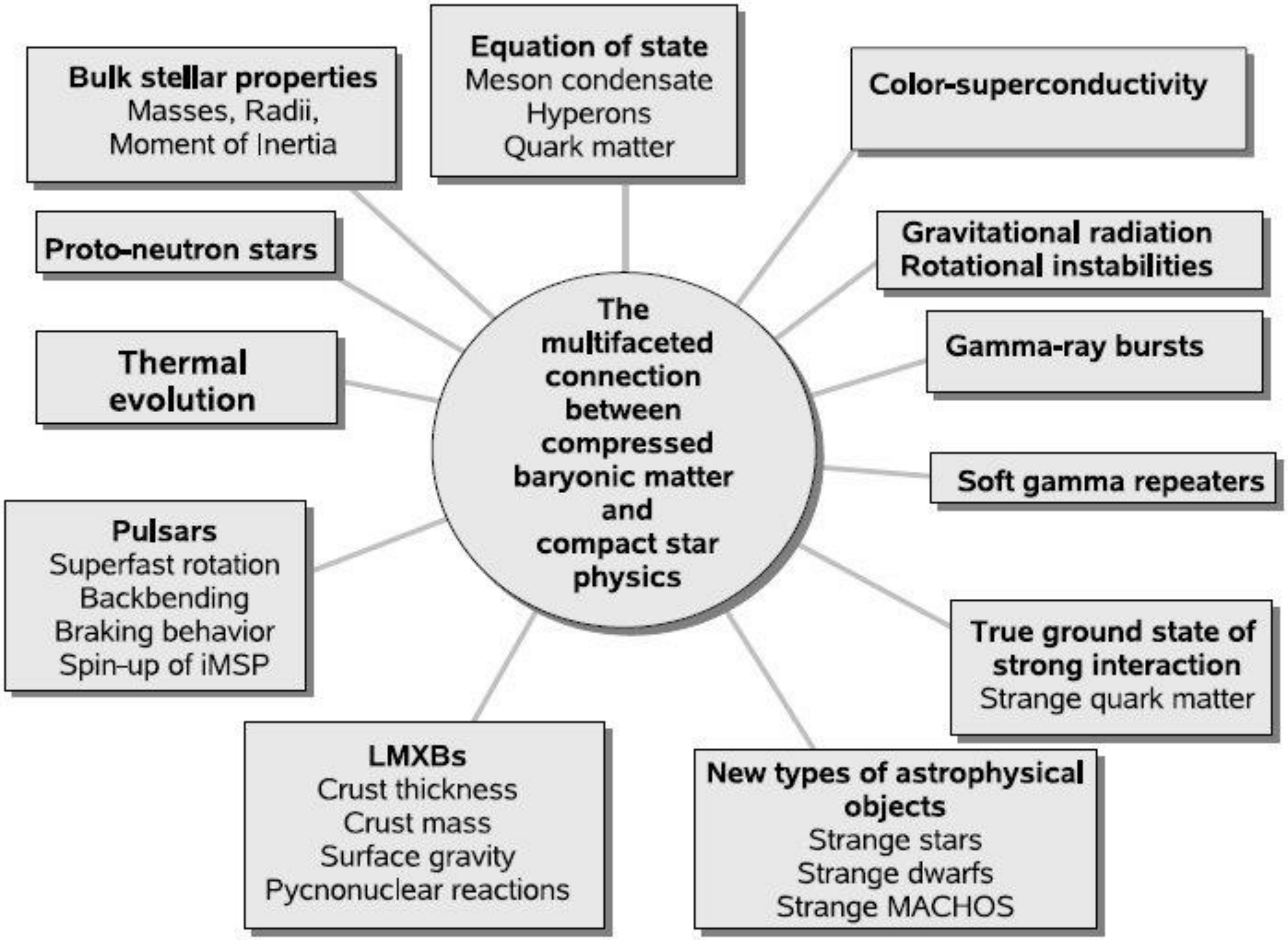
Internal structure of Neutron Stars

Artistic view

Astronomy meets QCD



arXiv: 0808.1279

Hydrostatic equilibrium for a star

$$\left\{\begin{array}{ll} (1) & \dfrac{dP}{dr} = -\dfrac{Gm\rho}{r^2} \qquad m = m(r) \\ (2) & \dfrac{dm}{dr} = 4\pi\rho\, r^2 \\ (3) & \cancel{\dfrac{dS}{dt} = Q} \end{array}\right.$$

$$(4) \qquad P = P(\rho)$$

For NSs we can take T=0
and neglect the third equation

For a NS effects of GR are also important.

$$r_g = \frac{2GM}{c^2} \approx 2.95 \ \frac{M}{M_{SUN}} \ \mathrm{km}$$

M/R ~ 0.15 $(M/M_{\odot})(R/10\ \mathrm{km})^{-1}$
J/M ~ 0.25 (1 ms/P) $(M/M_{\odot})(R/10\mathrm{km})^2$

Lane-Emden equation. Polytrops.

$$P = K\rho^{\gamma}, \quad K, \gamma = \text{const}, \quad \gamma = 1 + \frac{1}{n}$$

$$\frac{dP}{dr} = -\frac{Gm\rho}{r^2} = g\rho, \quad g = -\frac{Gm}{r^2} = -\frac{d\varphi}{dr}$$

$$\frac{dP}{dr} = -\rho\,\frac{d\varphi}{dr}, \quad \Delta\varphi = 4\pi \mathrm{G}\rho$$

$$\rho = \rho_c\,\Theta^n, \quad \Theta = 1 \ \text{при} \ r = 0$$

$$P = K\rho_c^{1+1/n}\,\Theta^{1+n}, \quad \frac{dP}{dr} = (n+1)K\rho_c^{1+1/n}\,\Theta^n\,\frac{d\Theta}{dr}$$

$$\frac{d\varphi}{dr} = -(n+1)K\rho_c^{1/n}\,\frac{d\Theta}{dr}$$

$$\Delta\Theta = -\frac{4\pi G\rho_c^{1-1/n}}{(n+1)K}\,\Theta^n$$

$$\xi = r/a, \quad a^2 = (n+1)K\rho_c^{1/n-1}/(4\pi G)$$

$$\boxed{\frac{1}{\xi^2}\frac{d}{d\xi}\xi^2\frac{d}{d\xi}\Theta = -\Theta^n}$$

$$\Theta = \Theta(\xi)$$

$$0 \le \xi \le \xi_1$$

$$\Theta(0) = 1, \quad \Theta'(0) = 0$$

$$\Theta(\xi_1) = 0$$

Properties of polytropic stars

Analytic solutions:

$$n=0 \qquad \Theta = 1-\frac{\xi^2}{6} \qquad \xi_1 = \sqrt{6}$$

$$n=1 \qquad \Theta = \frac{\sin\xi}{\xi} \qquad \xi_1 = \pi$$

$$n=5 \qquad \Theta = \frac{1}{\sqrt{1+\xi^2/3}} \qquad \xi_1 = \infty$$

$$M = 4\pi\int_0^R dr\, r^2\rho = 4\pi\rho_c a^3 \xi_1^2\,|\Theta'(\xi_1)|$$

$$\frac{\rho_c}{\bar{\rho}} = \frac{4\pi R^3\rho_c}{3M} = \frac{\xi_1}{3|\Theta'(\xi_1)|}$$

γ=5/3 γ=4/3

n	0	1	1.5	2	3
ξ_1	**2.449**	**3.142**	**3.654**	**4.353**	**6.897**
$\lvert\Theta'_1\rvert$	**0.7789**	**0.3183**	**0.2033**	**0.1272**	**0.04243**
$\rho_c/\bar{\rho}$	**1**	**3.290**	**5.991**	**11.41**	**54.04**

$$M \sim \rho_c^{(3-n)/(2n)}$$

$$R \sim \rho_c^{(1-n)/(2n)}$$

$$M \sim R^{(3-n)/(1-n)}$$

$$n=0 \qquad M \sim R^3$$

$$n=1 \qquad M \sim \rho_c \qquad R = \text{const}$$

$$n=1.5 \qquad M \sim \sqrt{\rho_c} \sim R^{-3}$$

$$n=3 \qquad M = \text{const} \qquad R \sim \rho_c^{-1/3}$$

Useful equations

White dwarfs

1. Non-relativistic electrons
   $\gamma=5/3$, $K=(3^{2/3}\,\pi^{4/3}/5)\,(\hbar^2/m_e m_u^{5/3}\mu_e^{5/3})$;
   μ_e-mean molecular weight per one electron
   $K=1.0036\;10^{13}\;\mu_e^{-5/3}$ (CGS)

2. Relativistic electrons
   $\gamma=4/3$, $K=(3^{1/3}\,\pi^{2/3}/4)\,(\hbar c/m_u^{4/3}\mu_e^{4/3})$;
   $K=1.2435\;10^{15}\;\mu_e^{-4/3}$ (CGS)

Neutron stars

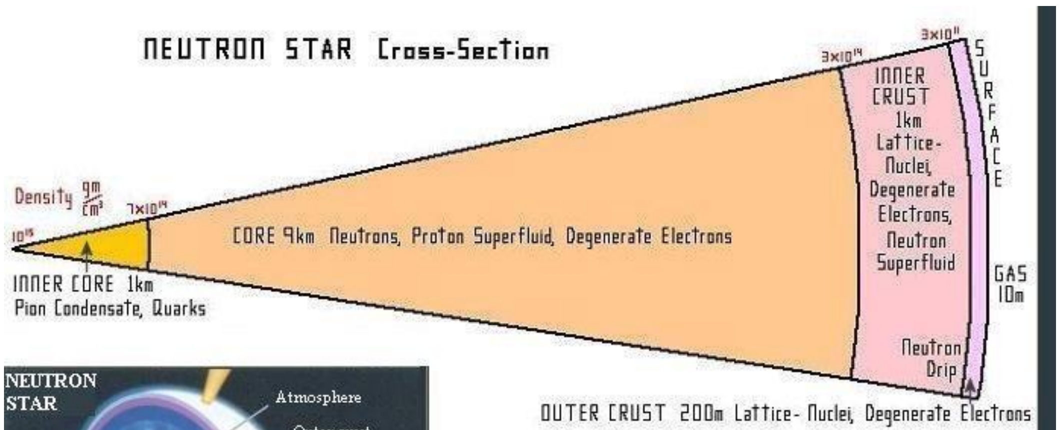
1. Non-relativistic neutrons
   $\gamma=5/3$, $K=(3^{2/3}\,\pi^{4/3}/5)\,(\hbar^2/m_n^{8/3})$;
   $K=5.3802\;10^{9}$ (CGS)

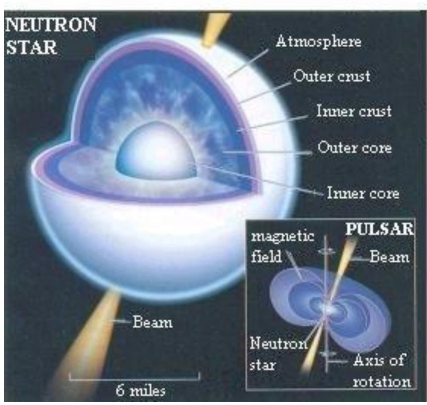
2. Relativistic neutrons
   $\gamma=4/3$, $K=(3^{1/3}\,\pi^{2/3}/4)\,(\hbar c/m_n^{4/3})$;
   $K=1.2293\;10^{15}$ (CGS)

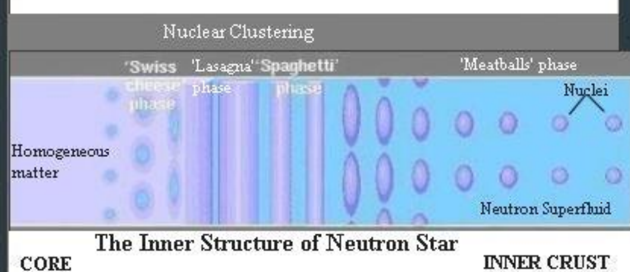
[Shapiro, Teukolsky]

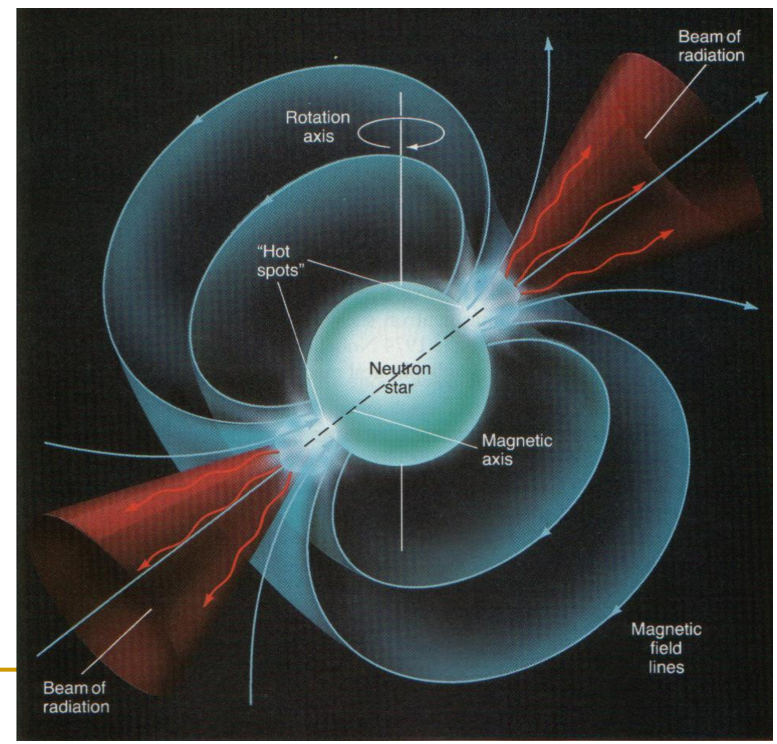
Neutron stars

Superdense matter and superstrong magnetic fields

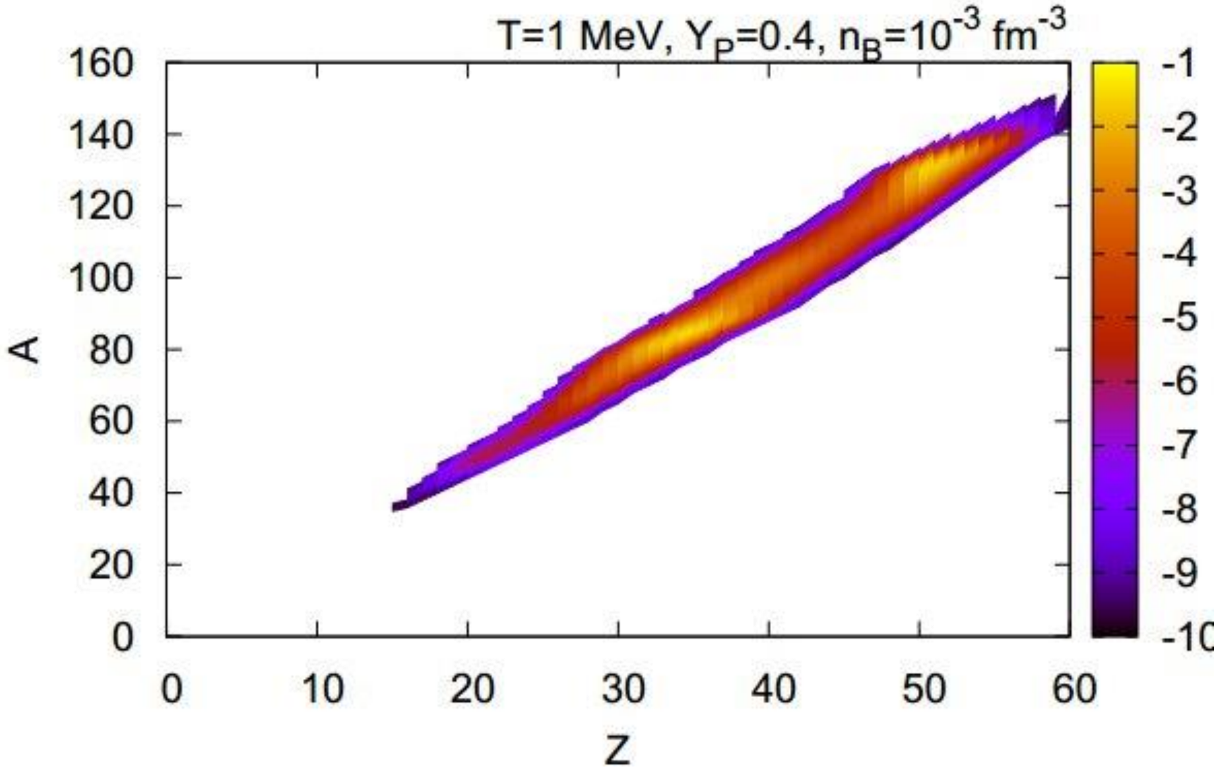








Proto-neutron stars



Mass fraction of nuclei in the nuclear chart for matter at T = 1 MeV, $n_B = 10^{-3}$ fm^{-3}, and $Y_P = 0.4$ (proton fraction). Different colors indicate mass fraction in Log_{10} scale.

1202.5791

NS EoS are also important for SN explosion calculation, see 1207.2184

EoS for core-collapse, proto-NS and NS-NS mergers

	Core-collapse supernovae	Proto-neutron stars	Mergers of compact binary stars
n/n_s	10^{-8} - 10	10^{-8} - 10	10^{-8} - 10
T(MeV)	0 - 30	0 - 50	0 - 100
Y_e	0.35 - 0.45	0.01 - 0.3	0.01 - 0.6
$S(k_B)$	0.5 - 10	0 - 10	0 - 100

Wide ranges of parameters

1512.07820

Astrophysical point of view

Astrophysical appearence of NSs is mainly determined by:

- **Spin**
- **Magnetic field**
- **Temperature**
- **Velocity**
- **Environment**

The first four are related to the NS structure!

Equator and radius

$$ds^2=c^2dt^2e^{2\Phi}-e^{2\lambda}dr^2-r^2[d\theta^2+\sin^2\theta d\varphi^2]$$

In flat space $\Phi(r)$ and $\lambda(r)$ are equal to zero.

