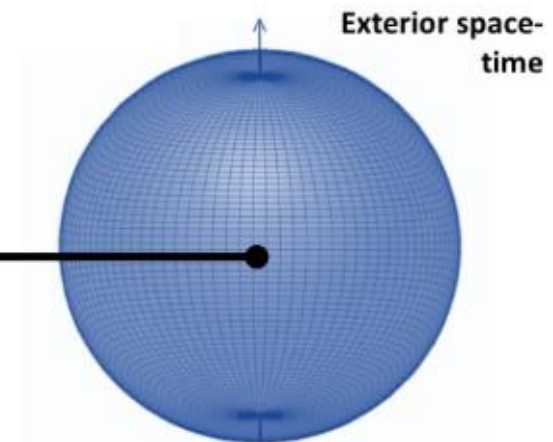
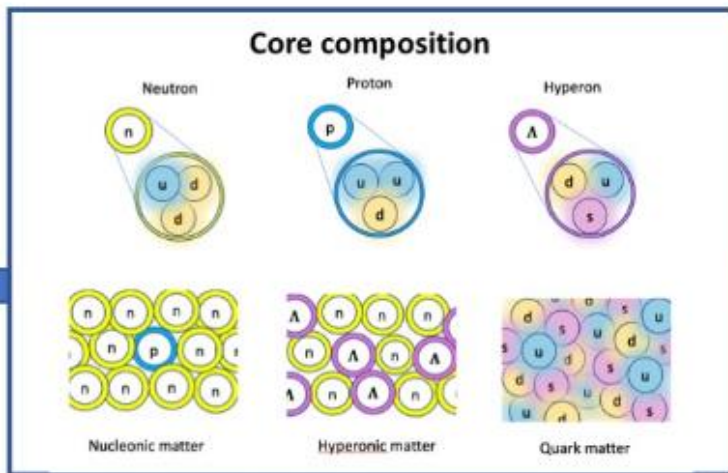
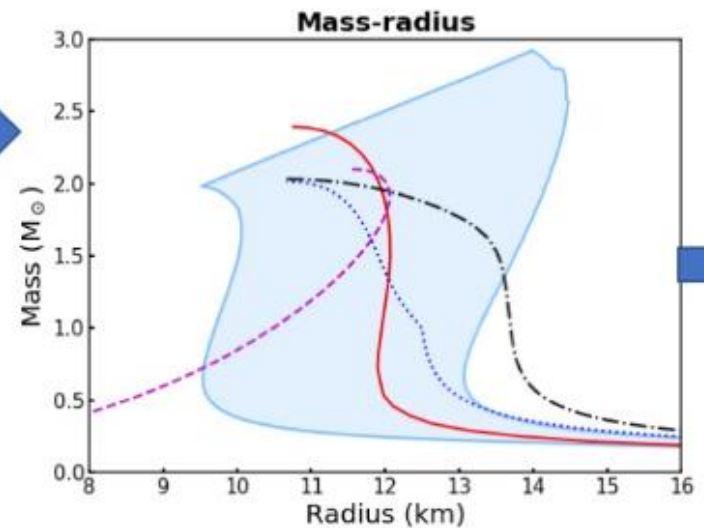
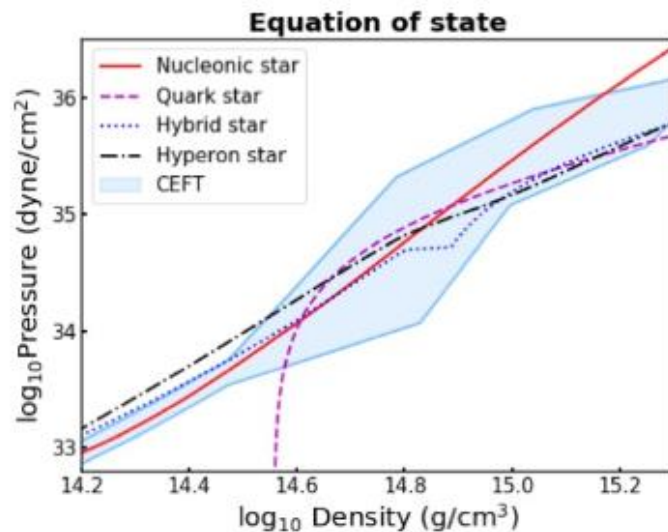


Neutron Star masses and radii

Why important?



NS Masses

- Stellar masses are directly measured in binary systems
- Accurate NS mass determination for PSRs in relativistic systems by measuring PK corrections
- Gravitational redshift may provide M/R in NSs by detecting a *known* spectral line,

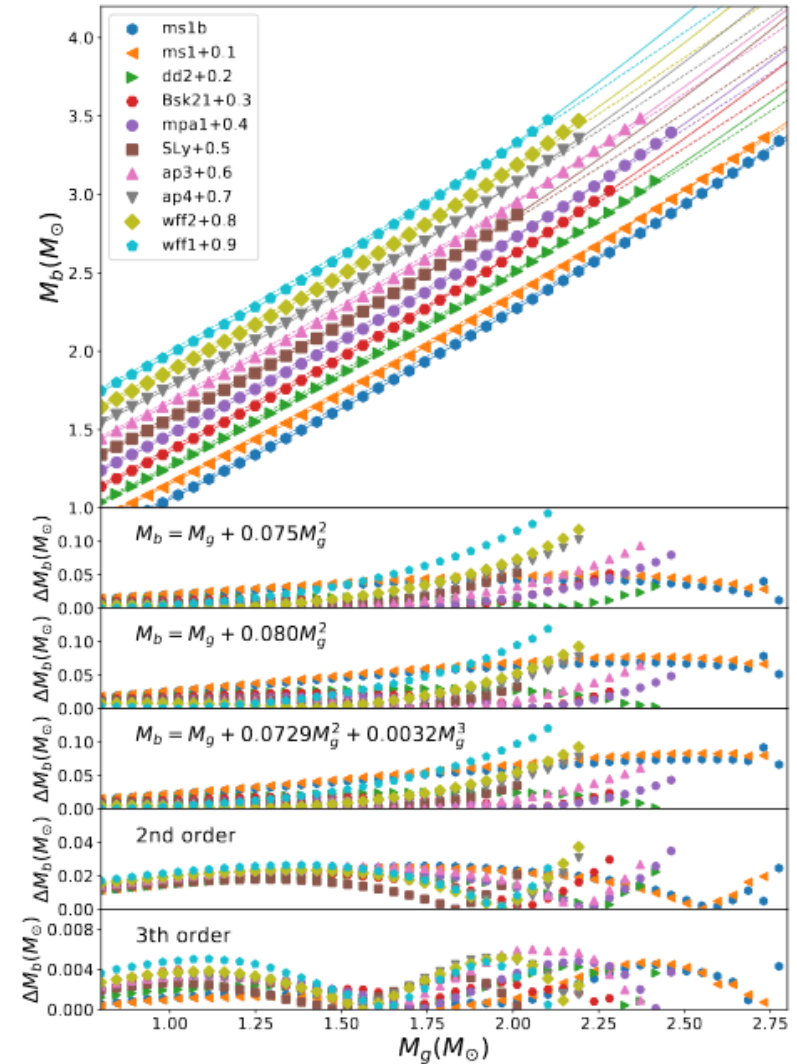
$$E_{\infty} = E(1-2GM/Rc^2)^{1/2}$$

Baryonic vs. gravitational

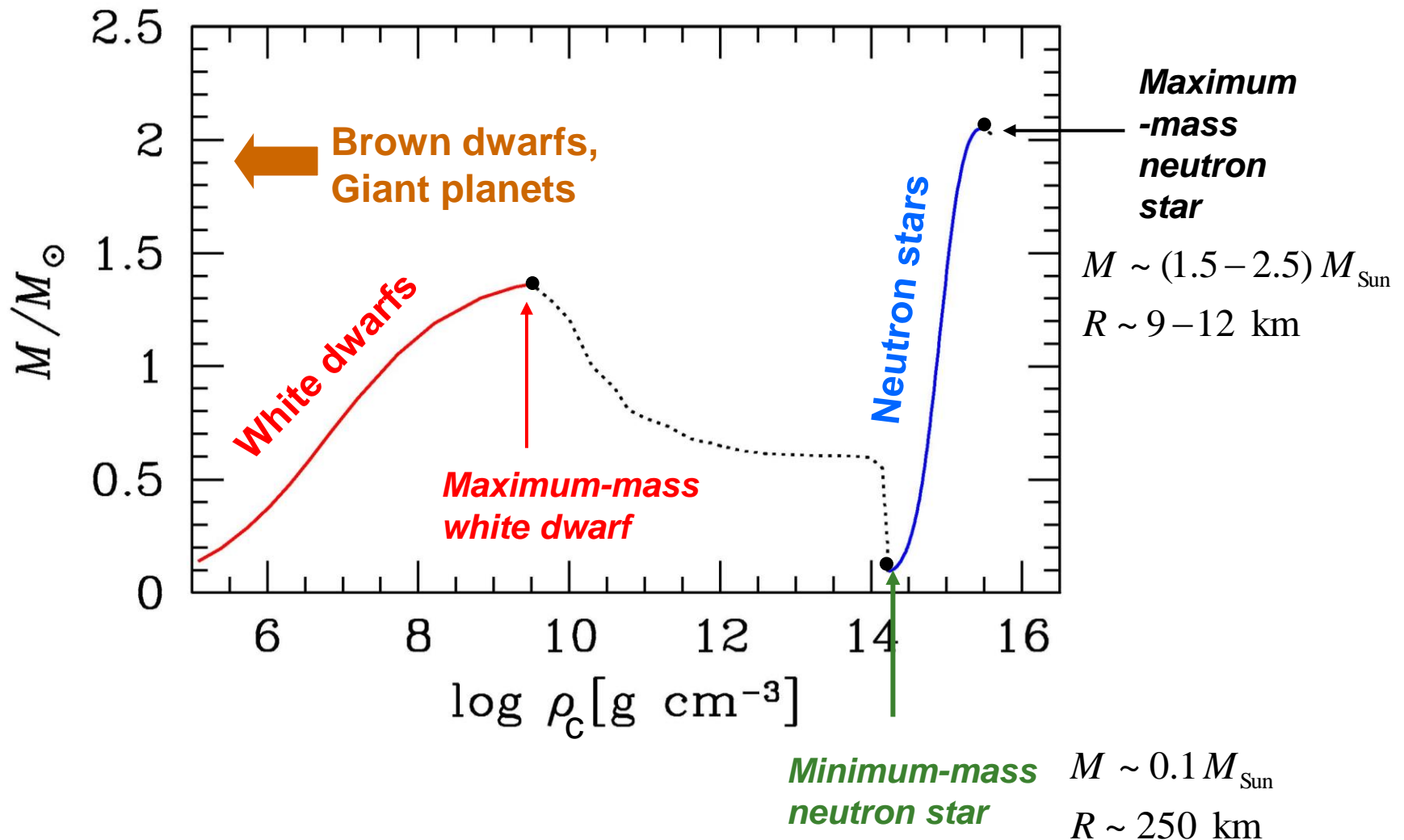
$$M_b = M_g + A_1 \times M_g^2 + A_2 \times M_g^3$$

$A_1 \ 0.0729 \quad A_2 \ 0.0032$

$$M_b = M_g + R_{1.4}^{-1} \times M_g^2$$



Neutron stars and white dwarfs



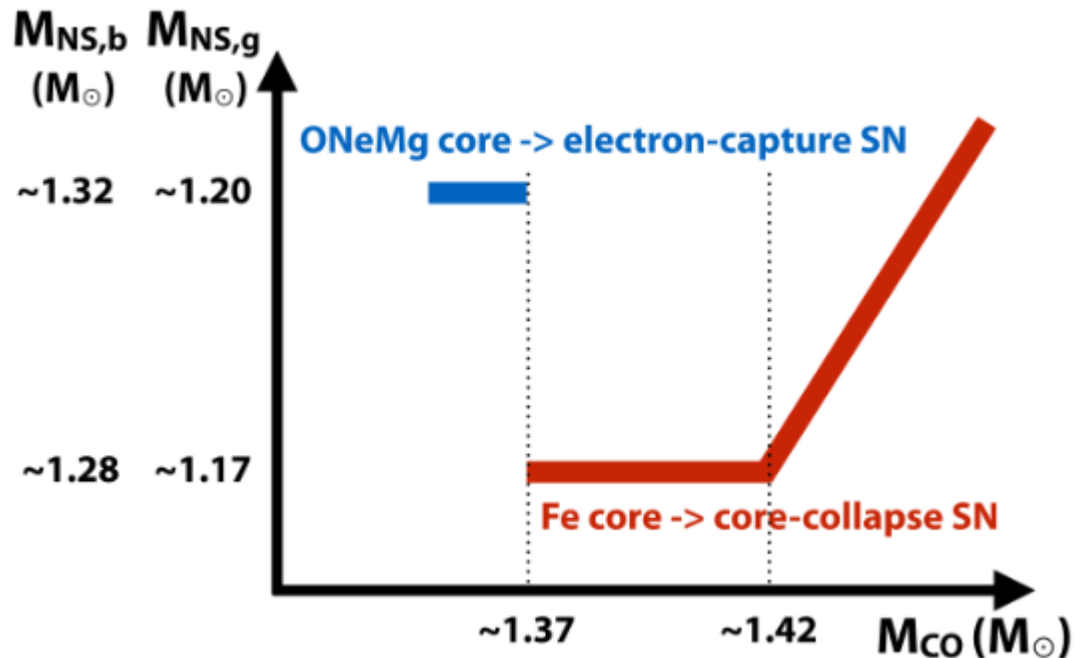
Remember about the difference between baryonic and gravitational masses in the case of neutron stars!

Minimal mass

In reality, minimal mass is determined by properties of protoNSs.
Being hot, lepton rich they have much higher limit: about 0.7 solar mass.

Stellar evolution does not produce
NSs with baryonic mass
less than about 1.1-1.2 solar.

