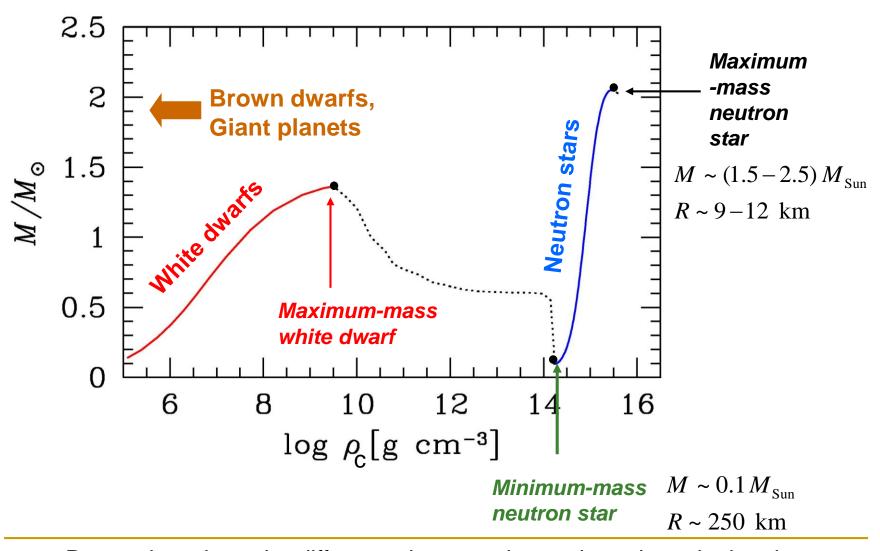
## Neutron Star masses and radii

#### NS Masses

- Stellar masses are directly measured only in binary systems
- Accurate NS mass determination for PSRs in relativistic systems by measuring PK corrections
- Gravitational redshift may provide M/R in NSs by detecting a known spectral line,

$$E_{\infty} = E(1-2GM/Rc^2)^{1/2}$$

#### Neutron stars and white dwarfs



Remember about the difference between baryonic and gravitational masses in the case of neutron stars!

#### Minimal mass

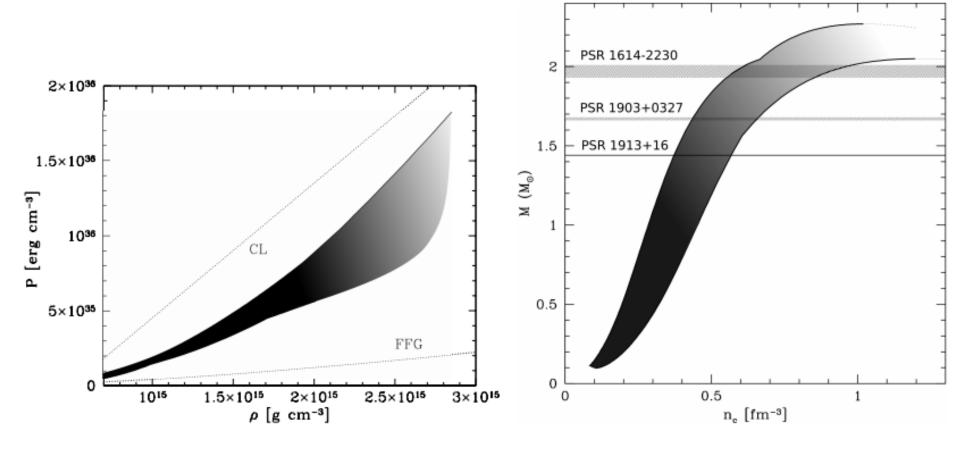
In reality, minimal mass is determined by properties of protoNSs. Being hot, lepton rich they have much higher limit: about 0.7 solar mass.

Stellar evolution does not produce NSs with baryonic mass less than about 1.1-1.2 solar mass.

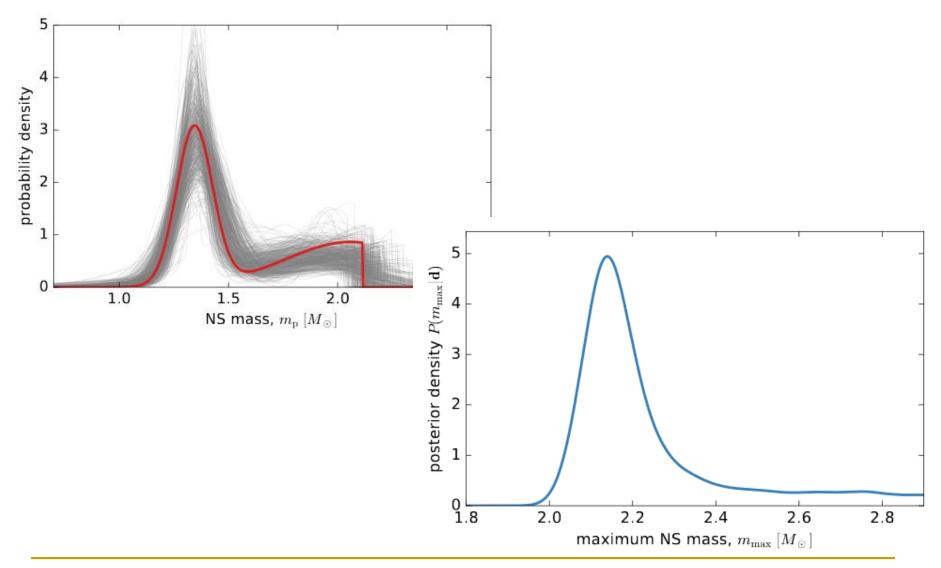
Fragmentation of a core due to rapid rotation potentially can lead to smaller masses, but not as small as the limit for cold NSs.

#### Maximum mass

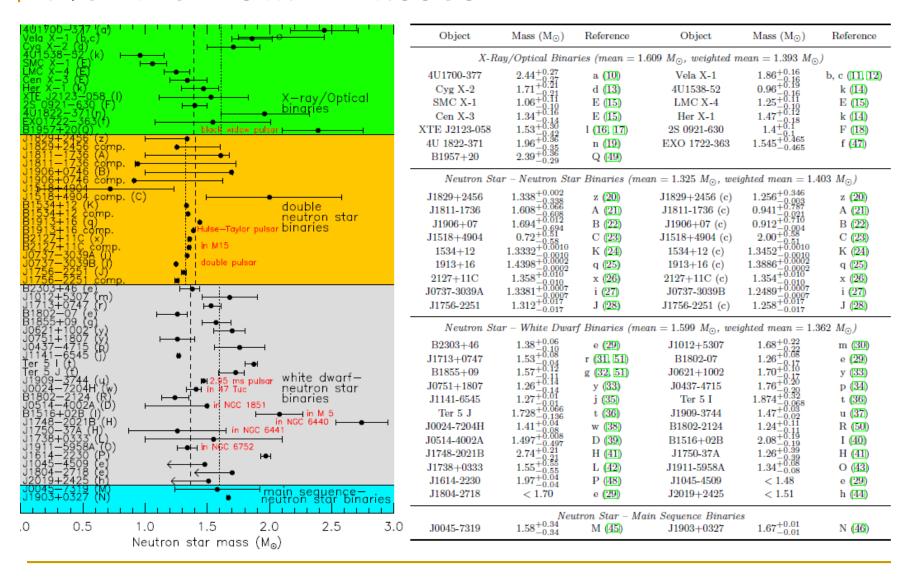
Detailed discussion about the maximum mass is given in 1307.3995



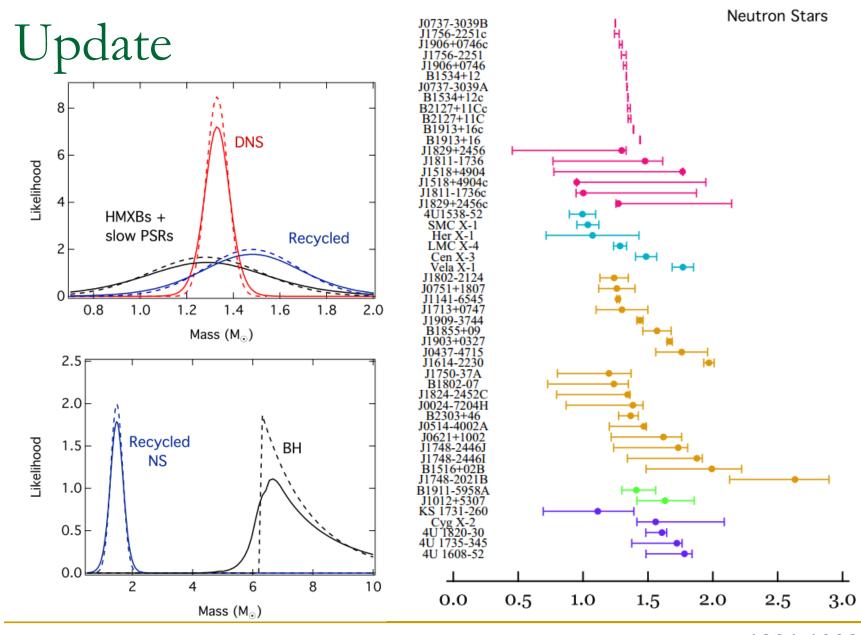
#### Maximum mass and cut-off

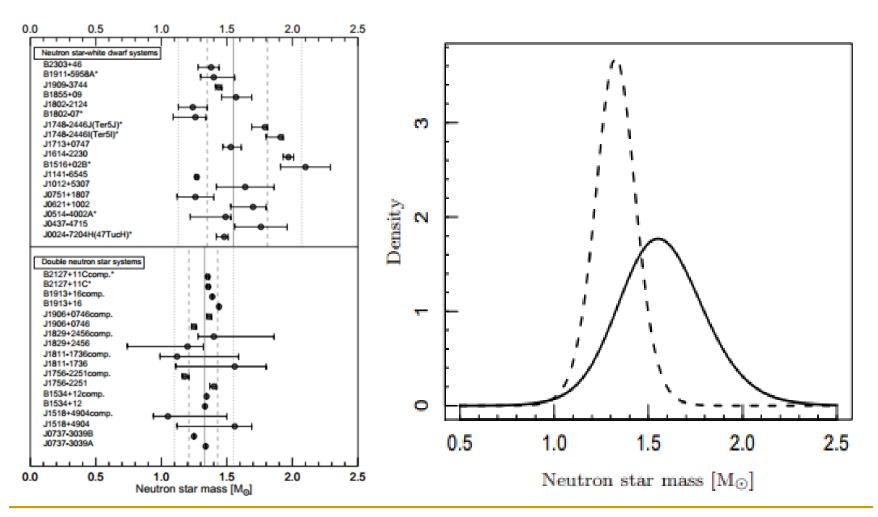


#### Neutron star masses

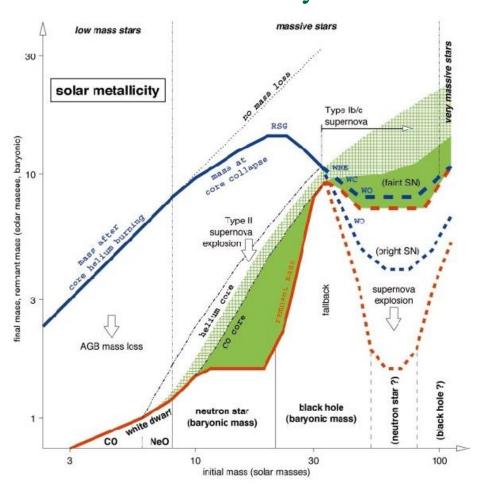


arXiv: 1012.3208





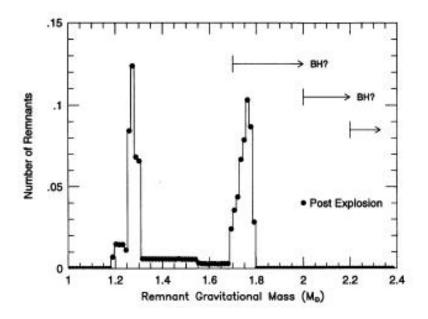
## Compact objects and progenitors. Solar metallicity.