- $t=const,\ r=const,\ \theta=\pi/2,\ 0<\Phi<2\pi$ $\Rightarrow$ $l=2\pi r$
- $t=const,\ \theta=const,\ \varphi=const,\ 0<r<r_0$ $\Rightarrow$ $dl=e^{\lambda}dr$ $\Rightarrow$ $l=\int_0^{r_0} e^{\lambda}dr \neq r_0$

Gravitational redshift

<1

$$d\tau = dt\,e^{\Phi},\qquad \nu_r = \frac{dN}{d\tau} = e^{-\Phi}\frac{dN}{dt}$$

Frequency emitted at r

$$r \to \infty \qquad \Phi \to 0 \qquad \nu_\infty = \frac{dN}{dt}$$

Frequency detected by an observer at infinity

$$\nu_\infty = \nu_r\,e^{\Phi} \quad \Rightarrow \quad \Phi(r)$$

This function determines gravitational redshift

$$e^{2\lambda} \equiv \frac{1}{1-\dfrac{2Gm}{c^2 r}}$$

It is useful to use m(r) – gravitational mass inside r – instead of λ(r)

Outside of the star

$$r > R \Rightarrow m(r) = M = \text{const}$$

$$e^{2\Phi} = 1 - \frac{2GM}{c^2 r} = 1 - \frac{r_g}{r}, \qquad r_g = \frac{2GM}{c^2}$$

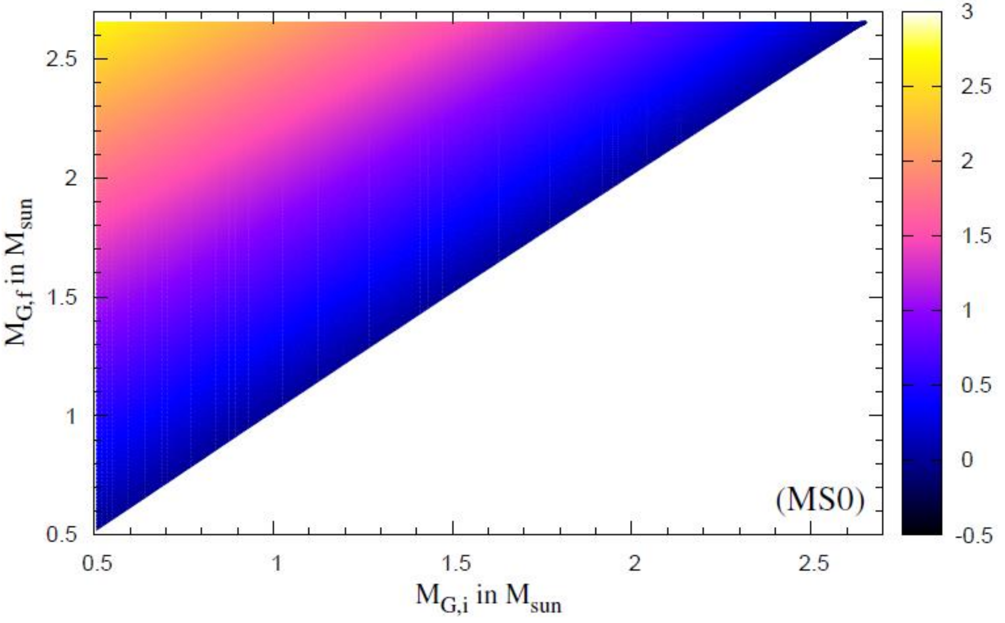
$$ds^2 = \left(1 - \frac{r_g}{r}\right) c^2\, dt^2 - \left(1 - \frac{r_g}{r}\right)^{-1} dr^2 - r^2\, d\Omega^2$$

$$\nu_\infty = \nu_r \sqrt{1 - \frac{r_g}{r}}$$ → redshift

Bounding energy → $\Delta M = M_b - M \sim 0.2\ M_{\mathrm{sun}}$

Apparent radius → $R_\infty = R / \sqrt{1 - r_g / R}$

Bounding energy



$M_{G,i}$ $(M_\odot)$	ΔM_G $(M_\odot)$	$M_{B,i}$ $(M_\odot)$		M_{acc} (ΔM_B) $(M_\odot)$	
		APR	MS0	APR	MS0
1.4	0.57	1.554	1.525	0.768	0.712
1.5	0.47	1.681	1.647	0.641	0.591
1.6	0.37	1.811	1.767	0.511	0.470
1.7	0.27	1.943	1.892	0.379	0.345
1.8	0.17	2.080	2.018	0.242	0.219
1.9	0.07	2.221	2.146	0.101	0.091

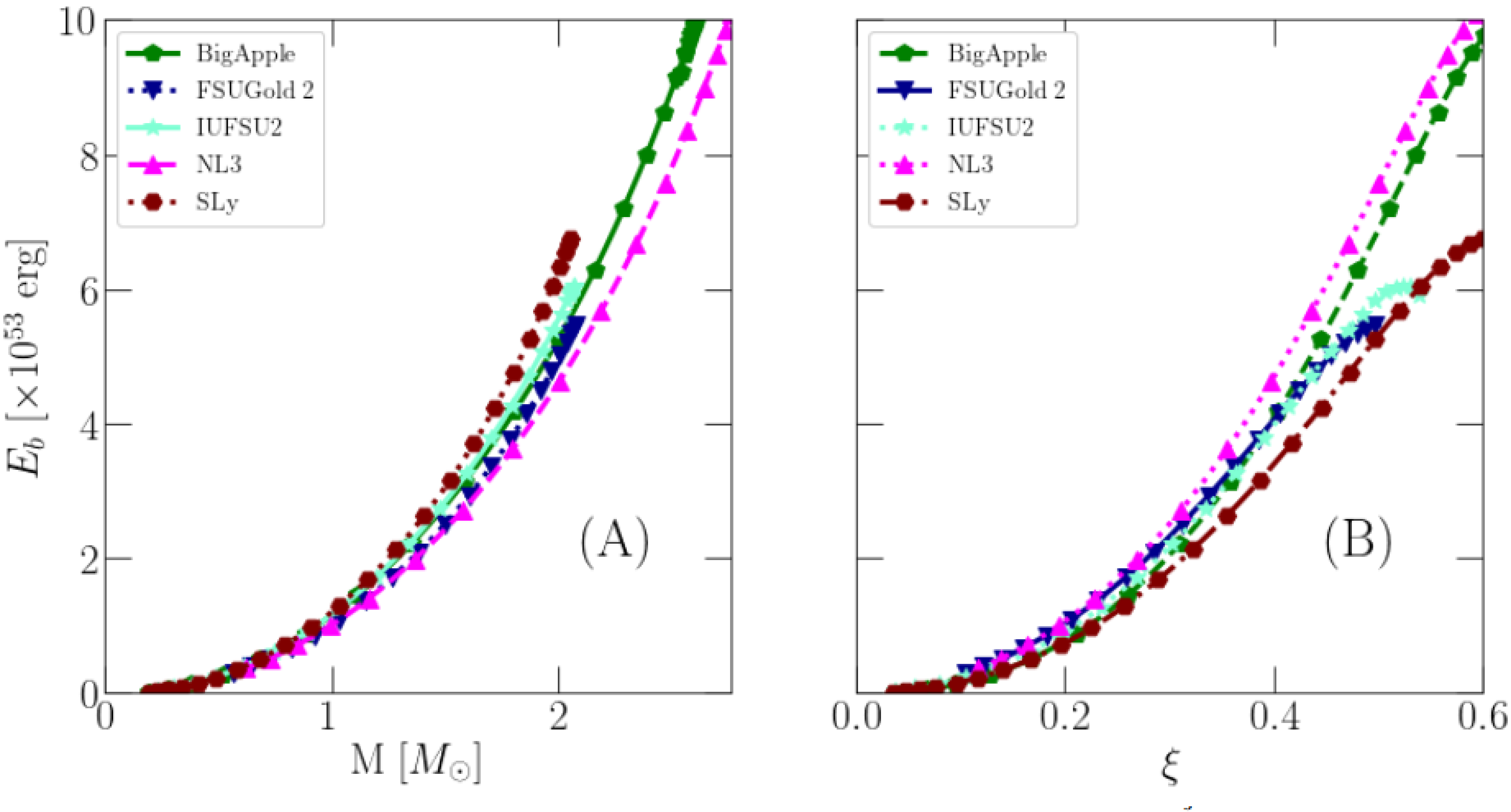
If you drop a kilo on a NS, then you increase its mass for < kilo

M_{acc} is shown with color

$M_{acc}=\Delta M_G+\Delta BE/c^2=\Delta M_B$
BE- binding energy
$BE=(M_B-M_G)c^2$

1102.2912

Binding energy vs. mass



$\xi = 2GM/R$

2008.06108

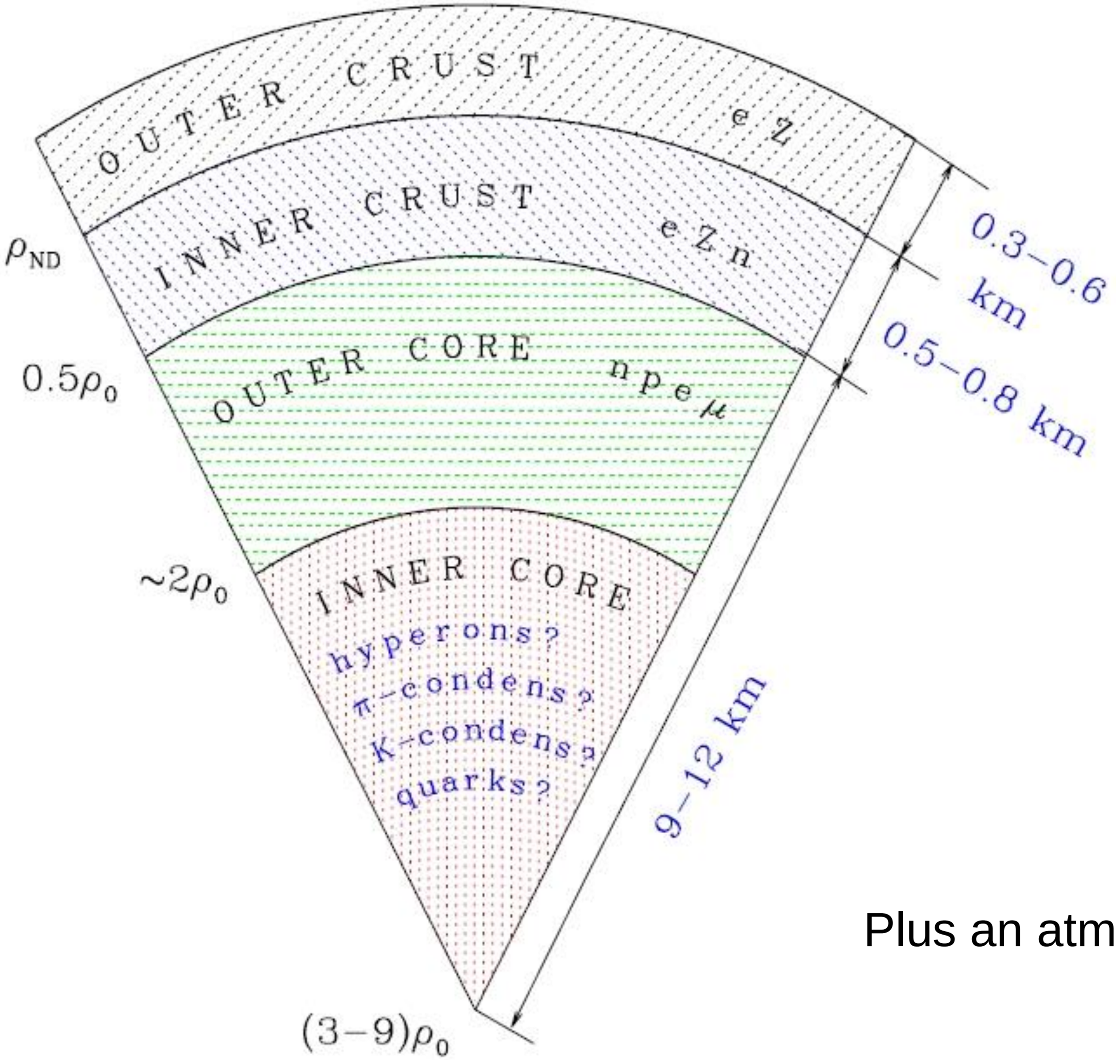
TOV equation

$$\boxed{R_{ik} - \frac{1}{2} g_{ik} R = \frac{8\pi G}{c^4} T_{ik}}$$

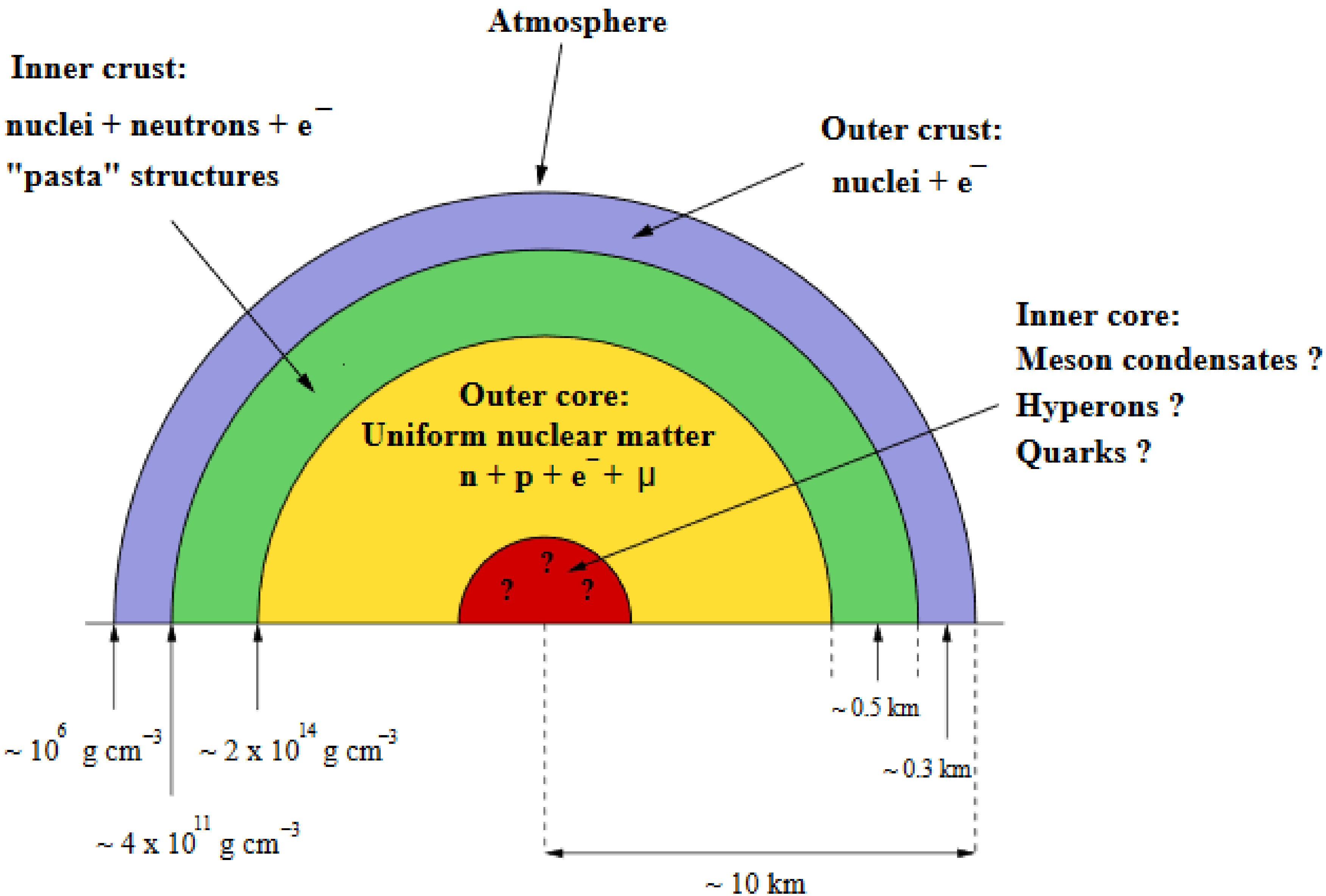
$$\begin{cases} (1) & \boxed{\dfrac{dP}{dr} = -\dfrac{G\rho m}{r^2}\left(1 + \dfrac{P}{\rho c^2}\right)\left(1 + \dfrac{4\pi r^3 P}{mc^2}\right)\left(1 - \dfrac{2Gm}{rc^2}\right)^{-1}} \\ (2) & \dfrac{dm}{dr} = 4\pi r^2 \rho \\ (3) & \dfrac{d\Phi}{dr} = -\dfrac{1}{\rho c^2}\dfrac{dP}{dr}\left(1 + \dfrac{P}{\rho c^2}\right)^{-1} \\ (4) & P = P(\rho) \end{cases}$$

Tolman (1939)
Oppenheimer-Volkoff (1939)

Structure and layers



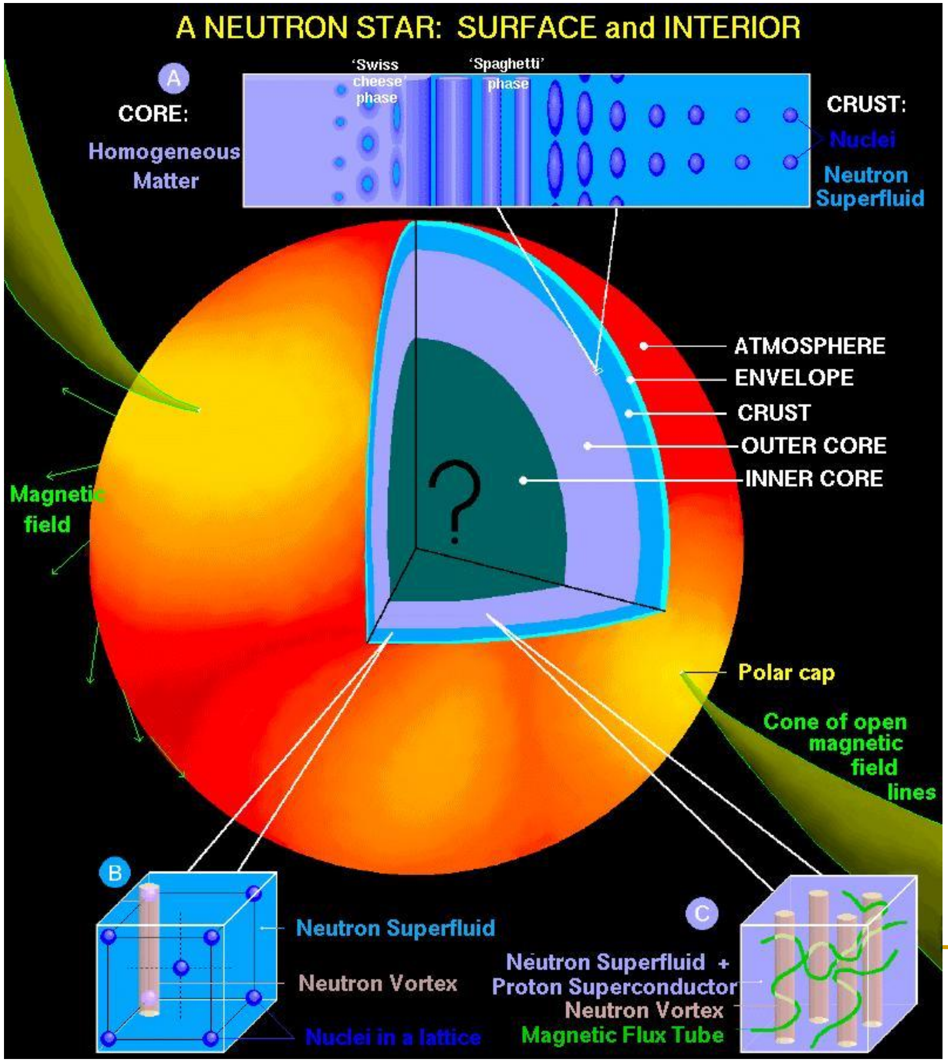
Plus an atmosphere...