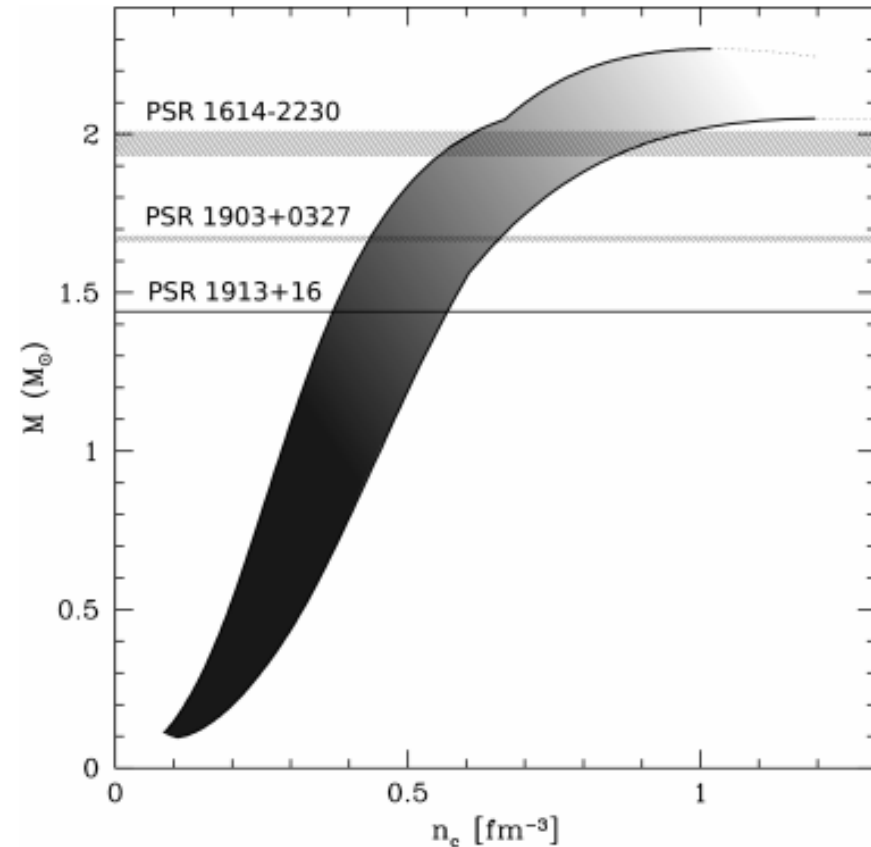
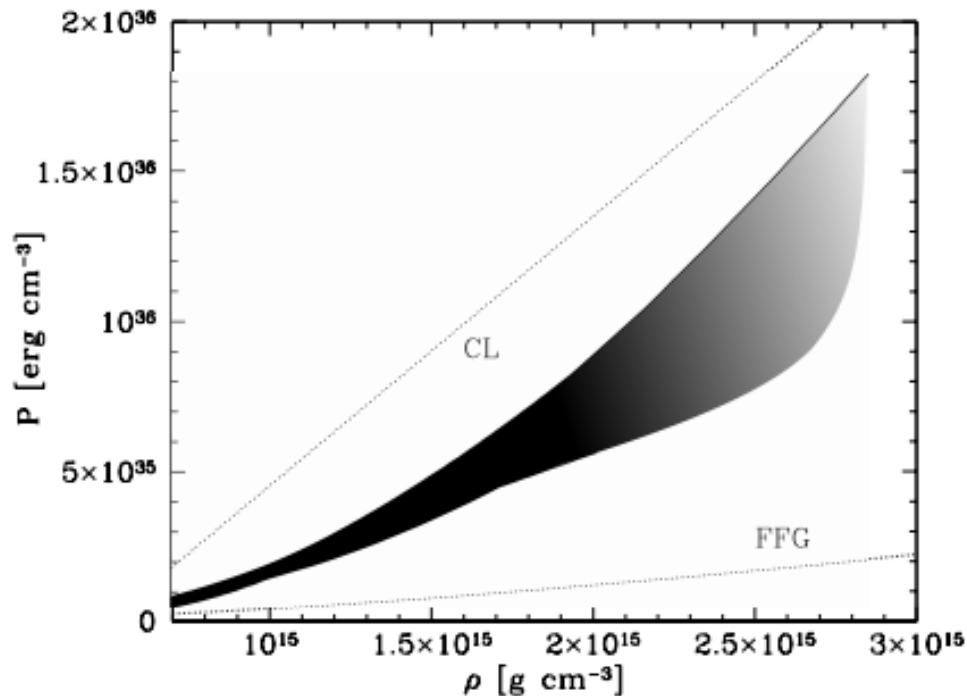
Fragmentation of a core
due to rapid rotation
potentially can lead
to smaller masses,
but not as small as
the limit for cold NSs.



1808.02328

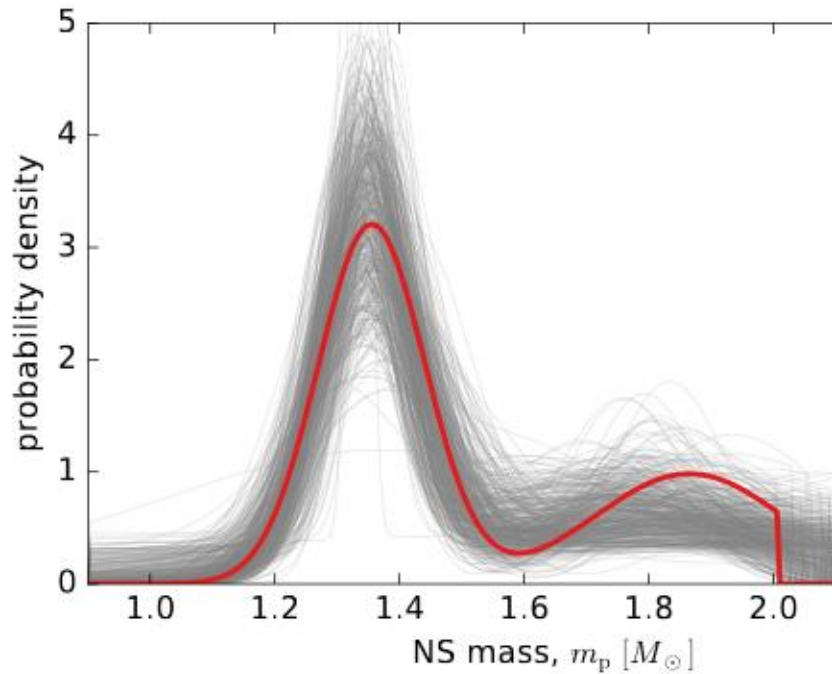
Maximum mass

Detailed discussion about the maximum mass is given in 1307.3995

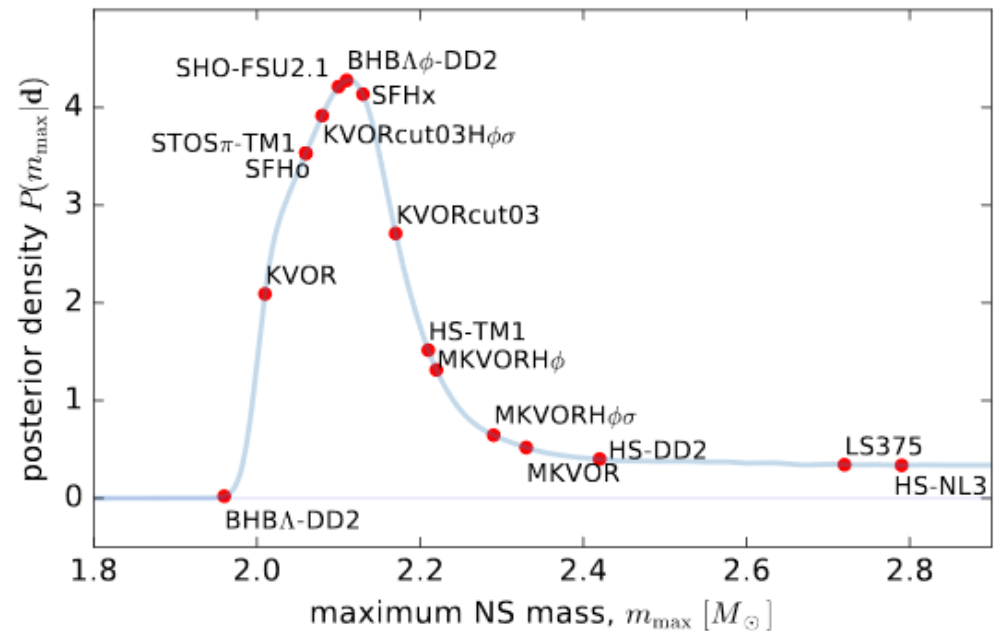


Correlations between macroscopic parameters for M-Mmax are discussed in 2005.03549