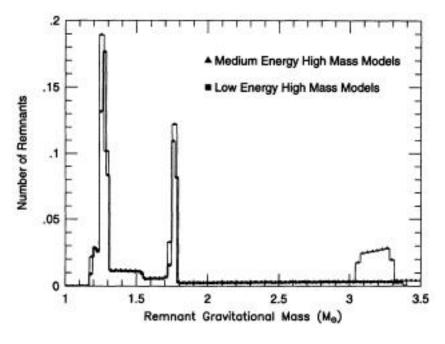
There can be a range of progenitor masses in which NSs are formed, however, for smaller and larger progenitors masses BHs appear.

(Woosley et al. 2002)

## Mass spectrum of compact objects

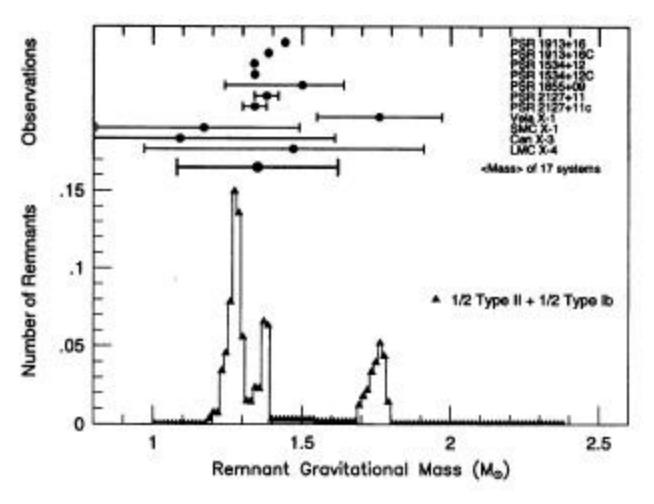


Results of calculations (depend on the assumed model of explosion)



(Timmes et al. 1996, astro-ph/9510136)

## Mass spectrum of compact objects

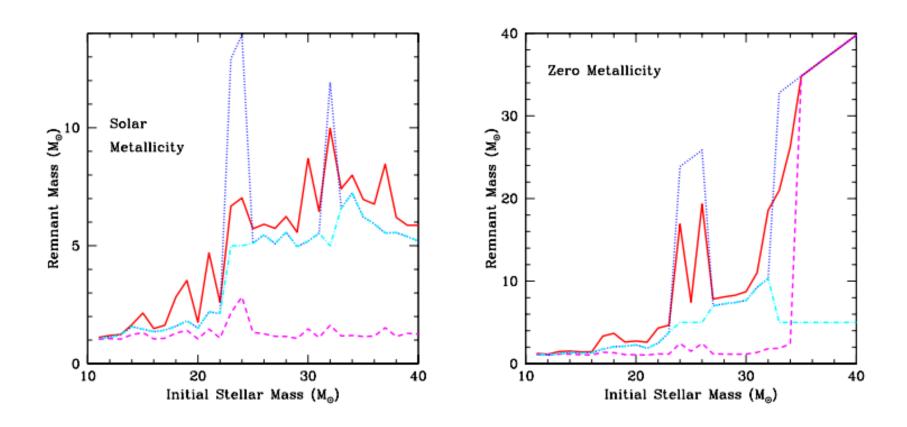


Comparison of one of the model with observations.

However, selection effects can be important as observed NSs a all in binaries.

(Timmes et al. 1996, astro-ph/9510136)

## New calculations of the mass spectrum

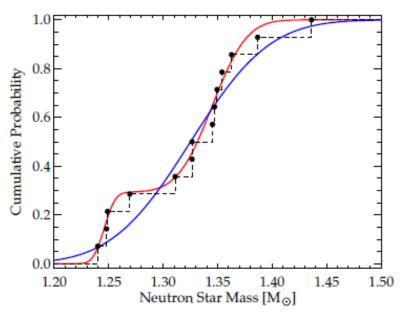


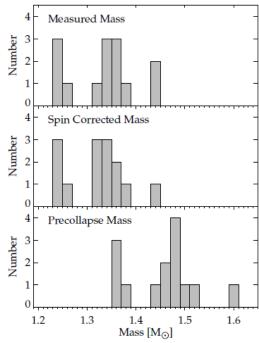
Different curves are plotted for different models of explosion: dashed – with a magnetar

## Bi-modal mass spectrum?

Pulsar Name	Mass of Recycled Neutron Star $(M_{\odot})$	Mass of Young Neutron Star (M <sub>☉</sub> )	P <sub>orb</sub> (hours)	Eccentricity	Pulse Period (ms)	Reference
J0737-3039A/B	$1.3381 \pm 0.0007$	$1.2489 \pm 0.0007$	2.4	$0.088$ $0.273$ $0.18$ $0.085$ $0.617$ $0.681$ $\leq 10^{-6}$ $0.172$	23	Kramer et al. (2006)
B1534+12	$1.3332 \pm 0.0010$	$1.3452 \pm 0.0010$	10.1		38	Stairs et al. (2002)
J1756-2251	$1.32 \pm 0.02$	$1.24 \pm 0.02$	7.67		28	Stairs (2008)
J1906+0746	$1.365 \pm 0.018$	$1.248 \pm 0.018$	3.98		144 <sup>†</sup>	Kasian (2008)
B1913+16	$1.4414 \pm 0.0002$	$1.3867 \pm 0.0002$	7.92		59	Weisberg & Taylor (2005)
B2127+11C	$1.358 \pm 0.010$	$1.354 \pm 0.010$	8.05		30	Jacoby et al. (2006)
J1909-3744	$1.438 \pm 0.024$	white dwarf	36.7		2.9	Jacoby et al. (2005)
J1141-6545	white dwarf	$1.27 \pm 0.01$	4.74		393 <sup>†</sup>	Bhat et al. (2008)

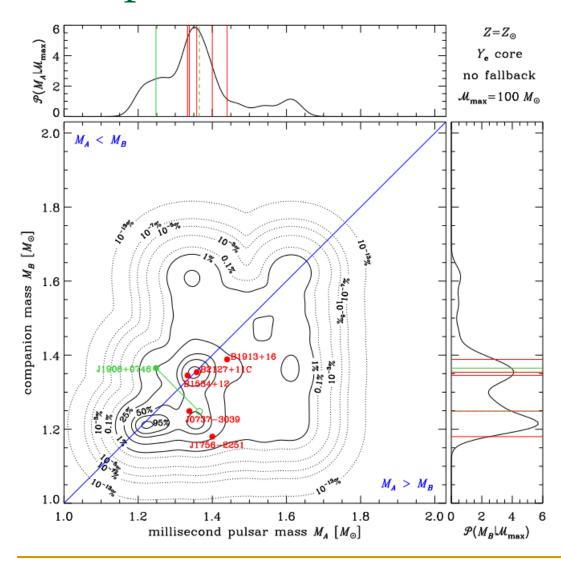
The low-mass peak the authors relate to e<sup>-</sup>-capture SN.



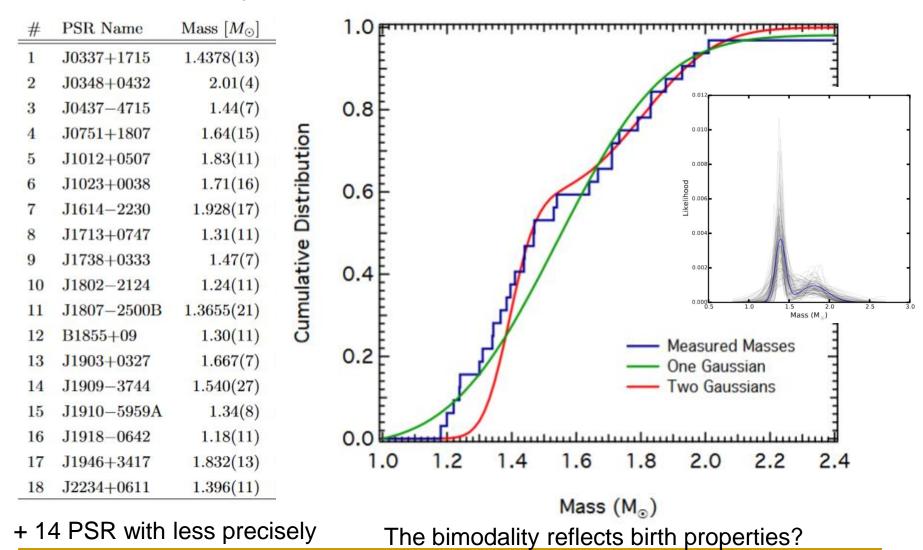


Based on 14 observed systems

#### Comparison of observations with theory



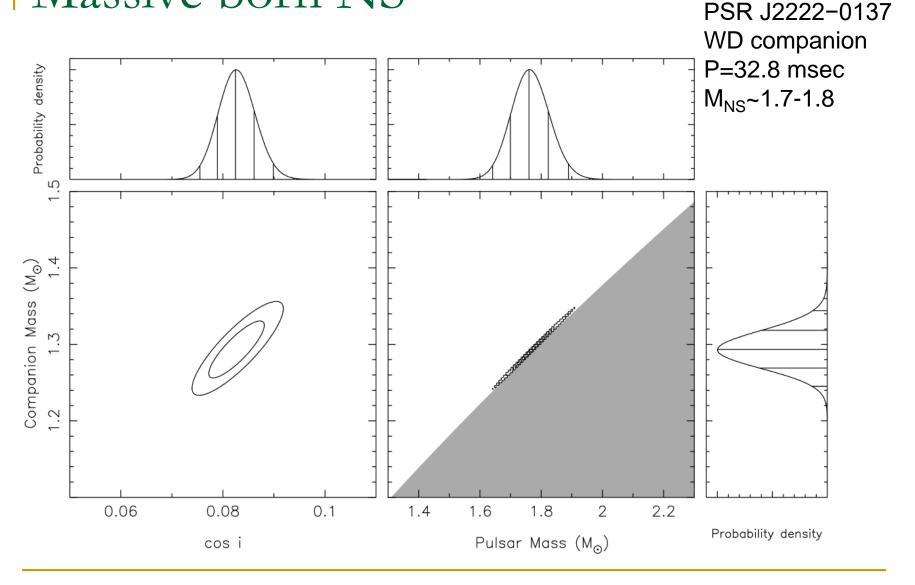
## Bimodality in mPSR mass distribution



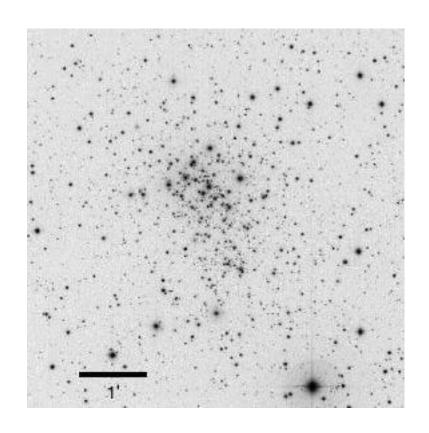
determined masses

1605.01665

#### Massive born NS



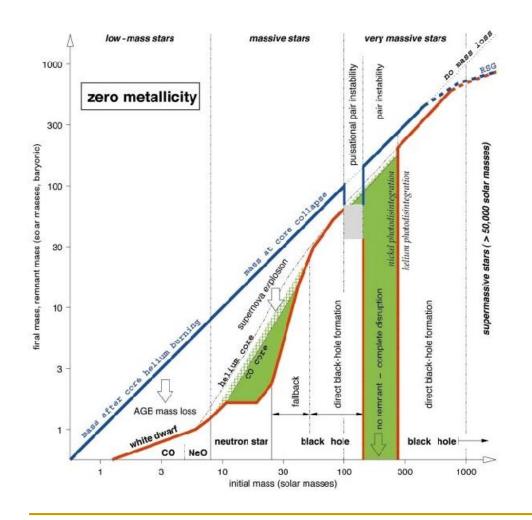
## A NS from a massive progenitor



Anomalous X-ray pulsar in the association Westerlund1 most probably has a very massive progenitor, >40 M<sub>O</sub>.