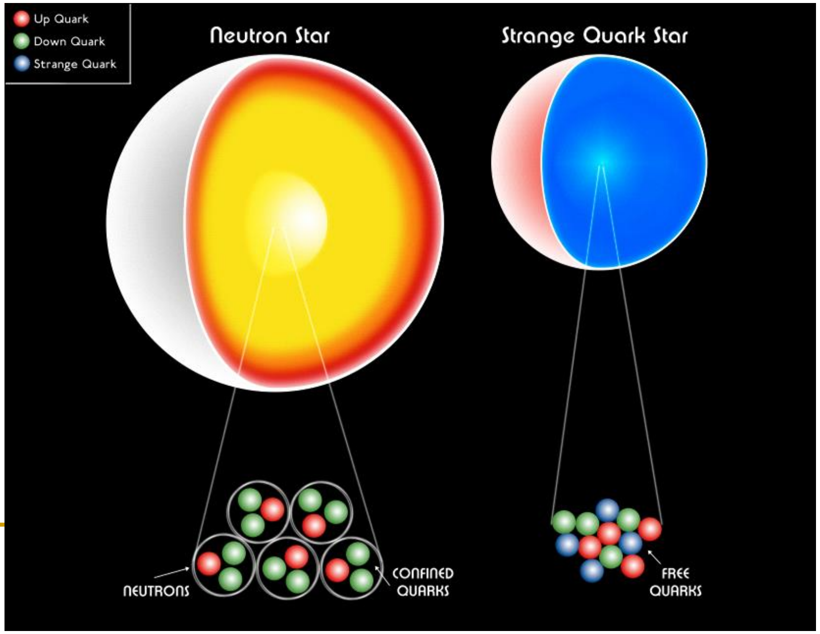
Atmosphere
Inner crust:
nuclei + neutrons + e⁻
"pasta" structures
Outer crust:
nuclei + e⁻
Inner core:
Meson condensates ?
Hyperons ?
Quarks ?
Outer core:
Uniform nuclear matter
n + p + e⁻ + μ
? ? ?
~ 10^6 g cm^-3
~ 2 x 10^14 g cm^-3
~ 4 x 10^11 g cm^-3
~ 0.5 km
~ 0.3 km
~ 10 km

2007.04427

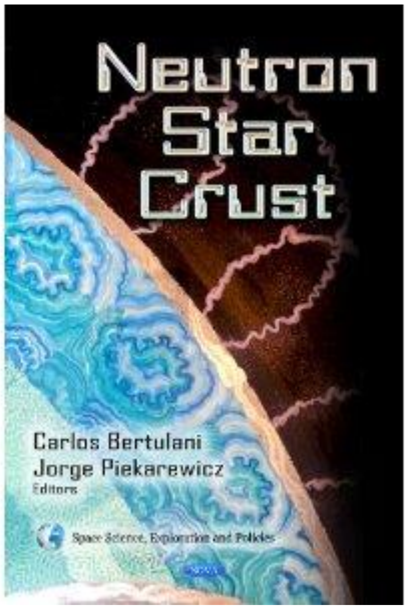
Neutron star interiors

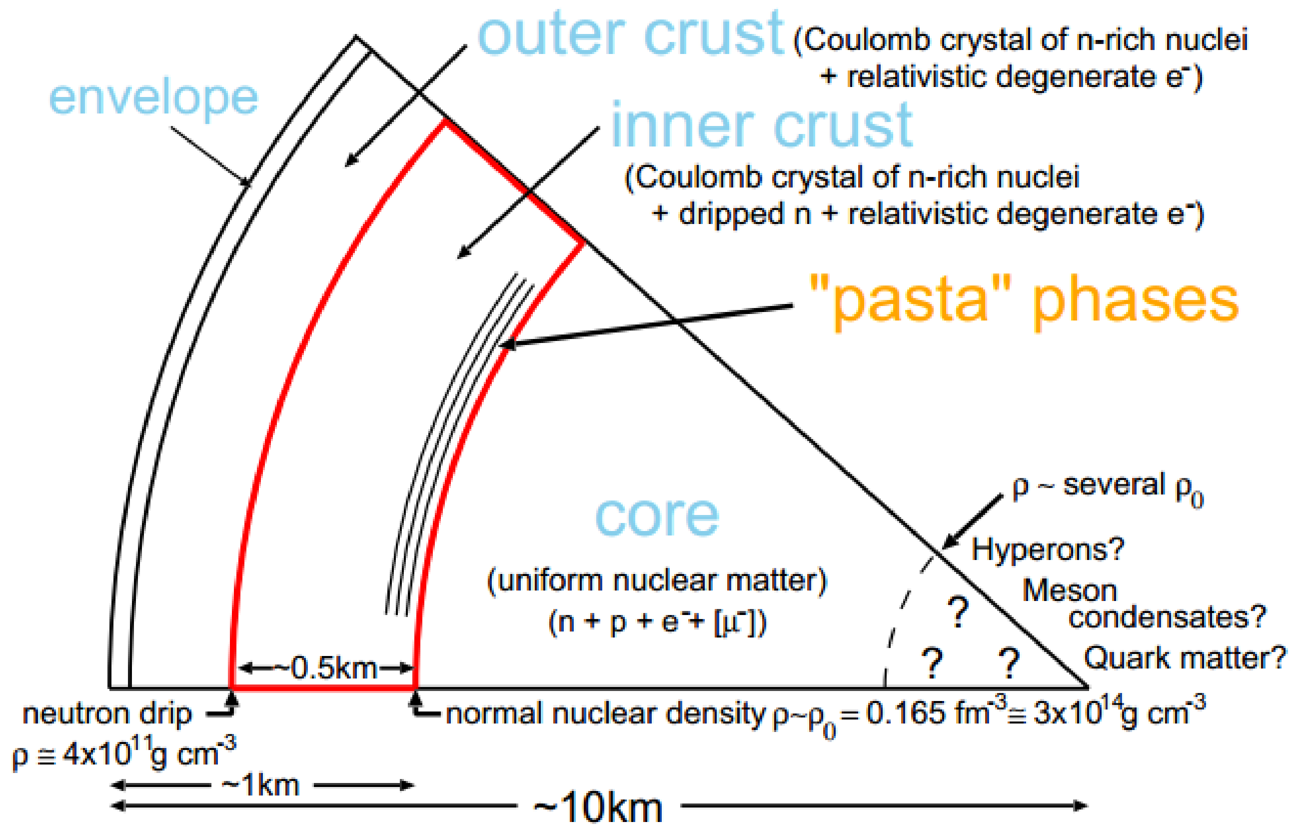


Radius: 10 km
Mass: 1-2 solar
Density: above the nuclear
Strong magnetic fields



Neutron star crust

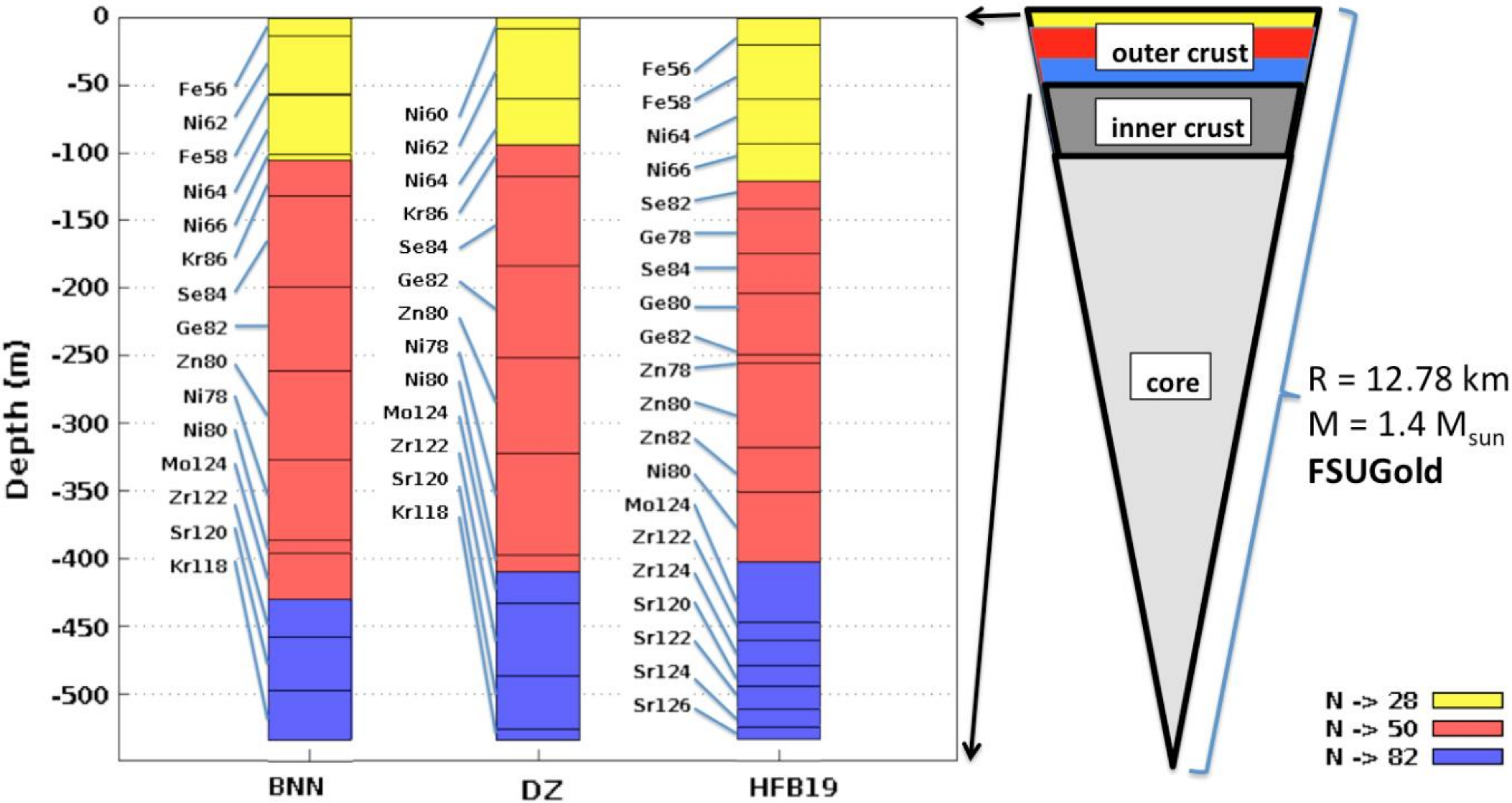




Many contributions to the book are available in the arXiv.

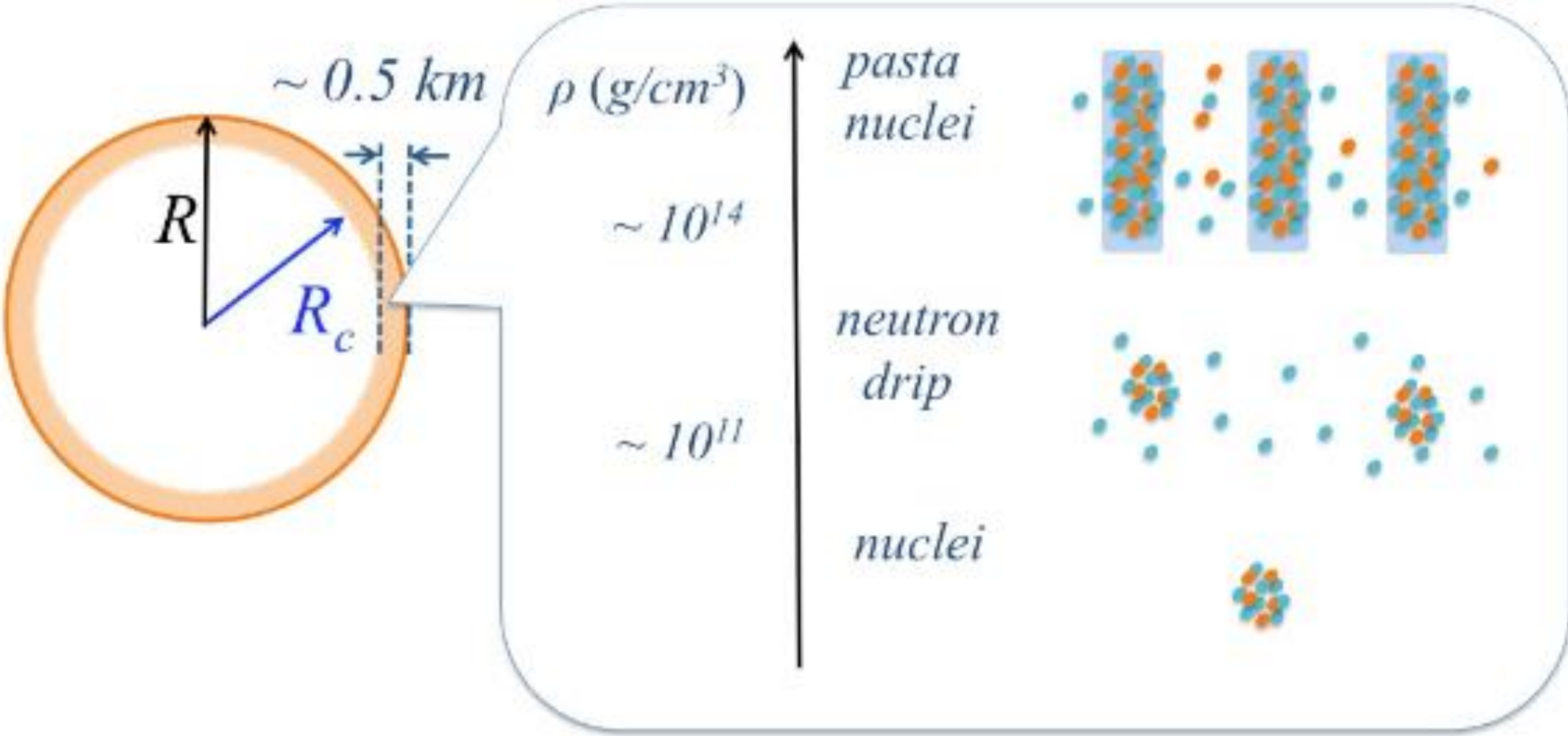
Mechanical properties of crusts are continuously discussed,
see 1208.3258, 1808.06415, 2010.08398

Element composition of the crust



1805.04780

Inner crust properties

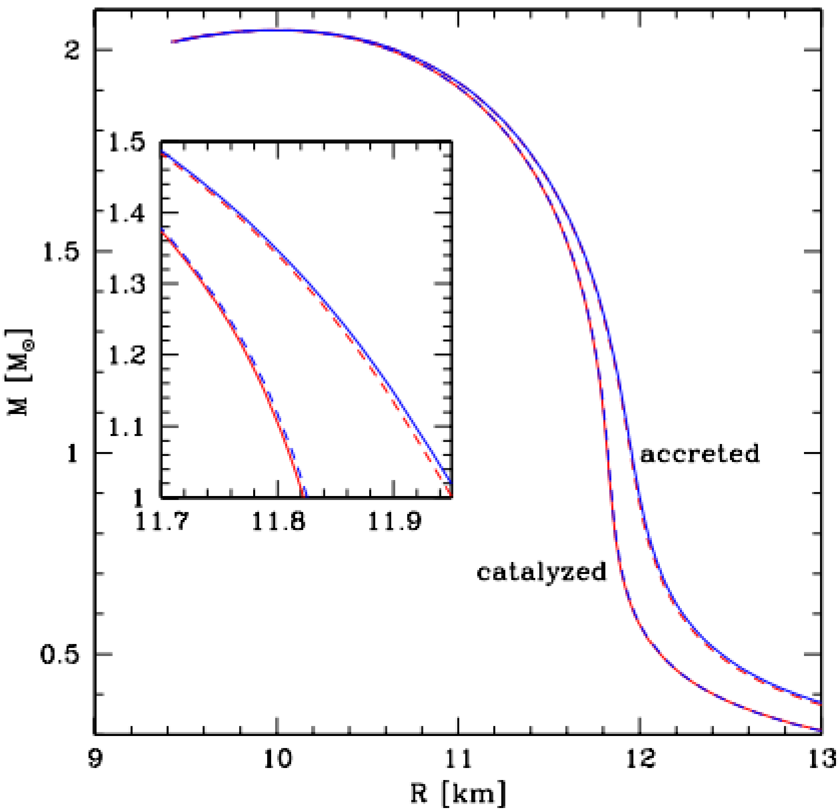


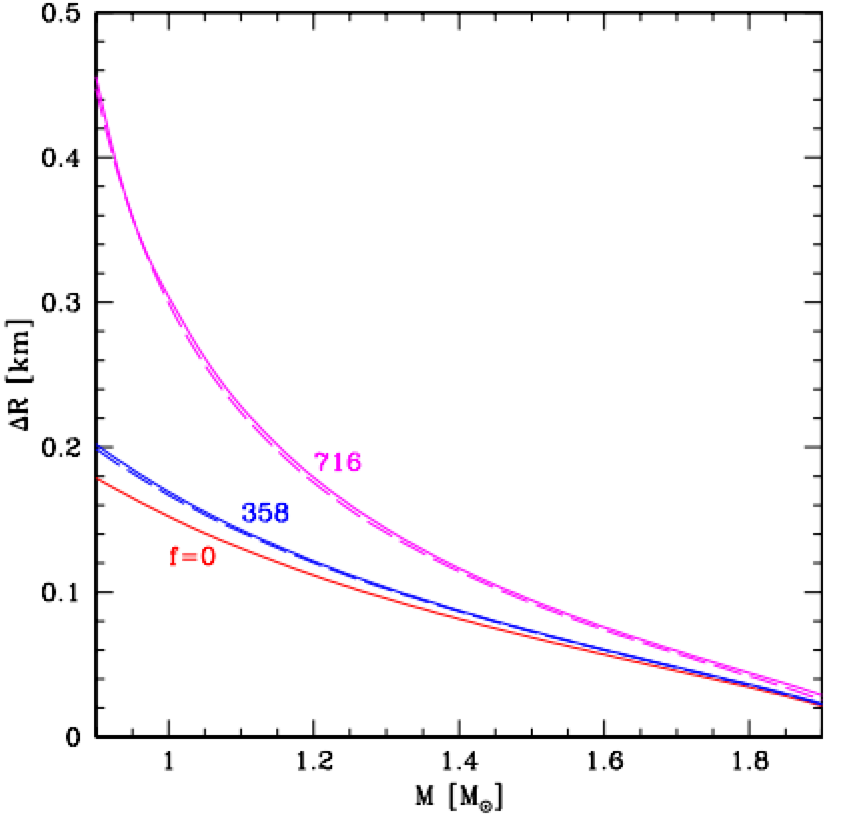
1707.04966

See new results in 2001.09739

Accreted crust

It is interesting that the crust formed by accreted matter differs from the crust formed from catalyzed matter. The former is thicker.