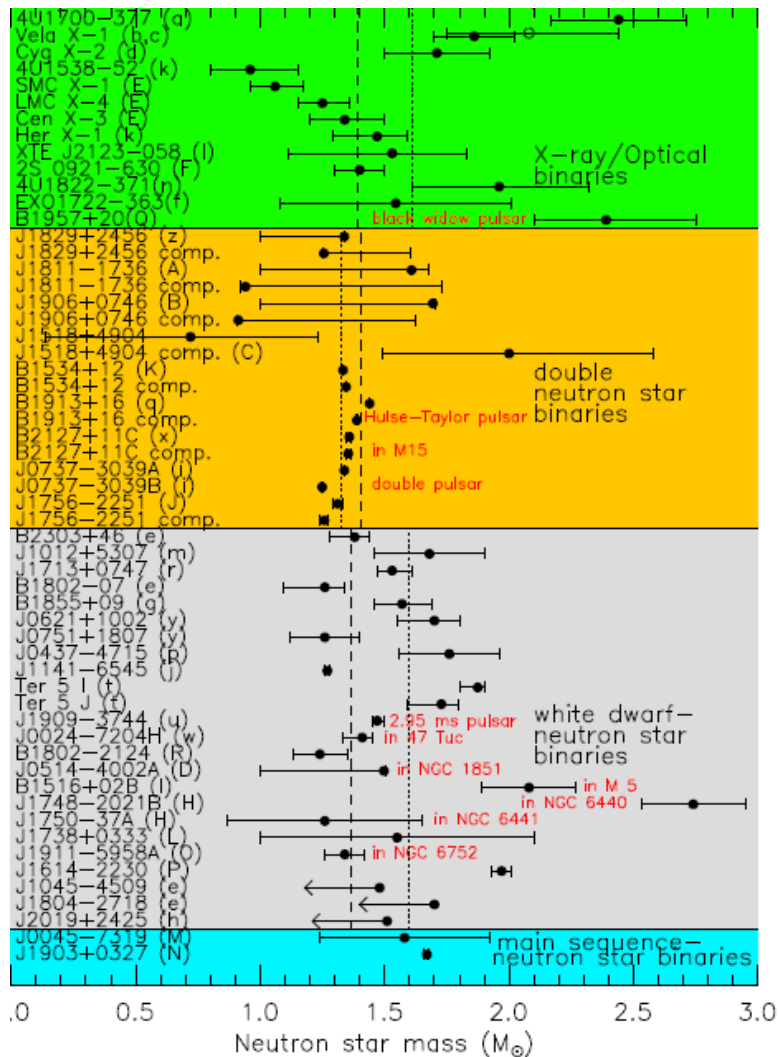
Maximum mass and cut-off



Two gaussians and a hard cut at M_{\max}



Neutron star masses



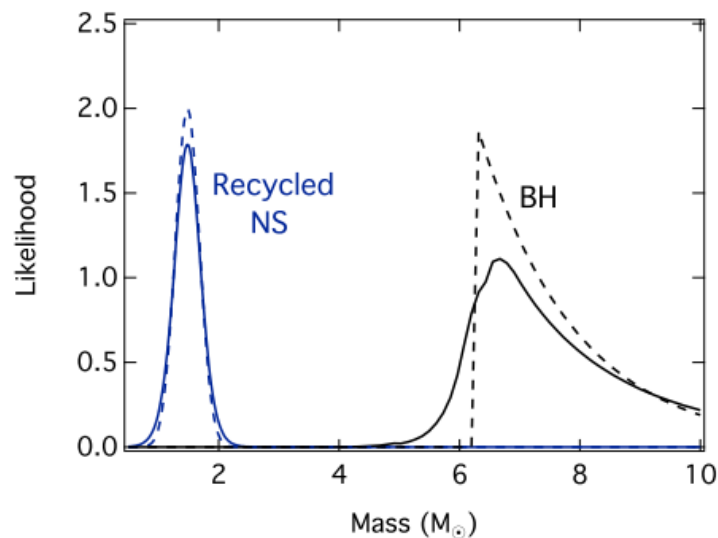
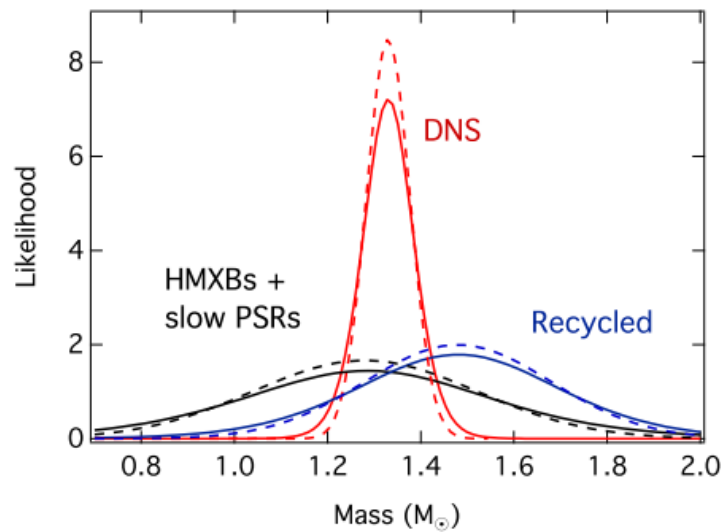
Object	Mass (M_{\odot})	Reference	Object	Mass (M_{\odot})	Reference
<i>X-Ray/Optical Binaries (mean = $1.609 M_{\odot}$, weighted mean = $1.393 M_{\odot}$)</i>					
4U1700-377	$2.44^{+0.27}_{-0.27}$	a (10)	Vela X-1	$1.86^{+0.16}_{-0.16}$	b, c (11) (12)
Cyg X-2	$1.71^{+0.21}_{-0.21}$	d (13)	4U1538-52	$0.96^{+0.19}_{-0.16}$	k (14)
SMC X-1	$1.06^{+0.11}_{-0.10}$	E (15)	LMC X-4	$1.25^{+0.11}_{-0.12}$	E (15)
Cen X-3	$1.34^{+0.18}_{-0.14}$	E (15)	Her X-1	$1.47^{+0.12}_{-0.18}$	k (14)
XTE J2123-058	$1.53^{+0.30}_{-0.42}$	l (16) (17)	2S 0921-630	$1.4^{+0.1}_{-0.1}$	F (18)
4U 1822-371	$1.96^{+0.36}_{-0.32}$	n (19)	EXO 1722-363	$1.545^{+0.465}_{-0.465}$	f (47)
B1957+20	$2.39^{+0.32}_{-0.29}$	Q (49)			
<i>Neutron Star - Neutron Star Binaries (mean = $1.325 M_{\odot}$, weighted mean = $1.403 M_{\odot}$)</i>					
J1829+2456	$1.338^{+0.002}_{-0.002}$	z (20)	J1829+2456 (c)	$1.256^{+0.346}_{-0.003}$	z (20)
J1811-1736	$1.608^{+0.066}_{-0.068}$	A (21)	J1811-1736 (c)	$0.941^{+0.787}_{-0.021}$	A (21)
J1906+07	$1.694^{+0.012}_{-0.012}$	B (22)	J1906+07 (c)	$0.912^{+0.710}_{-0.004}$	B (22)
J1518+4904	$0.72^{+0.58}_{-0.58}$	C (23)	J1518+4904 (c)	$2.00^{+0.51}_{-0.51}$	C (23)
1534+12	$1.3332^{+0.0010}_{-0.0010}$	K (24)	1534+12 (c)	$1.3452^{+0.0010}_{-0.0010}$	K (24)
1913+16	$1.4398^{+0.0002}_{-0.0002}$	q (25)	1913+16 (c)	$1.3886^{+0.0002}_{-0.0002}$	q (25)
2127+11C	$1.358^{+0.010}_{-0.010}$	x (26)	2127+11C (c)	$1.354^{+0.010}_{-0.010}$	x (26)
J0737-3039A	$1.3381^{+0.0007}_{-0.0007}$	i (27)	J0737-3039B	$1.2489^{+0.0007}_{-0.0007}$	i (27)
J1756-2251	$1.312^{+0.017}_{-0.017}$	J (28)	J1756-2251 (c)	$1.258^{+0.017}_{-0.017}$	J (28)
<i>Neutron Star - White Dwarf Binaries (mean = $1.599 M_{\odot}$, weighted mean = $1.362 M_{\odot}$)</i>					
B2303+46	$1.38^{+0.06}_{-0.10}$	e (29)	J1012+5307	$1.68^{+0.22}_{-0.22}$	m (30)
J1713+0747	$1.53^{+0.08}_{-0.08}$	r (31) (51)	B1802-07	$1.26^{+0.08}_{-0.17}$	e (29)
B1855+09	$1.57^{+0.04}_{-0.11}$	g (32) (51)	J0621+1002	$1.70^{+0.10}_{-0.17}$	y (33)
J0751+1807	$1.26^{+0.14}_{-0.14}$	y (33)	J0437-4715	$1.76^{+0.20}_{-0.20}$	p (34)
J1141-6545	$1.27^{+0.01}_{-0.01}$	j (35)	Ter 5 I	$1.874^{+0.32}_{-0.068}$	t (36)
Ter 5 J	$1.728^{+0.066}_{-0.136}$	t (36)	J1909-3744	$1.47^{+0.02}_{-0.02}$	u (37)
J0024-7204H	$1.41^{+0.04}_{-0.08}$	w (38)	B1802-2124	$1.24^{+0.11}_{-0.11}$	R (50)
J0514-4002A	$1.497^{+0.008}_{-0.497}$	D (39)	B1516+02B	$2.08^{+0.19}_{-0.19}$	I (40)
J1748-2021B	$2.74^{+0.21}_{-0.21}$	H (41)	J1750-37A	$1.26^{+0.39}_{-0.39}$	H (41)
J1738+0333	$1.55^{+0.55}_{-0.55}$	L (42)	J1911-5958A	$1.34^{+0.08}_{-0.08}$	O (43)
J1614-2230	$1.97^{+0.04}_{-0.04}$	P (48)	J1045-4509	< 1.48	e (29)
J1804-2718	< 1.70	e (29)	J2019+2425	< 1.51	h (44)
<i>Neutron Star - Main Sequence Binaries</i>					
J0045-7319	$1.58^{+0.34}_{-0.34}$	M (45)	J1903+0327	$1.67^{+0.01}_{-0.01}$	N (46)

Follow updates at <https://stellarcollapse.org/nsmasses>

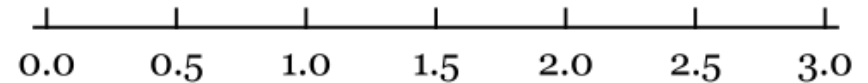
arXiv: 1012.3208

Update - 2012

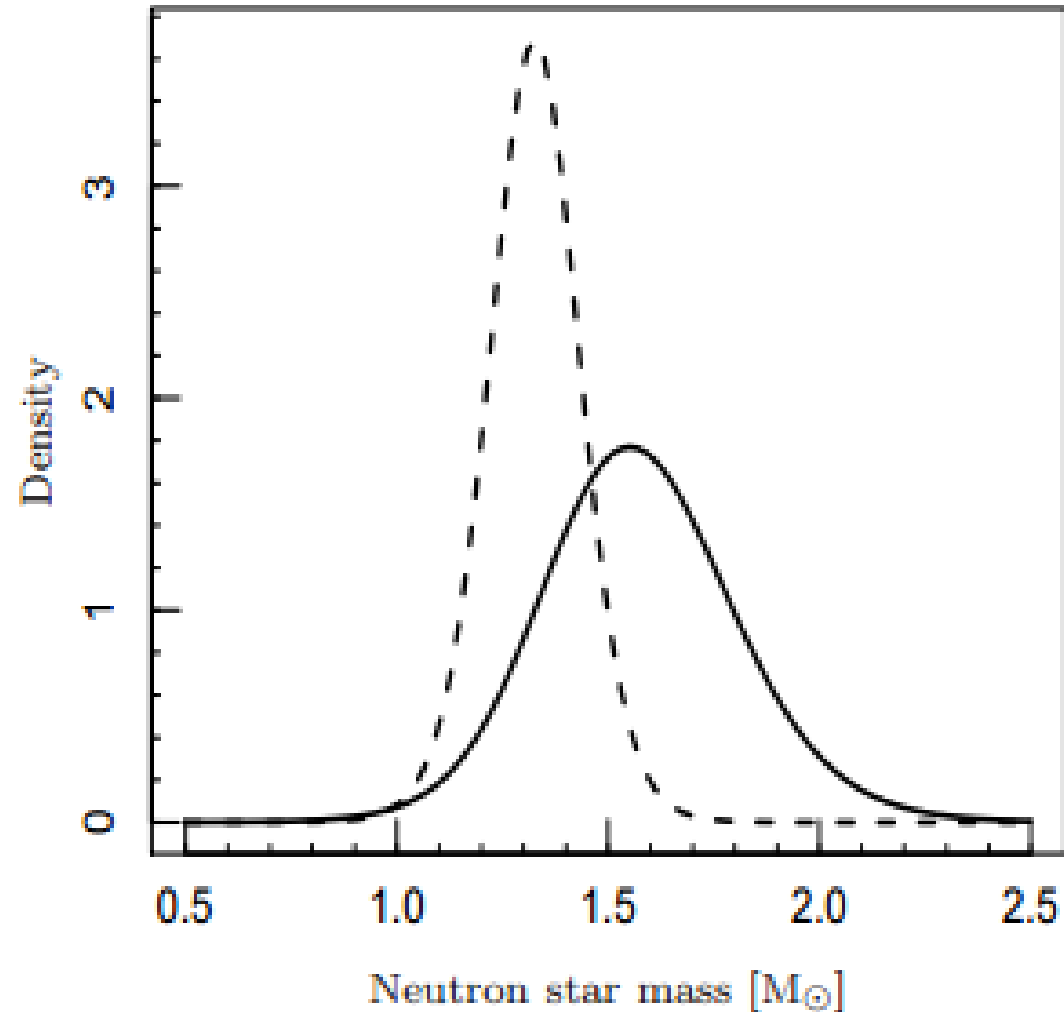
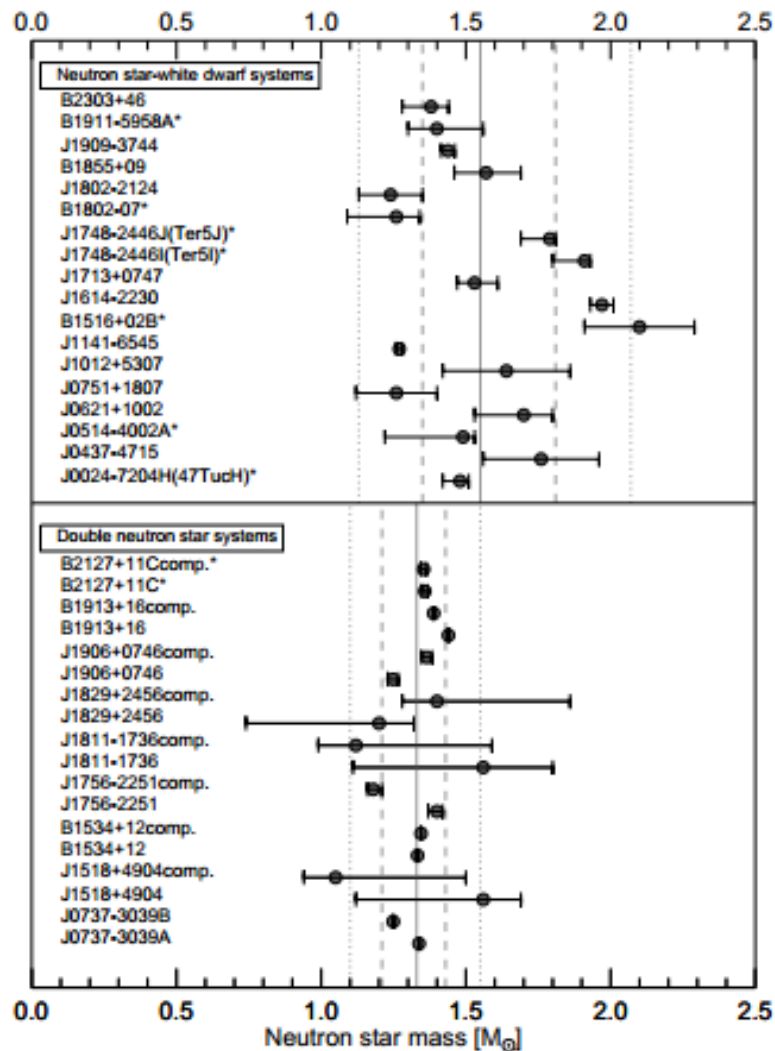
Neutron Stars



J0737-3039B
J1756-2251c
J1906+0746c
J1756-2251
J1906+0746
B1534+12
J0737-3039A
B1534+12c
B2127+11Cc
B2127+11C
B1913+16c
B1913+16
J1829+2456
J1811-1736
J1518+4904
J1518+4904c
J1811-1736c
J1829+2456c
4U1538-52
SMC X-1
Her X-1
LMC X-4
Cen X-3
Vela X-1
J1802-2124
J0751+1807
J1141-6545
J1713+0747
J1909-3744
B1855+09
J1903+0327
J0437-4715
J1614-2230
J1750-37A
B1802-07
J1824-2452C
J0024-7204H
B2303+46
J0514-4002A
J0621+1002
J1748-2446J
J1748-2446I
B1516+02B
J1748-2021B
B1911-5958A
J1012+5307
KS 1731-260
Cyg X-2
4U 1820-30
4U 1735-345
4U 1608-52



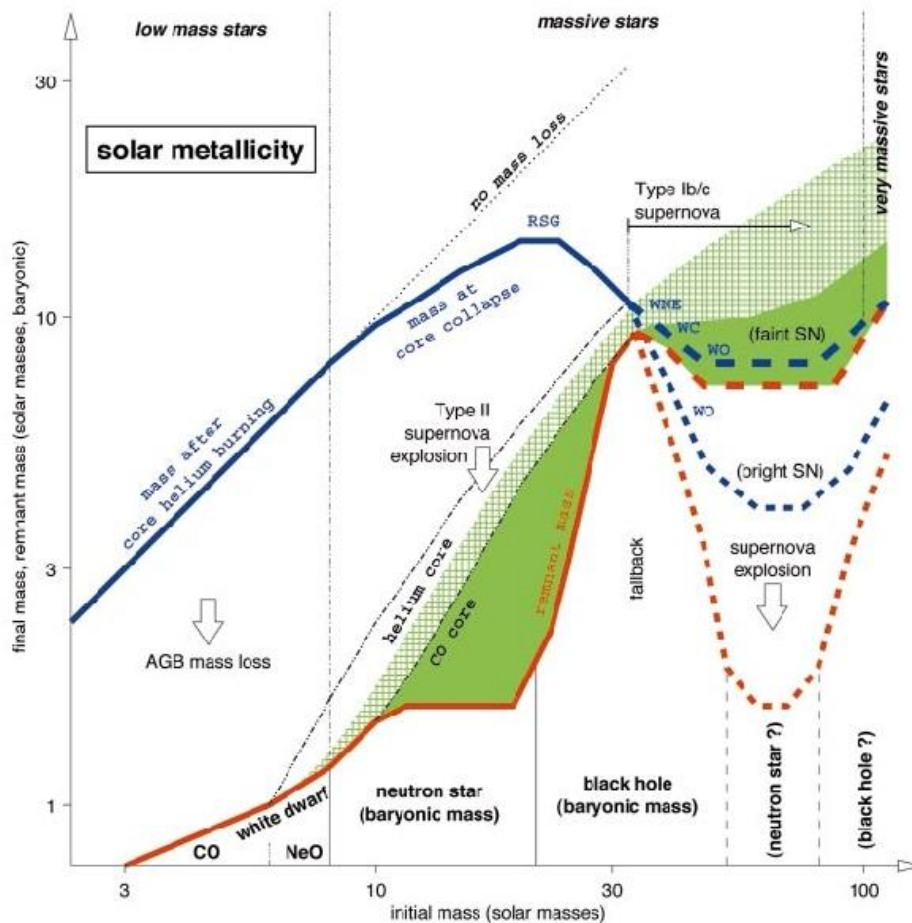
Update - 2013



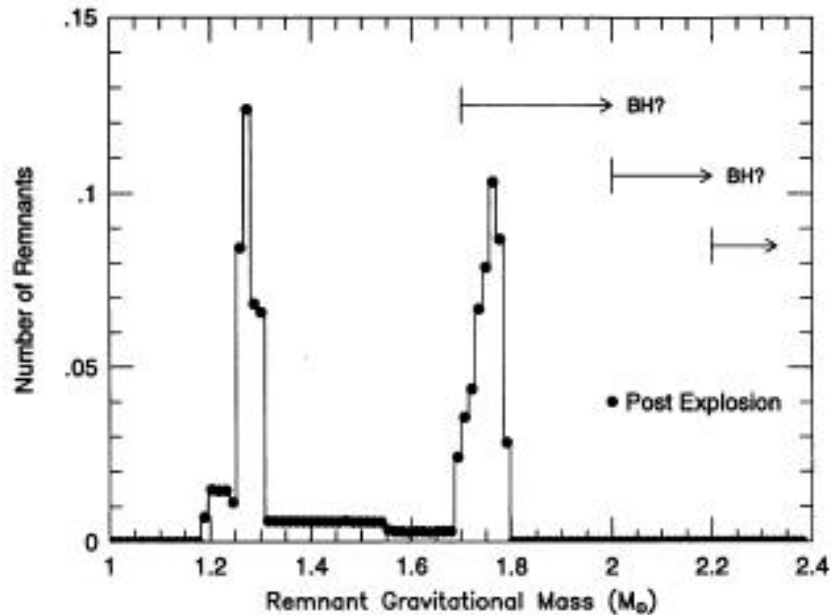
Compact objects and progenitors.