(astro-ph/0611589)

#### The case of zero metallicity

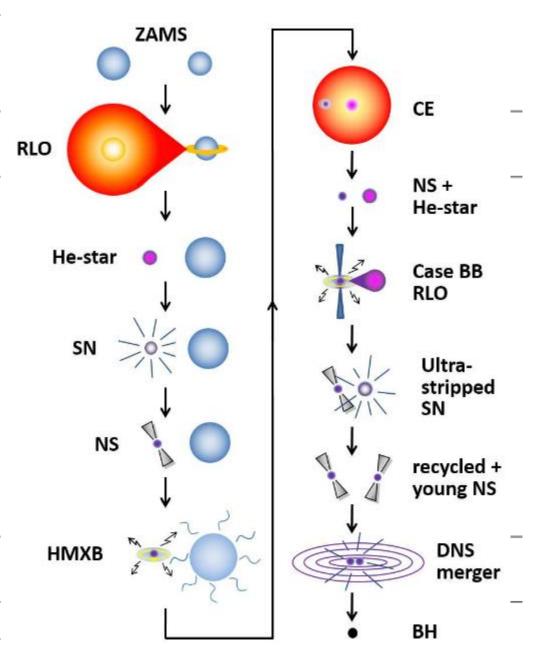


No intermediate mass range for NS formation.

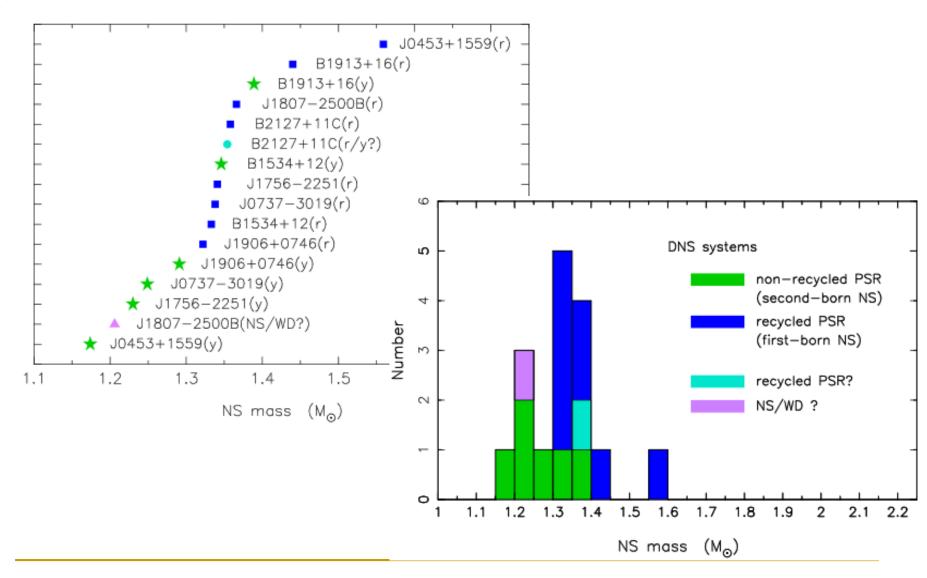
(Woosley et al. 2002)

## DNS

Radio Pulsar	Туре	P (ms)
70.1001000		
$J0453+1559^a$	recycled	45.8
$J0737 - 3039A^{b}$	recycled	22.7
$J0737 - 3039B^{b}$	young	2773.5
$J1518+4904^{c}$	recycled	40.9
$B1534+12^d$	recycled	37.9
$J1753-2240^e$	recycled	95.1
$J1755-2550^{f}*$	young	315.2
$J1756-2251^{g}$	recycled	28.5
$J1811-1736^{h}$	recycled	104.2
$J1829+2456^{i}$	recycled	41.0
$J1906+0746^{j*}$	young	144.1
$J1913+1102^k$	recycled	27.3
$B1913+16^{l}$	recycled	59.0
$J1930-1852^m$	recycled	185.5
$J1807-2500B^{n*}$	GC	4.2
$B2127+11C^{p}$	GC	30.5



#### Mass distribution



## PSR J1518+4904

[Janssen et al. arXiv: 0808.2292]

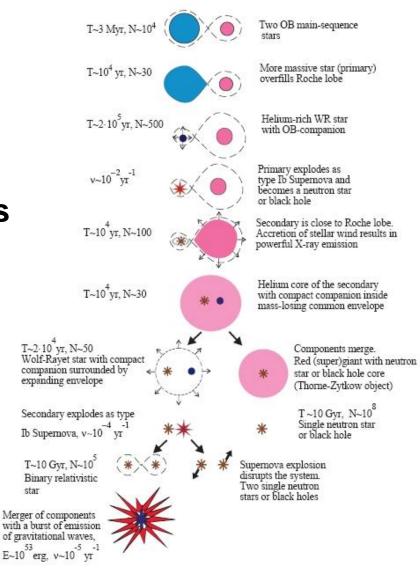
#### **Surprising results !!!**

Mass of the recycled pulsar is <1.17 solar masses
Mass of its component is >1.55 solar masses

Central values are even more shocking:

 $0.72^{+0.51}_{-0.58}$  and  $2.00^{+0.58}_{-0.51}$ 

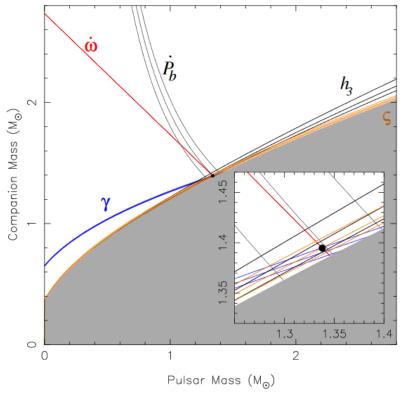
V~25 km/s, e~0.25 The second SN was e<sup>-</sup>-capture?



New measurements show less extreme values, see table 1 in 1603.02698: <1.768 and >0.95 solar masses. Total mass is the same 2.7183 solar masses.

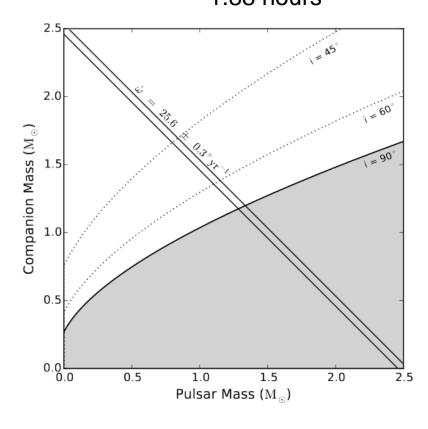
#### Recent discoveries with records

PSR J1757-1854 4.4 hours



 $\dot{\omega} \simeq 10.37^{\circ} \,\mathrm{yr}^{-1}$ 

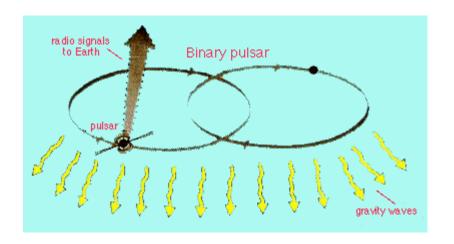
PSR J1946+2052 1.88 hours



$$\dot{\omega} = 25.6 \pm 0.3 \,\mathrm{deg}\,\mathrm{yr}^{-1},$$

1711.07697 1802.01707

## Binary pulsars



$$\frac{d\Delta_{\rm E\odot}}{dt} = \sum_{i} \frac{Gm_i}{c^2 r_i} + \frac{v_{\oplus}^2}{2c^2} - {\rm constant} \ .$$

$$\Delta_{S\odot} = -\frac{2GM_{\odot}}{c^3}\log(1+\cos\theta),\,$$

$$\begin{split} T = t_{\text{obs}} - t_0 + \Delta_C - D/f^2 + \Delta_{R\odot}(\alpha, \delta, \mu_{\alpha}, \mu_{\delta}, \pi) \\ + \Delta_{E\odot} - \Delta_{S\odot}(\alpha, \delta) \\ - \Delta_R(x, e, P_b, T_0, \omega, \dot{\omega}, \dot{P}_b) - \Delta_E(\gamma) - \Delta_S(r, s) \end{split}$$

See 1502.05474 for a recent detailed review

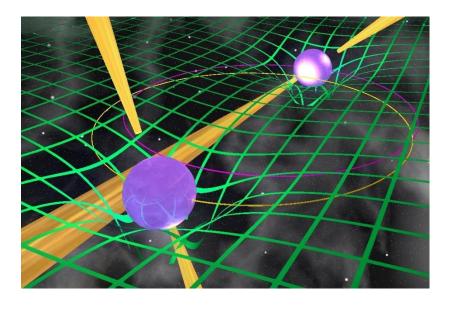
## Relativistic corrections and measurable parameters

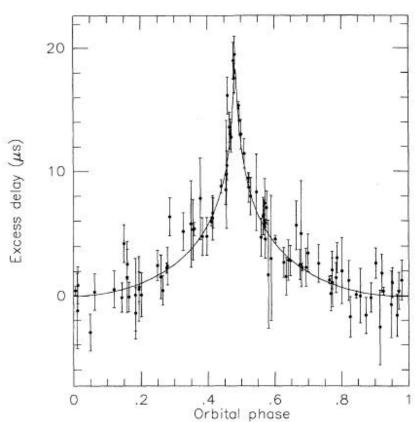
$$\begin{split} \dot{\omega} &= 3 \left[ \frac{P_b}{2\pi} \right]^{-5/3} (T_{\odot} M)^{2/3} (1 - e^2)^{-1} , \\ \gamma &= e \left[ \frac{P_b}{2\pi} \right]^{1/3} T_{\odot}^{2/3} M^{-4/3} m_2 (m_1 + 2m_2) , \\ \dot{P}_b &= -\frac{192\pi}{5} \left[ \frac{P_b}{2\pi} \right]^{-5/3} \left[ 1 + \frac{73}{24} e^2 + \frac{37}{96} e^4 \right] \\ &\qquad \times (1 - e^2)^{-7/2} T_{\odot}^{5/3} m_1 m_2 M^{-1/3} , \\ r &= T_{\odot} m_2 , \\ s &= x \left[ \frac{P_b}{2\pi} \right]^{-2/3} T_{\odot}^{-1/3} M^{2/3} m_2^{-1} . \end{split}$$

For details see Taylor, Weisberg 1989 ApJ 345, 434

## Shapiro delay

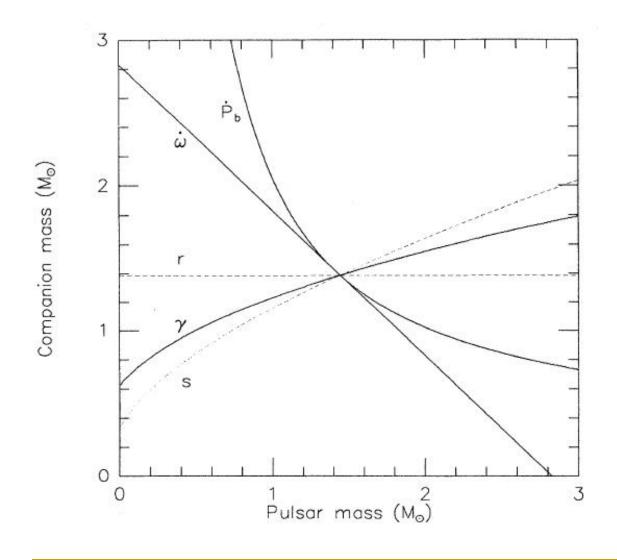
$$\Delta_S = -2r \log(1 - s \cos[2\pi(\phi - \phi_0)])$$





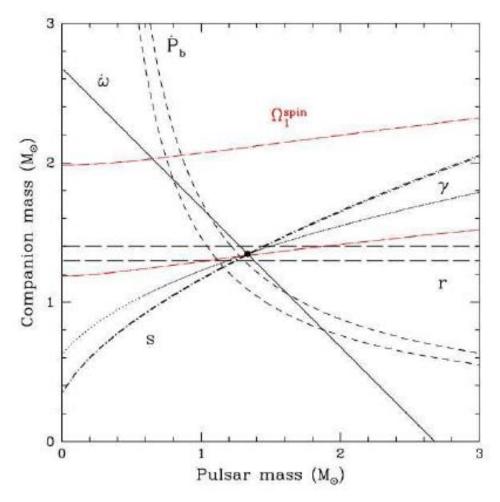
PSR 1855+09 (Taylor, Nobel lecture)

#### Mass measurements



PSR 1913+16

## Uncertainties and inverse problems

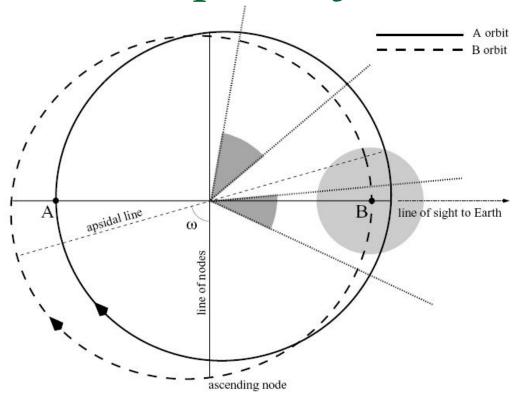


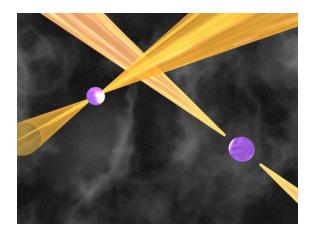
P<sub>b</sub>dot depends on the Shklovskii effect. So, if distance is not certain, it is difficult to have a good measurement of this parameter.

