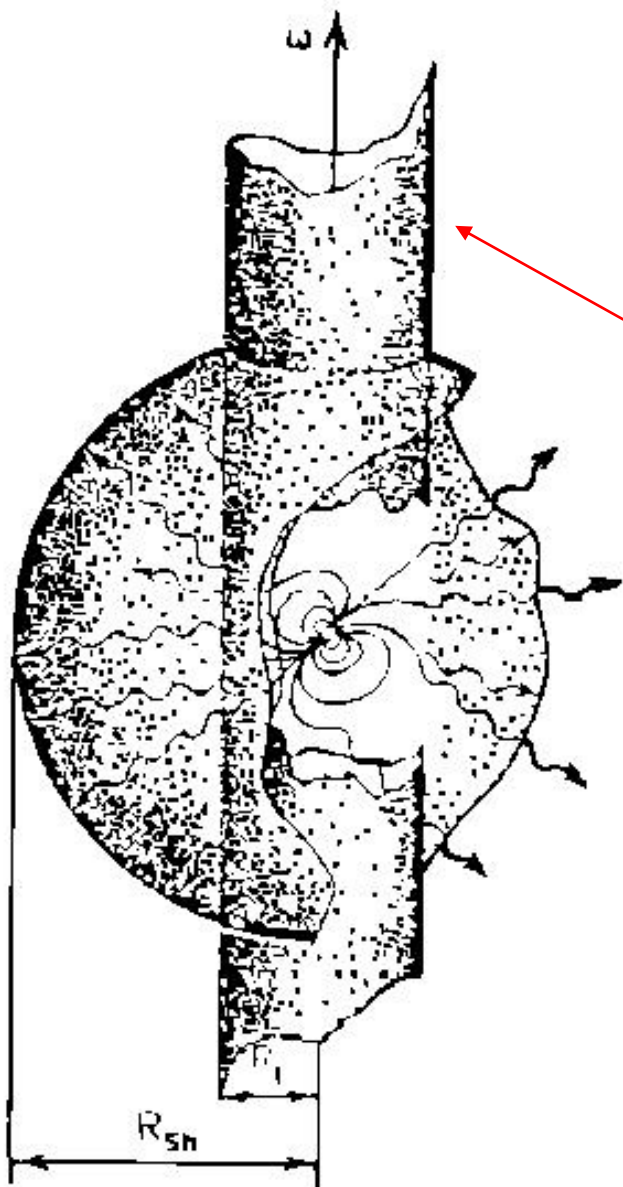
$$L_m = \kappa_t \frac{\mu^2}{R_t^3} \omega$$

We can define a stopping radius  $R_{\text{st}}$ , at which external and internal pressures are equal.

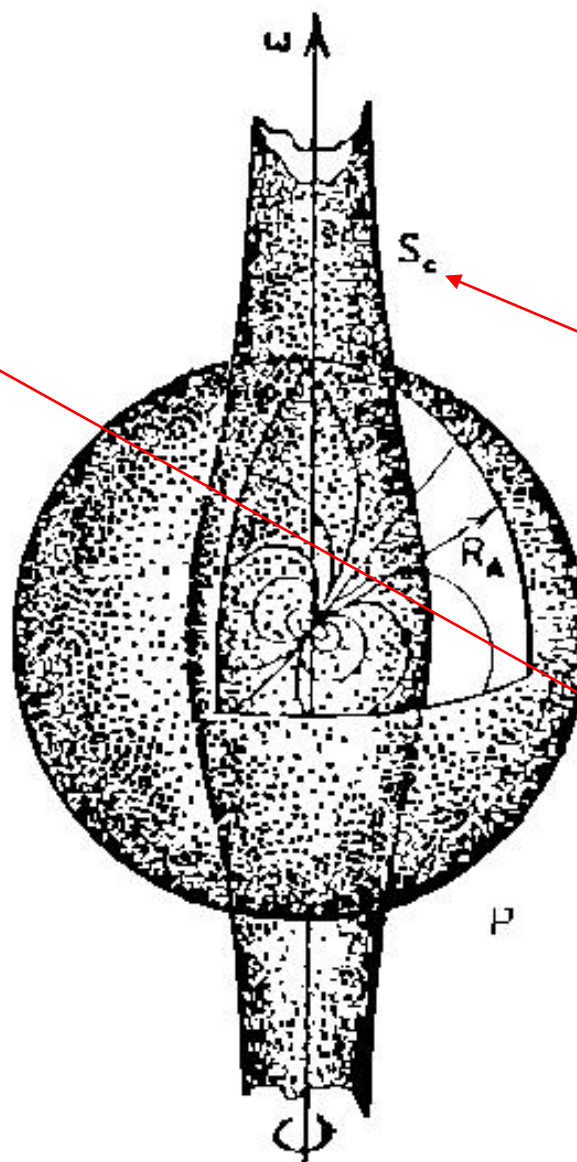
The stage is determined by relation of this radius to other critical radii.