Solar metallicity.

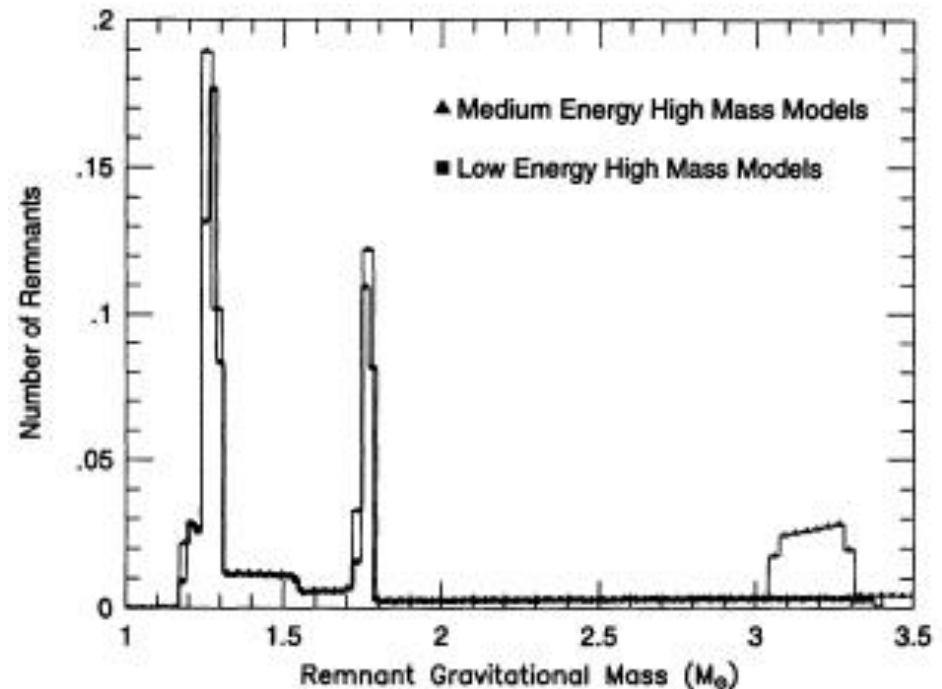
There can be a range of progenitor masses in which NSs are formed, however, for smaller and larger progenitors masses BHs appear.



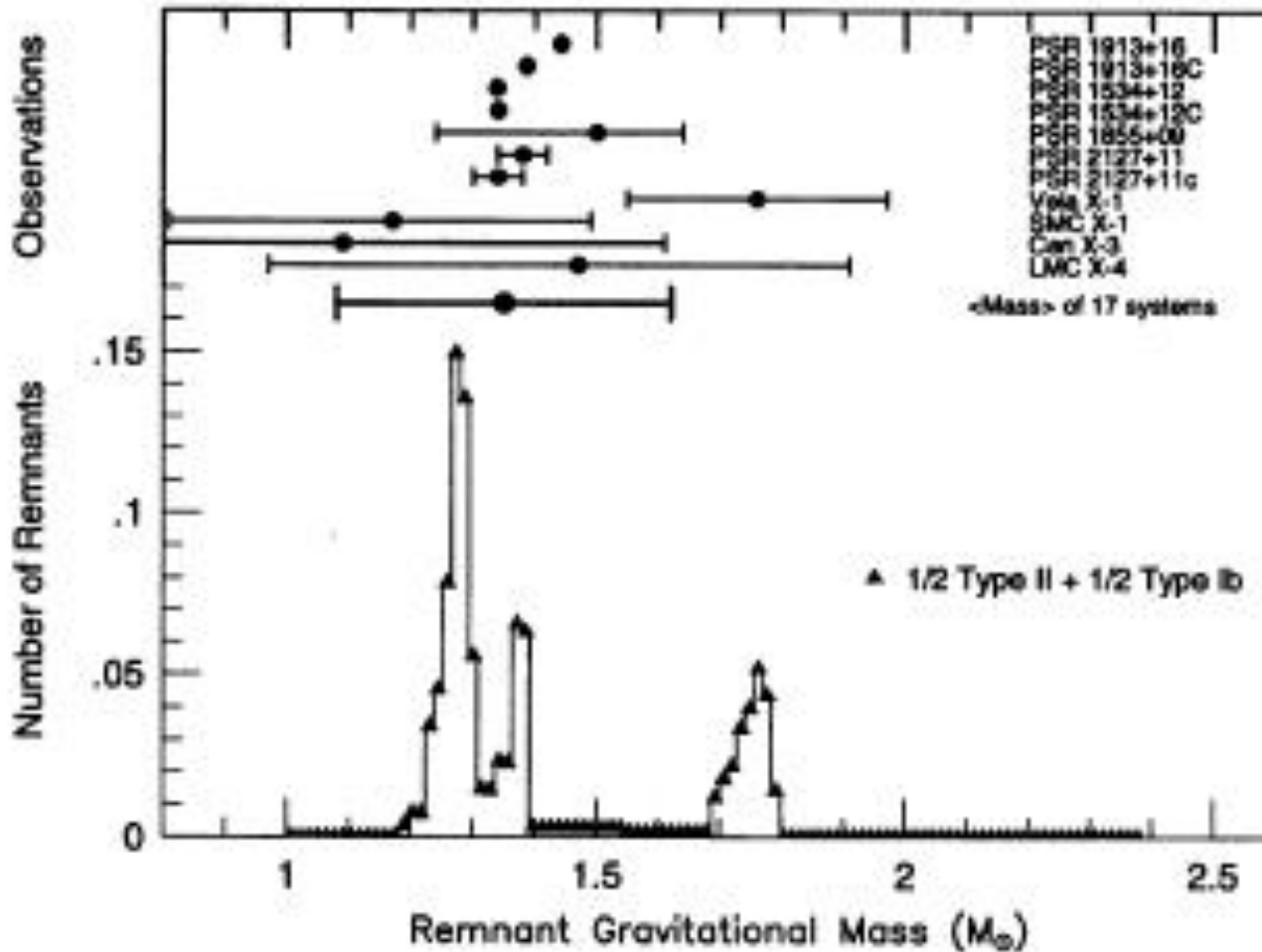
Mass spectrum of compact objects



Results of calculations
(depend on the assumed model
of explosion)



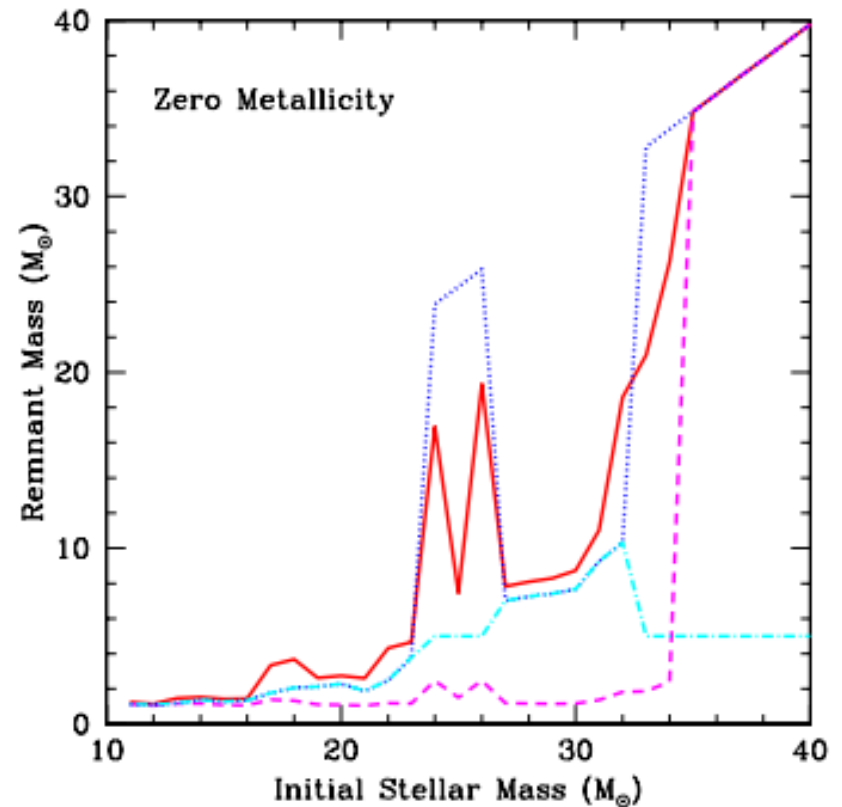
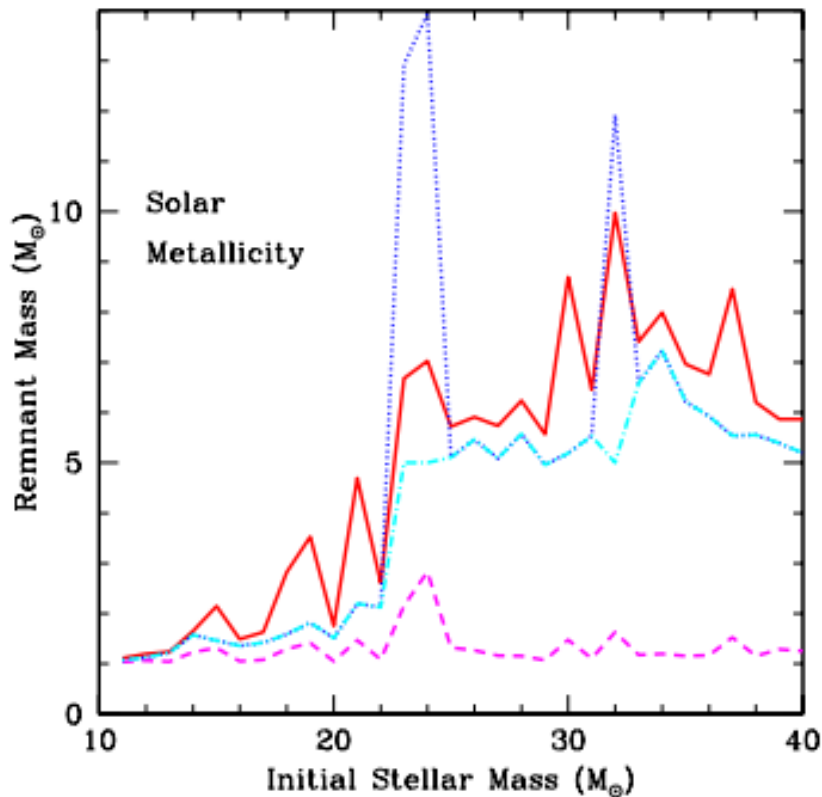
Mass spectrum of compact objects



Comparison of one of the model with observations.

However, selection effects can be important as observed NSs are all in binaries.

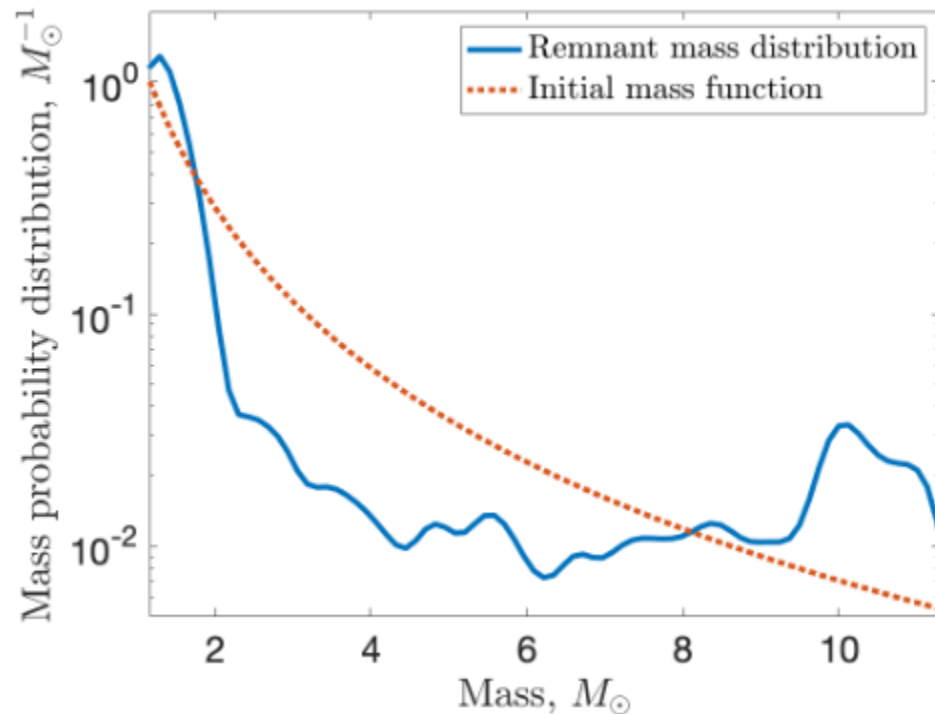
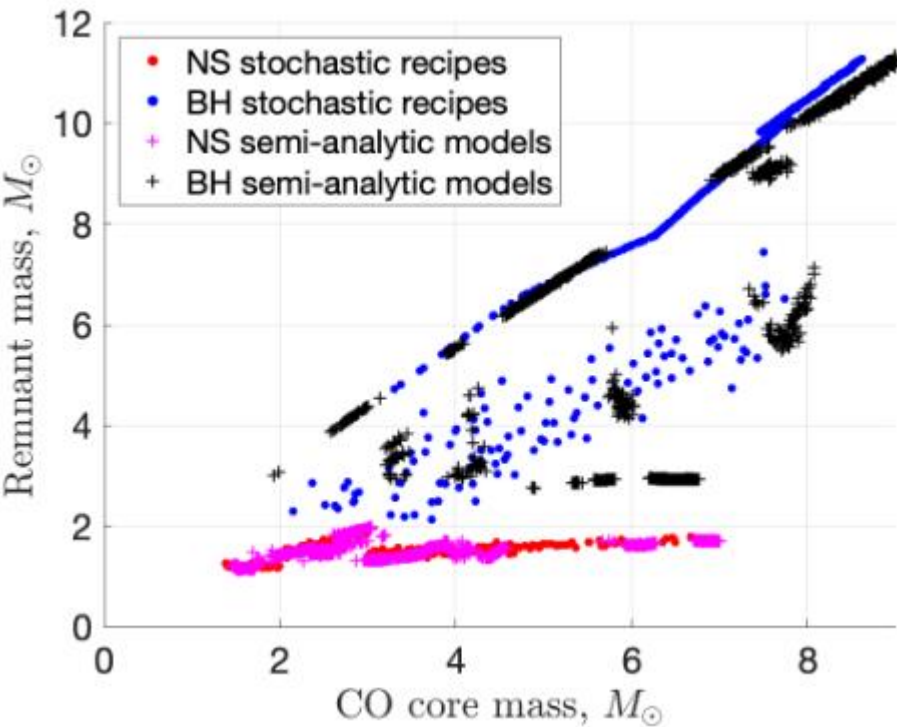
Newer calculations of the mass spectrum