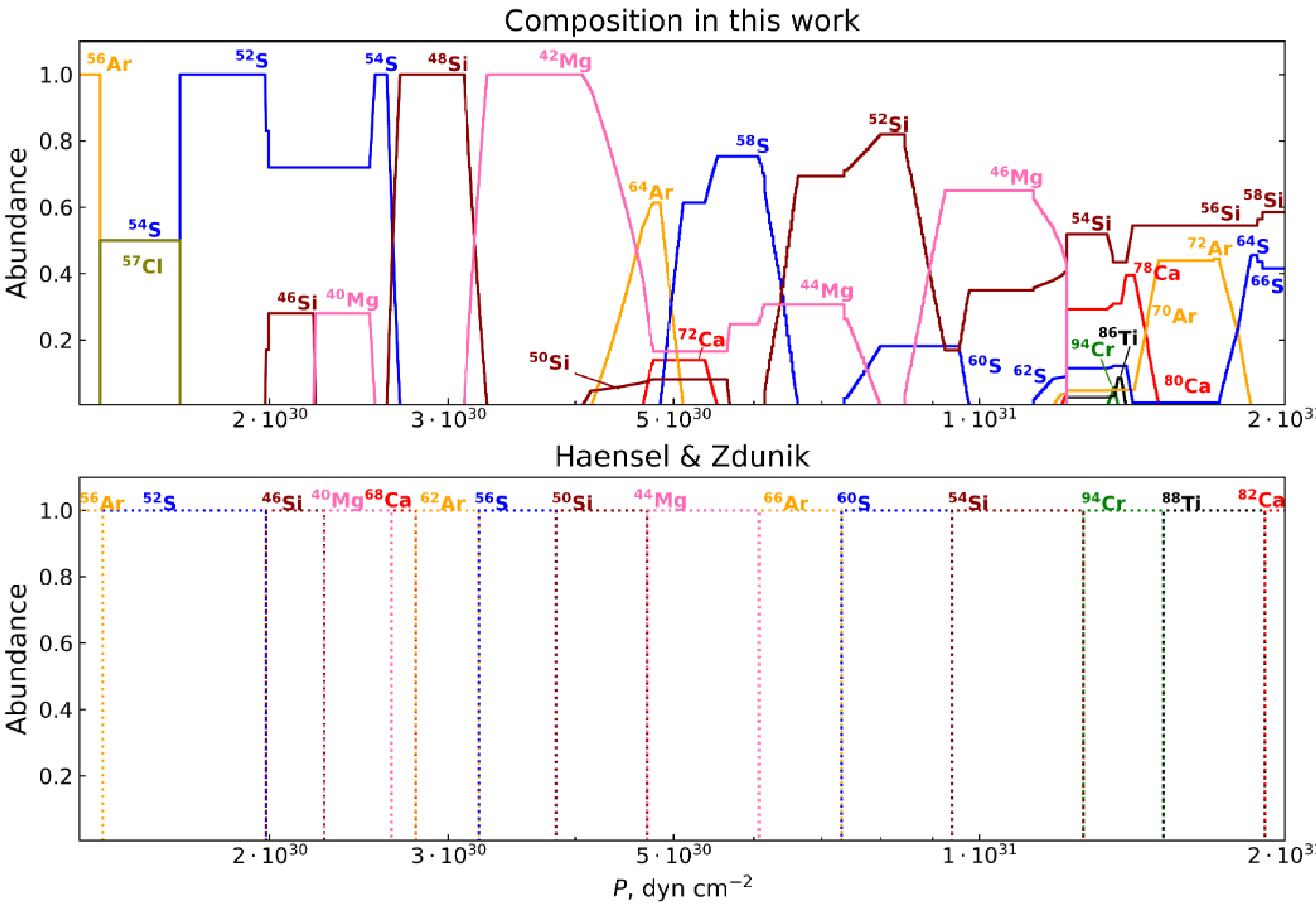




1104.0385

See new results in 1910.03932

Composition of accreted crust



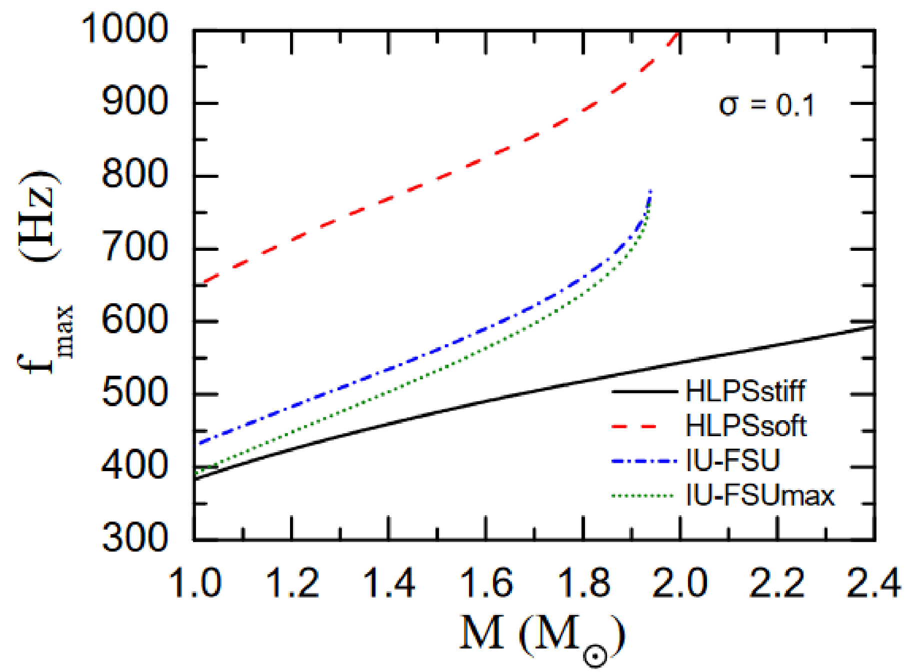
2001.09739

Crust and limiting rotation

Model	σ	$f_{\rm in}^{1.4}$ (Hz)	$f_{\rm fin}^{1.4}$ (Hz)	$f_{\rm in}^{1.8}$ (Hz)	$f_{\rm fin}^{1.8}$ (Hz)
HLPSStiff	0.05	0	326	35	368
	0.10	136	479	236	569
IU-FSU	0.05	349	515	909	1022
	0.10	781	947	1875	1988
IU-FSUmax	0.05	35	358	374	586
	0.10	232	555	854	1066

The maximum initial rotational frequency of a neutron star whose crust will never fail from slowing down, assuming a breaking strain of σ= 0.1.

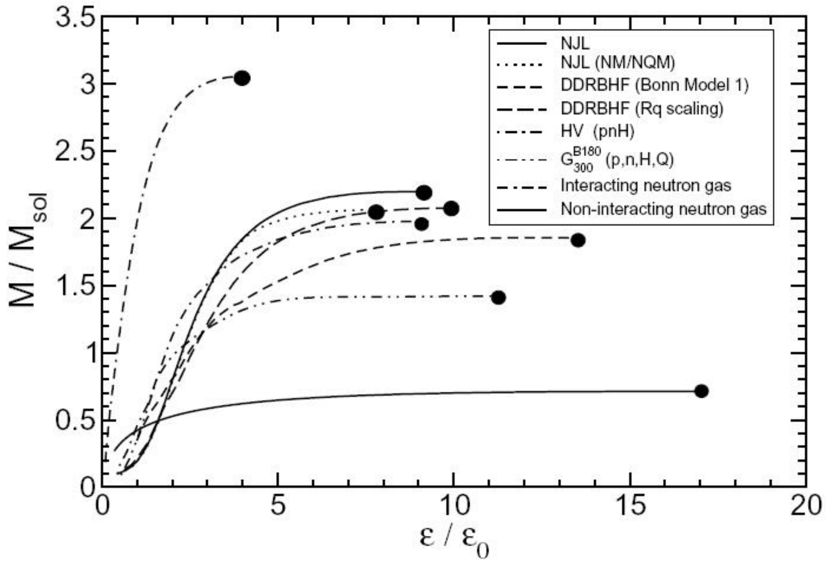
Failure of the crust can be the reason of the limiting frequency.
Spinning-up of a NS due to accretion can result in crust failure.
Then the shape of the star is deformed, it gains ellipticity.
So, GWs are emitted which slow down the compact object.



1804.04952

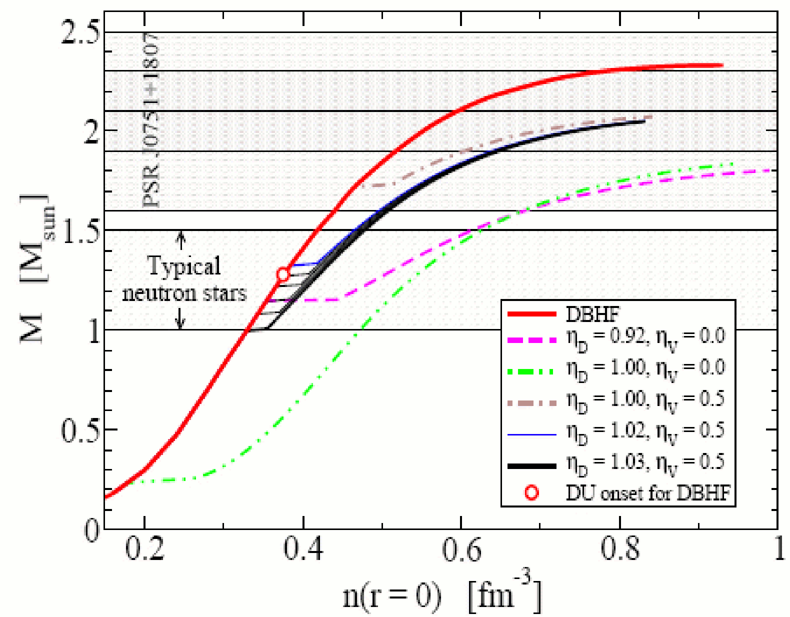
(about limits based on EoS see 1805.11277)

Configurations



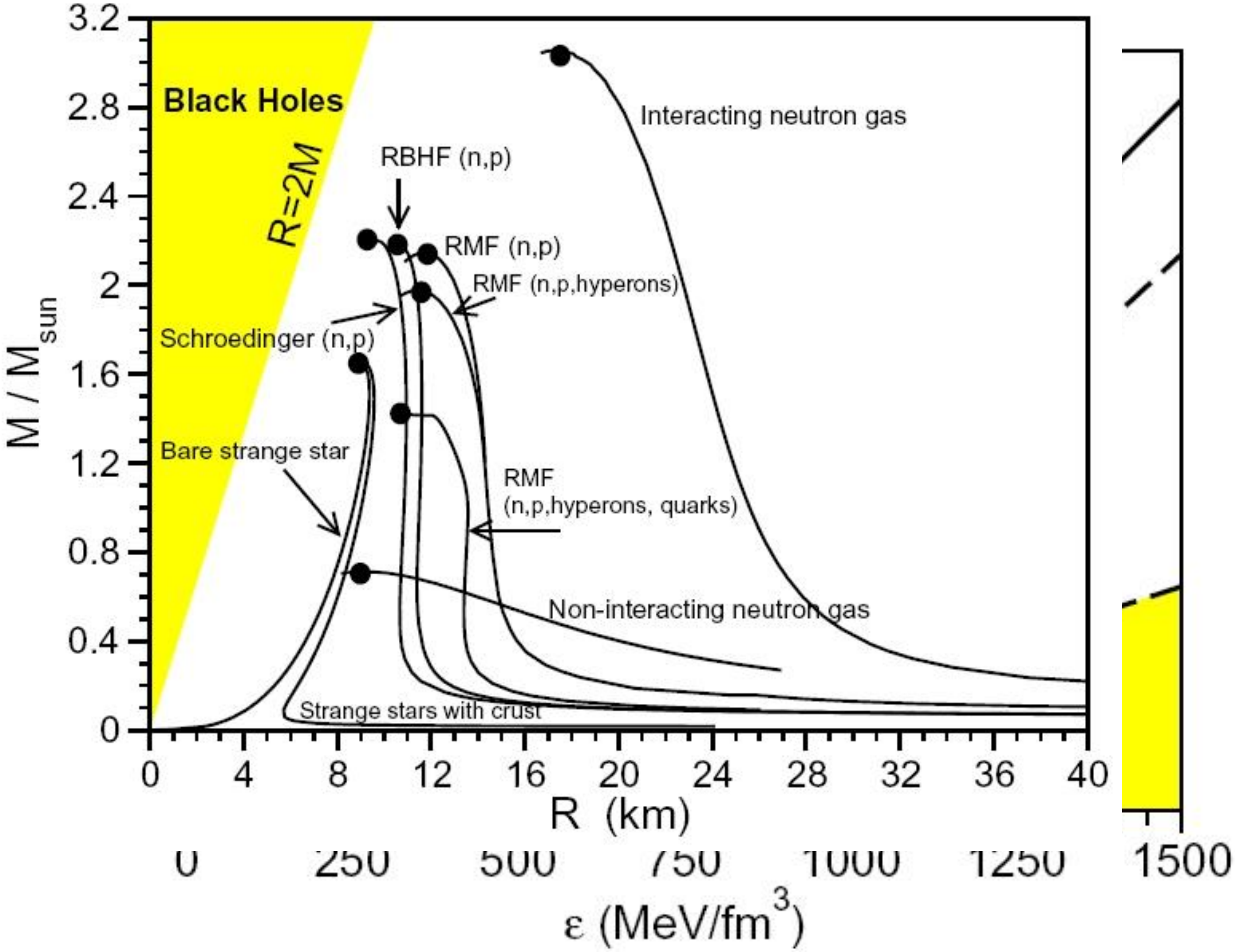
NS mass vs. central density (Weber et al. arXiv: 0705.2708)

Stable configurations for neutron stars and hybrid stars (astro-ph/0611595).



A RNS code is developed and made available to the public by Sterligioulas and Friedman ApJ 444, 306 (1995) http://www.gravity.phys.uwm.edu/rns/

EoS

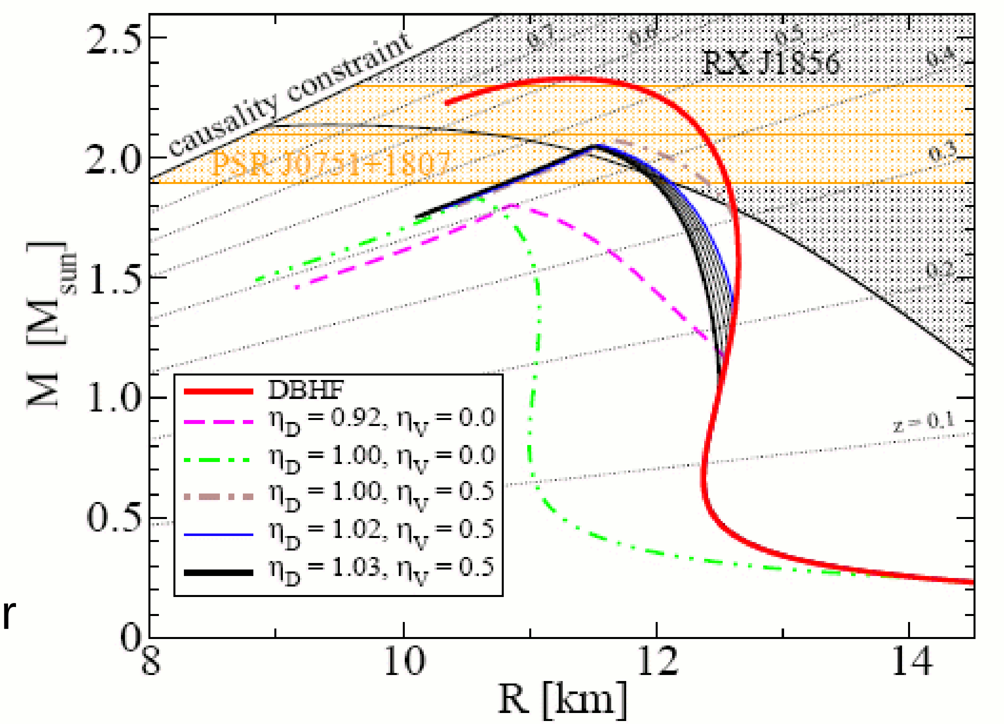


(Weber et al. ArXiv: 0705.2708)

Mass-radius

Mass and radius
are marcoscopical
potentially measured
parameters.
Thus, it is important
to formulate EoS
in terms of these
two parameters.

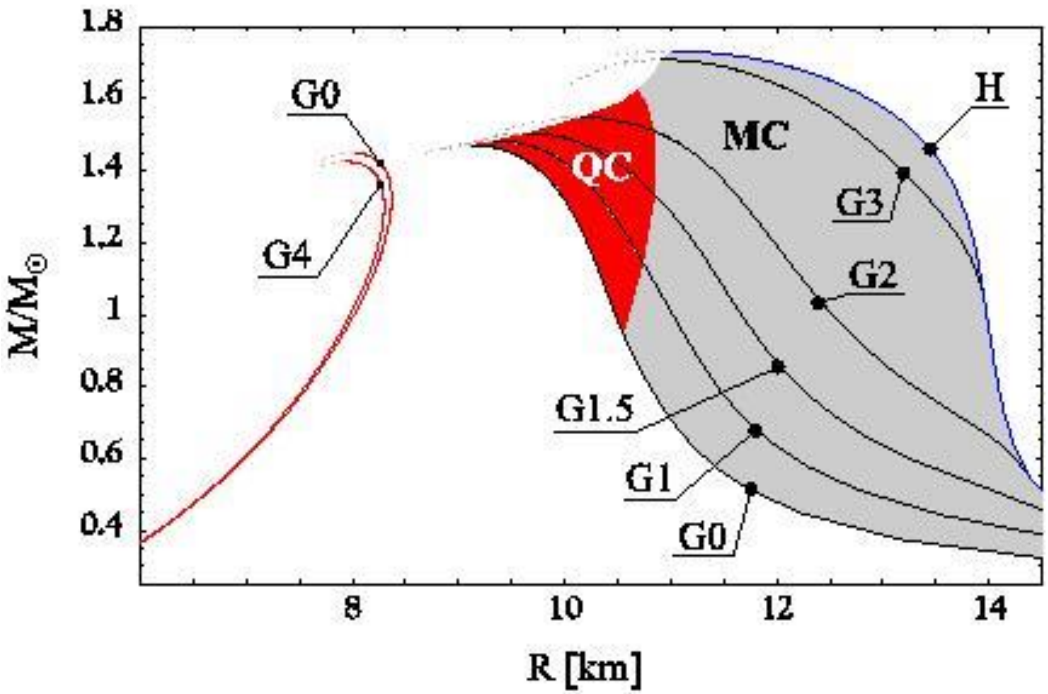
Mass-radius relations for CSs
with possible phase transition
to deconfined quark matter.



About hyperon stars see a
review in 1002.1658.
About strange stars and some other
exotic options – 1002.1793

astro-ph/0611595

Mass-radius relation

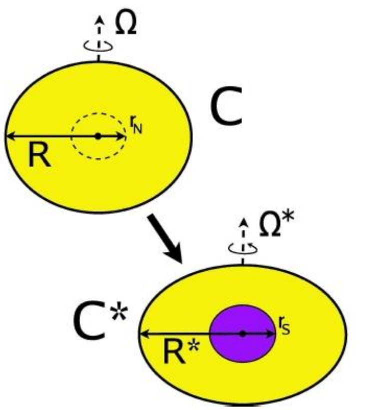


Main features

- Max. mass
- Diff. branches (quark and normal)
- Stiff and soft EoS
- Small differences for realistic parameters
- Softening of an EoS with growing mass

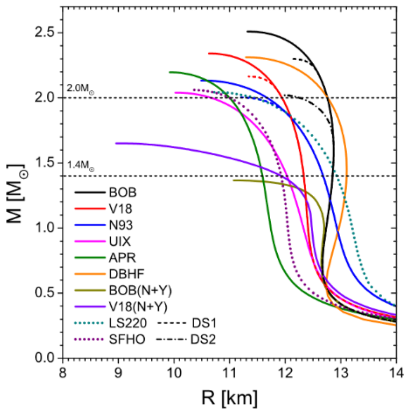
Rotation is neglected here.
Obviously, rotation results in:
- larger max. mass
- larger equatorial radius

Spin-down can result in phase transition, as well as spin-up (due to accreted mass), see 1109.1179



Haensel, Zdunik
astro-ph/0610549

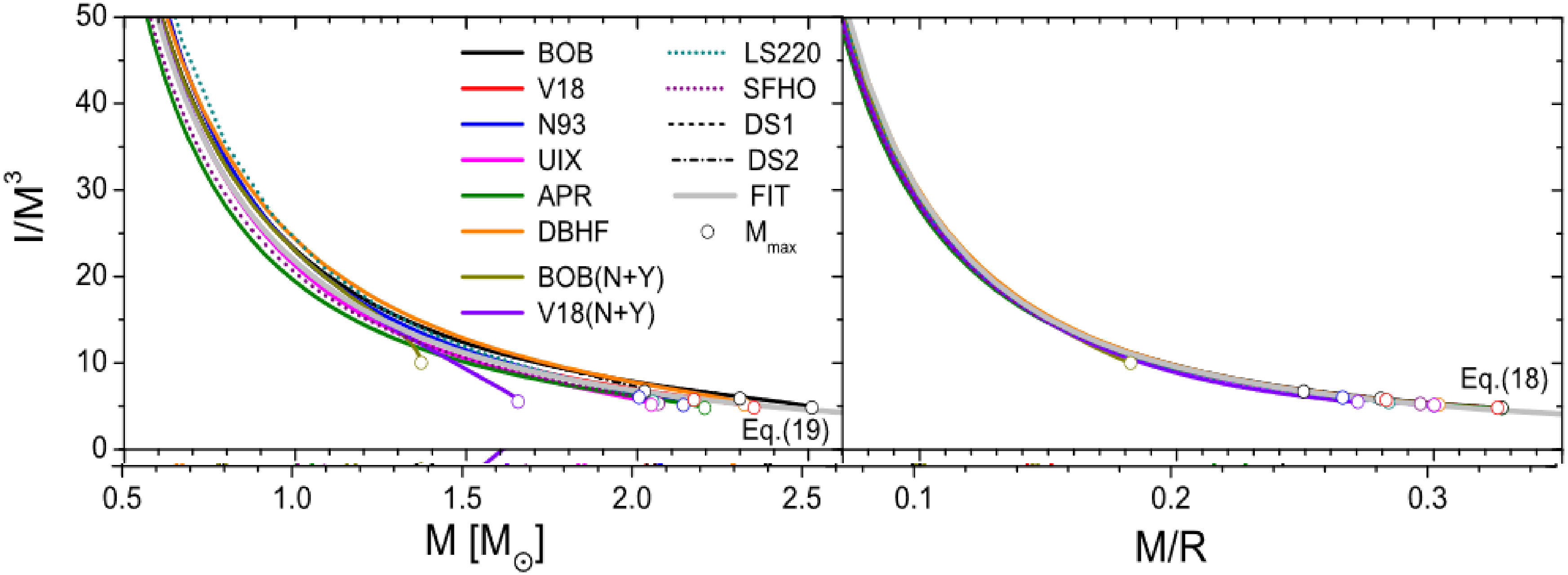
Fitting formulae for moment of inertia



$$\frac{I}{M^3} \equiv 0.8134\,\beta^{-1} + 0.2101\,\beta^{-2} + 0.003175\,\beta^{-3} - 0.0002717\,\beta^{-4} \qquad (18)$$