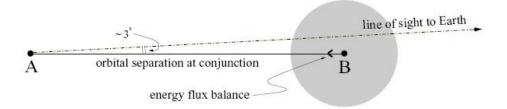
It is possible to invert the problem. Assuming that GR is correct, one can improve the distance estimate for the given source.

PSR B1534+12.

## Double pulsar J0737-3039

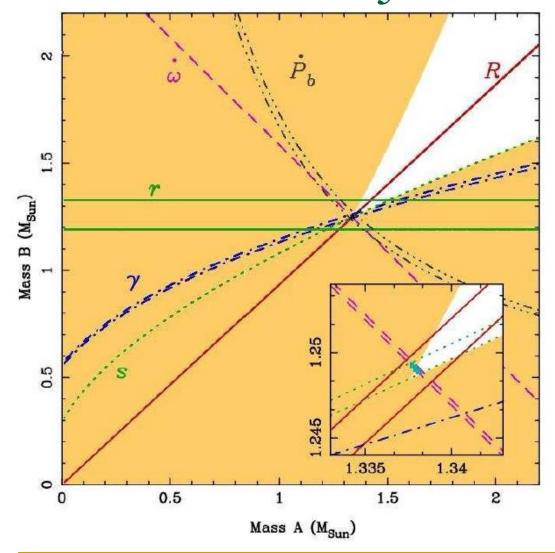






(Lyne et al. astro-ph/0401086)

## Masses for PSR J0737-3039

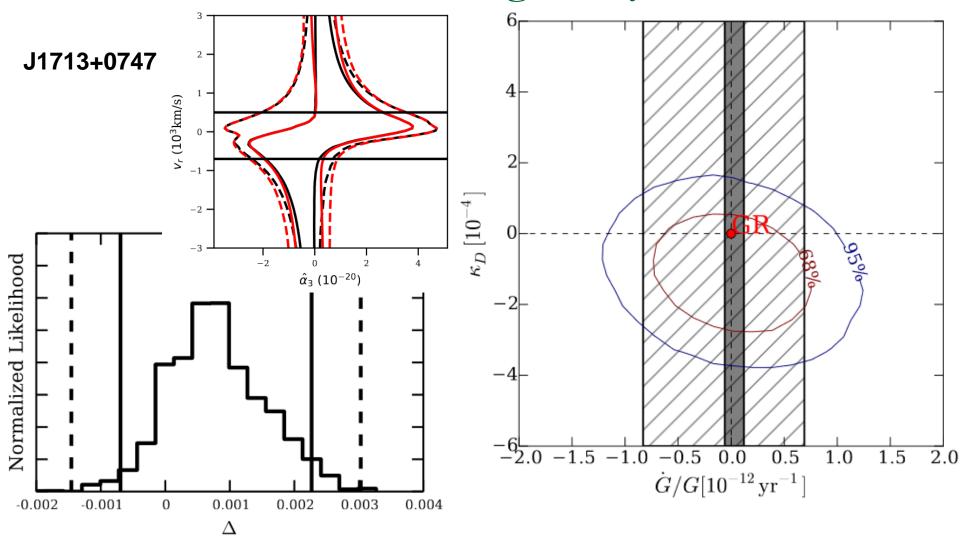


The most precise values.

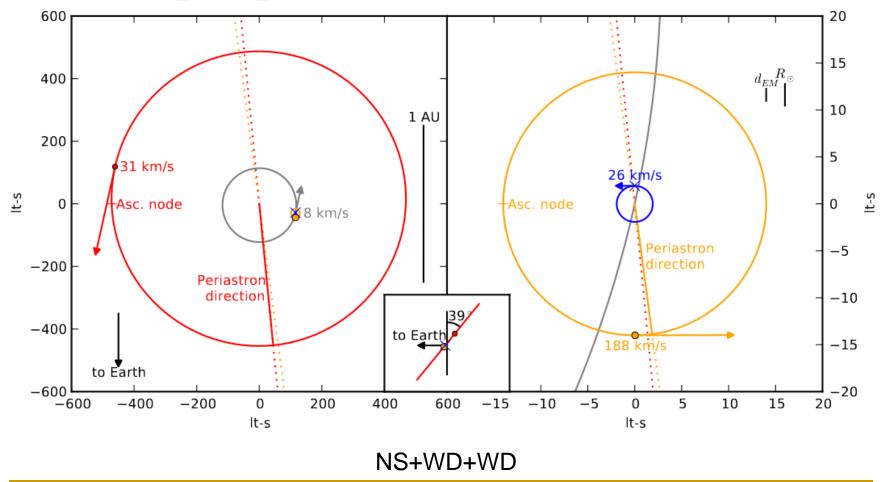
New mass estimates have uncertainties < 0.001

(Kramer et al. astro-ph/0609417)

## Tests of theories of gravity



# Testing strong equivalence principle with triple pulsar PSR J0337+1715



#### NS+WD binaries

#### Some examples

PSR J0437-4715. WD companion [0801.2589, 0808.1594]. The closest millisecond PSR.  $M_{NS}$ =1.76+/-0.2 solar.

The case of PSR J0751+1807.

Initially, it was announced that it has a mass ~2.1 solar [astro-ph/0508050]. However, then in 2007 at a conference the authors announced that the result was incorrect. Actually, the initial value was 2.1+/-0.2 (1 sigma error).

New result: 1.26 +/- 0.14 solar

[Nice et al. 2008, Proc. of the conf. "40 Years of pulsars"]

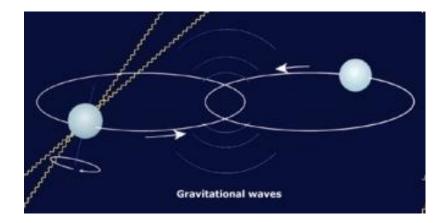
It is expected that most massive NSs get their additional "kilos" due to accretion from WD companions [astro-ph/0412327].

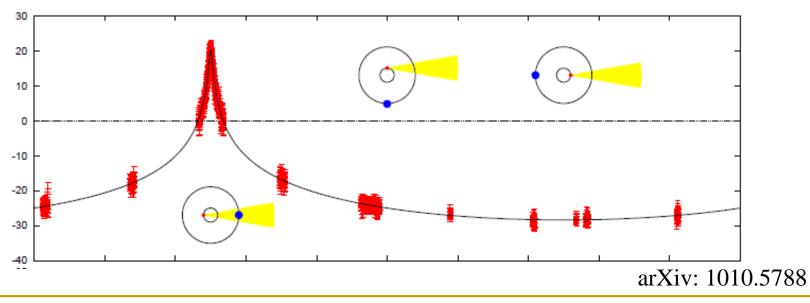
## Very massive neutron star

Binary system: pulsar + white dwarf PSR 1614-2230

Mass ~ 2 solar

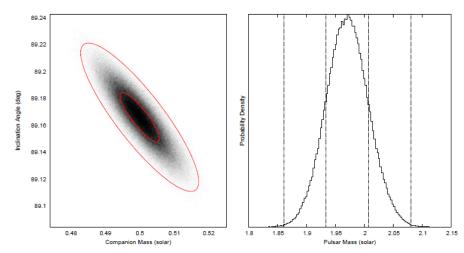
About the WD see 1106.5497. The object was identified in optics.





About formation of this objects see 1103.4996

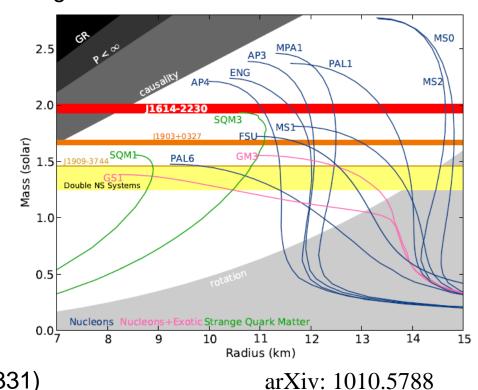
#### Why is it so important?



Collapse happens earlier for softer EoSs, see however, 1111.6929 about quark and hybrid stars to explain these data.

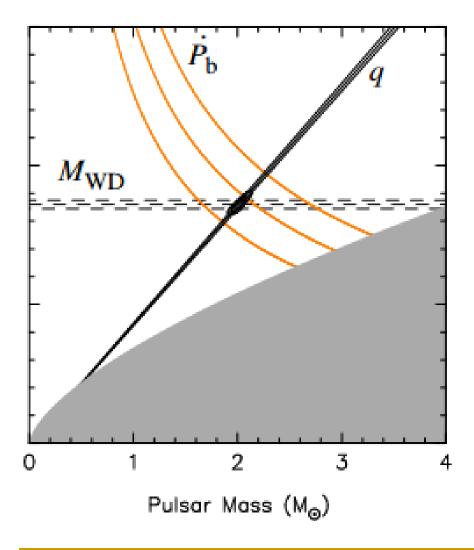
Interestingly, it was suggested that just <0.1 solar masses was accreted (1210.8331)

The maximum mass is a crucial property of a given EoS



In the future specific X-ray sources (eclipsing msec PSR like SWIFT J1749.4-2807) can show Shapiro delay and help to obtain masses for a different kind of systems, see 1005.3527, 1005.3479.

#### 2.01 solar masses NS

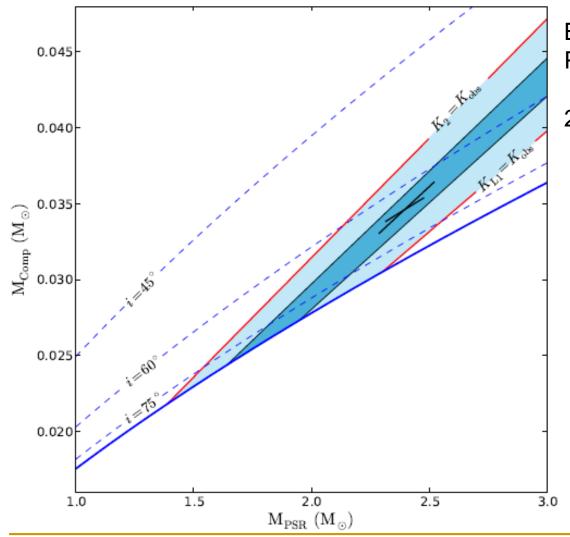


PSR J0348+0432 39 ms, 2.46 h orbit WD companion

The NS mass is estimated to be: 1.97 – 2.05 solar mass at 68.27% 1.90 – 2.18 solar mass at 99.73% confidence level.

System is perfect for probing theories of gravity as it is very compact.