# Classification

Abbrevi- ation	Type	Characteristic radii relation	Accretion rate	Observational appearances
E	Ejector	$R_{\text{st}} > R_G$ $R_{\text{st}} > R_l$	$\dot{M}_c \leq \dot{M}_{\text{cr}}$	Radiopulsars, Soft $\gamma$ -ray repeaters, Cyg X-3? LSI+61 303?
P	Propeller	$R_c < R_{\text{st}}$ $R_{\text{st}} \leq R_G$ $R_{\text{st}} \leq R_l$	$\dot{M}_c \leq \dot{M}_{\text{cr}}$	X-ray transients? Rapid burster? $\gamma$ -bursters?? Magnetic Ap-stars
A	Accretor	$R_{\text{st}} \leq R_G$ $R_{\text{st}} \leq R_l$	$\dot{M}_c \leq \dot{M}_{\text{cr}}$	X-ray pulsars, bursters, cataclysmic variables, intermediate polars
G	Gcorotator	$R_G < R_{\text{st}}$ $R_{\text{st}} \leq R_c$	$\dot{M}_c \leq \dot{M}_{\text{cr}}$	Earth, Jupiter
M	Magnetor	$R_{\text{st}} > a$ $R_c > a$ ???	$\dot{M}_c \leq \dot{M}_{\text{cr}}$	AM Her, polars



Ejector



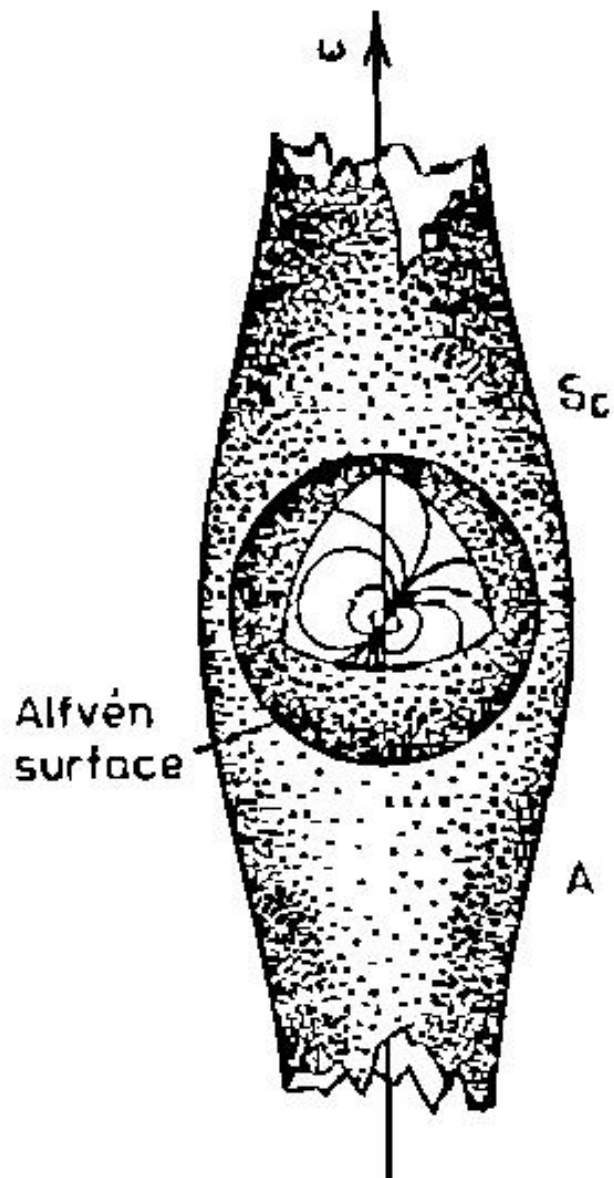
Propeller

$$R = R_{co} \cos^{-2/3} \theta$$

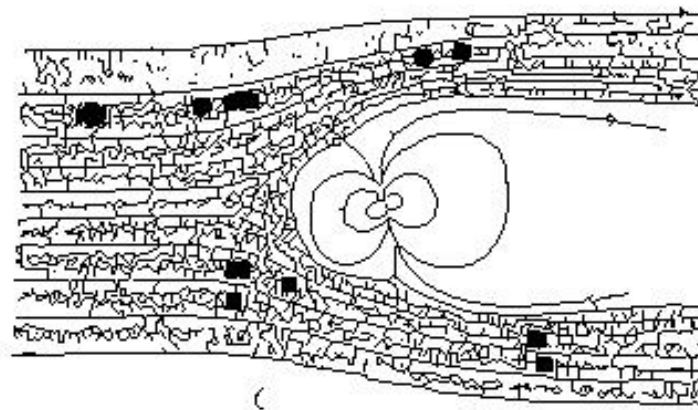
$$R_{co} = (GM/\omega^2)^{1/3}$$

Light cylinder  
 $R_l = \omega/c$





**Accretor**



**Georotator**

# Critical periods for isolated NSs

$$P_E(E \rightarrow P) \simeq 10 \mu_{30}^{1/2} n^{-1/4} v_{10}^{1/2} \text{ s}$$

Transition from Ejector to Propeller (supersonic)

$$t_E \simeq 10^9 \mu_{30}^{-1} n^{-1/2} v_{10} \text{ yr}$$

Duration of the ejector stage

$$P_A(P \rightarrow A) \simeq 420 \mu_{30}^{6/7} n^{-3/7} v_{10}^{9/7} \text{ s}$$

Transition from supersonic Propeller to subsonic Propeller or Accretor

$$P_{eq} = 2.6 \times 10^3 v_{(t)10}^{-2/3} \mu_{30}^{2/3} n^{-2/3} v_{10}^{13/3} \text{ s}$$

A kind of equilibrium period for the case of accretion from turbulent medium

$$v < 410 n^{1/10} \mu_{30}^{-1/5} \text{ km s}^{-1}$$

Condition for the Georotator formation (instead of Propeller or Accretor)

(see, for example, astro-ph/9910114)

# Spin-up/down at the stage of accretion

$$\frac{dI\omega}{dt} = \dot{M}k_{\text{su}} - \kappa_t \frac{\mu^2}{R_t^3},$$

$$k_{\text{su}} = \begin{cases} (GM_x R_d)^{1/2}, & \text{Keplerian disk accretion,} \\ \eta_t \Omega R_G^2, & \text{wind accretion in a binary,} \\ \sim 0, & \text{a single magnetic rotator.} \end{cases}$$

For a single rotator (i.e. an isolated NS) spin-up can be possible due to turbulence in the interstellar medium.