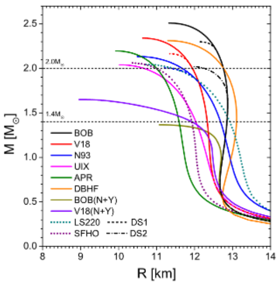
$$\frac{I}{M^3} \equiv 1.0334\,M^{-1} + 30.7271\,M^{-2} - 12.8839\,M^{-3} + 2.8841\,M^{-4} \qquad (19)$$

$$\beta = Gm/(Rc^2)$$



1809.04315

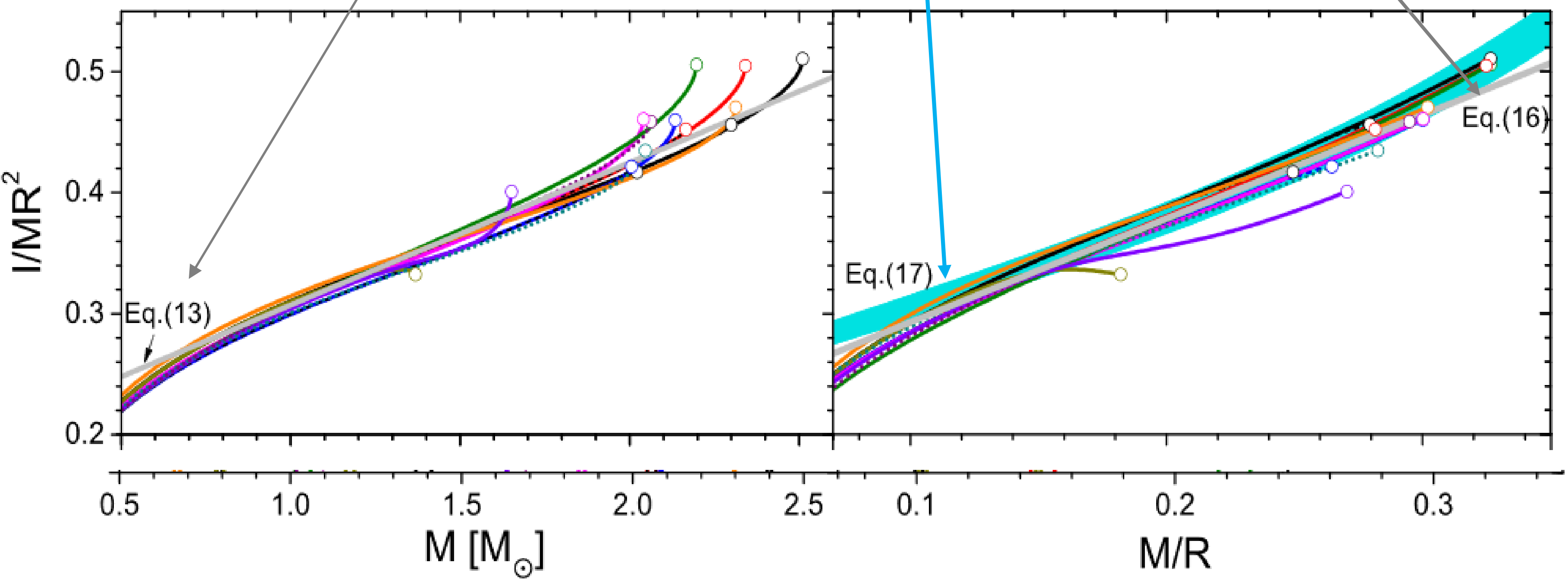
Fits for I/MR^2



$$\frac{I}{MR^2} \approx (0.237 \pm 0.008)(1 + 2.844\beta + 18.91\beta^4)$$

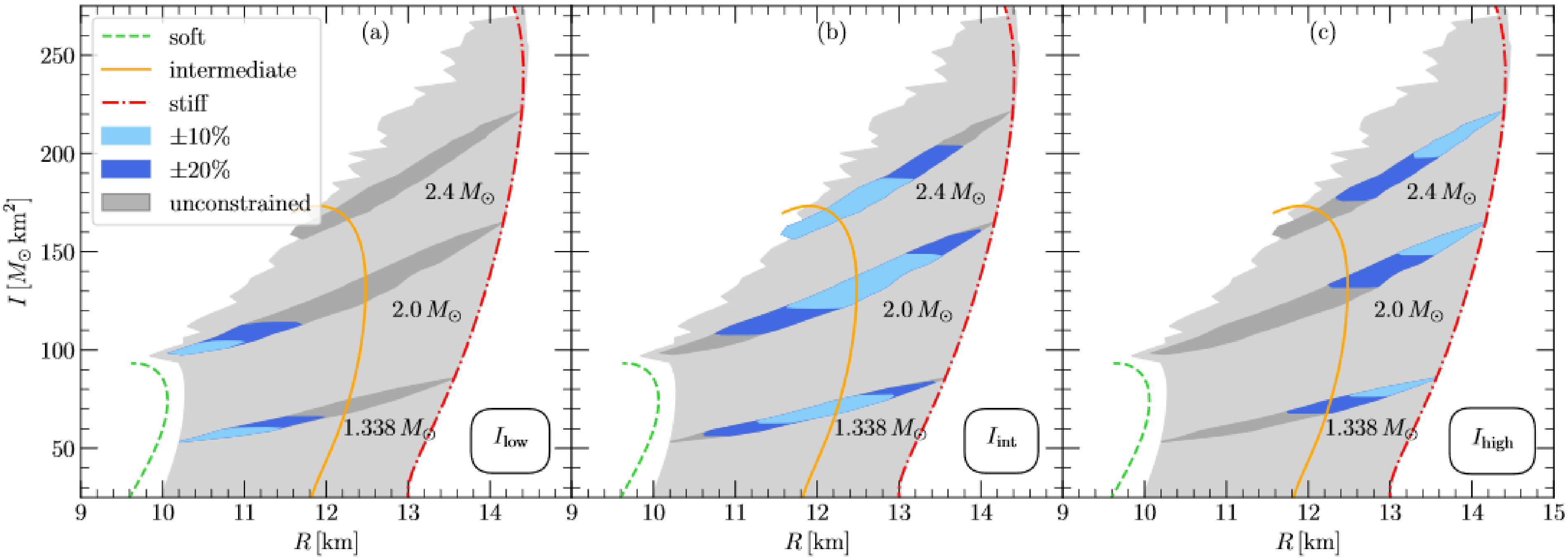
$$\frac{I}{MR^2} \equiv 0.207 + 0.857\beta \pm 0.011$$

$$\frac{I}{MR^2} \approx 0.189 + 0.118\frac{M}{M_\odot} \pm 0.016\,.$$



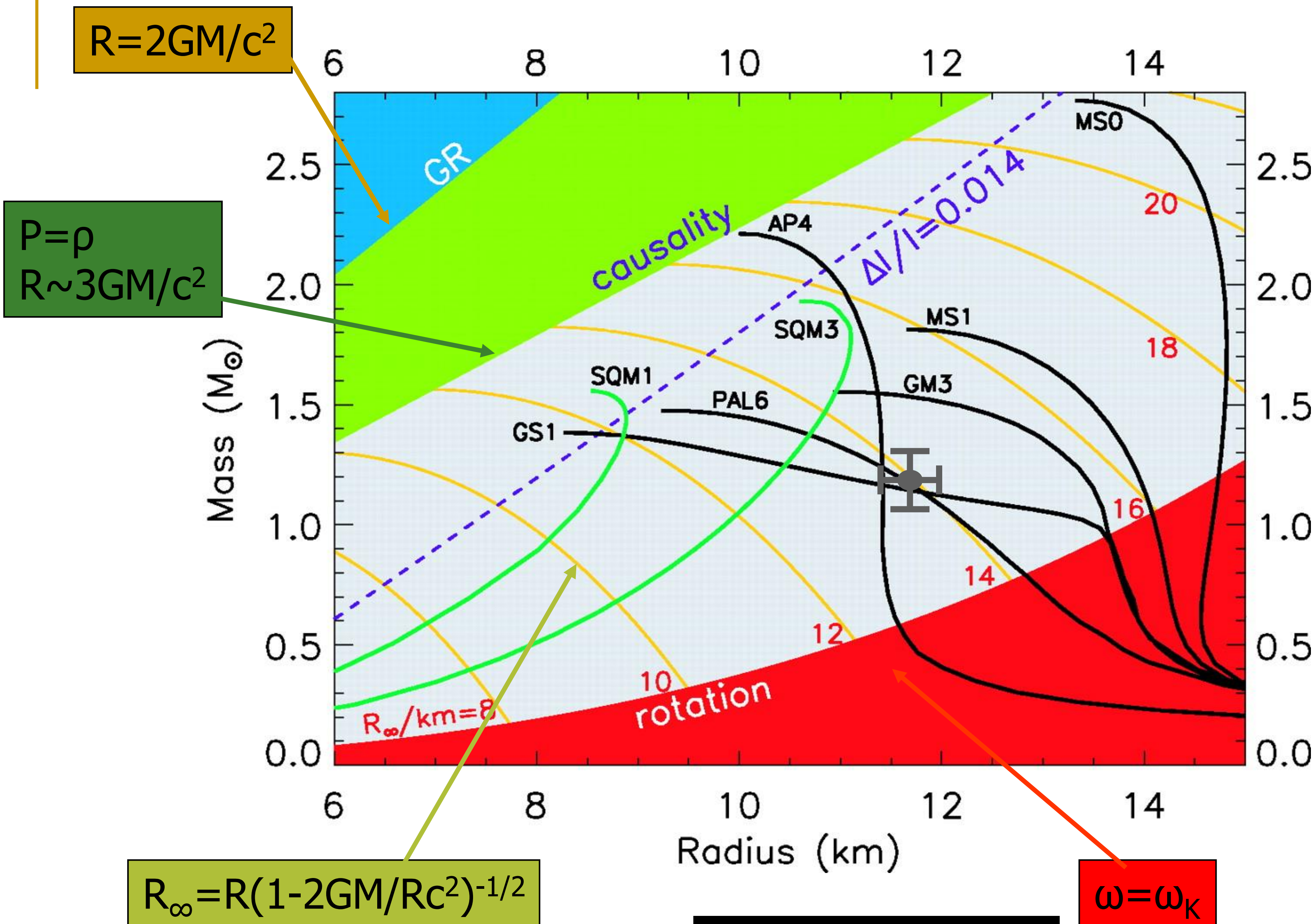
1809.04315

Moment of inertia measurements to constrain EoS



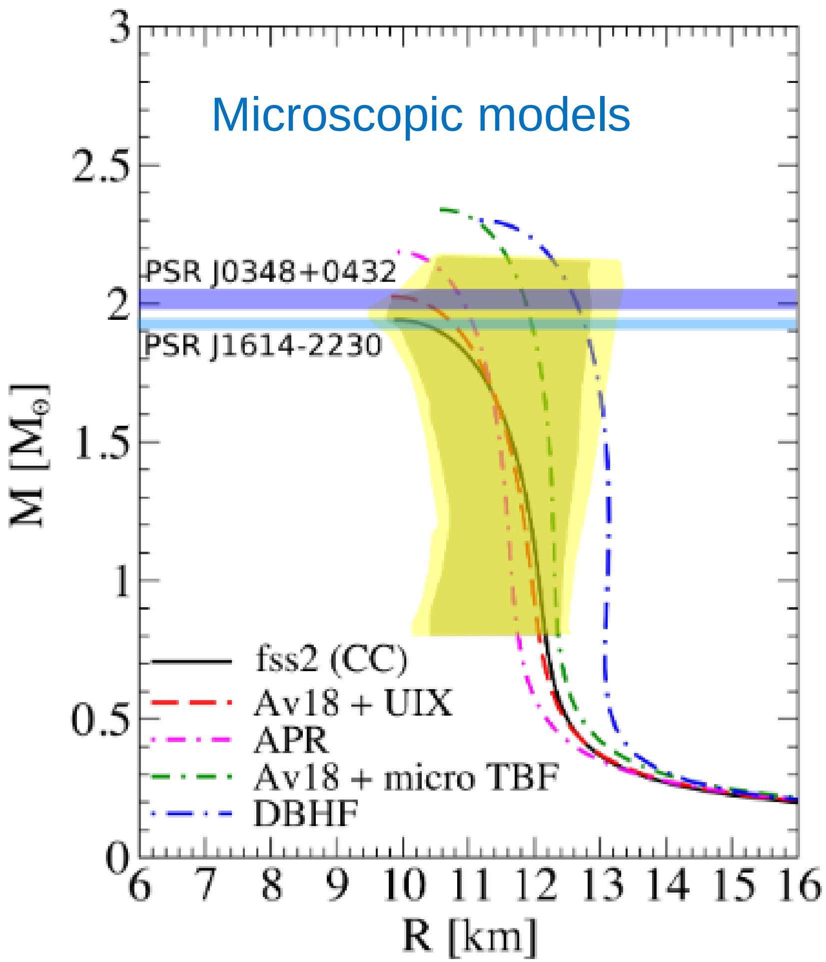
Precise (+/-10%) measurements of a moment of inertia can significantly constrain the range of allowed radii for a give EoS.

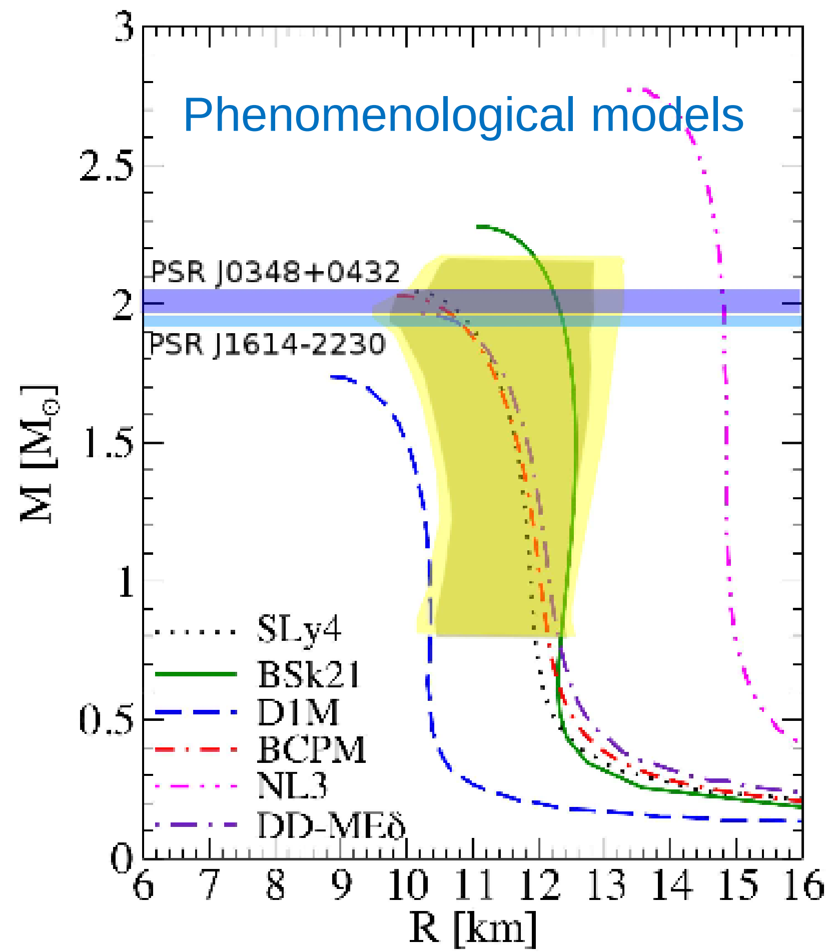
2005.14164



Lattimer & Prakash (2004)