## The most extreme (but unclear) example



BLACK WIDOW PULSAR PSR B1957+20

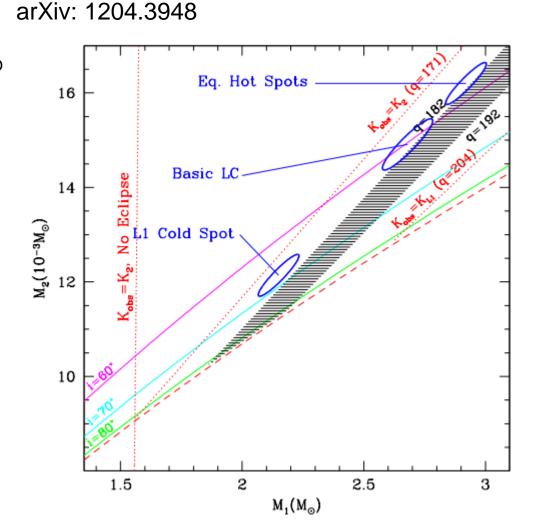
2.4+/-0.12 solar masses

### More measurements

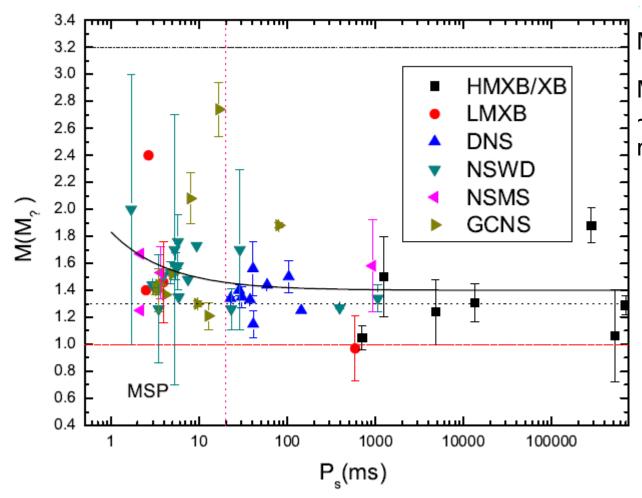
PSR J1738+0333 NS+WD

 $M_{WD} = 0.181 + 0.007 - 0.005 M_{O}$  $M_{PSR} = 1.47 + 0.07 - 0.06 M_{O}$ 

PSR J1311-3430 arXiv: 1210.6884 MPSR>2.1 at least!



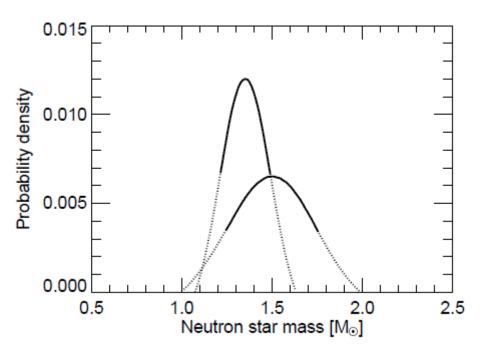
### How much do PSRs accrete?



 $M=1.4+0.43(P/ms)^{-2/3}$ 

Millisecond pulsars are ~0.2 solar masses more massive than the rest ones.

### DNS and NS+WD binaries



1.35+/-0.13 and 1.5+/-0.25

Cut-off at ~2.1 solar masses can be mainly due to evolution in a binary, not due to nuclear physics (see 1309.6635)

## Neutron stars in binaries

Study of close binary systems gives an opportunity to obtain mass estimate for progenitors of NSs (see for example, Ergma, van den Heuvel 1998 A&A 331, L29). For example, an interesting estimate was obtained for GX 301-2.

The progenitor mass is >50 solar masses.

On the other hand, for several other systems with both NSs and BHs progenitor masses a smaller: from 20 up to 50.

Finally, for the BH binary LMC X-3 the progenitor mass is estimated as >60 solar. So, the situation is tricky.

Most probably, in some range of masses, at least in binary systems, stars can produce both types of compact objects: NSs and BHs.



# Mass determination in binaries: mass function

$$f_v(m) \frac{m_x^3 \sin i^3}{(m_x + m_v)^2} = 1,038 \cdot 10^{-7} K_v^3 P(1 - e^2)^{3/2},$$

 $m_x$ ,  $m_v$  - masses of a compact object and of a normal star (in solar units),  $K_v$  - observed semi-amplitude of line of sight velocity of the normal star (in km/s), P - orbital period (in days), e - orbital eccentricity, i - orbital inclination (the angle between the orbital plane and line of sight).

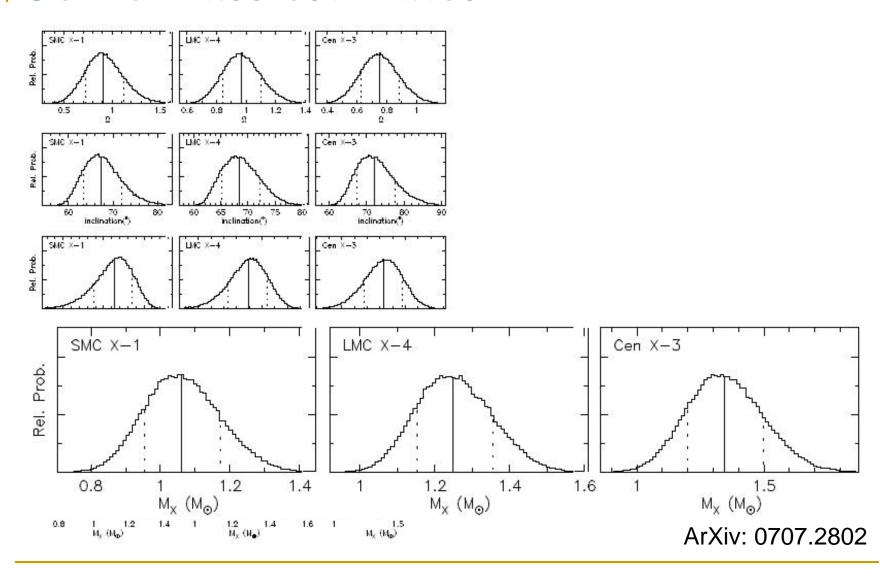
One can see that the mass function is the lower limit for the mass of a compact star.

The mass of a compact object can be calculated as:

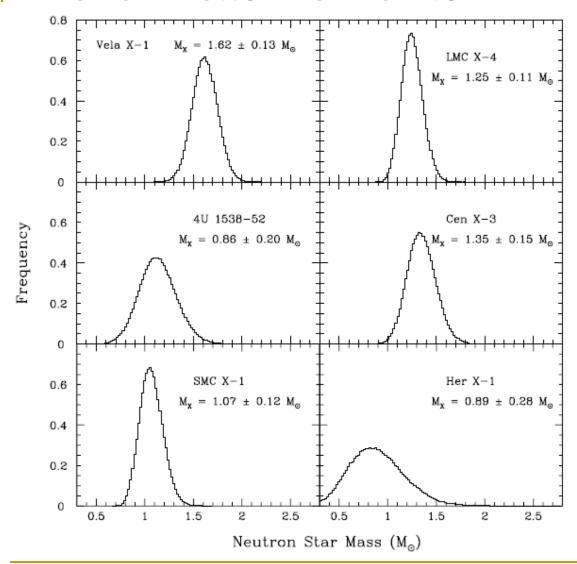
$$m_x = f_v(m) \left( 1 + \frac{m_v}{m_x} \right)^2 \frac{1}{\sin i^3} \,.$$

So, to derive the mass it is necessary to know (besides the line of sight velocity) independently two more parameters: mass ration  $q=m_x/m_v$ , and orbital inclination i.

### Some mass estimates

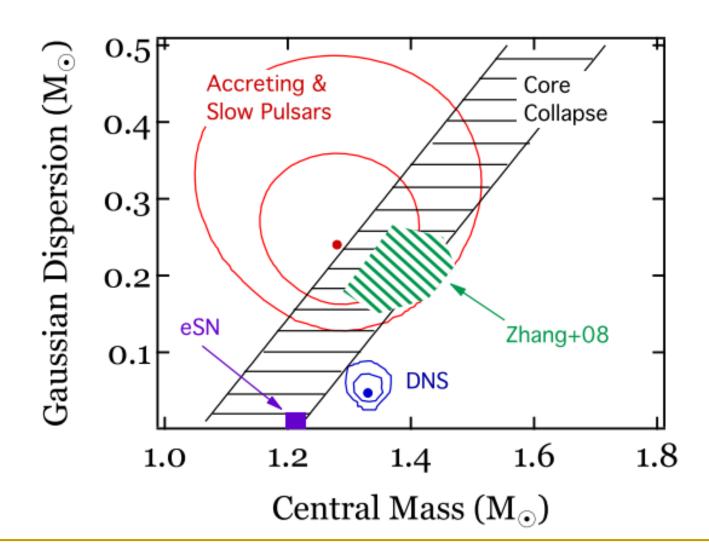


### More measurements



Six X-ray binary systems. All are eclipsing pulsars.

## Altogether



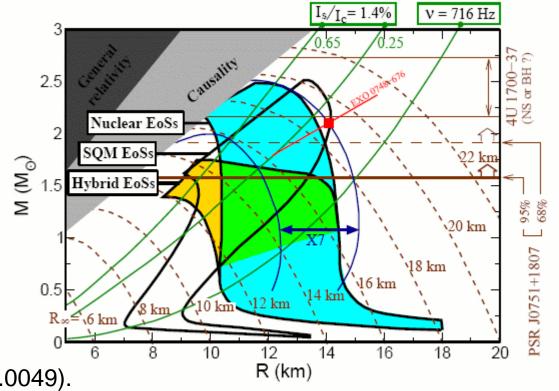
## Mass-radius diagram and constraints

Unfortunately, there are no good data on independent measurements of masses and radii of NSs.

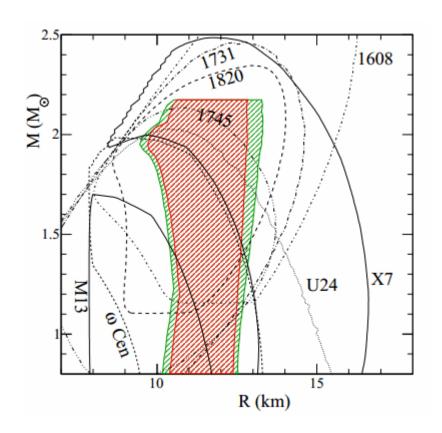
Still, it is possible to put important constraints.

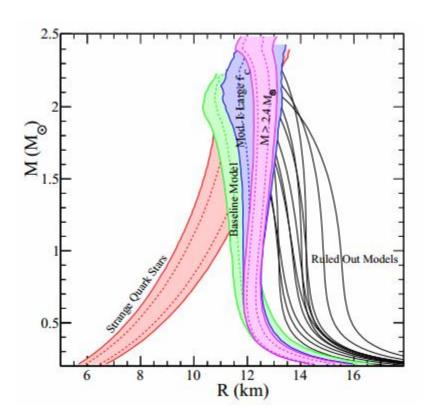
Most of recent observations favour stiff EoS.

Useful analytical estimates for EoS can be found in 1310.0049).