Different curves are plotted for different models of explosion:
dashed – with a magnetar

Stochastic predictions

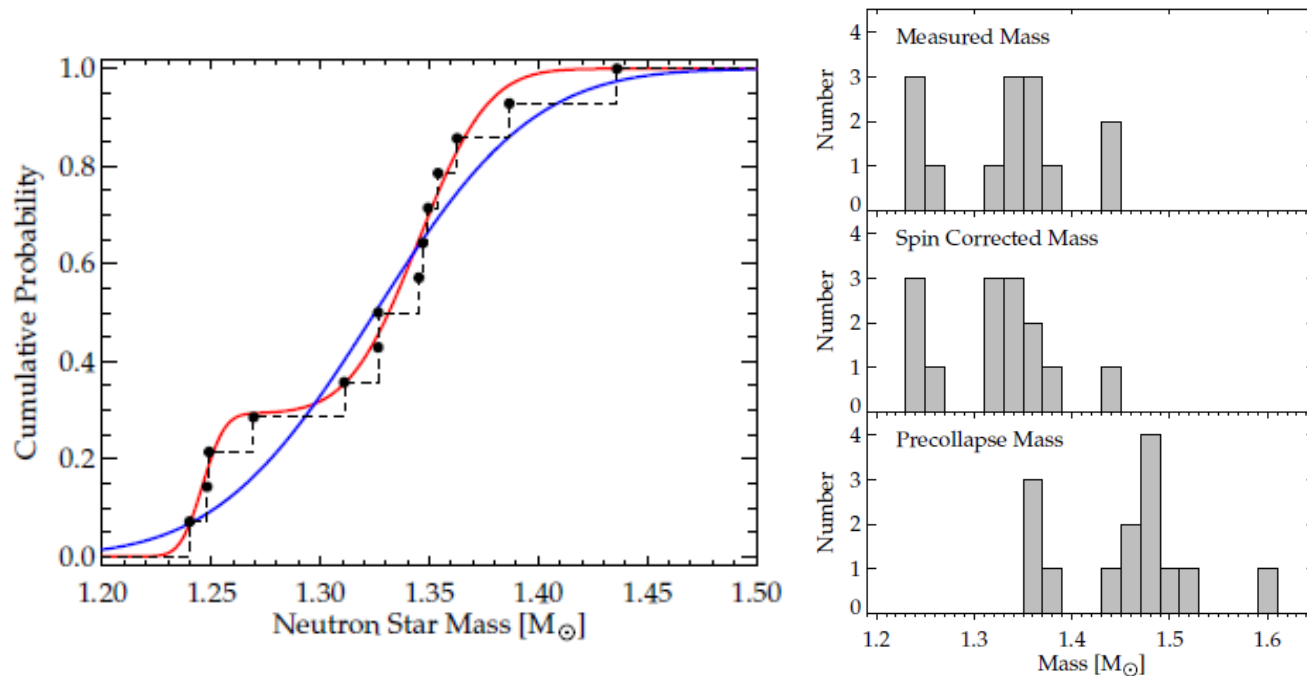
Prediction (semi-analytic) is based on detailed 3D models of single stars SN explosions. Stochastic recipe just assumes some general properties of pre-SN and SN applied to COMPAS stellar population synthesis code.



Bi-modal mass spectrum?

Pulsar Name	Mass of Recycled Neutron Star (M_{\odot})	Mass of Young Neutron Star (M_{\odot})	P_{orb} (hours)	Eccentricity	Pulse Period (ms)	Reference
J0737-3039A/B	1.3381 ± 0.0007	1.2489 ± 0.0007	2.4	0.088	23	Kramer et al. (2006)
B1534+12	1.3332 ± 0.0010	1.3452 ± 0.0010	10.1	0.273	38	Stairs et al. (2002)
J1756-2251	1.32 ± 0.02	1.24 ± 0.02	7.67	0.18	28	Stairs (2008)
J1906+0746	1.365 ± 0.018	1.248 ± 0.018	3.98	0.085	144 [†]	Kasian (2008)
B1913+16	1.4414 ± 0.0002	1.3867 ± 0.0002	7.92	0.617	59	Weisberg & Taylor (2005)
B2127+11C	1.358 ± 0.010	1.354 ± 0.010	8.05	0.681	30	Jacoby et al. (2006)
J1909-3744	1.438 ± 0.024	white dwarf	36.7	$\lesssim 10^{-6}$	2.9	Jacoby et al. (2005)
J1141-6545	white dwarf	1.27 ± 0.01	4.74	0.172	393 [†]	Bhat et al. (2008)

The low-mass peak the authors relate to e-capture SN.

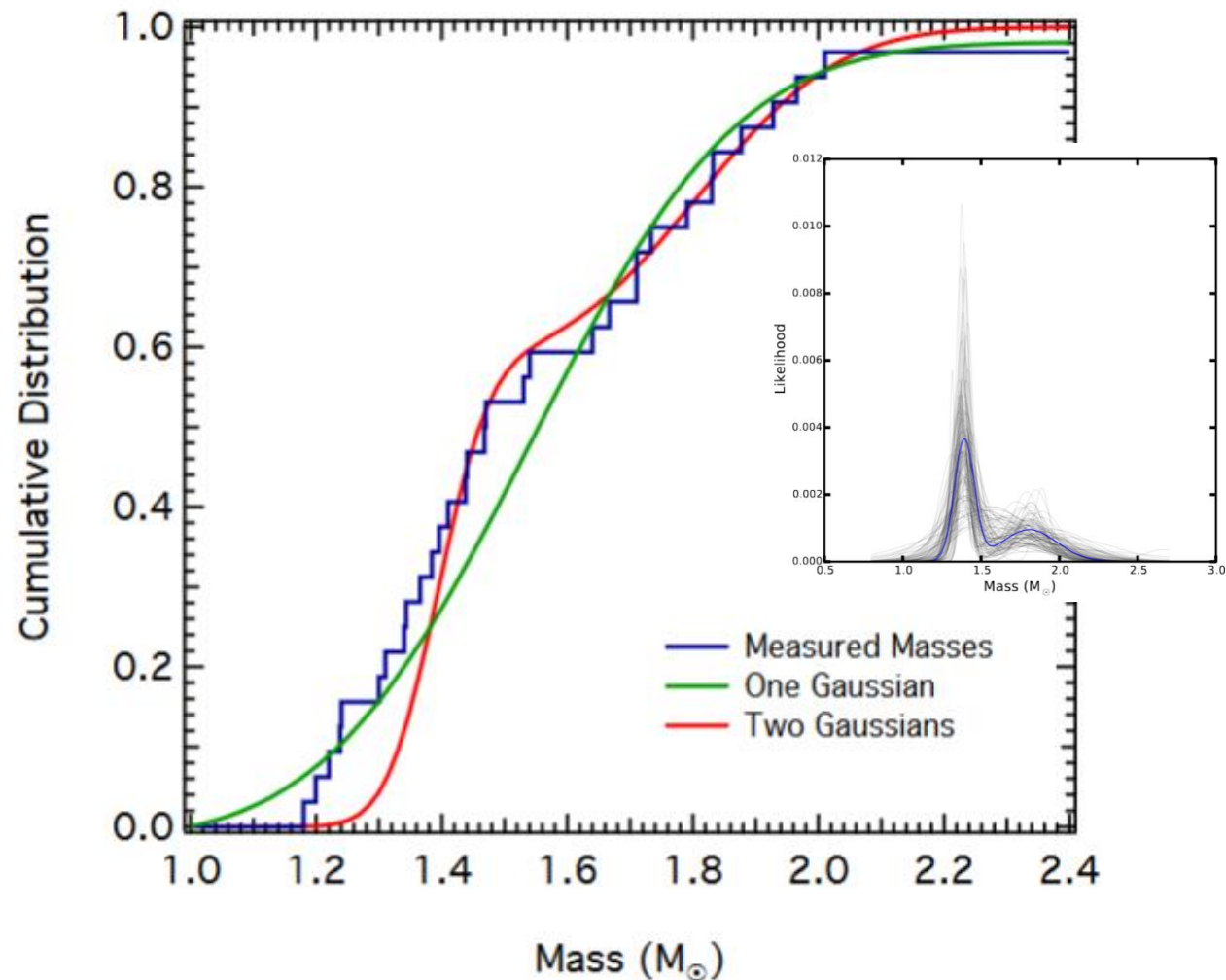


Based on 14 observed systems

Figure 1 consists of three panels. The top panel shows the probability density $\mathcal{P}(M_A | \mathcal{M}_{\max})$ as a function of the millisecond pulsar mass M_A in solar masses M_\odot . The x-axis ranges from 1.0 to 2.0, and the y-axis ranges from 0 to 6. A black curve shows the distribution, with a peak around 1.35 M_\odot . Vertical lines are drawn at $M_A \approx 1.25$ (green), $M_A \approx 1.35$ (red), and $M_A \approx 1.45$ (red). The bottom-left panel is a 2D contour plot of the joint probability density $\mathcal{P}(M_A, M_B)$ in the M_A - M_B plane. Both axes range from 1.0 to 2.0 M_\odot . A blue diagonal line represents $M_A = M_B$. Contours are labeled with percentages: $10^{-13}\%$, $10^{-9}\%$, $10^{-5}\%$, $10^{-3}\%$, 1% , 0.1% , 1% , 0.1% , $10^{-3}\%$, $10^{-9}\%$, and $10^{-13}\%$. Several pulsar systems are marked with red dots: J1906+0746 (green), B1913+16, B1534+12, J0737-3039, and J1756-2251. The bottom-right panel shows the probability density $\mathcal{P}(M_B | \mathcal{M}_{\max})$ as a function of the companion mass M_B in solar masses M_\odot . The x-axis ranges from 1.0 to 2.0, and the y-axis ranges from 0 to 6. A black curve shows the distribution, with a peak around 3.5 M_\odot . Vertical lines are drawn at $M_B \approx 1.25$ (green), $M_B \approx 1.35$ (red), and $M_B \approx 1.45$ (red). The right side of the bottom-right panel shows the probability density $\mathcal{P}(M_B | \mathcal{M}_{\max})$ as a function of M_B , with a peak around 3.5 M_\odot . The top right corner contains text: $Z=Z_\odot$, Y_e core, no fallback, $\mathcal{M}_{\max}=100 M_\odot$.

Bimodality in mPSR mass distribution

#	PSR Name	Mass [M_{\odot}]
1	J0337+1715	1.4378(13)
2	J0348+0432	2.01(4)
3	J0437-4715	1.44(7)
4	J0751+1807	1.64(15)
5	J1012+0507	1.83(11)
6	J1023+0038	1.71(16)
7	J1614-2230	1.928(17)
8	J1713+0747	1.31(11)
9	J1738+0333	1.47(7)
10	J1802-2124	1.24(11)
11	J1807-2500B	1.3655(21)
12	B1855+09	1.30(11)
13	J1903+0327	1.667(7)
14	J1909-3744	1.540(27)
15	J1910-5959A	1.34(8)
16	J1918-0642	1.18(11)
17	J1946+3417	1.832(13)
18	J2234+0611	1.396(11)



+ 14 PSR with less precisely
determined masses

The bimodality reflects birth properties?