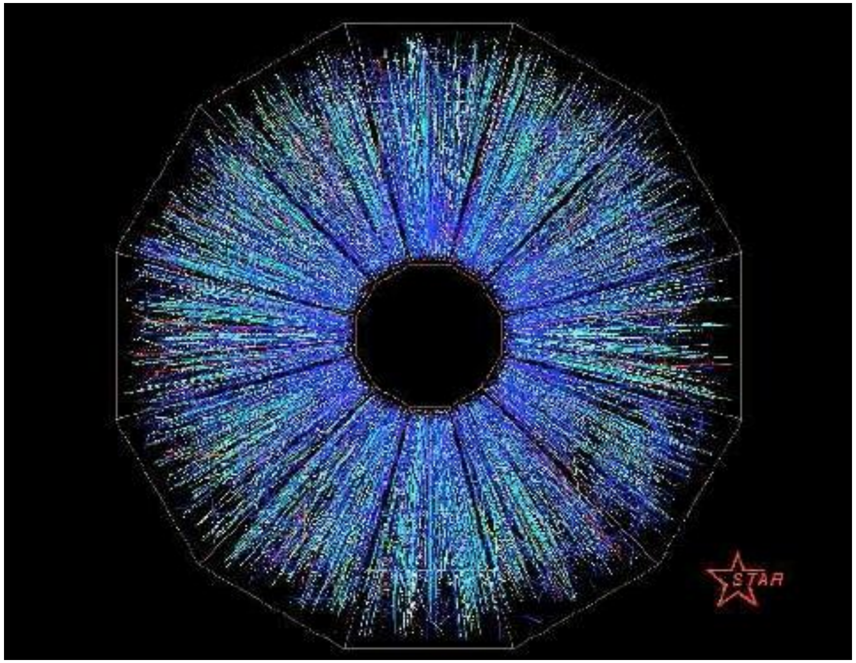
Theory vs. observations

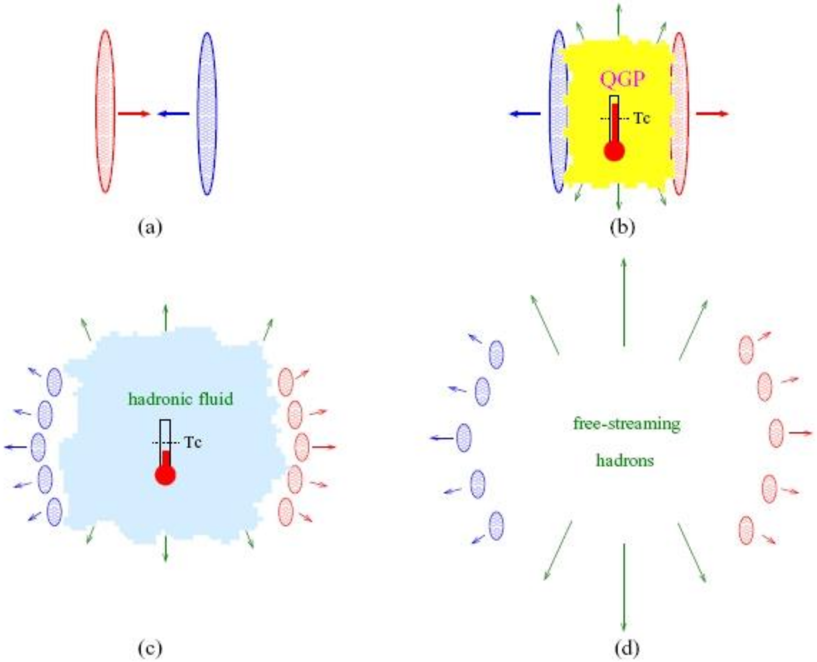




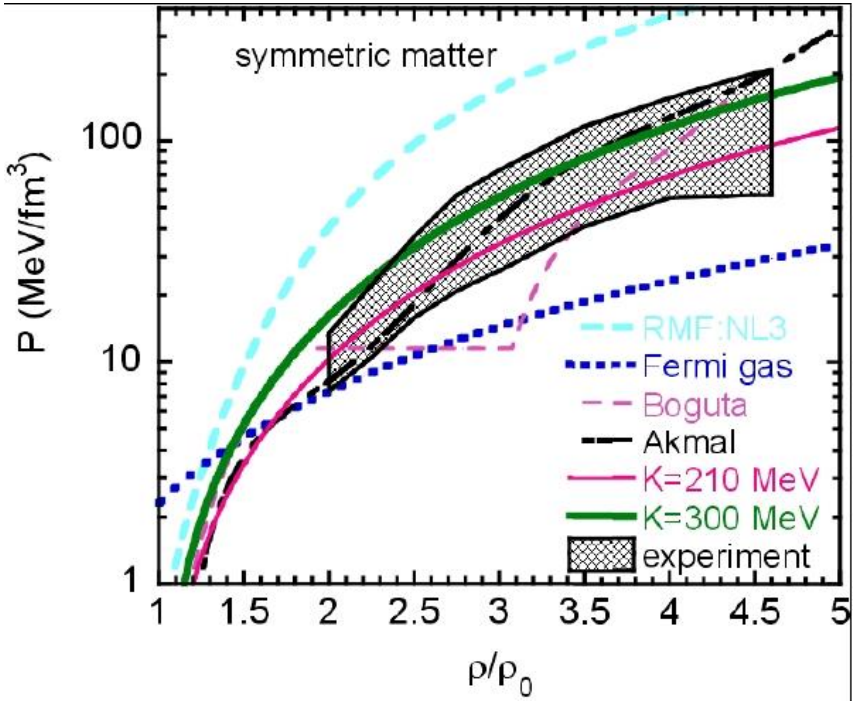
1804.03020

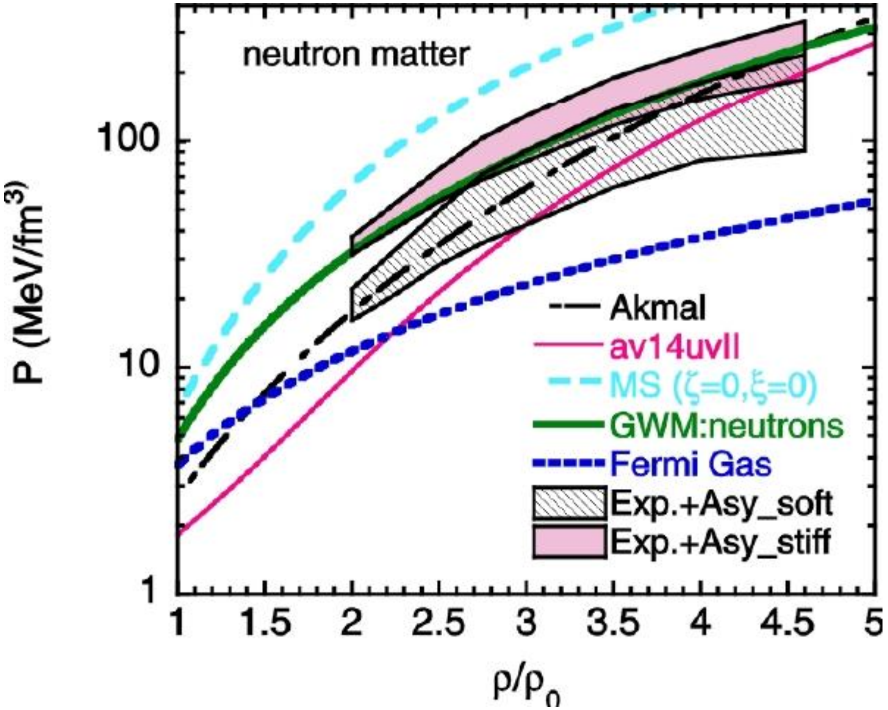
Au-Au collisions





Experimental results and comparison





1 Mev/fm^3 = 1.6 10^{32} Pa

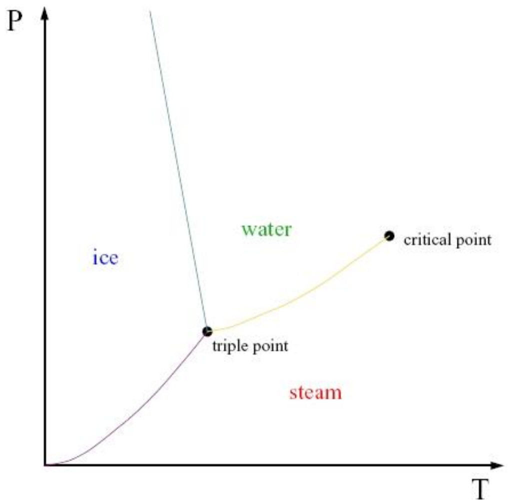
GSI-SIS and AGS data

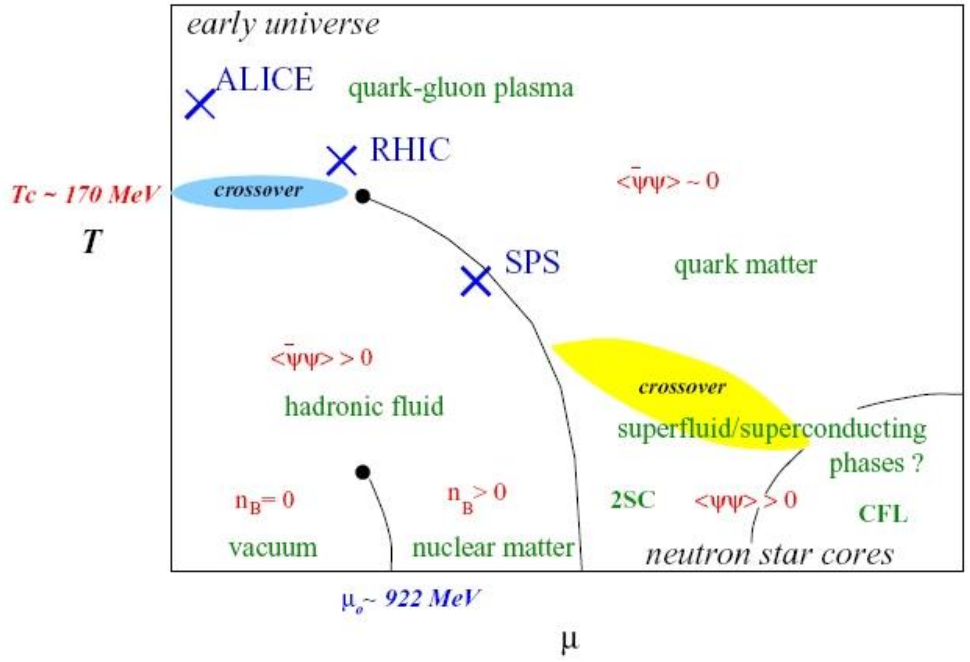
Danielewicz et al. nucl-th/0208016

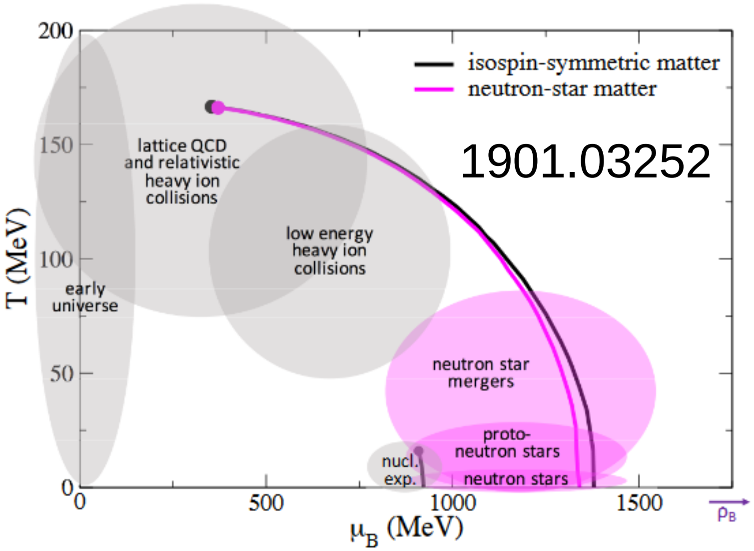
New heavy-ion data and discussion: 1211.0427

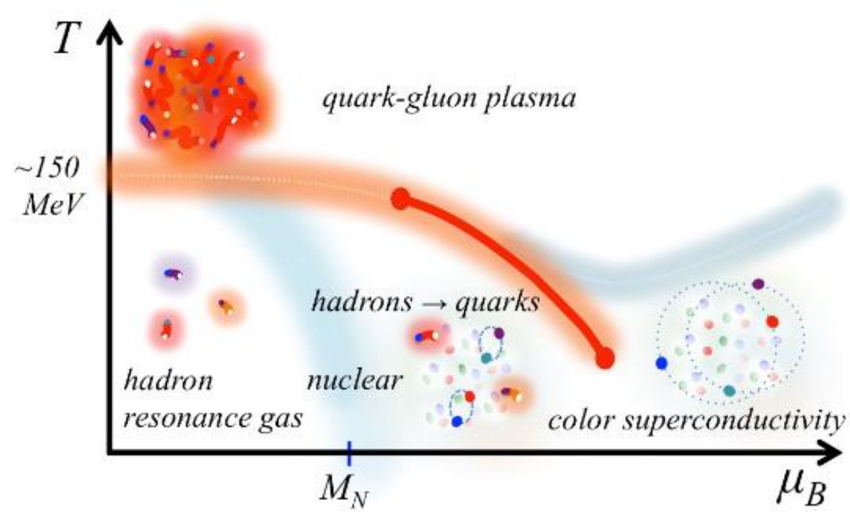
Also laboratory measurements of lead nuclei radius can be important, see 1202.5701

Phase diagram





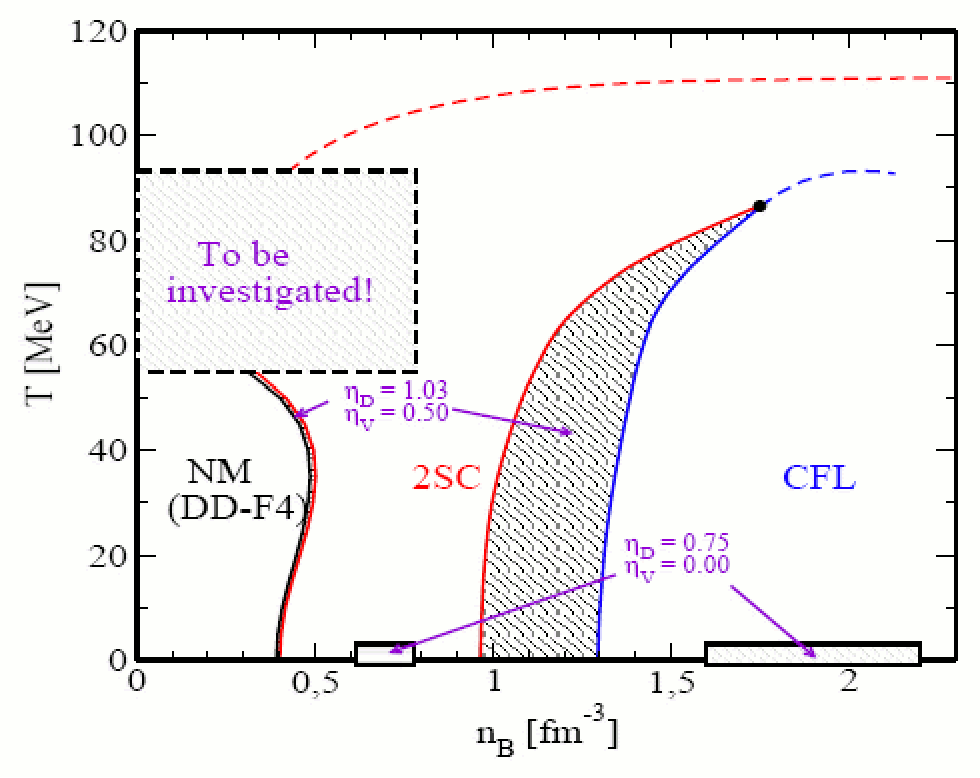




See 1803.01836

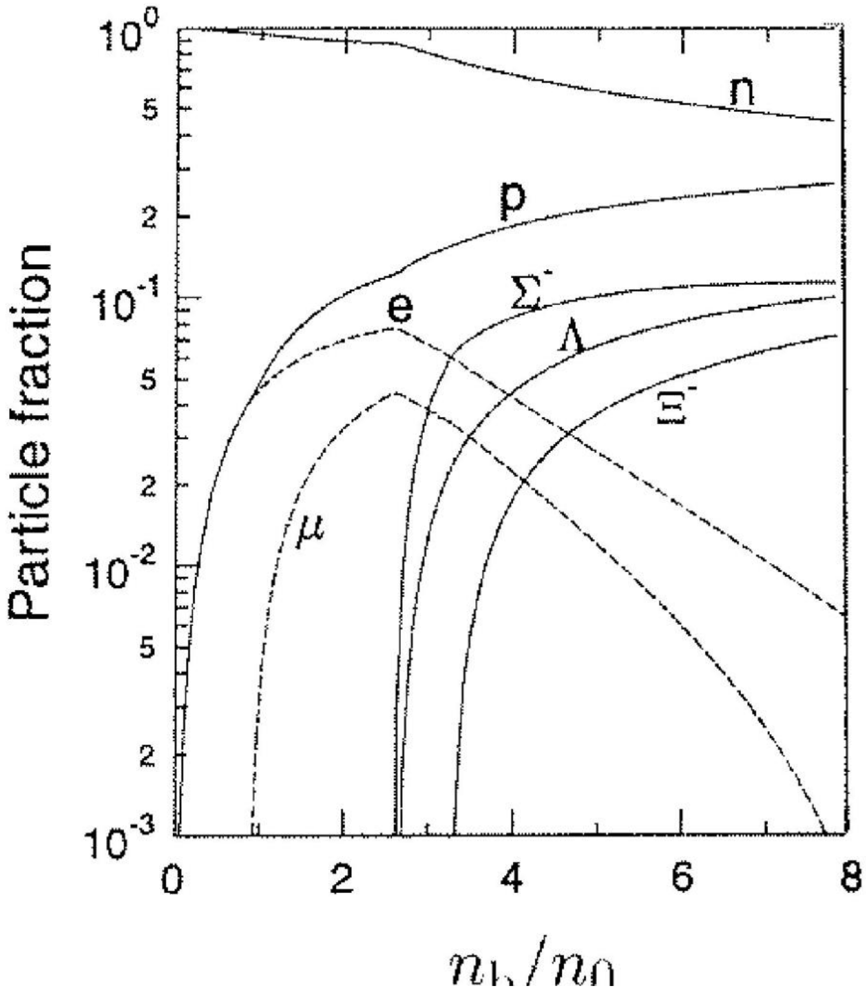
Phase diagram

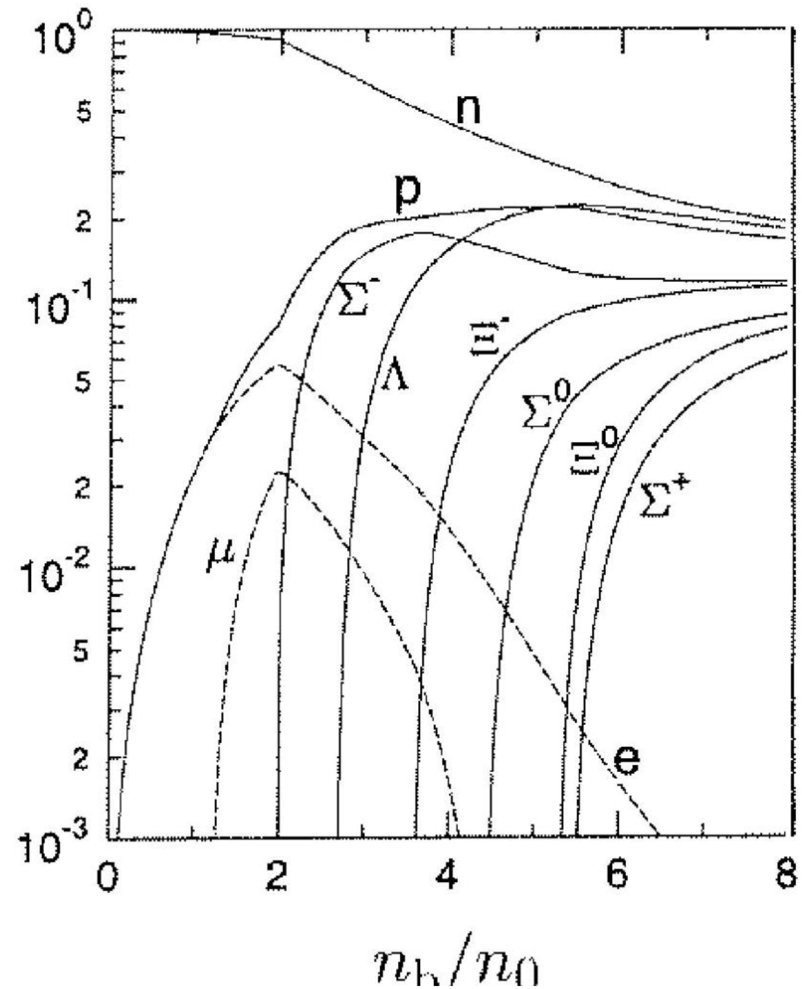
Phase diagram for isospin symmetry using the most favorable hybrid EoS studied in astro-ph/0611595.



(astro-ph/0611595)

Particle fractions

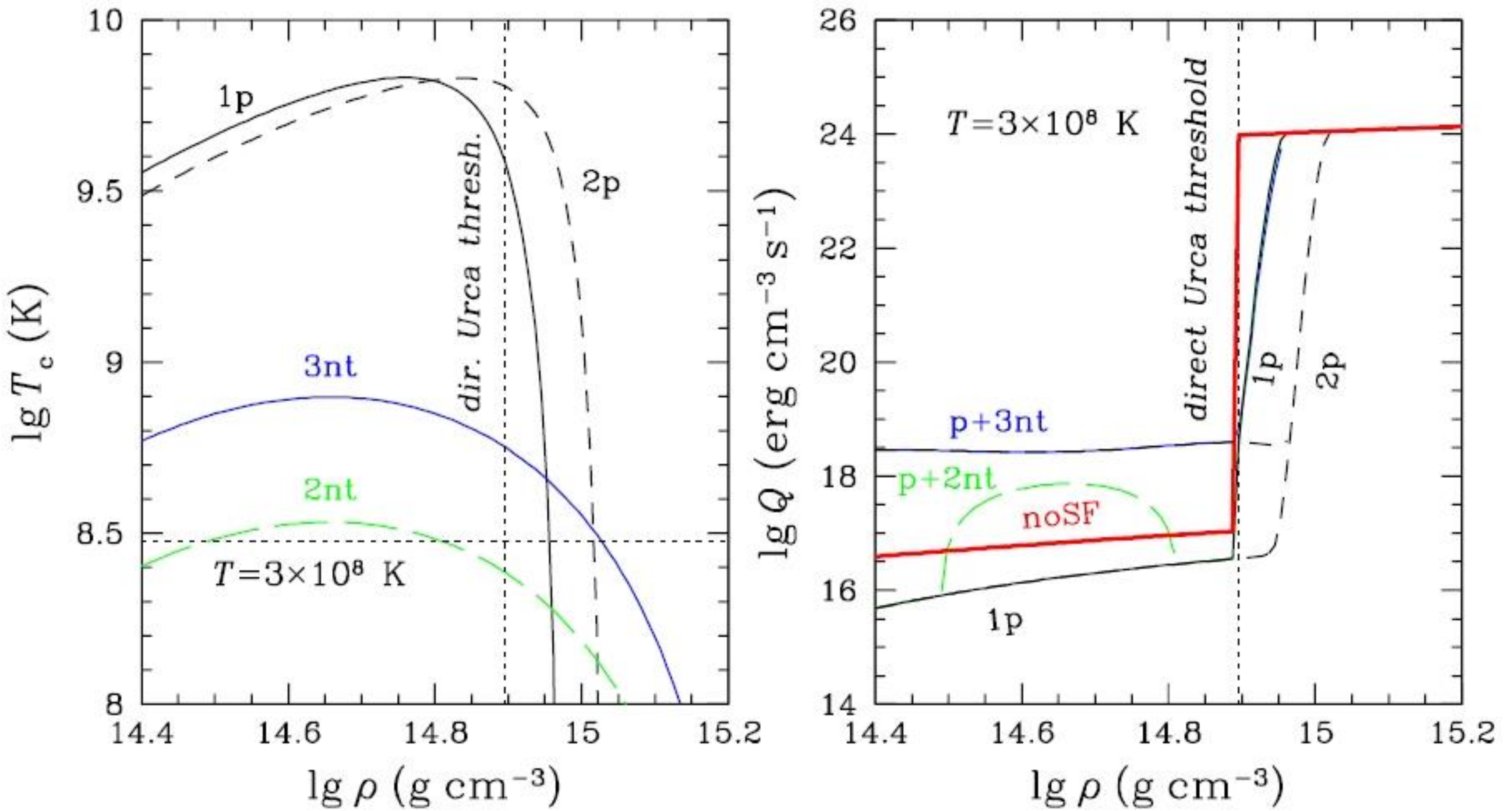




Effective chiral model of Hanauske et al. (2000)