### Observations vs. data





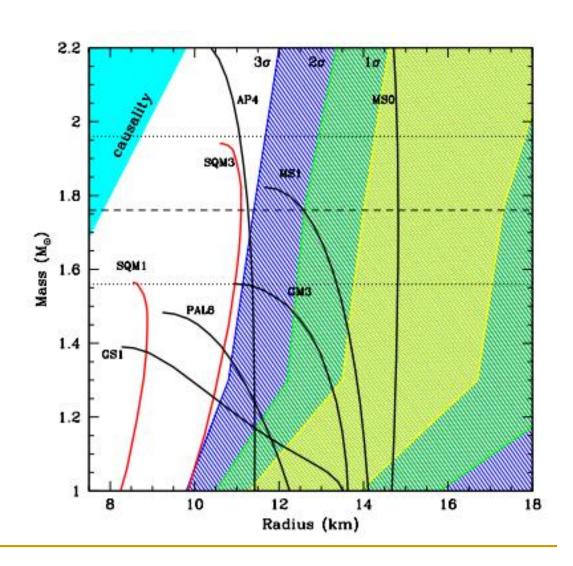
1205.6871

Some newer results by the same group are presented in 1305.3242

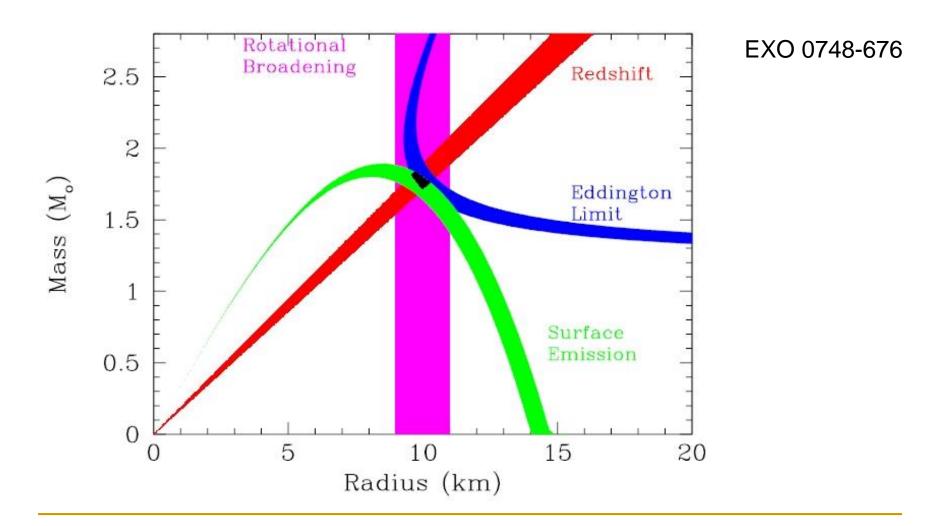
## Mass and radius for a pulsar!

PSR J0437-4715 NS+WD

The nearest known mPSR 155-158 pc

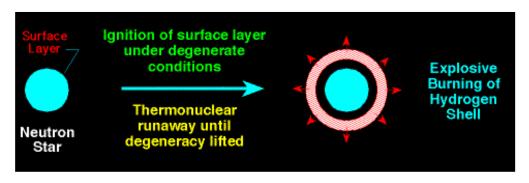


### Combination of different methods



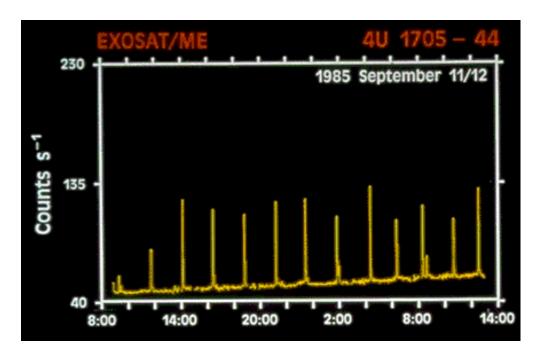
(Ozel astro-ph/0605106)

### Radius determination in bursters



Explosion with a ~ Eddington liminosity.

Modeling of the burst spectrum and its evolution.



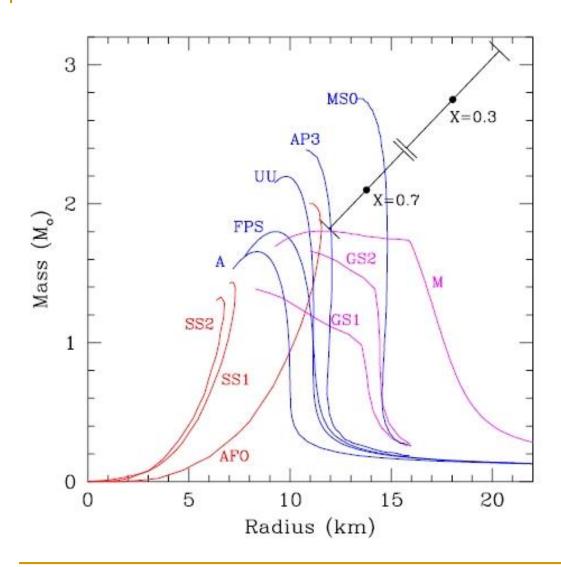
See, for example, Joss, Rappaport 1984, Haberl, Titarchuk 1995

### More measurements

#### Continuously new measurements, critics and discussion appears

- 1104.2602 Systematic Uncertainties in the Spectroscopic Measurements of Neutron-Star Masses and Radii from Thermonuclear X-ray Bursts. II. Eddington Limit
- 1104.5027 The Mass and Radius of the Neutron Star in the Bulge Low-Mass X-ray Binary KS 1731-260
- 1103.5767 Systematic Uncertainties in the Spectroscopic Measurements of Neutron-Star Masses and Radii from Thermonuclear X-ray Bursts. I. Apparent Radii
- 1105.1525 Mass and radius estimation for the neutron star in X-ray burster 4U 1820-30
- 1105.2030 New Method for Determining the Mass and Radius of Neutron Stars
- 1106.3131 Constraints on the Mass and Radius of the Neutron Star XTE J1807-294
- 1111.0347 Constraints on neutron star mass and radius in GS 1826-24 from sub-Eddington X-ray bursts
- 1201.1680 On the consistency of neutron-star radius measurements from thermonuclear bursts
- 1204.3627 Constraints on the mass and radius of the accreting neutron star in the Rapid Burster
- 1301.0831 The mass and the radius of the neutron star in the transient low mass X-ray binary SAX J1748.9-2021

### Limits on the EoS from EXO 0748-676



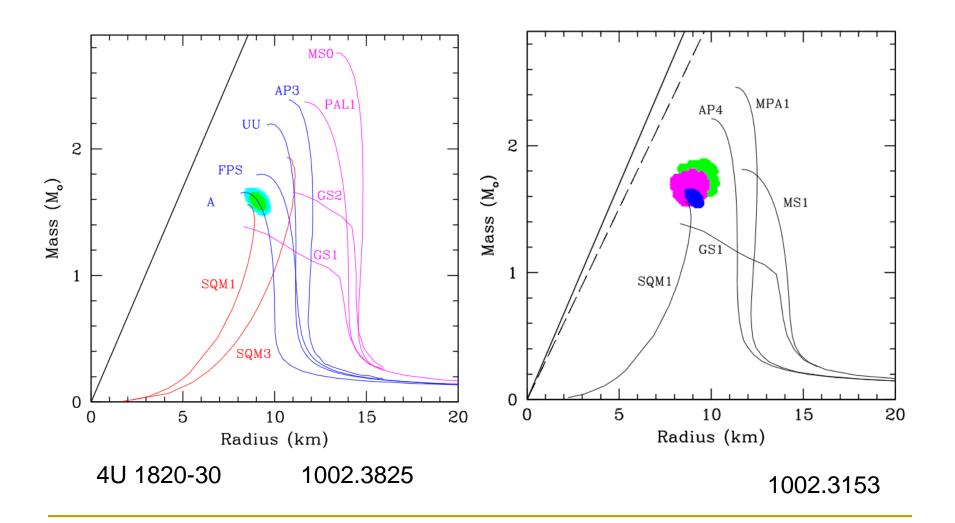
Stiff EoS are better.

Many EoS for strange
matter are rejected.

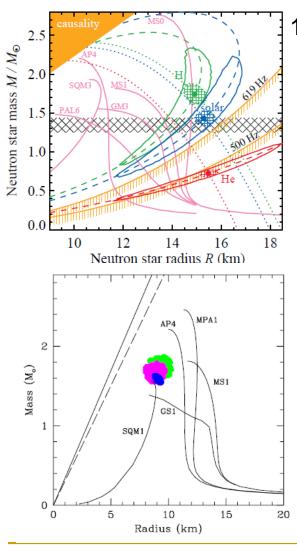
But no all! (see discussion in Nature).

X- hydrogen fraction in the accreted material

## Some optimistic estimates

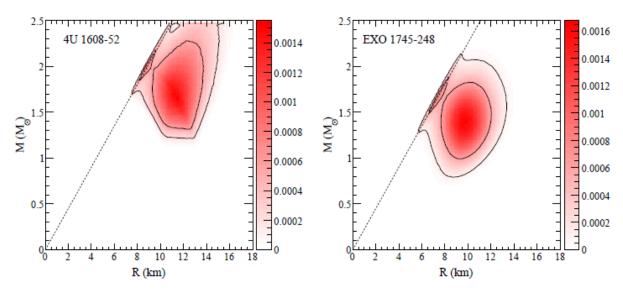


### Pessimistic estimates



1004.4871





It seems that Ozel et al. underestimate different uncertainties and make additional assumptions.

1002.3153

### Radius measurement

Fitting X-ray spectrum of a low-mass X-ray binary in quiescent state.

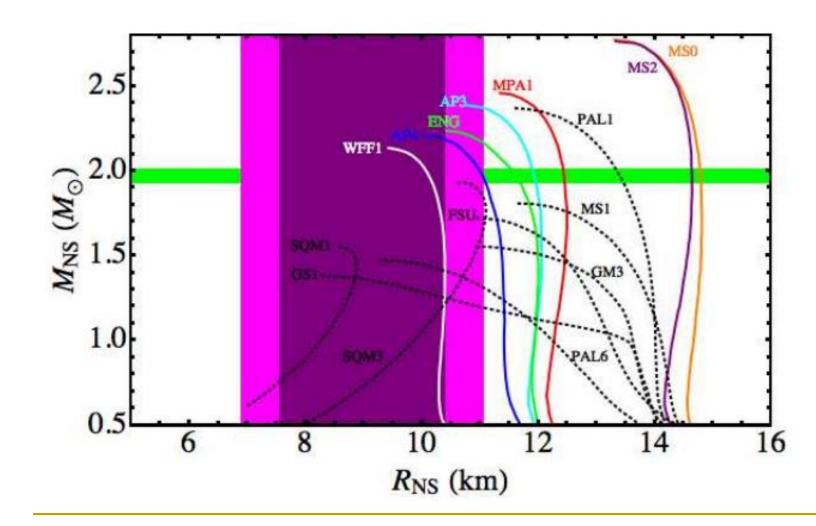
Mostly sources in globular clusters.

For 4 objects ~10% precision. But this is for fixed mass.

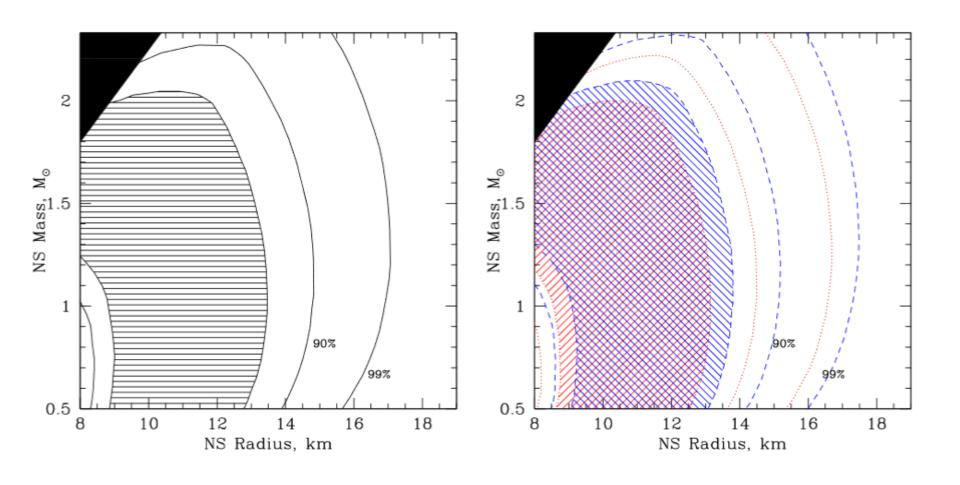
For U24 in NGC 6397  $R_{NS}$ =8.9<sup>+0.9</sup><sub>-0.6</sub> km for 1.4 solar masses. For the radius observed from infinity: 11.9<sup>+2.2</sup><sub>-2.5</sub> km