1605.01665

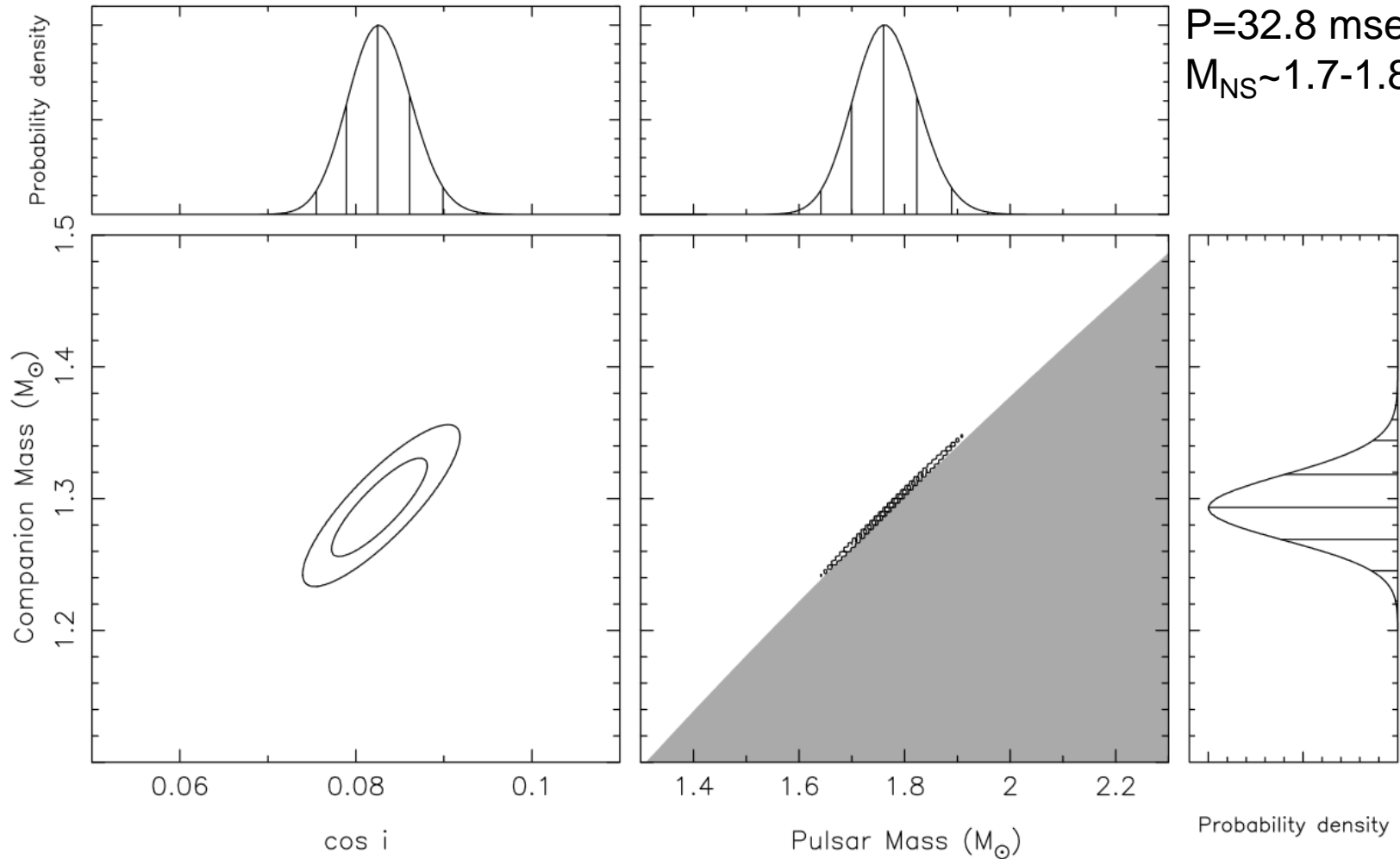
Massive born NS

PSR J2222-0137

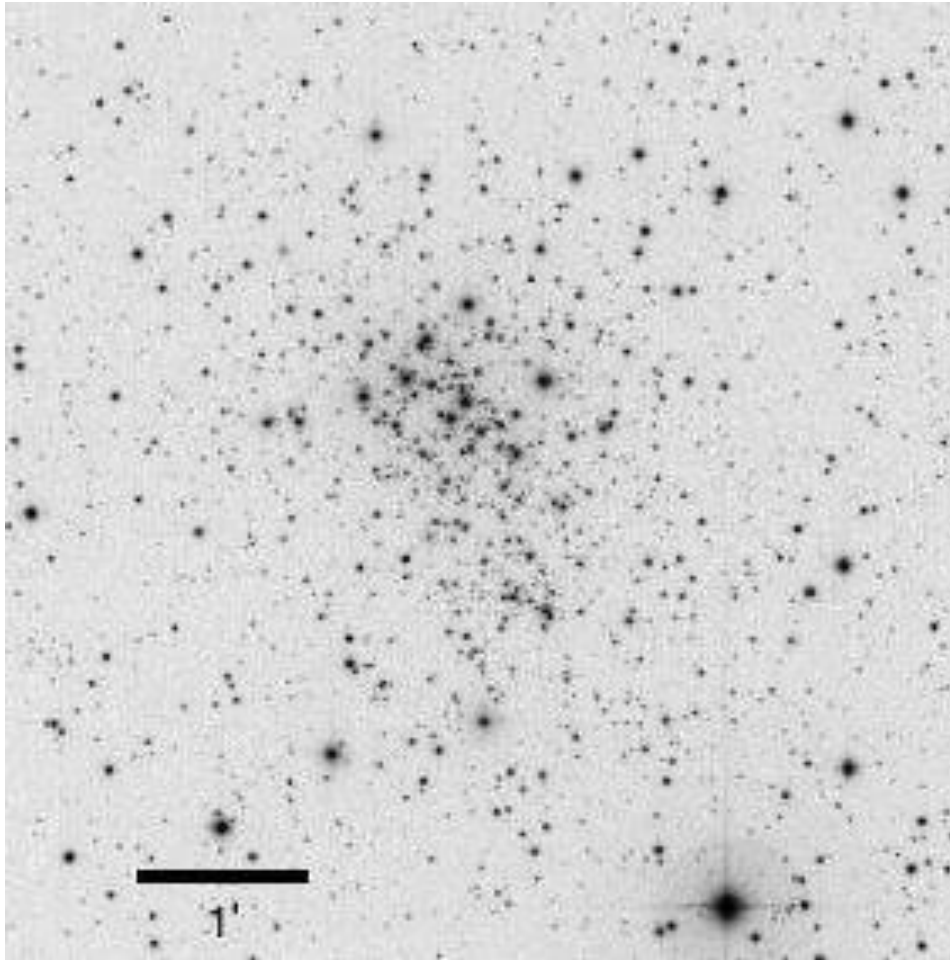
WD companion

$P=32.8$ msec

$M_{\text{NS}} \sim 1.7-1.8$



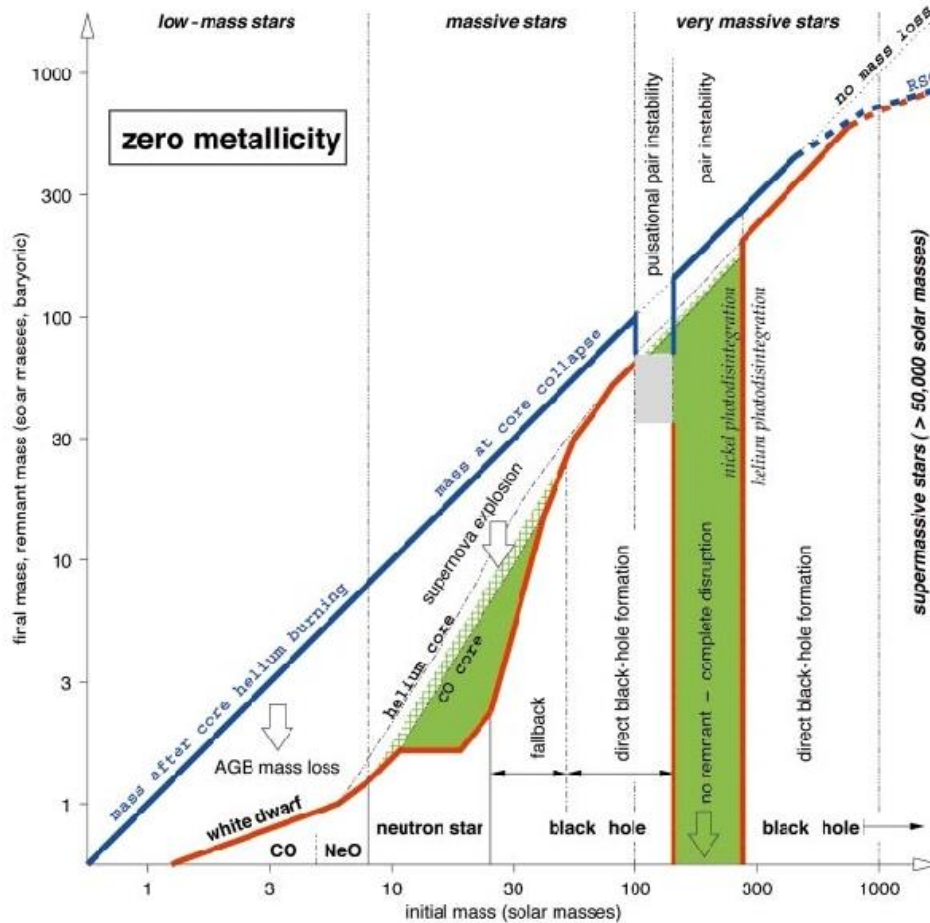
A NS from a massive progenitor



Anomalous X-ray pulsar
in the association

Westerlund1 most probably has
a very massive progenitor, $>40 M_{\odot}$.

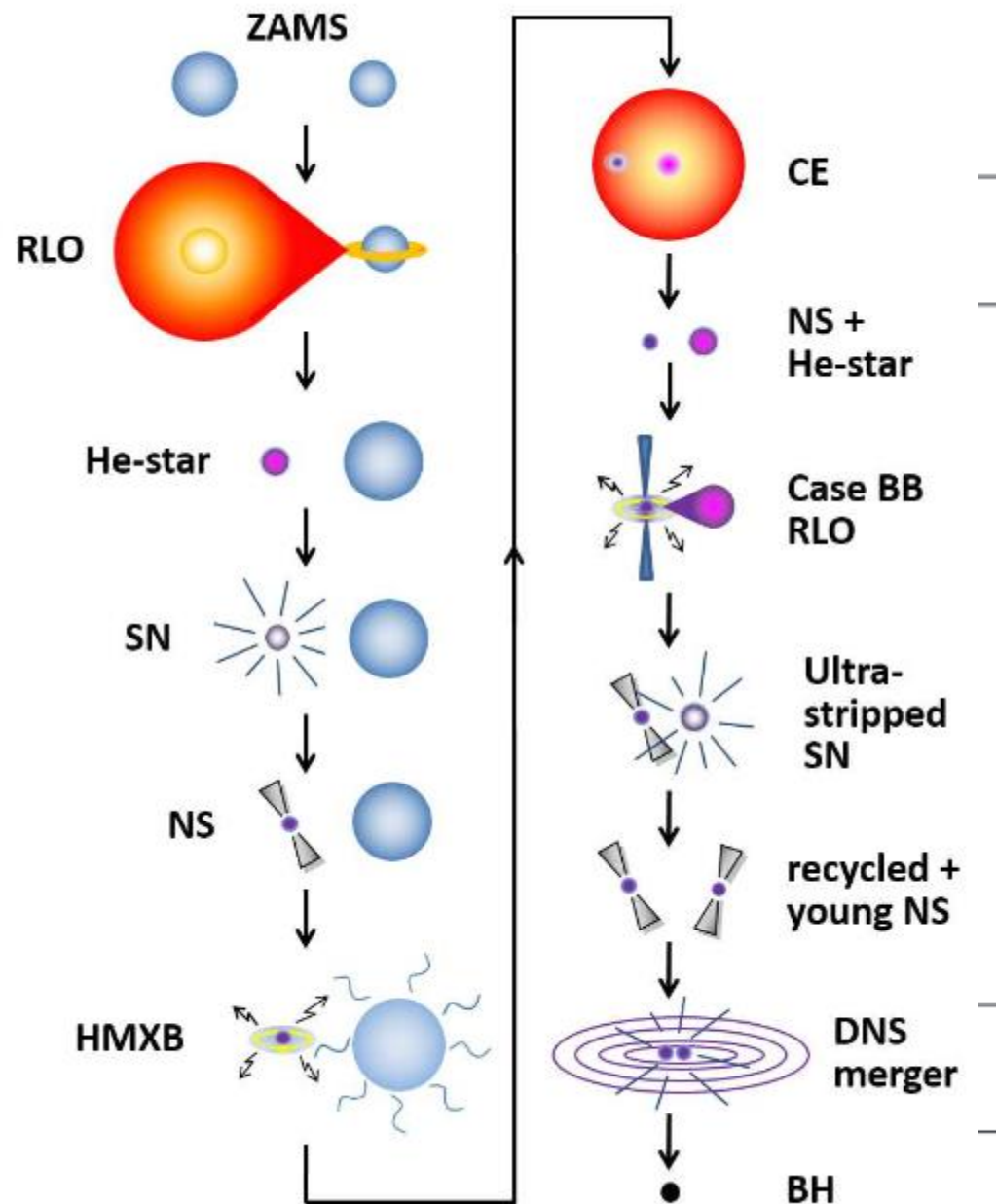
The case of zero metallicity



No intermediate mass range for NS formation.

DNS

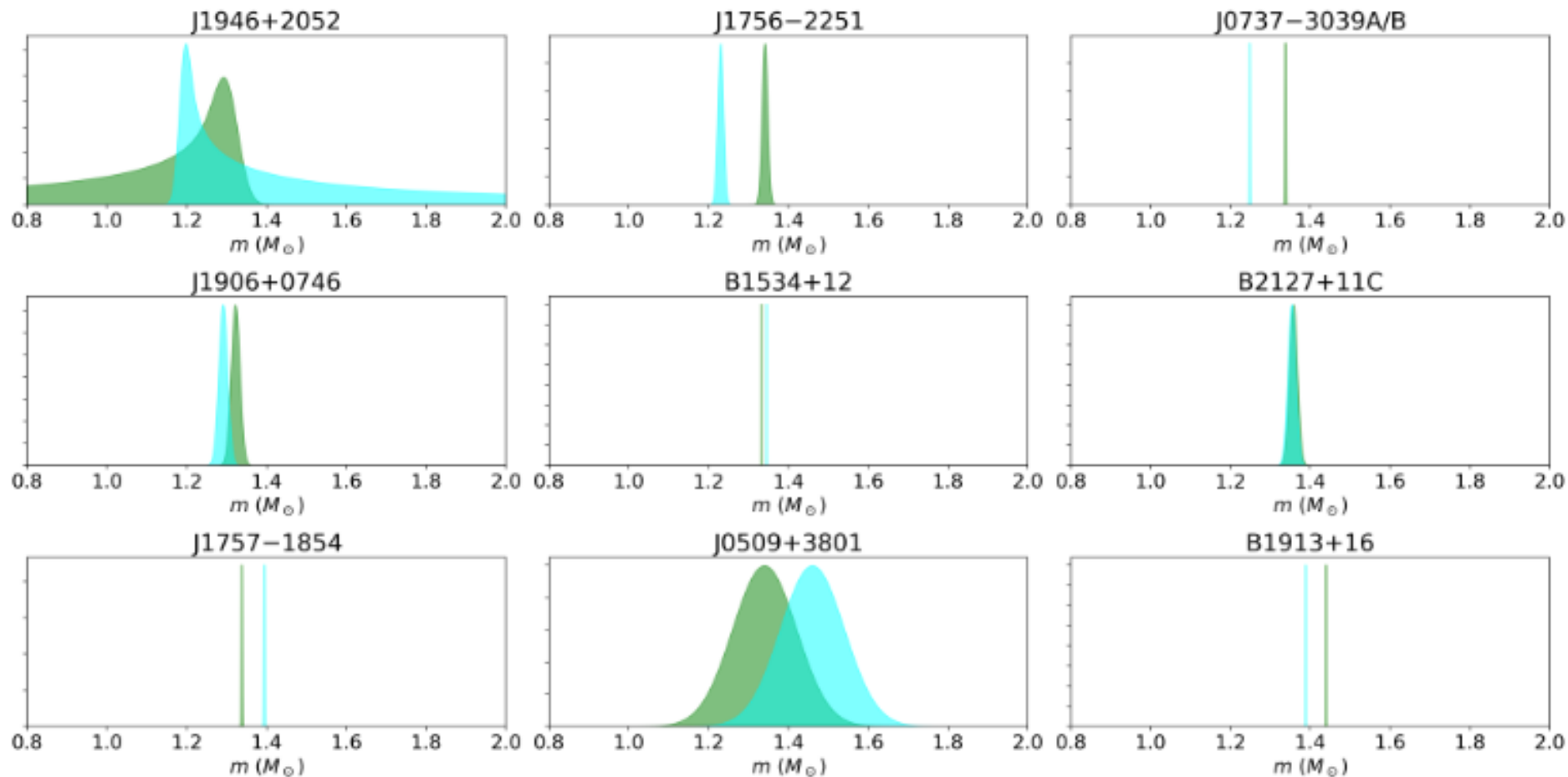
Radio Pulsar	Type	P (ms)
J0453+1559 ^a	recycled	45.8
J0737–3039A ^b	recycled	22.7
J0737–3039B ^b	young	2773.5
J1518+4904 ^c	recycled	40.9
B1534+12 ^d	recycled	37.9
J1753–2240 ^e	recycled	95.1
J1755–2550 ^{f*}	young	315.2
J1756–2251 ^g	recycled	28.5
J1811–1736 ^h	recycled	104.2
J1829+2456 ⁱ	recycled	41.0
J1906+0746 ^{j*}	young	144.1
J1913+1102 ^k	recycled	27.3
B1913+16 ^l	recycled	59.0
J1930–1852 ^m	recycled	185.5
J1807–2500B ^{n*}	GC	4.2
B2127+11C ^p	GC	30.5