In the case of  
isolated accreting NS  
one can estimate  
the accretion rate as:

$$\dot{M}_c = 4\pi R_G^2 \rho_\infty v_\infty$$

# Unified approach to spin-down

One can find it comfortable to represent the spin-down moment by such a formula

$$- \kappa_t \frac{\mu^2}{R_t^3}$$

$k_t$  and  $R_t$  are different for different stages.  
 $k_t$  can be also frequency dependent.

Parameter	Regime					
	E, SE	P, SP	A	SA	G	M
$\dot{M}$	0	0	$\dot{M}_c$	$\dot{M}_c(R_\Lambda/R_s)$	0	$\dot{M}_c$
$\kappa_t$	$\sim 2/3$	$\lesssim 1/3$	$\sim 1/3$	$\sim 1/3$	$\sim 1/3$	$\sim 1/3$
$R_t$	$R_t$	$R_m$	$R_c$	$R_c$	$R_\Lambda$	$a$

# Equilibrium period

$$\dot{M} k_{su} = -\kappa_t \frac{\mu^2}{R_t^3}$$

The hypothesis of equilibrium can be used to determine properties of a NS.

The corotation radius is decreasing as a NS is spinning up.

So, before equilibrium is reached the transition to the propeller stage can happen.

---

Looking at this formula (and remembering that for Accretors  $R_t=R_{co}$ ) it is easy to understand why millisecond PSRs have small magnetic field.

Spin-up can not be very large (Eddington rate).

So, to have small spin periods (and so small corotation radii), it is necessary to have small magnetic fields.

High magnetic field NS can not be spun-up to millisecond periods.

---

# Accreting isolated neutron stars

## Why are they so important?

- Can show us how old NSs look like
  1. Magnetic field decay
  2. Spin evolution
- Physics of accretion at low rates
- NS velocity distribution
- New probe of NS surface and interiors
- ISM probe

# Expected properties

## 1. Accretion rate

An upper limit can be given by the Bondi formula:

$$\dot{M} = \pi R_G^2 \rho v, \quad R_G \sim v^{-2}$$

$$\dot{M} = 10^{11} \text{ g/s} \quad (v/10 \text{ km/s})^{-3} n$$

$$L = 0.1 \dot{M} c^2 \sim 10^{31} \text{ erg/s}$$

However, accretion can be smaller due to the influence of a magnetosphere of a NS

## 2. Periods

Periods of old accreting NSs are uncertain, because we do not know evolution well enough.

a) 
$$p_A = 2^{5/14} \pi (GM)^{-5/7} (\mu^2 / \dot{M})^{3/7} \simeq$$

$$R_A = R_{\text{co}}$$

$$300 \mu_{30}^{6/7} (v/10 \text{ km s}^{-1})^{9/7} n^{-3/7} \text{ s.}$$

# Subsonic propeller

Even after  $R_{co} > R_A$  accretion can be inhibited.

This have been noted already in the pioneer papers by Davies et al.

Due to rapid (however, subsonic) rotation a hot envelope is formed around the magnetosphere. So, a new critical period appear.

$$P_{br} \simeq 450 \mu_{30}^{16/21} \dot{M}_{15}^{-5/7} m^{-4/21} \text{ s.}$$

(Ikhsanov astro-ph/0310076)

If this stage is realized (inefficient cooling) then

- accretion starts later
- accretors have longer periods



# Initial spin periods

Determination of initial spin periods is closely linked with models of magneto-rotational evolution of neutron stars.

Among thousands of known NSs just for a few tens there are estimates of initial spin periods. Just for a few such estimates a robust enough.

Typically, it is necessary to have a independent estimate of a NS age. Then, using some model of magneto-rotational evolution the initial spin period is reconstructed.

Independent ages:

- SNR
- Kinematic
- Cooling

# Sample of NSs+SNRs

Table 1. Sample of PSRs associated with SNRs

PSR	SNR	$\tau_{SNR}/10^3$ yrs	$\tau_{sd}/10^3$ yrs	Ref.
J0537-6910	N157B	as the PSR	4.9	Wang and Gotthelf (1998)
J1119-6127	G292.2-0.5	as the PSR	1.6	Pivovarov et al. (2001)
J1747-2809	G0.9+0.1	as the PSR	5.3	Aharonian and et al. (2005)
				Porquet et al. (2003)
J1747-2958	G359.23-0.82	as the PSR	25.5	Camilo et al. (2002b)
J1846-0258	Kes75	as the PSR	0.73	Leahy and Tian (2008)
J1930+1852	G54.1+0.3	as the PSR	2.9	Camilo et al. (2002a)
J0007+7303	CTA 1	10.2-15.8	13.9	Slane et al. (2004)
J0205+6449	3C58	4.3-7	5.4	Slane et al. (2008)
J0538+2817	S147	40-200	618.1	Anderson et al. (1996)
				Ng et al. (2007)
B0540-69	0540-693	0.66-1.1	1.67	Williams et al. (2008)
B0656+14	Monogem Ring	86-170	110.9	Thorsett et al. (2003)
J0821-4300	Puppis A	3.3-4.1	1489.	Gotthelf and Halpern (2009)
B0833-45	Vela	11-27	11.3	Aschenbach et al. (1995)
J1124-5916	G292.0+1.8	2.4-2.85	2.85	Gonzalez and Safi-Harb (2003)
B1509-58	G320.4-1.2	6-20	1.6	Yatsu et al. (2005)
J1809-2332	G7.5-1.7	10-100	67.6	Roberts and Brogan (2008)
J1813-1749	G12.8-0.0	0.285-2.5	4.7	Brogan et al. (2005)
J1833-1034	G21.5-0.9	0.8-40.	4.9	Safi-Harb et al. (2001)