Relativistic mean-field model TM1 of Sugahara & Toki (1971)

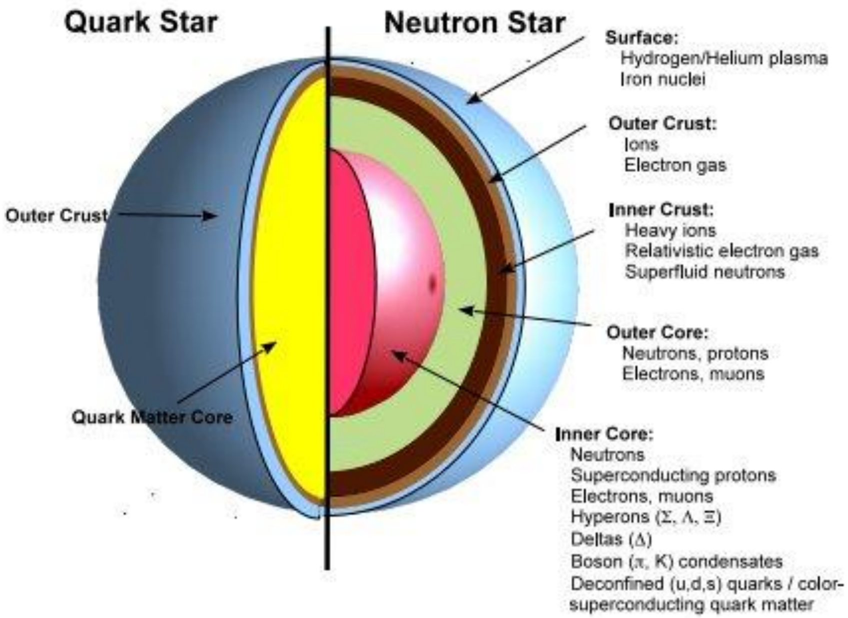
Superfluidity in NSs

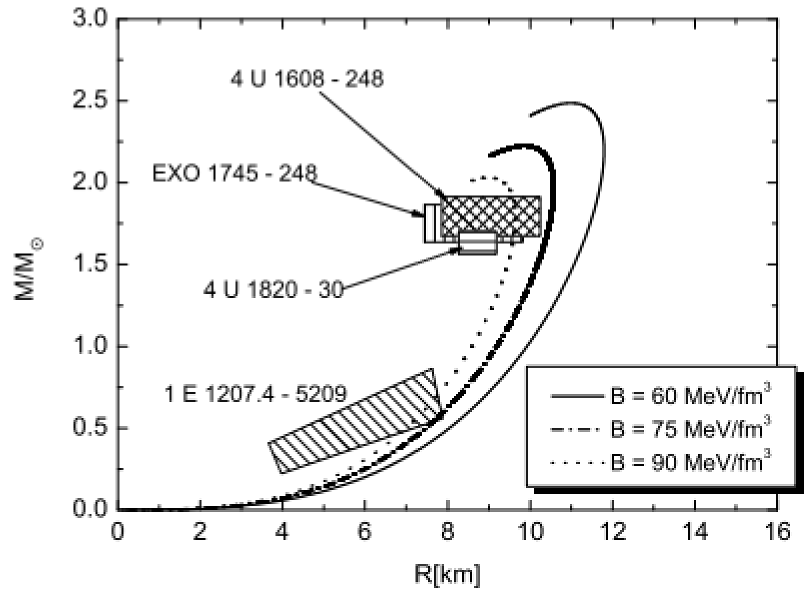


(Yakovlev)

Яковлев и др. УФН 1999

Quark stars

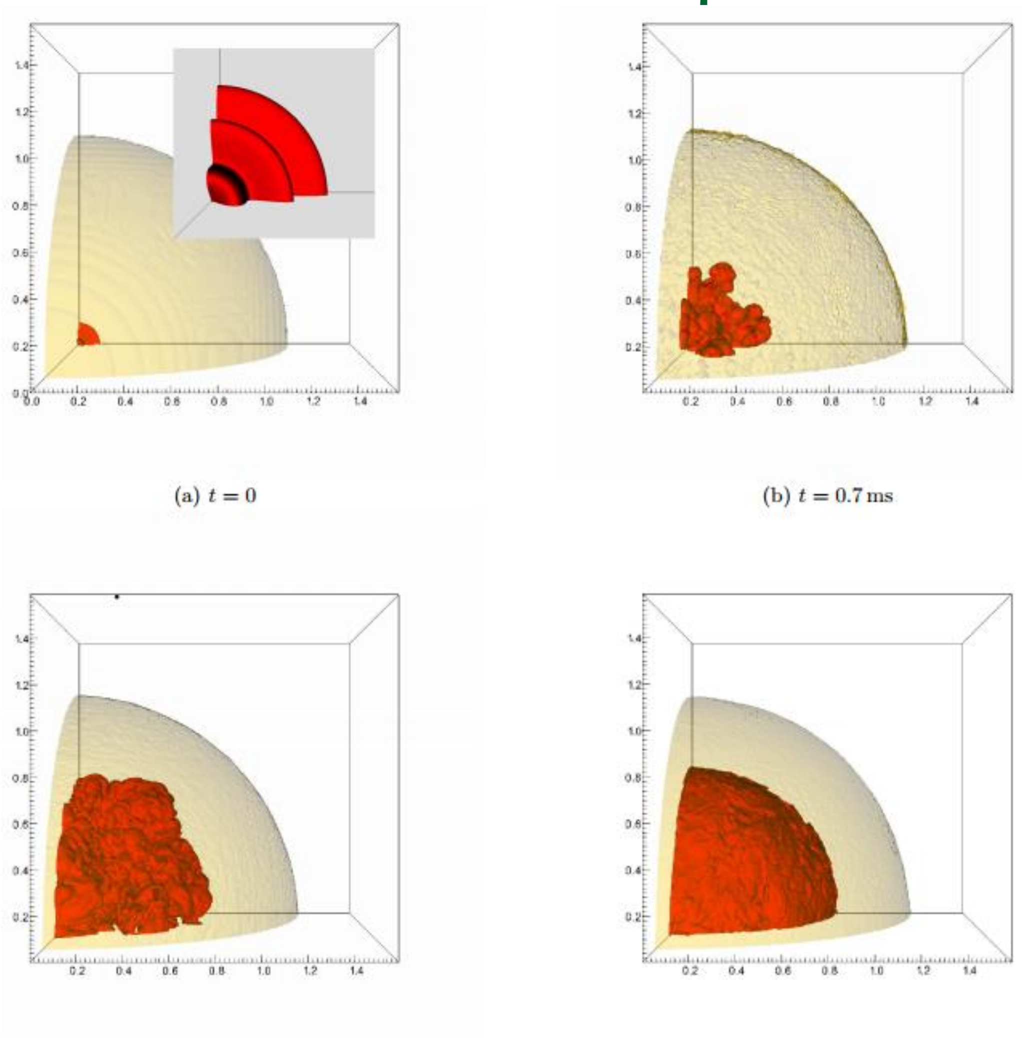




1210.1910

See also 1112.6430

Formation of quark stars



(a) $t = 0$ (b) $t = 0.7\,\mathrm{ms}$ (c) $t = 1.2\,\mathrm{ms}$ (d) $t = 4.0\,\mathrm{ms}$

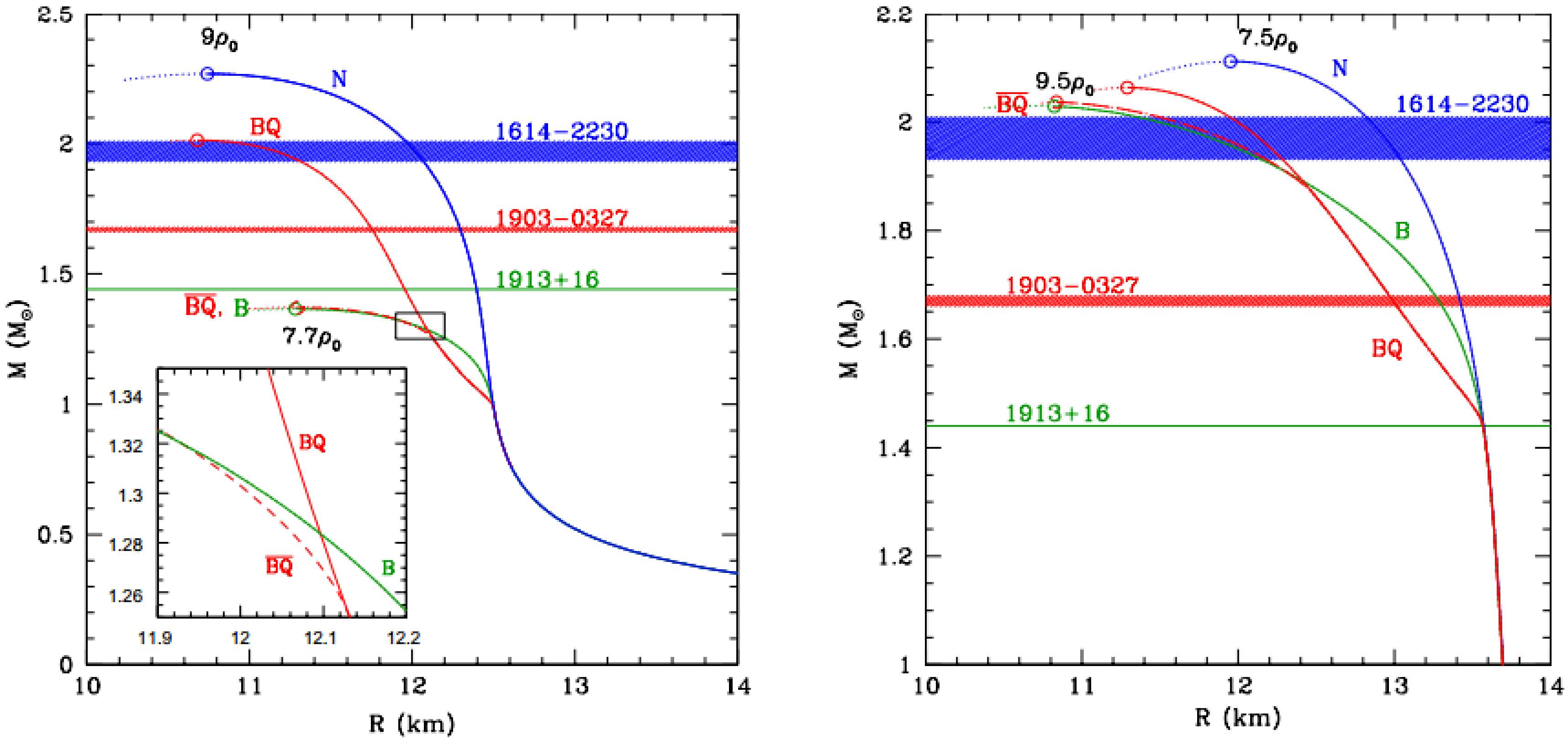
Turbulent deflagration,
as in SNIa.

Neutrino signal due to
conversion of a NS into
a quark star was calculated
in 1304.6884

1109.0539

Hybrid stars

1211.1231

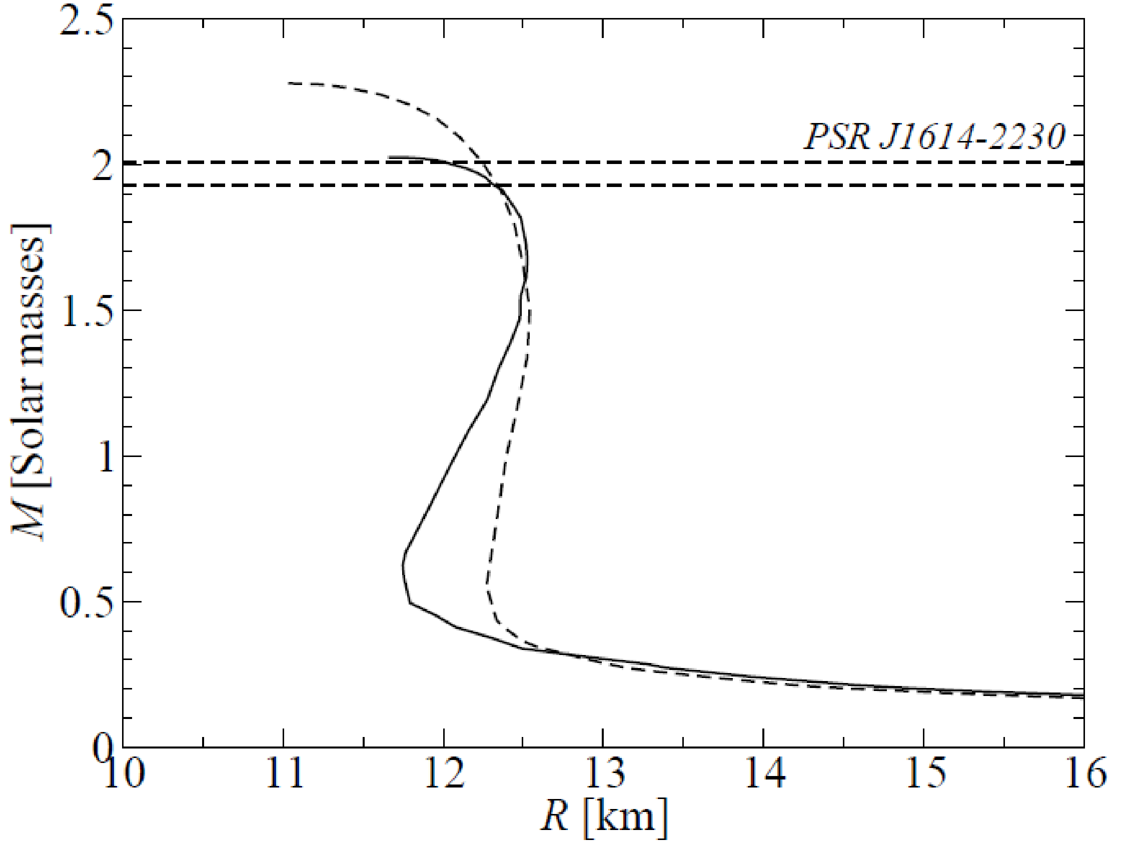


Stars with quark cores are reviewed in 1904.05471

See also **1302.4732, 1903.08963, 1903.09121**

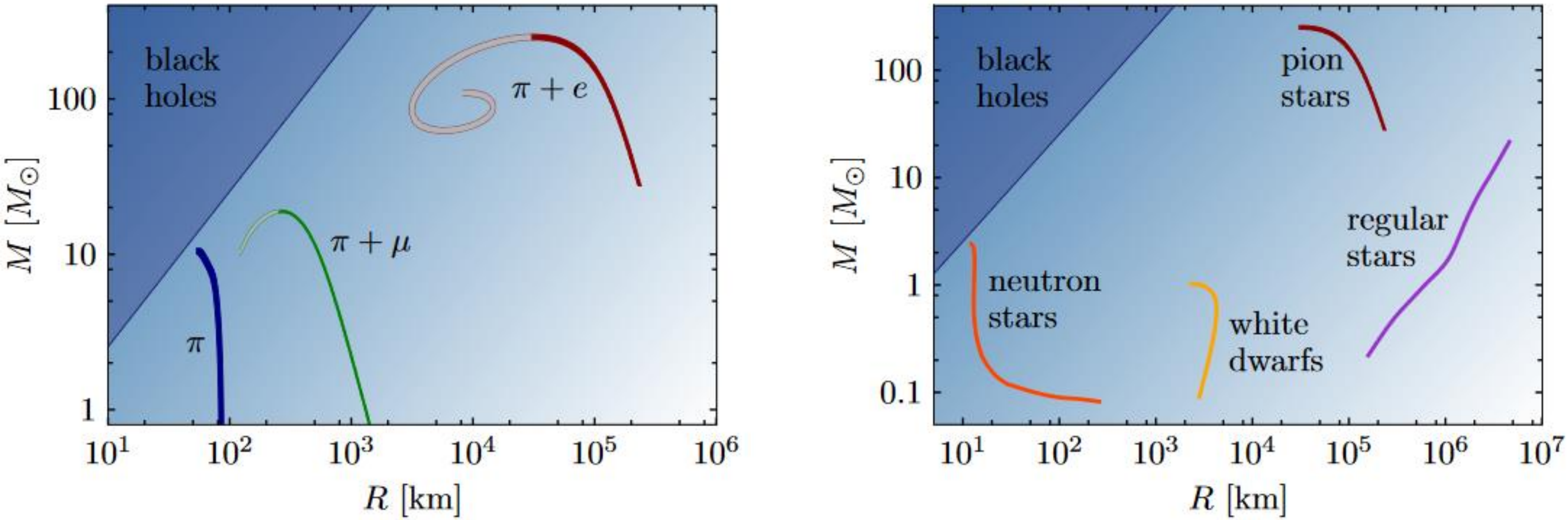
Massive hybrid stars

Stars with quark cores can be massive, and so this hypothesis is compatible with existence of pulsars with M>2 Msolar



1304.6907 New solutions also allow such objects, see **1903.08963, 1903.09121**

Pion stars

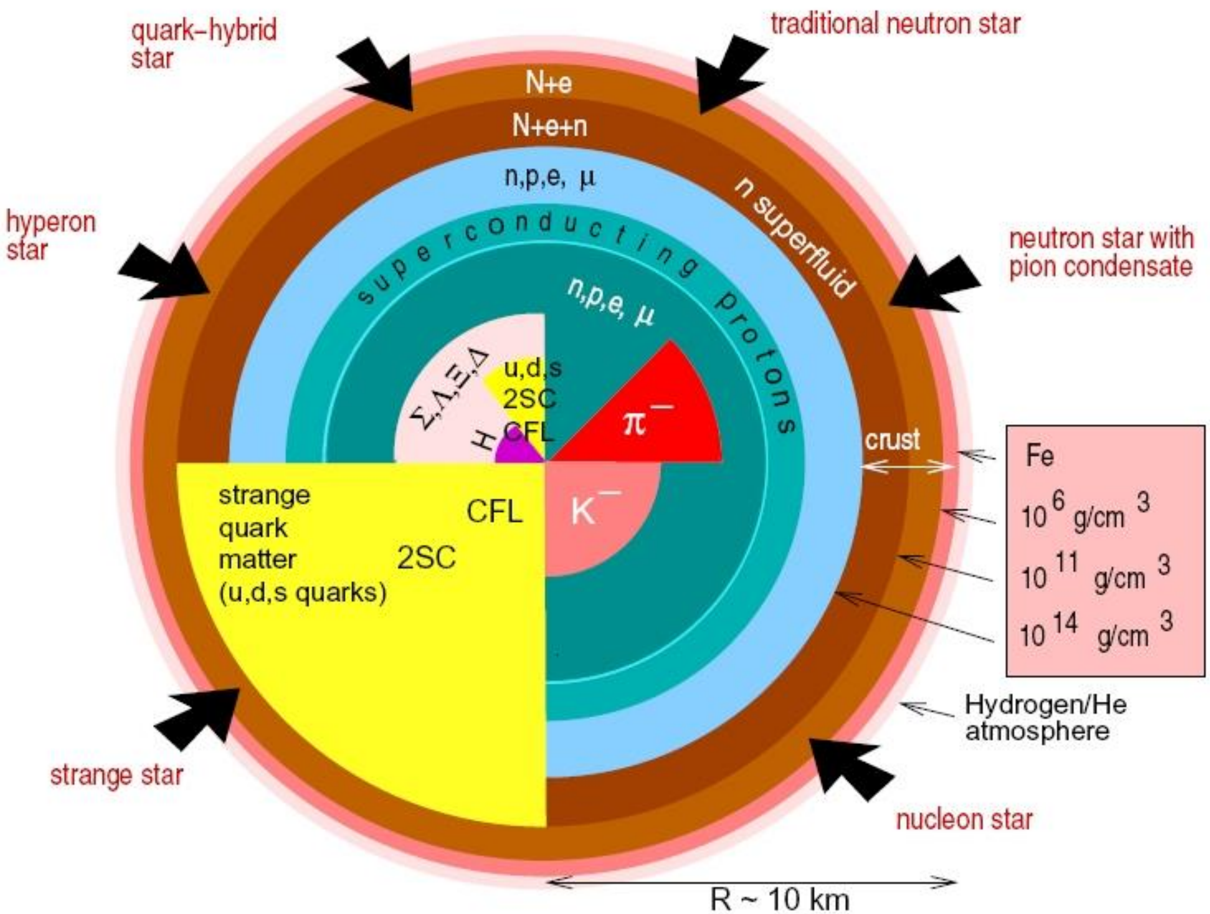


New exotic solution.