## Radii measurements for qLMXBs in GCs

5 sources

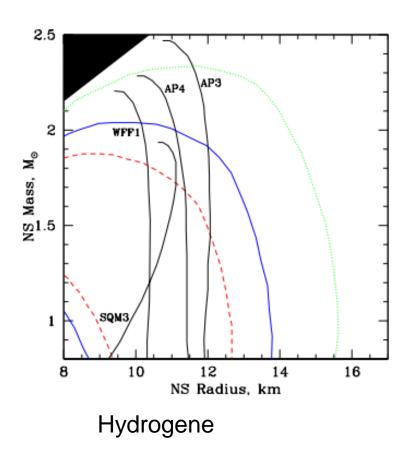


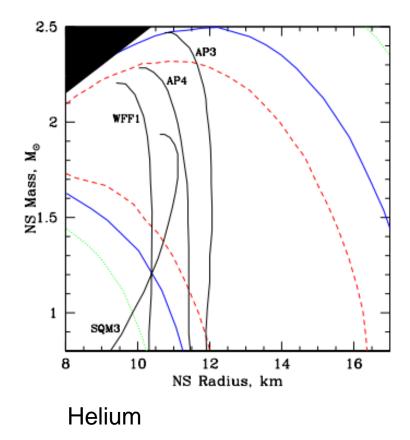
## Distance uncertainty



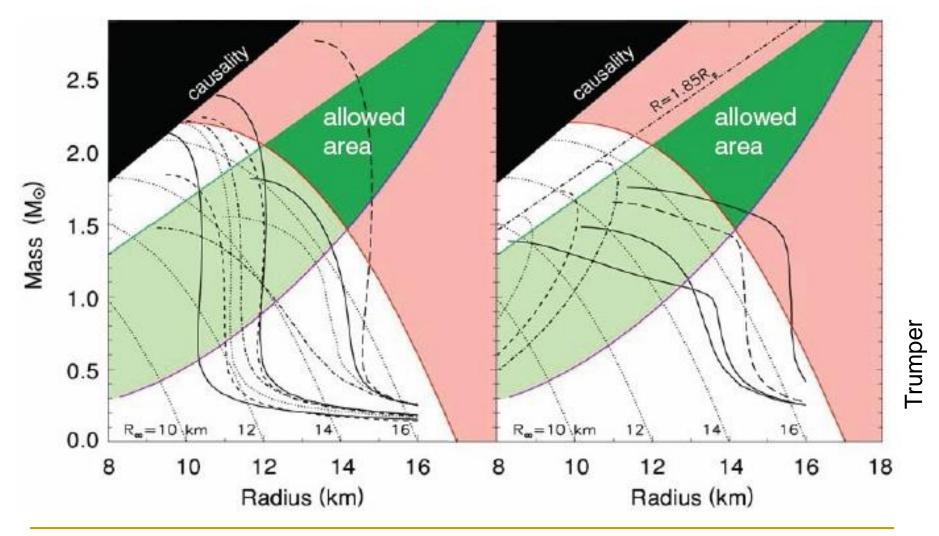
## Atmospheric uncertainties

#### qLMXB in M13



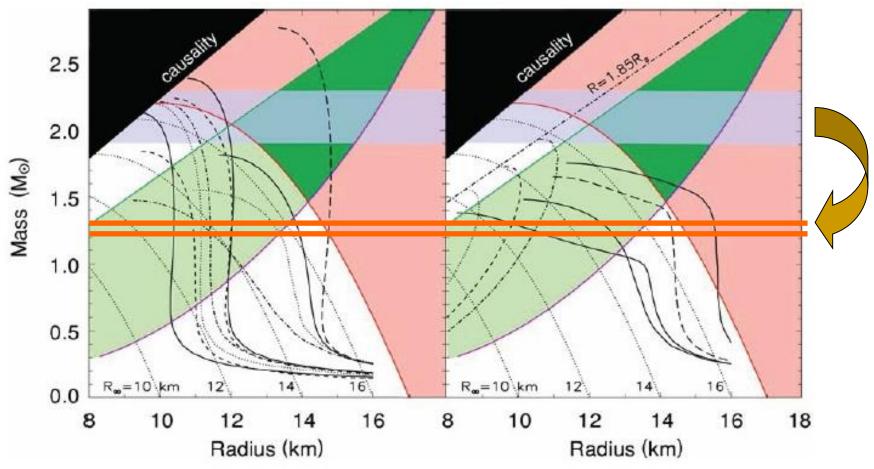


## Limits from RX J1856



About M7 for constraints on the EoS see 1111.0447

### PSR 0751+1807

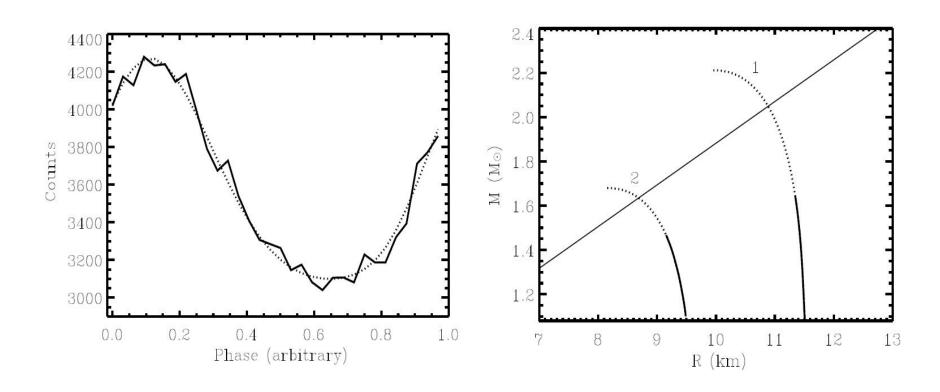


Massive NS: 2.1+/-0.3 solar masses – Now shown to be wrong (!) [see Nice et al. 2008]

(Trumper)

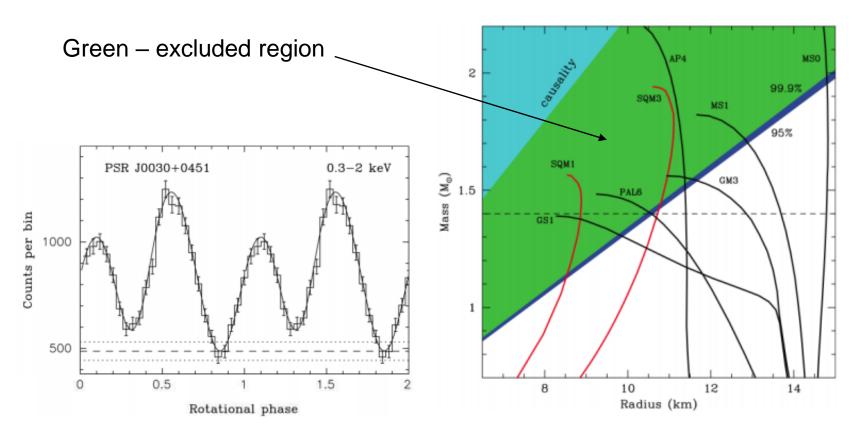
## Burst oscillations

Fitting light curves of X-ray bursts. Rc<sup>2</sup>/GM > 4.2 for the neutron star in XTE J1814-338



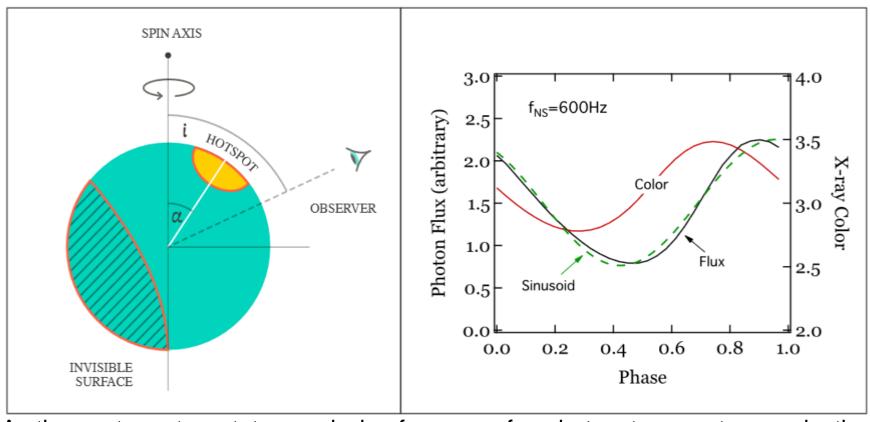
## Pulse profile constraints

The idea is that: sharp pulses are possible only in the case of a large star



Based on Bogdanov, Grindlay 2009

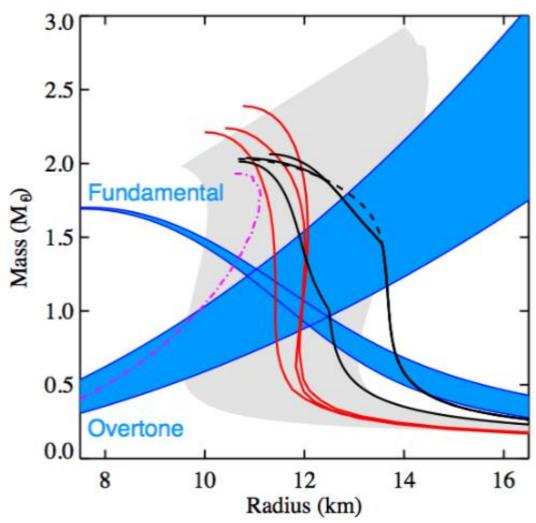
## Hot spots and pulse profiles



As the neutron star rotates, emission from a surface hotspot generates a pulsation. The figure shows observer inclination i, and hotspot inclination  $\alpha$ .

The invisible surface is smaller than a hemisphere due to relativistic light-bending.

## Astroseismology



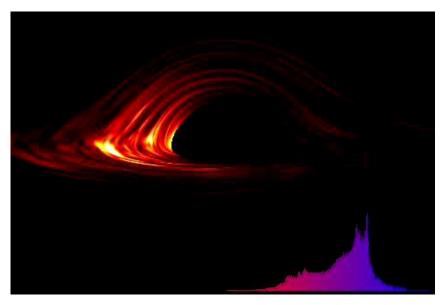
M – R diagram showing the seismological constraints for the soft gamma-ray repeater SGR 1806-20 using the relativistic torsional crust oscillation model of Samuelsson and Andersson (2007), in which the 29 Hz QPO is identified as the fundamental and the 625 Hz QPO as the first radial overtone. The neutron star lies in the box where the constraints from the two frequency bands overlap.

This is a simplified model.

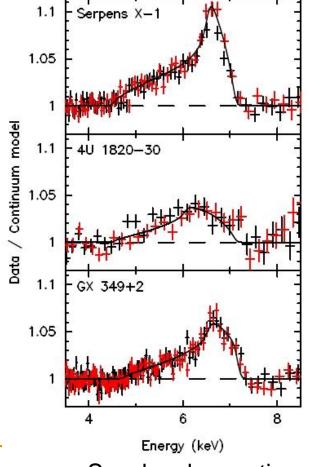
More detailed are in progress.

### Fe K lines from accretion discs

Measurements of the inner disc radius provide upper limits on the NS radius.



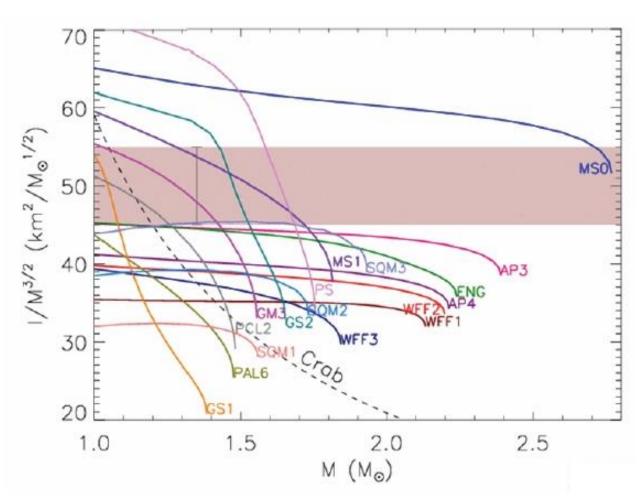
Ser X-1 <15.9+/-1 4U 1820-30 <13.8+2.9-1.4 GX 349+2 <16.5+/-0.8 (all estimates for 1.4 solar mass NS) [Cackett et al. arXiv: 0708.3615]



Suzaku observations

See also Papito et al. arXiv: 0812.1149, a review in Cackett et al. 0908.1098, and theory in 1109.2068.