Table 1—Continued

PSR	SNR	$\tau_{SNR}/10^3$ yrs	$\tau_{sd}/10^3$ yrs	Ref.
B1853+01	W44	6.5-20	20.3	Harrus et al. (1997)
J1957+2831	G65.1+0.6	40-140	1568.	Tian and Leahy (2006)
B1951+32	CTB80	> 18	107.	Castelletti et al. (2003)
B1338-62	G308.8-0.1	< 32.5	12.1	Caswell et al. (1992)
J2229+6114	G106.6+2.9	> 3.9	10.5	Kothes et al. (2006)
B0531+21	Crab	0.957	1.24	Stephenson and Green (2002)
J1210-5226	G296.5+10.0	10-20	101817.	Vasisht et al. (1997)
J1437-5959	G315.9-0.0	22	114.	Camilo et al. (2009)
J1811-1925	G11.2-0.3	1.6	23.2	Torii et al. (1999)
J1852+0040	Kes79	6	191502.	Sun et al. (2004)
J2021+4026	G78.2+2.1	6.6	76.9	Uchiyama et al. (2002)
B2334+61	G114.3+0.3	7.7	40.6	Yar-Uyaniker et al. (2004)

30 pairs: PSR+SNR  
Popov, Turolla arXiv: 1204.0632

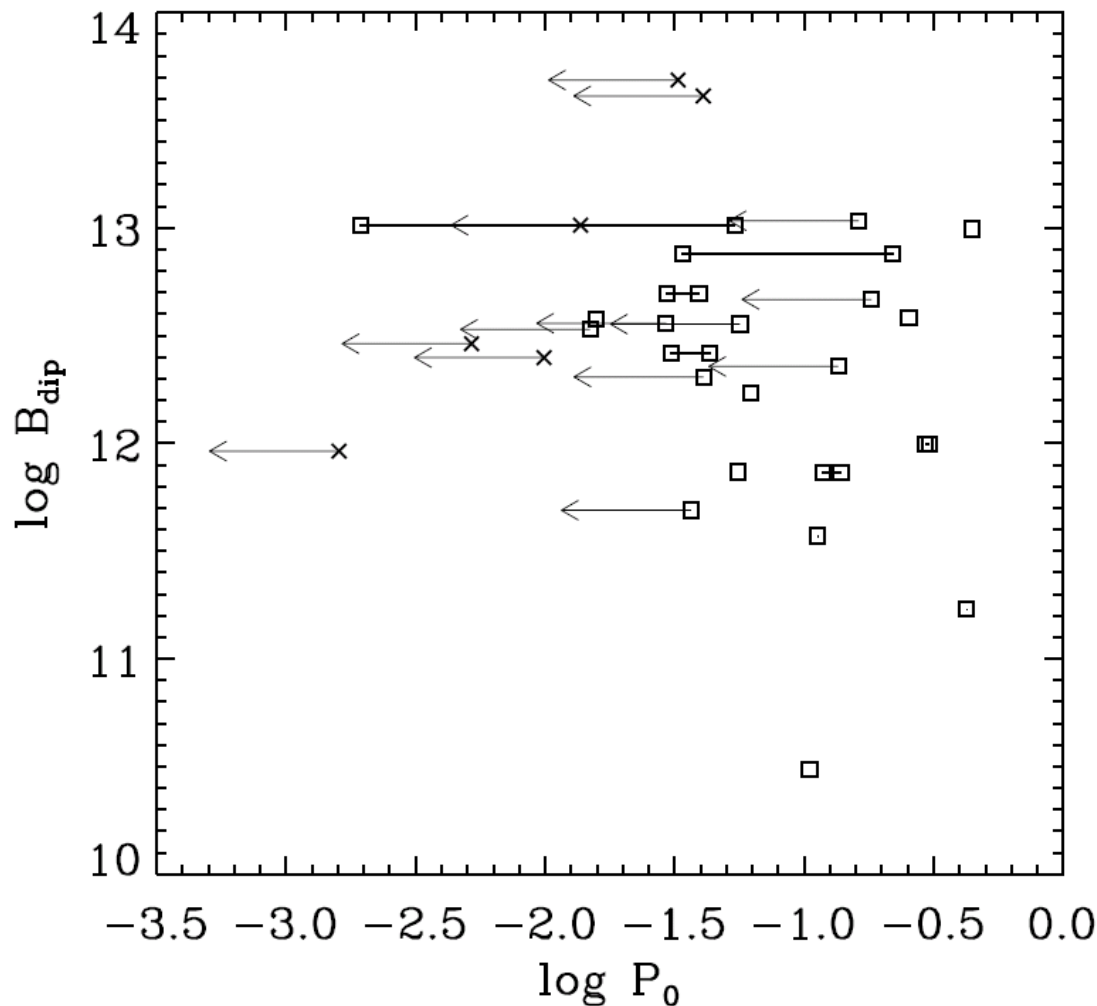
Table 2. Spin parameters of PSRs in the sample

PSR	$P$ s	$\dot{P}$	$B/10^{12}$ G	$P_0$ s	$P_0/P$
J0537-6910	0.016	5.18E-14	0.92	$\ll P$	$\sim 0$
J1119-6127	0.408	4.02E-12	41.	$\ll P$	$\sim 0$
J1747-2809	0.052	1.56E-13	2.9	$\ll P$	$\sim 0$
J1747-2958	0.099	6.13E-14	2.5	$\ll P$	$\sim 0$
J1846-0258	0.326	7.08E-12	48.6	$\ll P$	$\sim 0$
J1930+1852	0.137	7.51E-13	10.3	$\ll P$	$\sim 0$
J0007+7303	0.316	3.6E-13	10.8	$< 0.163$	$< 0.52$
J0205+6449	0.066	1.94E-13	3.6	$< 0.029$	$< 0.45$
J0538+2817	0.143	3.67E-15	0.73	$< 0.134$	$< 0.93$
	0.143	3.67E-15	0.73	$> 0.118$	$> 0.82$
B0540-69	0.05	4.79E-13	5.0	$< 0.039$	$< 0.78$
	0.05	4.79E-13	5.0	$> 0.03$	$> 0.59$
B0656+14	0.385	5.5E-14	4.7	$< 0.183$	$< 0.48$
J0821-4300	0.113	1.2E-15	0.37	$< 0.113$	$\sim 1$
	0.113	1.2E-15	0.37	$> 0.113$	$\sim 1$
B0833-45	0.089	1.25E-13	3.4	$< 0.016$	$< 0.2$
J1124-5916	0.135	7.53E-13	10.2	$< 0.054$	$< 0.40$
	0.135	7.53E-13	10.2	$> 0.004$	$> 0.03$
J1210-5226	0.424	6.6E-17	0.17	0.424	$\sim 1$
B1509-58	0.151	1.54E-12	15.4	—	—