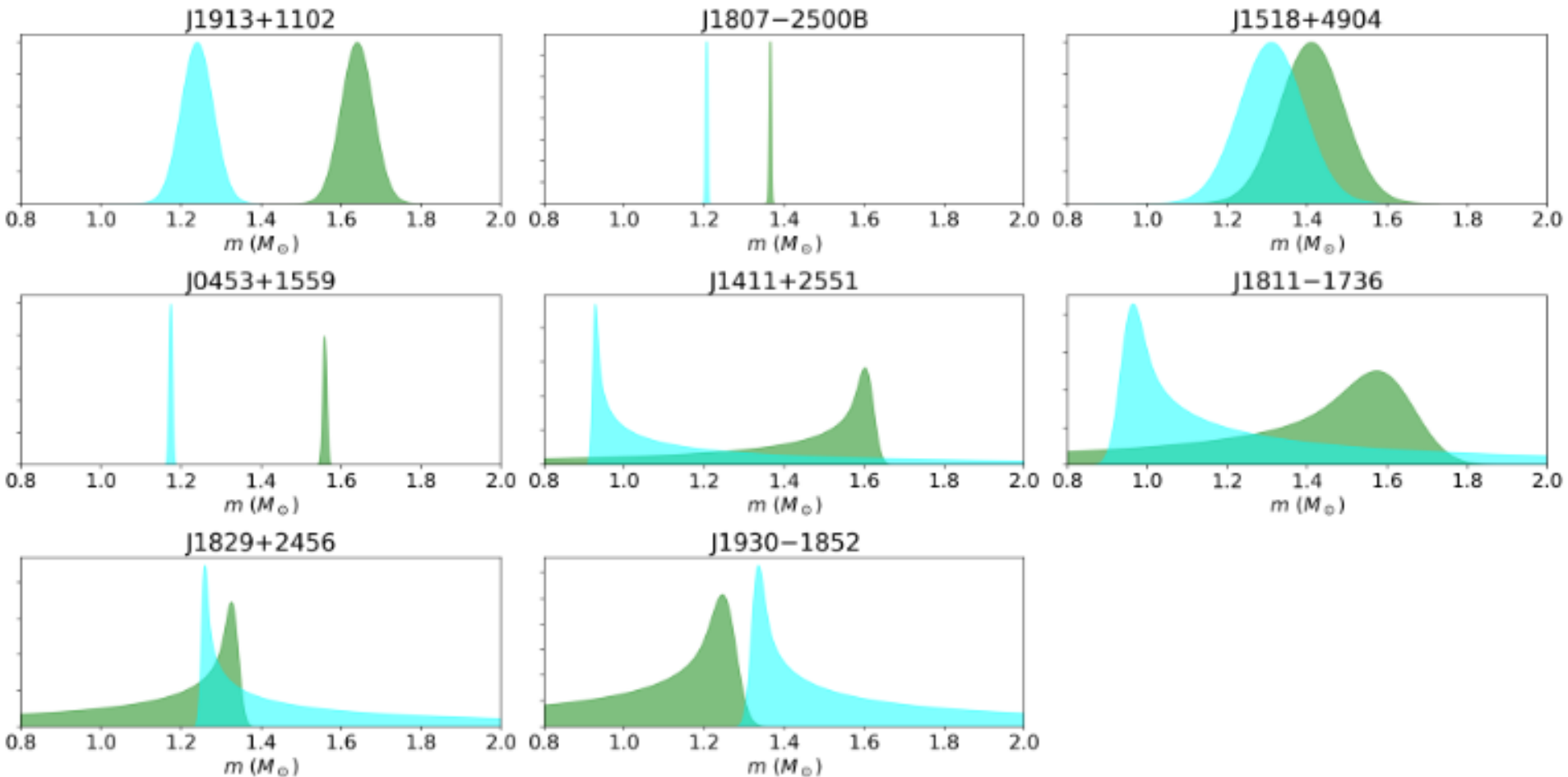
DNS parameters

Pulsar Name	M_T (M_\odot)	m_r (M_\odot)	m_s (M_\odot)	\mathcal{M}_c (M_\odot)	q	P_b (day)	T_c (Gyr)
Systems will merge within a Hubble time							
J1946+2052	2.50(4)	< 1.35	> 1.17	(1.05, 1.11)	(0.68, 1)	0.078	0.046
J1756–2251	2.56999(6)	1.341(7)	1.230(7)	1.1178(3)	0.92(1)	0.320	1.656
J0737–3039A/B	2.58708(16)	1.3381(7)	1.2489(7)	1.1253(1)	0.933(1)	0.102	0.086
J1906+0746	2.6134(3)	1.322(11)	1.291(11)	1.1372(2)	(0.956, 1)	0.166	0.308
B1534+12	2.678463(4)	1.3330(2)	1.3455(2)	1.165870(2)	0.9907(3)	0.421	2.734
B2127+11C	2.71279(13)	1.358(10)	1.354(10)	1.18043(8)	(0.975, 1)	0.335	0.217
J1757–1854	2.73295(9)	1.3384(9)	1.3946(9)	1.18930(4)	0.960(1)	0.184	0.076
J0509+3801	2.805(3)	1.34(8)	1.46(8)	1.215(5)	(0.793, 1)	0.380	0.574
B1913+16	2.828378(7)	1.4398(2)	1.3886(2)	1.230891(5)	0.9644(3)	0.323	0.301
J1913+1102	2.886(1)	1.65(5)	1.24(5)	1.242(8)	0.75(5)	0.206	0.473
Systems will not merge within a Hubble time							
J1807–2500B	2.57190(73)	1.3655(21)	1.2064(21)	1.1169(3)	0.883(3)	9.957	1044
J1518+4904	2.7183(7)	1.41(8)	1.31(8)	1.181(5)	(0.794, 1)	8.634	8832
J0453+1559	2.733(4)	1.559(5)	1.174(4)	1.175(2)	0.753(5)	4.072	1453
J1411+2551	2.538(22)	< 1.64	> 0.92	(1.05, 1.11)	(0.57, 0.95)	2.616	466
J1811–1736	2.57(10)	< 1.75	> 0.91	(1.02, 1.17)	(0.58, 0.95)	18.78	1794
J1829+2456	2.59(2)	< 1.36	> 1.25	(1.08, 1.14)	(0.65, 1)	1.176	55
J1930–1852	2.59(4)	< 1.32	> 1.30	(1.07, 1.15)	(0.58, 0.96)	45.06	$\sim 10^5$

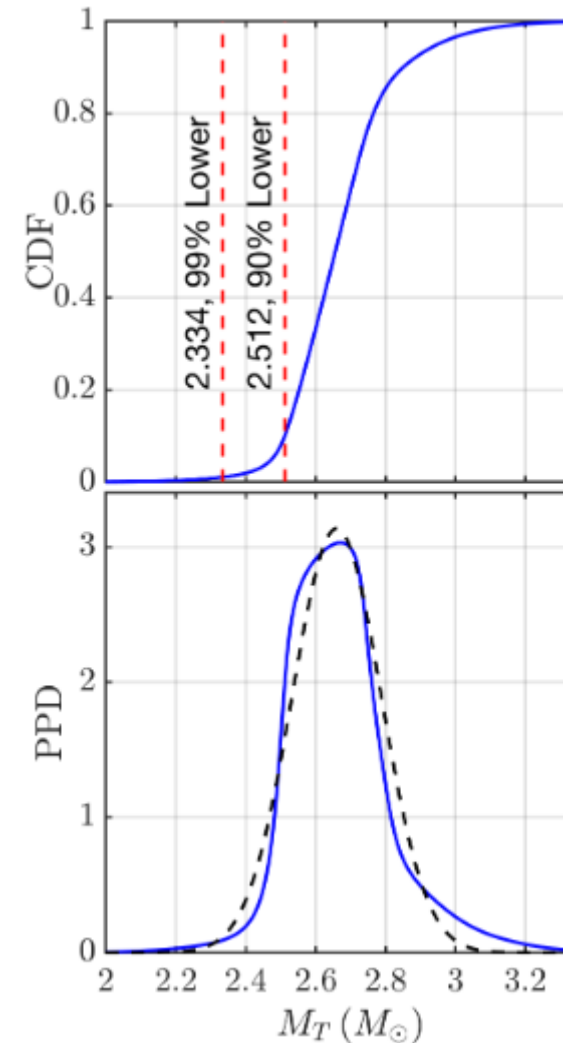
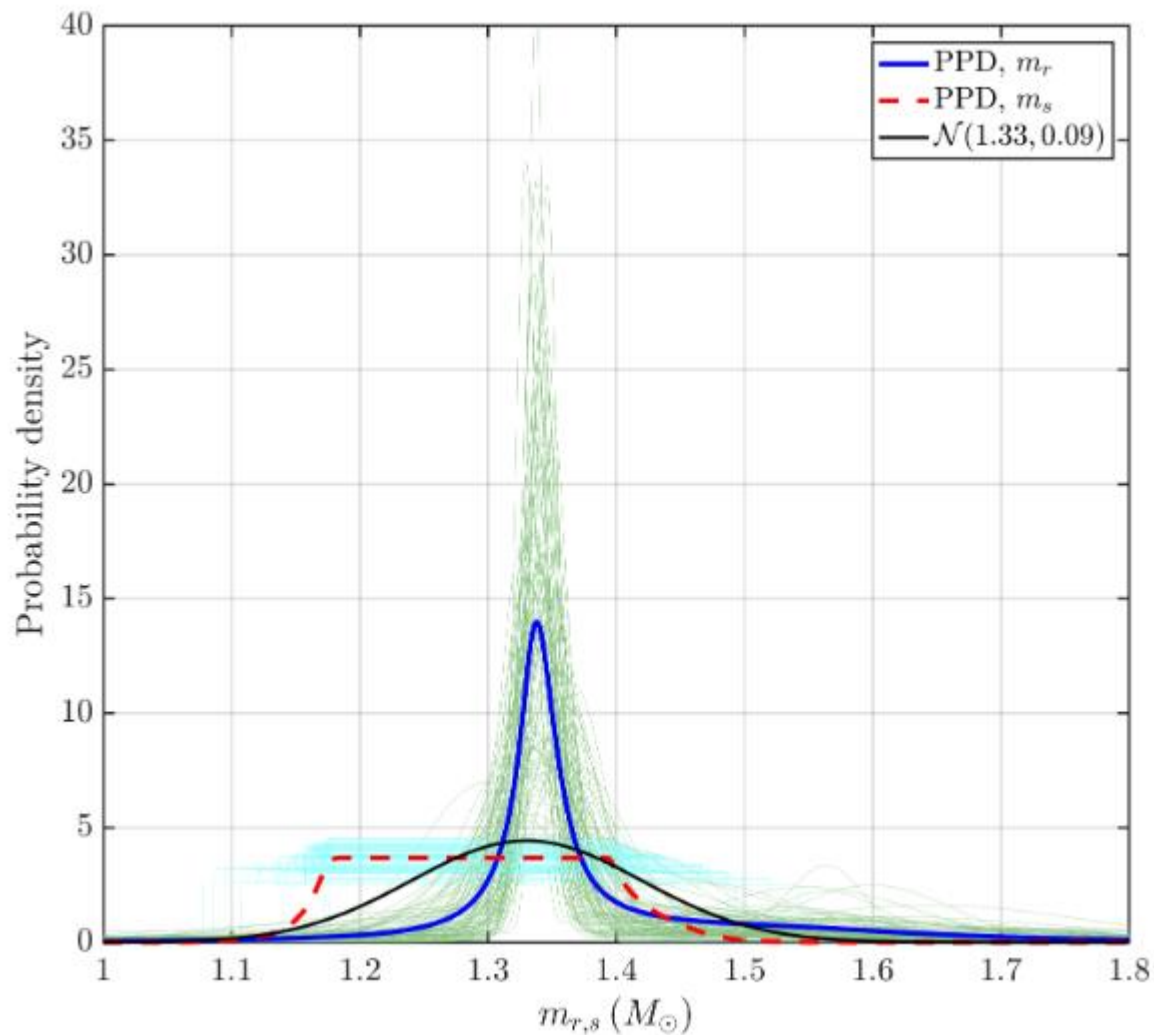
Individual masses of DNS