### Limits on the moment of inertia



Spin-orbital interaction

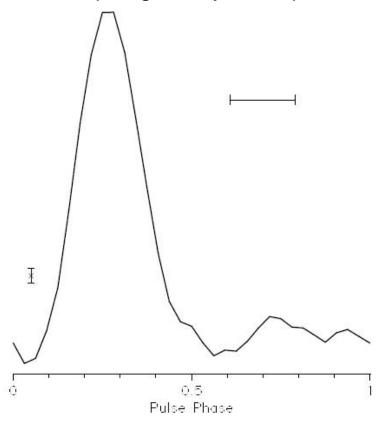
PSR J0737-3039 (see Lattimer, Schutz astro-ph/0411470)

The band refers to a hypothetical 10% error. This limit, hopefully, can be reached in several years of observ.

See a more detailed discussion in 1006.3758

## Most rapidly rotating PSR

716-Hz eclipsing binary radio pulsar in the globular cluster Terzan 5



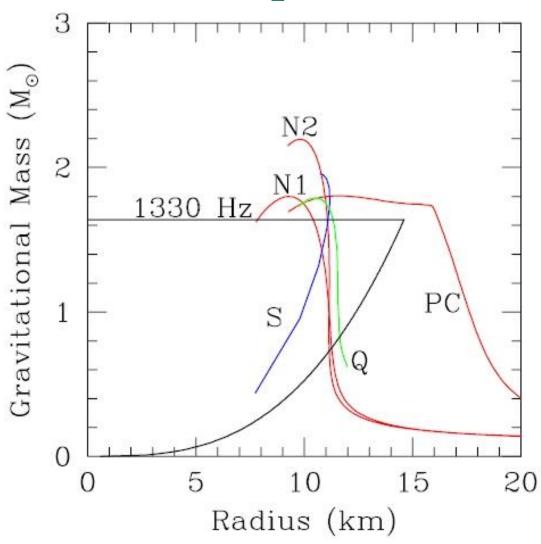
Previous record (642-Hz pulsar B1937+21) survived for more than 20 years.

Interesting calculations for rotating NS have been performed recently by Krastev et al. arXiv: 0709.3621

Rotation starts to be important from periods ~3 msec.

(Jason W.T. Hessels et al. astro-ph/0601337)

## QPO and rapid rotation



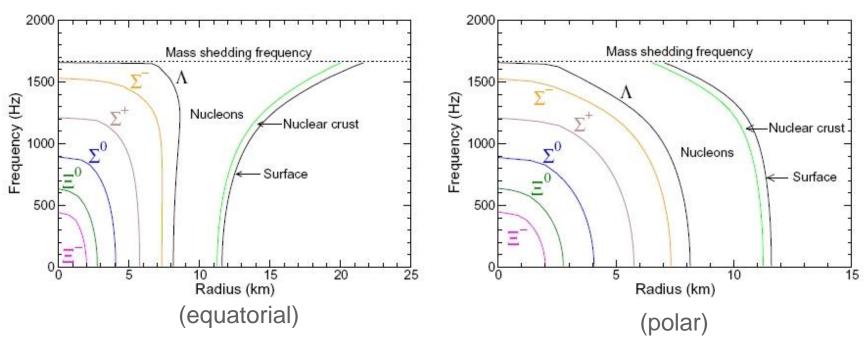
XTE J1739-285 1122 Hz P. Kaaret et al. astro-ph/0611716

1330 Hz – one of the highest QPO frequency

The line corresponds to the interpretation, that the frequency is that of the last stable orbit, 6GM/c<sup>2</sup>

(Miller astro-ph/0312449)

## Rotation and composition

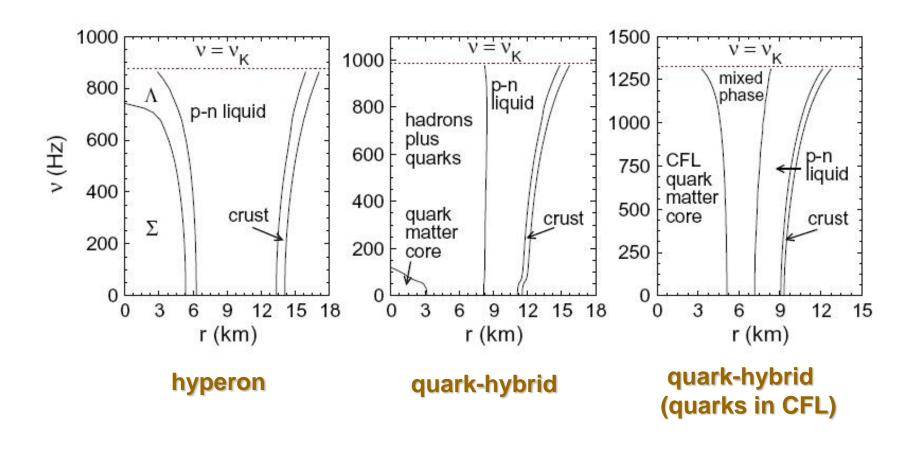


Computed for a particular model: density dependent relativistic Brueckner-Hartree-Fock (DD-RBHF)

(Weber et al. arXiv: 0705.2708)

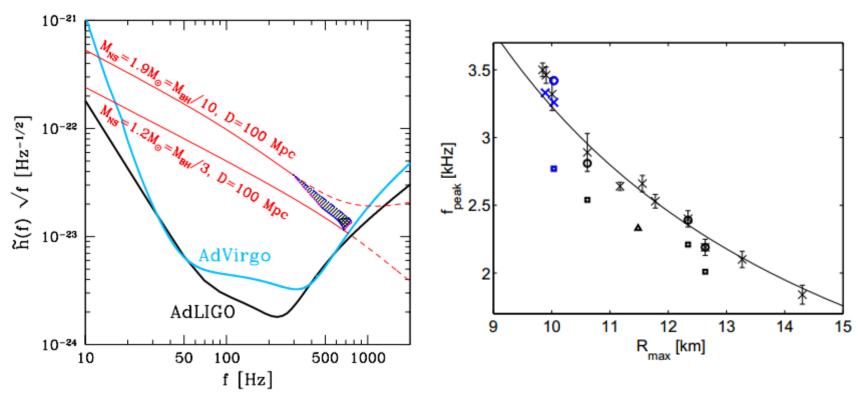
Detailed study of the influence of rotation onto structure and composition is given in 1307.1103

## Rotation and composition



(Weber et al. arXiv: 0705.2708) 1.4 solar mass NS (when non-rotating)

### Limits on the EoS from GW observations



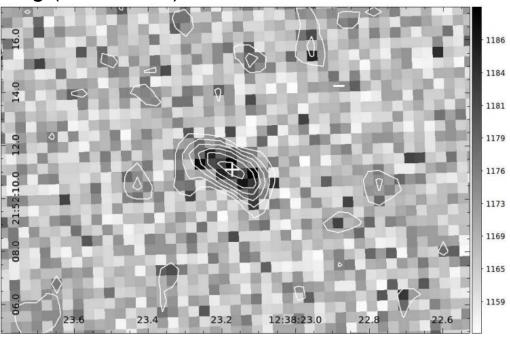
For stiff EoS AdLIGO and AdVIRGO can detect signatures in the GW signal during BH-NS mergers.

Measuring NS-NS mergers one can better constraint the EoS.

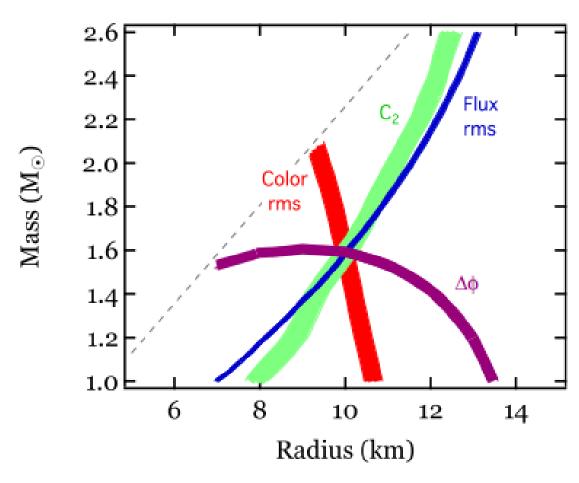
1103.3526 1106.1616

## Microlensing and weak lensing

In the future (maybe already with Gaia) it can possible to determine NS mass with lensing. Different techniques can be discussed: photometric (normal) microlensing (1009.0005), astrometric microlensing, weak lensing (1209.2249).



## Future X-ray measurements



Valid for future observations aboard NICER and LOFT space projects.

Data based on pulse profile.

The idea is to observe X-ray pulsars with spin periods ~few msec and to collect about 10<sup>6</sup> counts.

It allows to derive from the pulse profile a lot of info about a NS.

### References

- Observational Constraints on Neutron Star Masses and Radii 1604.03894
   The review is about X-ray systems
- Mass, radii and equation of state of neutron stars 1603.02698
   The review about different kinds of measurements, including radio pulsars.
   Recent lists of mass measurements for different NSs.
- Measuring the neutron star equation of state using X-ray timing 1602.01081
   The review about EoS and X-ray measurements
- The masses and spins of neutron stars and stellar-mass black holes 1408.4145
   The review covers several topics. Good brief description of radio pulsar mass measurements.
- Properties of DNS systems. 1706.09438
   The review covers all aspects of observations, formation and evolution.

### NS+NS binaries

Secondary companion in double NS binaries can give a good estimate of the initial mass if we can neglect effects of evolution in a binary system.

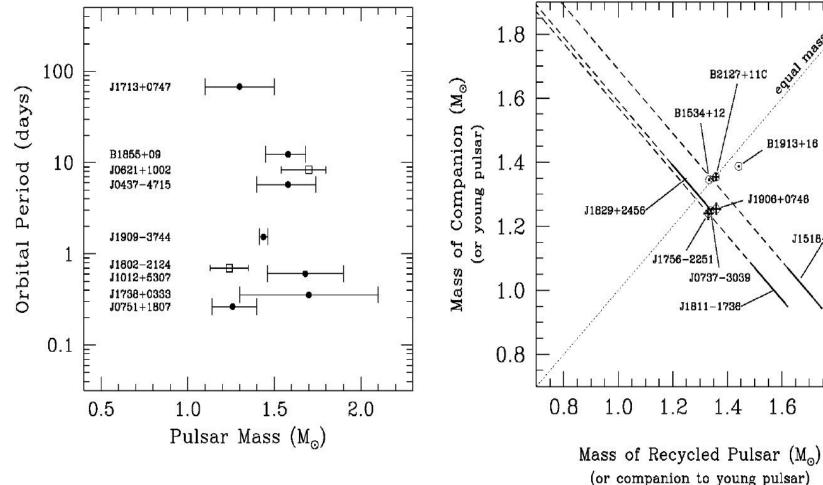
	Pulsar	Pulsar mass	Companion mas	SS
	B1913+16	1.44	1.39	
GC →	B2127+11C	1.36	1.35	
	B1534+12	1.33	1.35	
	J0737-3039	1.34	1.25	
	J1756-2251	1.40	1.18	
	J1518+4904	<1.17	>1.55 -	→ 0808.2292
Non- →	J1906+0746	1.25	1.37	
recycled	J1811-1736	1.56	1.12	
<u>-</u>	J1829+2456	1.2	1.4	Also there are

In NS-NS systems we can neglect all tidal effects etc.

candidates, for example PSR J1753-2240 arXiv:0811.2027

See a review on formation and evolution of DNS binaries in 1706.09438

### Pulsar masses



With WD companions

B2127+11C

B1913+16

J1518,+4904

1.8

J1906+0746

1.6

With NS companions

[Nice et al. 2008]