Table 2—Continued

PSR	$P$ s	$\dot{P}$	$B/10^{12}$ G	$P_0$ s	$P_0/P$
J1809-2332	0.147	3.44E-14	2.3	$< 0.136$	$< 0.92$
J1813-1749	0.045	1.5E-13	2.6	$< 0.043$	$< 0.97$
	0.045	1.5E-13	2.6	$> 0.031$	$> 0.69$
J1833-1034	0.062	2.02E-13	3.6	$< 0.057$	$< 0.91$
B1853+01	0.267	2.08E-13	7.5	$< 0.221$	$< 0.83$
	0.267	2.08E-13	7.5	$> 0.036$	$> 0.14$
J1957+2831	0.308	3.11E-15	0.99	$< 0.3$	$< 0.99$
	0.308	3.11E-15	0.99	$> 0.29$	$> 0.95$
B1951+32	0.04	5.84E-15	0.49	$< 0.036$	$< 0.91$
B1338-62	0.193	2.53E-13	7.1	—	—
J2229+6114	0.052	7.83E-14	2.0	$< 0.041$	$< 0.79$
B0531+21	0.033	4.23E-13	3.8	0.016	0.48
J1437-5959	0.062	8.59E-15	0.74	0.055	0.9
J1811-1925	0.065	4.40E-14	1.7	0.062	0.97
J1852+0040	0.105	8.68E-18	0.03	0.105	$\sim 1$
J2021+4026	0.265	5.47E-14	3.9	0.254	0.96
B2334+61	0.495	1.93E-13	9.9	0.45	0.91

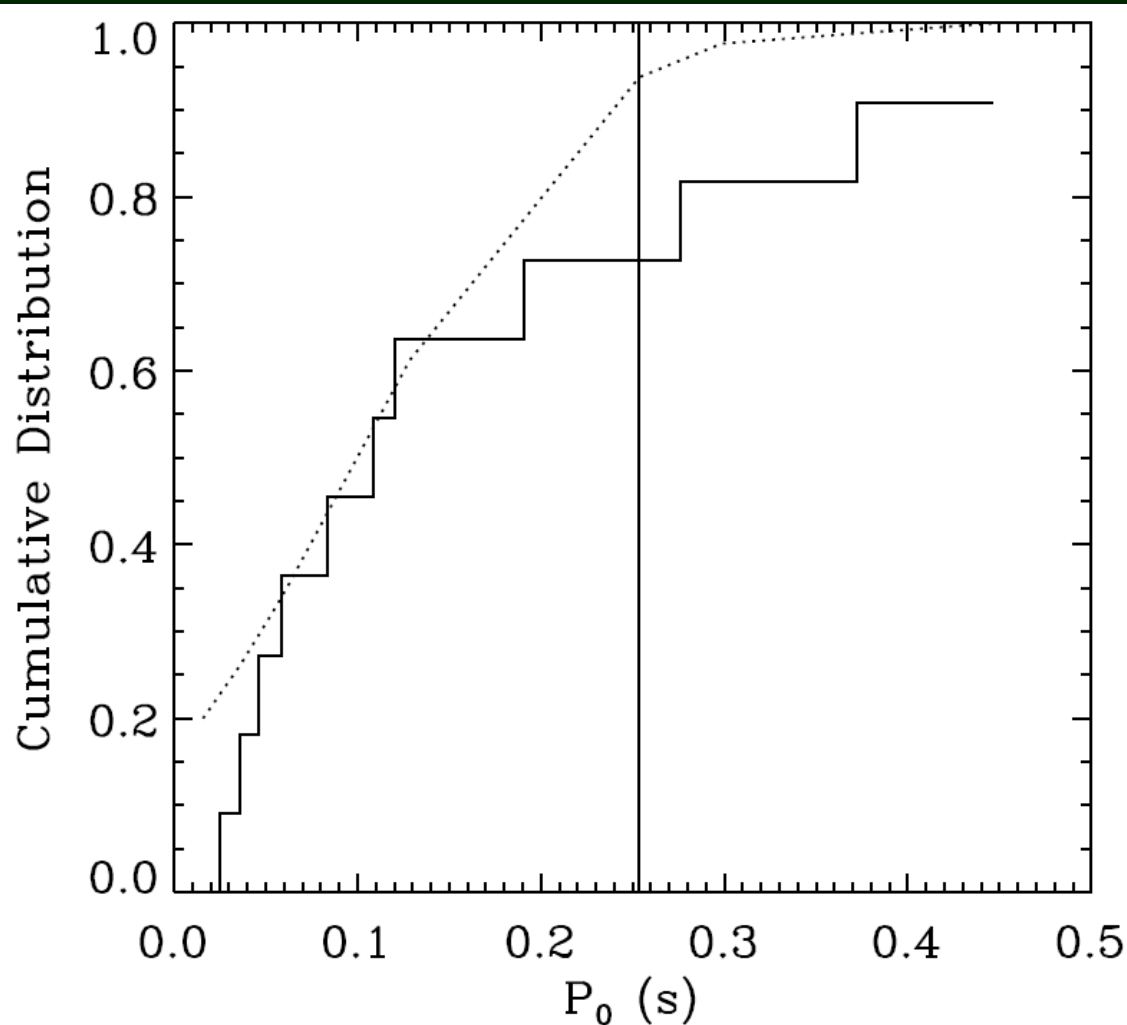
# B vs. $P_0$



All presented estimates are made for standard assumptions:  
 $n = \text{const} = 3$ .  
So, field is assumed to be constant, as well as the angle between spin and magnetic axis.