It is not clear if it can be applied to any known type of sources.

1802.06685

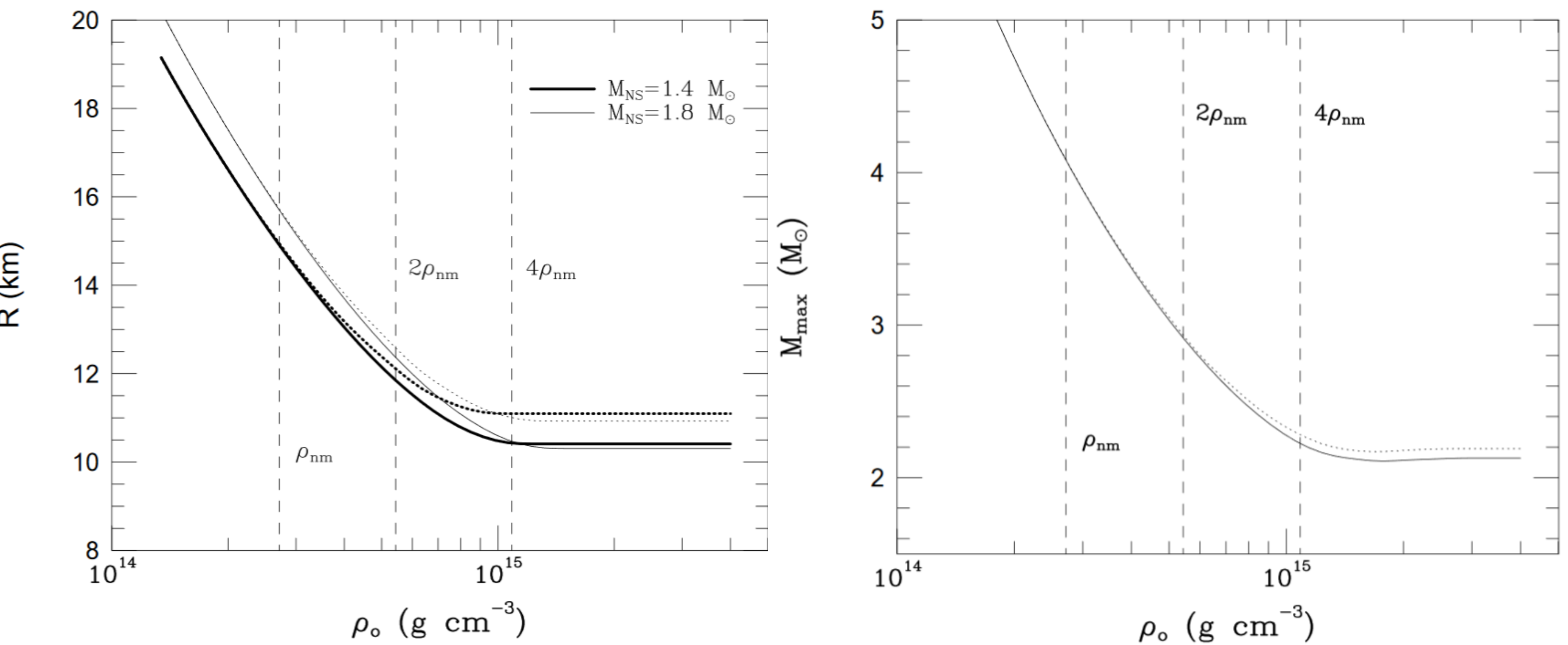
NS interiors: resume



(Weber et al. ArXiv: 0705.2708)

Maximum mass

Maximum mass of NSs depends on the EoS, however, it is possible to make calculations on the base of some fundamental assumptions.

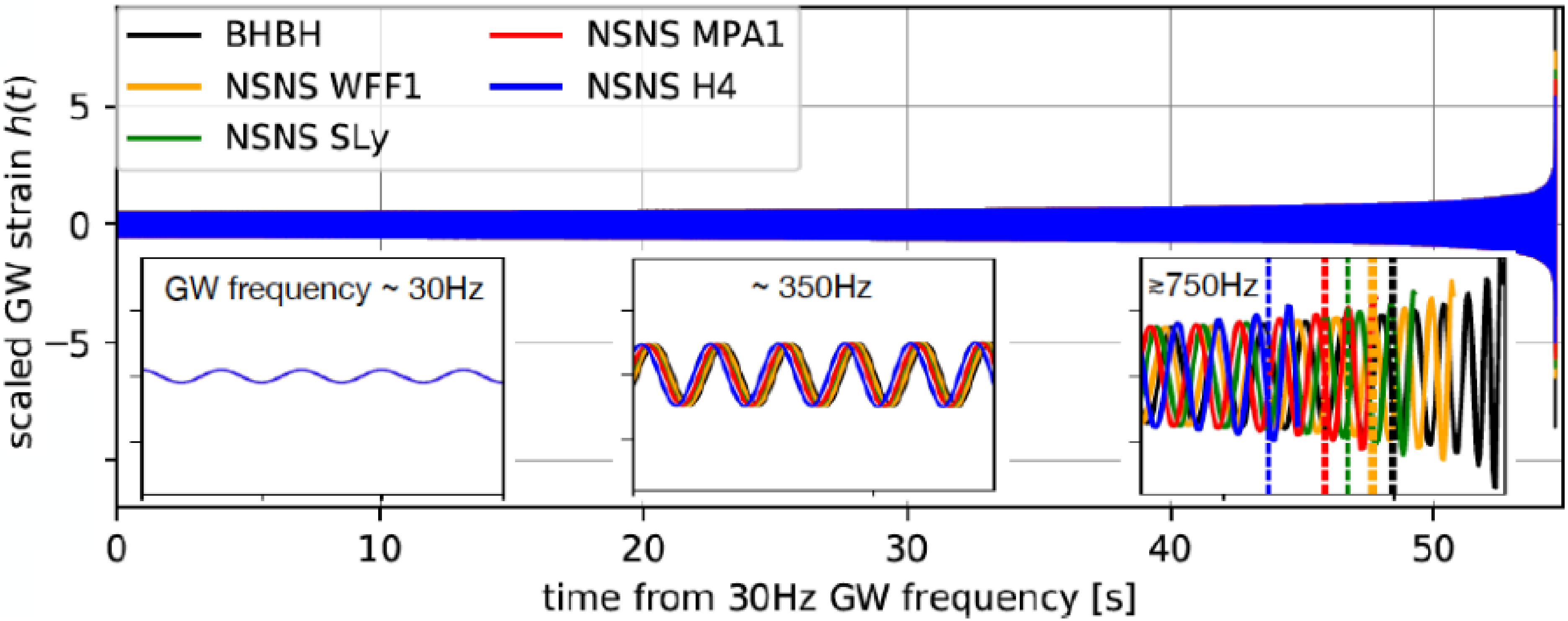


astro-ph/9608059

Seminal paper: Rhoades, Ruffini 1974
http://prl.aps.org/abstract/PRL/v32/i6/p324_1

$$c_s^2 = \frac{dP}{d\rho} = c^2.$$

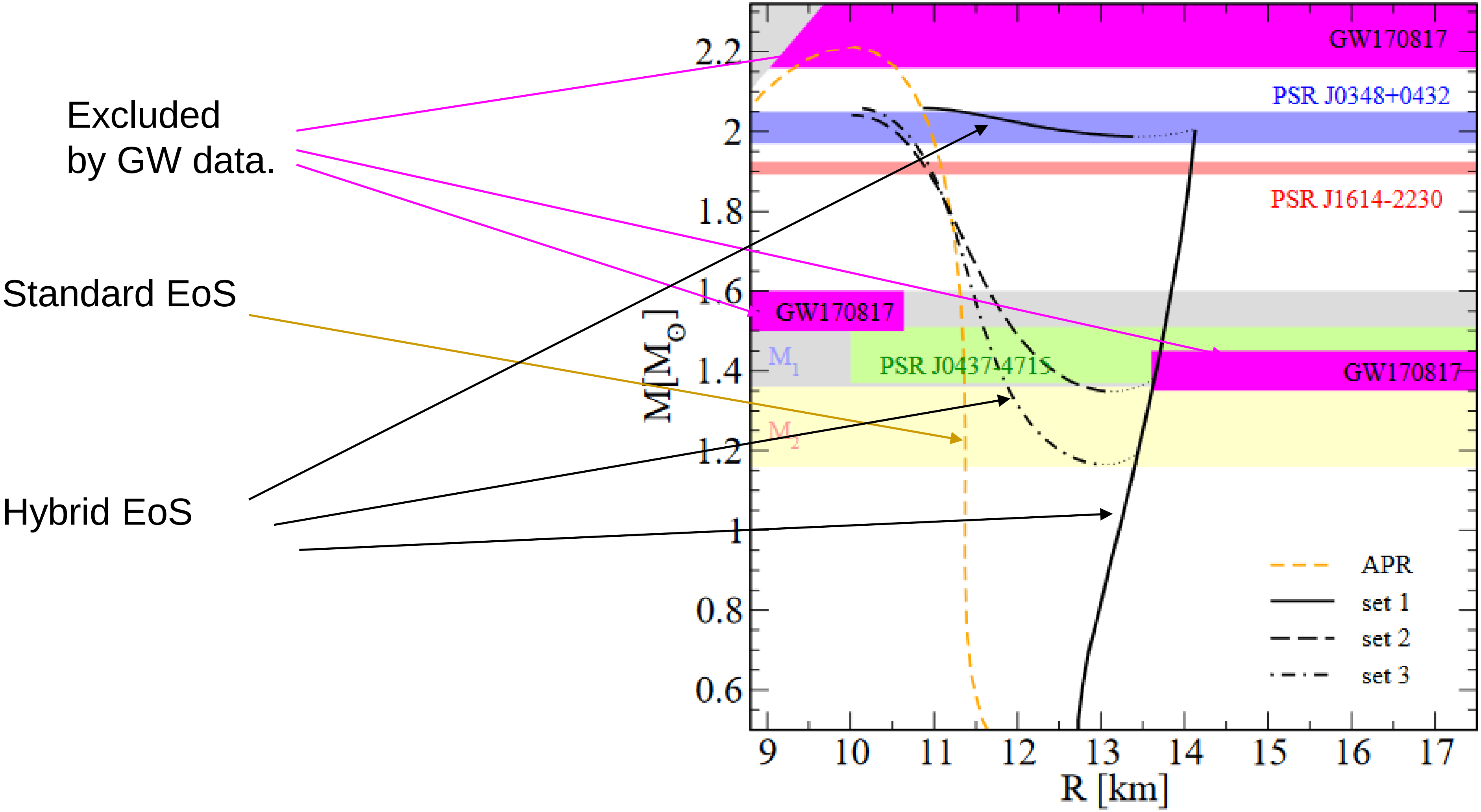
NS-NS coalescence and EoS



GW signals for different EoS for two non-spinning 1.4 solar mass NSs.
Vertical dashed line correspond to 1kHz frequency.
Differences are due to different tidal response.

1912.01461, see 1907.08534 on methods of such studies

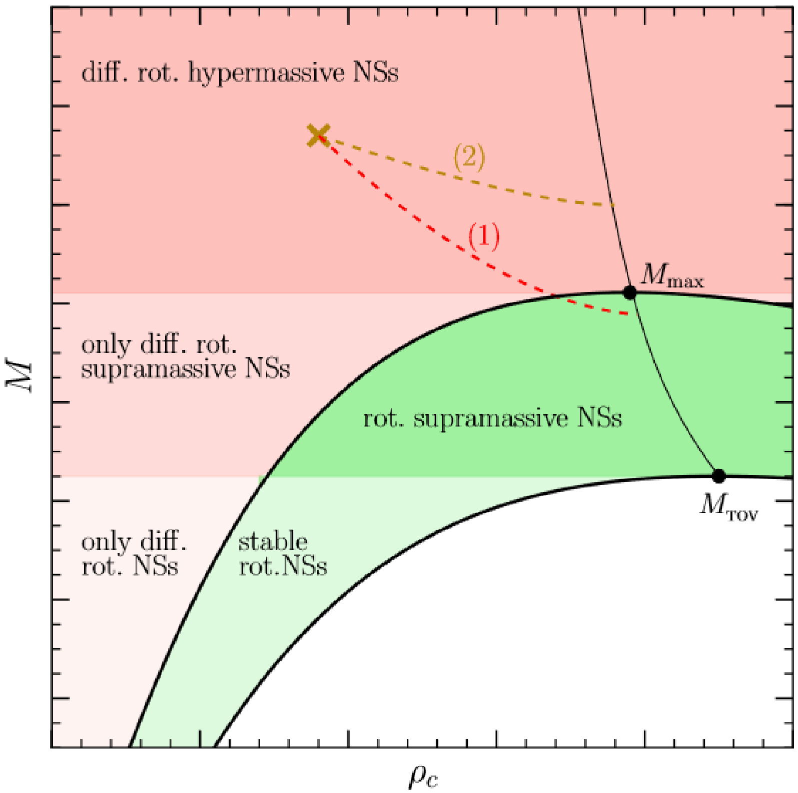
Constraints on the M-R plane

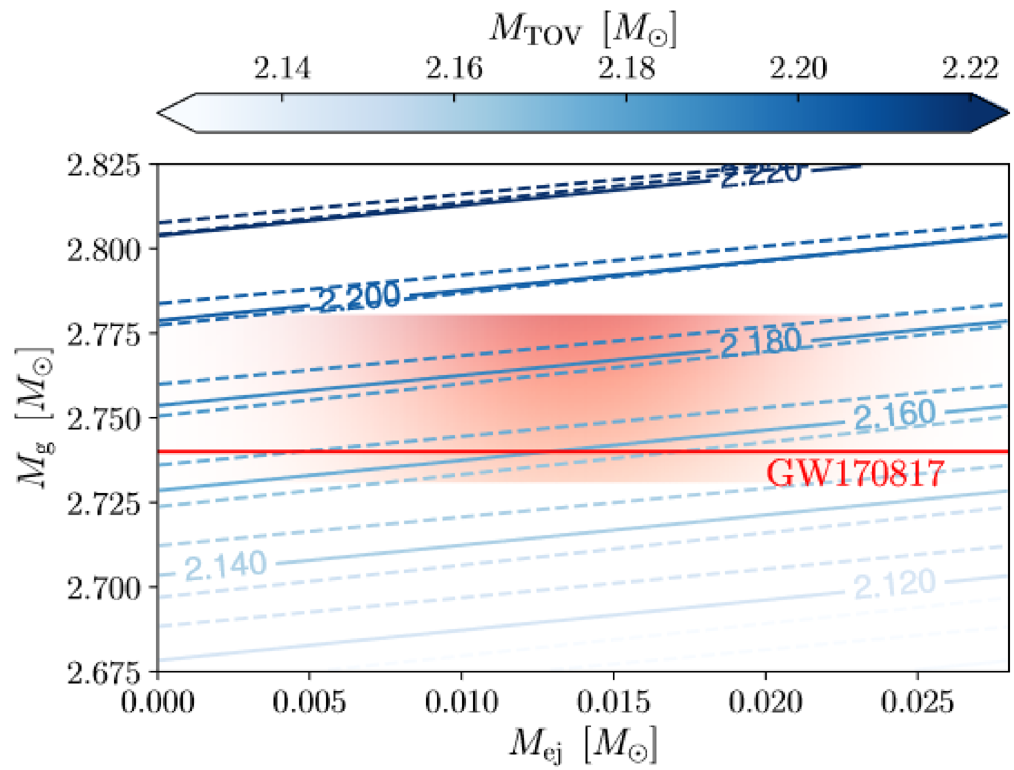


1907.08534, about physics of mergers see 2002.03863

Calculations based on recent data on NS-NS coalescence

What uniform rotation can give: $M_{\max} = \left(1.20^{+0.02}_{-0.02}\right) M_{\rm TOV}$ independently of the EOS





1711.00314

$M_{\rm TOV}$=2-2.3 solar masses

Another constraint from GW170817

$$M_{\rm NSNS} \approx 2.74 \lesssim M_{\rm thresh} \approx \alpha M_{\rm max}^{\rm sph}.$$

As there was no prompt collapse

Here $\alpha \approx 1.3 - 1.7$ is the ratio of the HMNS threshold mass limit to the NS spherical maximum mass as gleaned from multiple numerical experiments of merging NSNSs

$$M_{\rm NSNS} \approx 2.74 \gtrsim M_{\rm max}^{\rm sup} \approx \beta M_{\rm max}^{\rm sph},$$

where $\beta \approx 1.2$ is the ratio of the uniformly rotating supramassive NS limit to the nonrotating spherical maximum

$$M_{\rm max}^{\rm sph} = 4.8 \left(\frac{2 \times 10^{14}\ {\rm gr/cm^3}}{\rho_m/c^2}\right)^{1/2} M_\odot\,,$$

- TOV limit from causality

$$M_{\rm max}^{\rm sup} = 6.1 \left(\frac{2 \times 10^{14}\ {\rm gr/cm^3}}{\rho_m/c^2}\right)^{1/2} M_\odot\,,$$

$$\beta \approx 1.27.$$

$$2.74/\alpha \lesssim M_{\rm max}^{\rm sph} \lesssim 2.74/\beta$$

$$M_{\rm max}^{\rm sph} \lesssim 2.16. \quad \beta \approx 1.27.$$

$$M_{\rm max}^{\rm sph} \lesssim 2.28. \quad \beta = 1.2$$

1711.00473, see a review on the methods in 1904.04233 and 1912.01461

Papers to read

1. astro-ph/0405262 Lattimer, Prakash "Physics of neutron stars"
2. 0705.2708 Weber et al. "Neutron stars interiors and equation of state …"
3. physics/0503245 Baym, Lamb "Neutron stars"
4. 0901.4475 Piekarewicz "Nuclear physics of neutron stars" (first part)
5. 0904.0435 Paerels et al. "The Behavior of Matter Under Extreme Conditions"
6. 1512.07820 Lattimer, Prakash "The EoS of hot dense matter …."
7. 1001.3294 Schmitt "Dense matter in compact stars - A pedagogical introduction "
8. 1303.4662 Hebeler et al. "EoS and NS properties vs. nuclear phys. and observation"
9. 1210.1910 Weber et al. Structure of quark star
10. 1302.1928 Stone "High density matter "
11. 1707.04966 Baym et al. "From hadrons to quarks in neutron stars: a review"
12. 1804.03020. Burgio, Fantina "Nuclear EoS for Compact Stars and Supernovae"
13. 1803.01836 Blaschke, Chamel. "Phases of dense matter in compact stars"
14. 1904.05471 Alford et al. "Signatures of quark matter…"
15. 1904.08907 Coleman Miller et al. "Constraining the EoS …."
16. 1912.11876 Pethick "Dense matter and neutron stars"

+ the book by Haensel, Yakovlev, Potekhin

Lectures on the Web

Lectures can be found at my homepage:

http://xray.sai.msu.ru/~polar/html/presentations.html