Crosses – PSRs in SNRs (or PWN) with ages just consistent with spin-down ages. We assume that  $P_0 < 0.1P$

# Checking gaussian



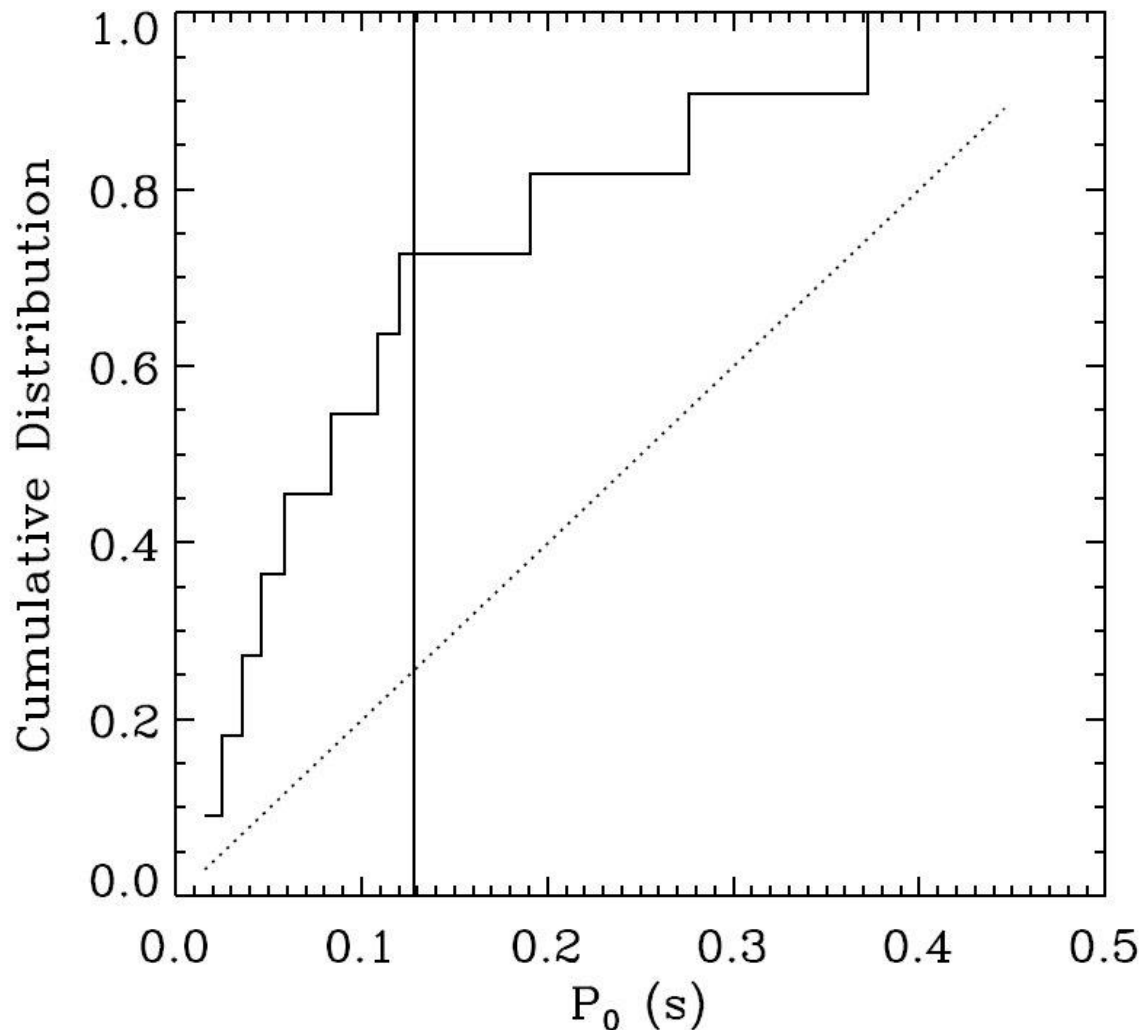
The data we have is not enough to derive the shape of the  $P_0$  distribution. However, we can exclude very wide and very narrow distributions, and also we can check if some specific distributions are compatible with our results.

Here we present a test for a gaussian distribution, which fits the data.

Still, we believe that the fine tuning is premature with such data.

$$P_0 = 0.1 \text{ s}; \sigma = 0.1 \text{ s}$$

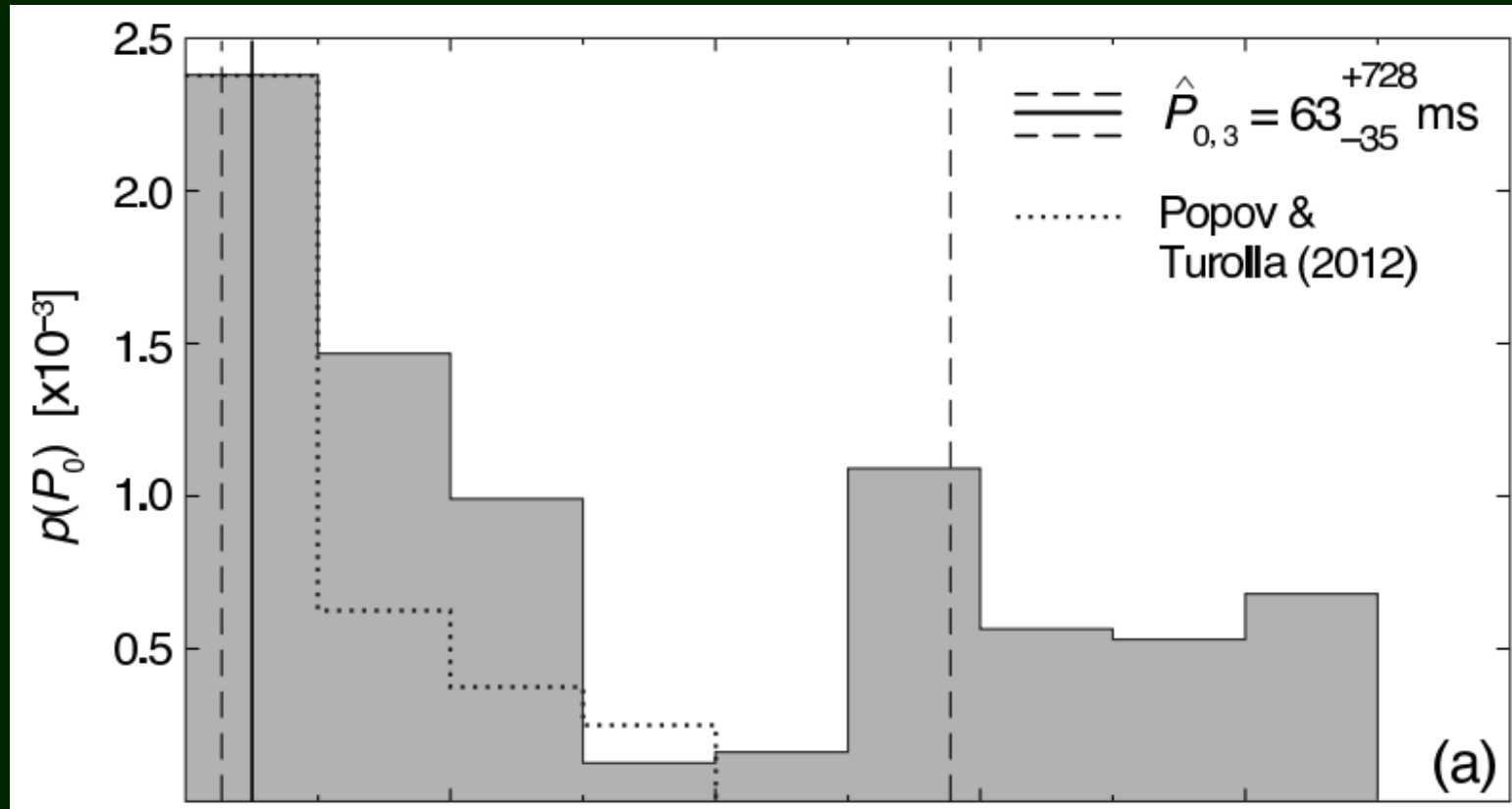
# Checking flat distribution



Flat between 0.001 and 0.5 s.

Very wide distributions  
in general do not fit  
the data we have.

# Wide initial spin period distribution



Based on kinematic ages. Mean age – few million years.  
Note, that in Popov & Turolla (2012) only NSs in SNRs  
were used, i.e. the sample is much younger!  
Can it explain the difference?

1301.1265

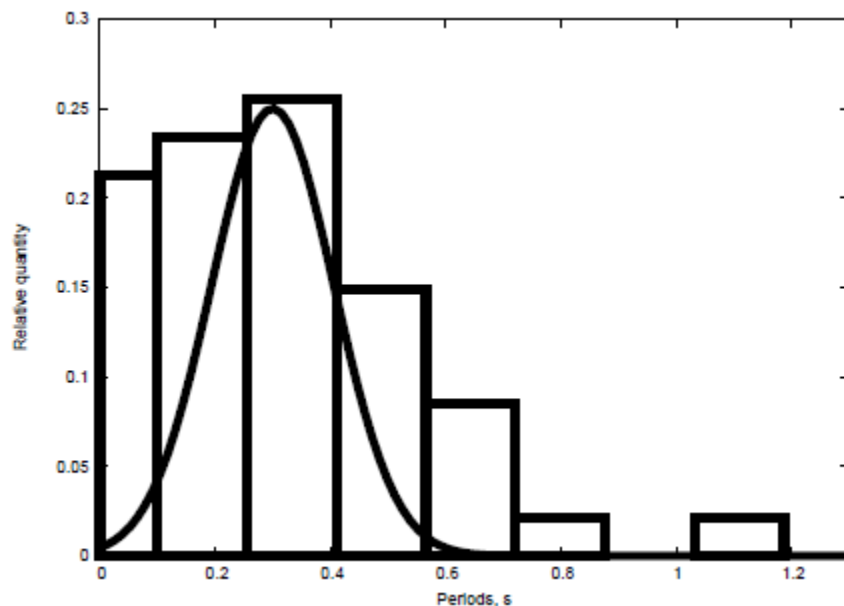
# Magnetic field decay and $P_0$

One can suspect that magnetic field decay can influence the reconstruction of the initial spin period distribution.

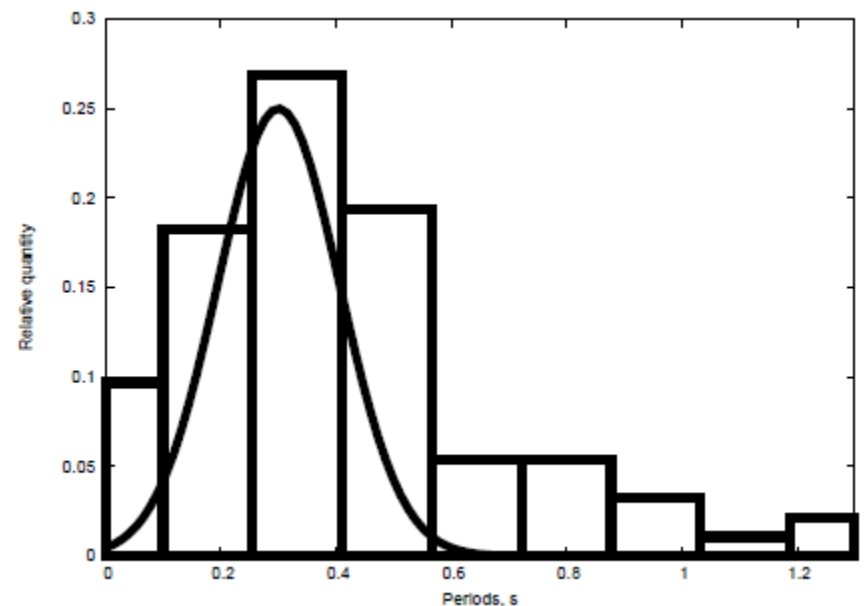
Exponential field decay with  $\tau=5$  Myrs.

$\langle P_0 \rangle = 0.3$  s,  $\sigma_P = 0.15$  s;  $\langle \log B_0 [\text{G}] \rangle = 12.65$ ,  $\sigma_B = 0.55$

$$P_0 = P \sqrt{1 - \frac{t}{\tau}}$$



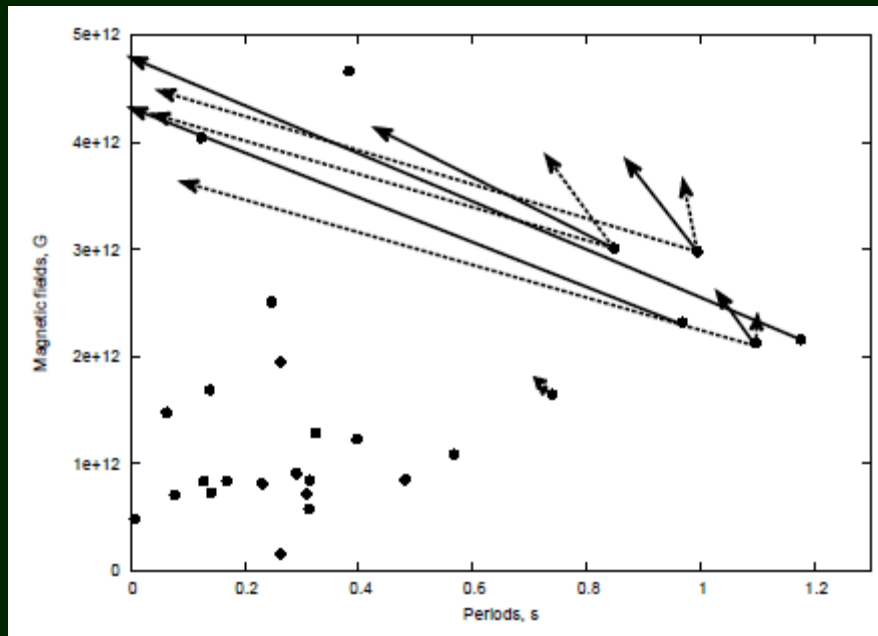
$\tau < 10^7$  yrs,  $10^5 < t$



$10^5 < t < 10^7$  yrs



# Real vs. reconstructed $P_0$



How much the reconstructed initial periods are changed due to not taking into account the exponential field decay?

Igoshev, Popov 2013

# Complications for magneto-rotational evolution

1. Internal structure can be important.

For example, neutron vortices can pin magnetic flux tubes (1106.5997).

Estimates indicate that this can be important for magnetars.

2. In young NSs a core can rotate faster than the crust (1210.5872).

3. Non-trivial topology of the magnetosphere can be important.

In magnetars a twisted magnetosphere can result in a different spin-down rate (1201.3635, and see the lecture on magnetars)

4. Magnetic field can have a very non-trivial evolution (see the next lecture)

5. Initial spin-periods can depend on additional phenomena.

Gravitational wave emission (1302.2649).

Neutrino emission (1301.7495).

Different instabilities (1110.3937).

# Conclusions

- We have some framework for spin evolution of NSs.  
They are expected to pass several well-defined stages:  
Ejector (including radio pulsar),  
Propeller (probably, with subsonic substage), Accretor.  
NSs with large velocities (or fields) after the Ejector stage  
can appear as Georotators.
- In binaries we observe Ejectors, Propellers and Accretor.  
For isolated NSs – only Ejectors (even, mostly radiopulsars).
- There are still many uncertainties related to the spin evolution:
  1. Spin-down rate and angle evolution for radio pulsars
  2. Subsonic propeller stage for isolated NSs
  3. Inhibition of accretion at low rates
  4. The role of the field decay

# Conclusions-2

- Observations of isolated accreting NSs can help a lot to understand all unknown questions of NS spin evolution and low-rate accretion.
- Magnetic field decay can be important also for young NSs, especially for highly magnetized ones, as a source of energy.

So, we have some coherent picture ..... But .....

**A lot of funny thing a still waitng for us!**



# Papers and books to read

- Lipunov V.M. “**Astrophysics of neutron stars**” (1992)
- Lipunov, Postnov, Prokhorov “**The Scenario Machine: Binary Star Population Synthesis**”  
**Astrophysics and Space Science Reviews** (1996)  
<http://xray.sai.msu.ru/~mystery/articles/review/>
- Ikhsanov “**The origin of long-period X-ray pulsars**” [astro-ph/0611442](https://arxiv.org/abs/astro-ph/0611442)