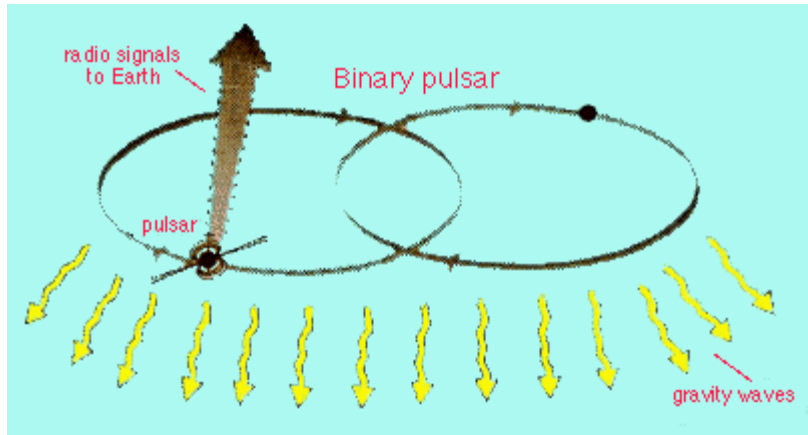
Individual masses of DNS



DNS mass distributions



Binary pulsars



$$\frac{d\Delta_{E\odot}}{dt} = \sum_i \frac{Gm_i}{c^2 r_i} + \frac{v_{\oplus}^2}{2c^2} - \text{constant} .$$

$$\Delta_{S\odot} = -\frac{2GM_{\odot}}{c^3} \log(1 + \cos \theta) ,$$

$$\begin{aligned} T = & t_{\text{obs}} - t_0 + \Delta_C - D/f^2 + \Delta_{R\odot}(\alpha, \delta, \mu_{\alpha}, \mu_{\delta}, \pi) \\ & + \Delta_{E\odot} - \Delta_{S\odot}(\alpha, \delta) \\ & - \Delta_R(x, e, P_b, T_0, \omega, \dot{\omega}, \dot{P}_b) - \Delta_E(\gamma) - \Delta_S(r, s) \end{aligned}$$

See 1502.05474 for a recent detailed review

Relativistic corrections and measurable parameters

$$\begin{aligned}\dot{\omega} &= 3 \left[\frac{P_b}{2\pi} \right]^{-5/3} (T_{\odot} M)^{2/3} (1-e^2)^{-1} , \\ \gamma &= e \left[\frac{P_b}{2\pi} \right]^{1/3} T_{\odot}^{2/3} M^{-4/3} m_2 (m_1 + 2m_2) , \\ \dot{P}_b &= -\frac{192\pi}{5} \left[\frac{P_b}{2\pi} \right]^{-5/3} \left[1 + \frac{73}{24}e^2 + \frac{37}{96}e^4 \right] \\ &\quad \times (1-e^2)^{-7/2} T_{\odot}^{5/3} m_1 m_2 M^{-1/3} ,\end{aligned}$$

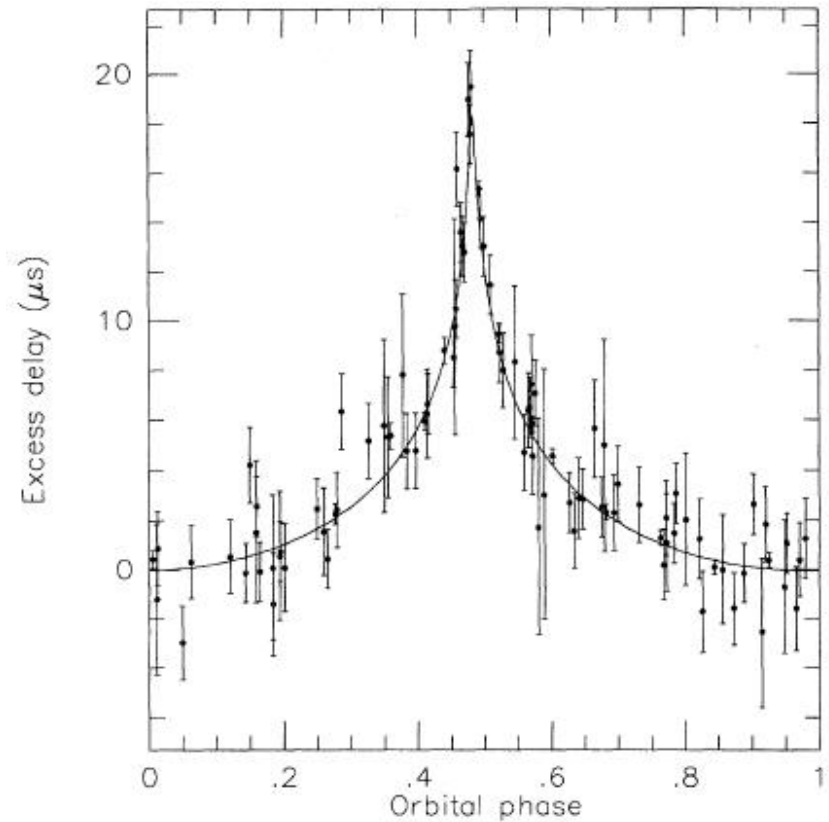
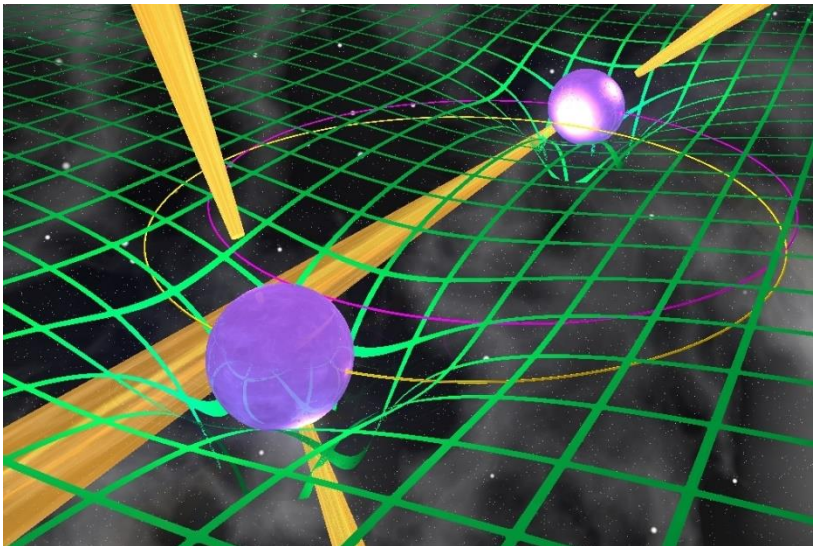
$$r = T_{\odot} m_2 ,$$

$$s = x \left[\frac{P_b}{2\pi} \right]^{-2/3} T_{\odot}^{-1/3} M^{2/3} m_2^{-1} .$$

For details see
Taylor, Weisberg 1989
ApJ 345, 434

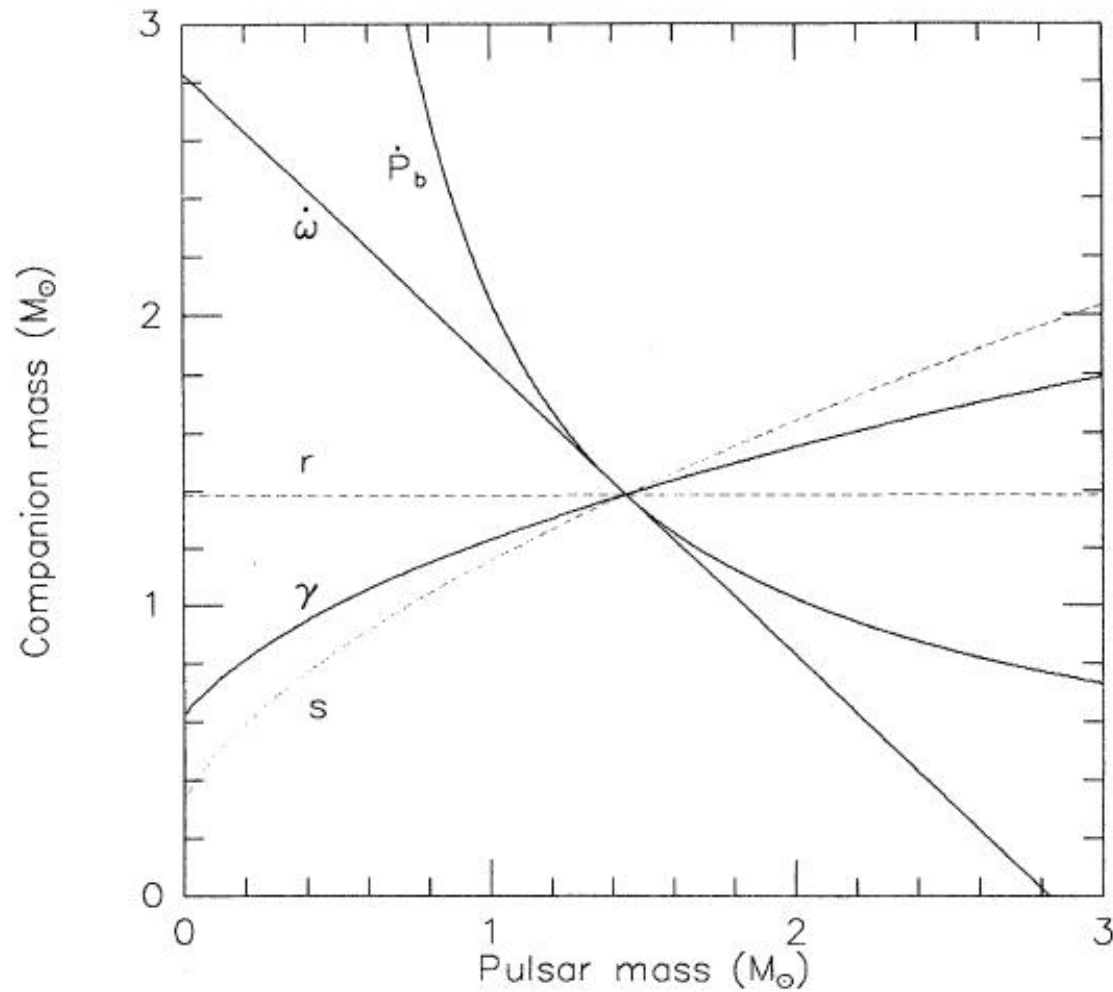
Shapiro delay

$$\Delta_S = -2r \log(1 - s \cos[2\pi(\phi - \phi_0)])$$



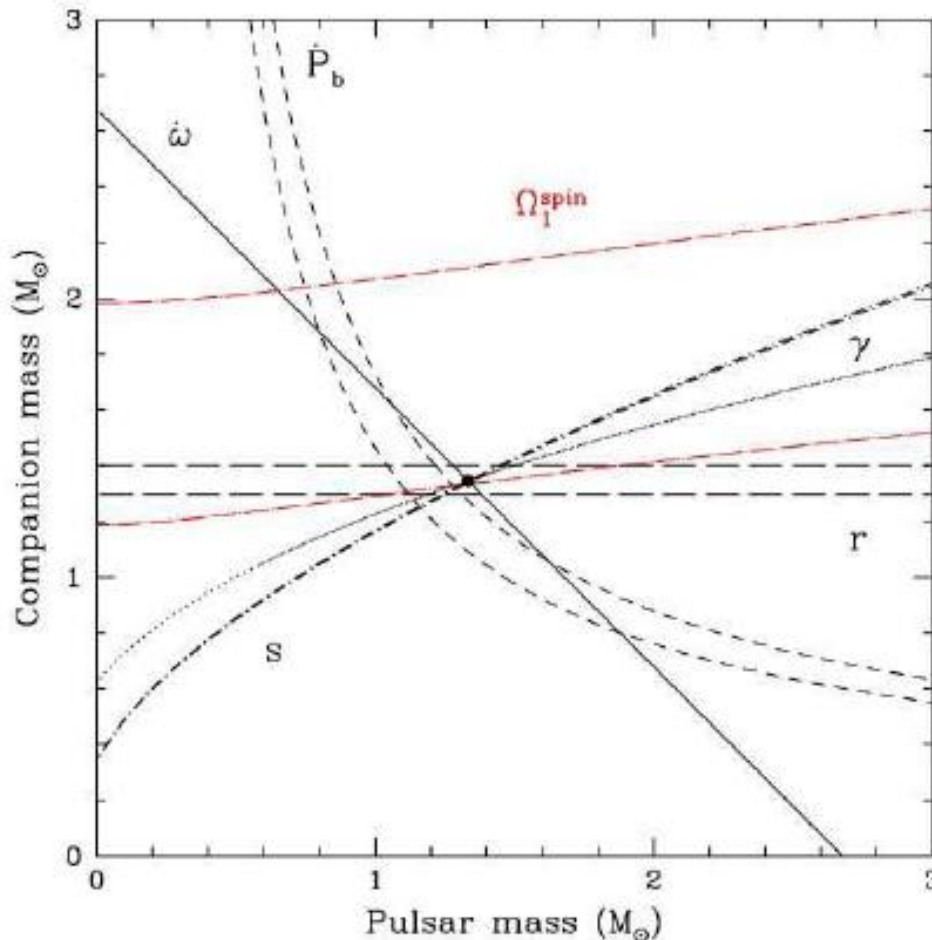
PSR 1855+09 (Taylor, Nobel lecture)

Mass measurements



PSR 1913+16

Uncertainties and inverse problems

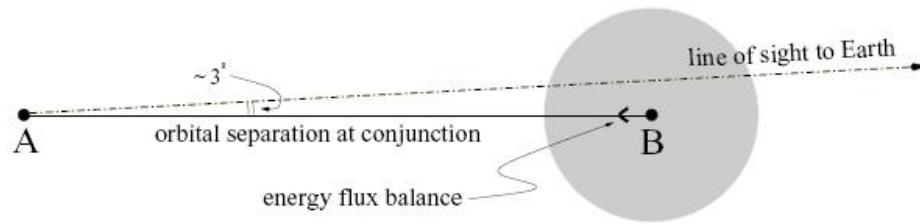
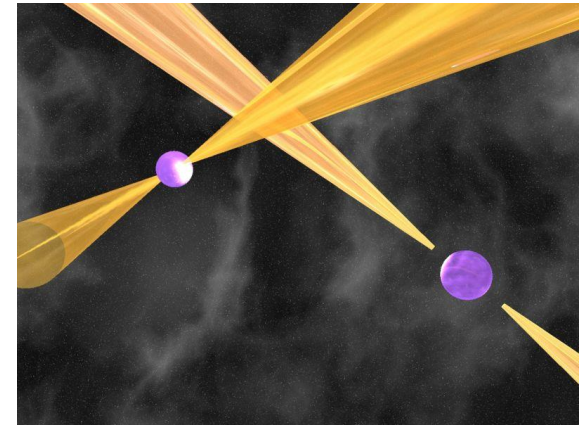
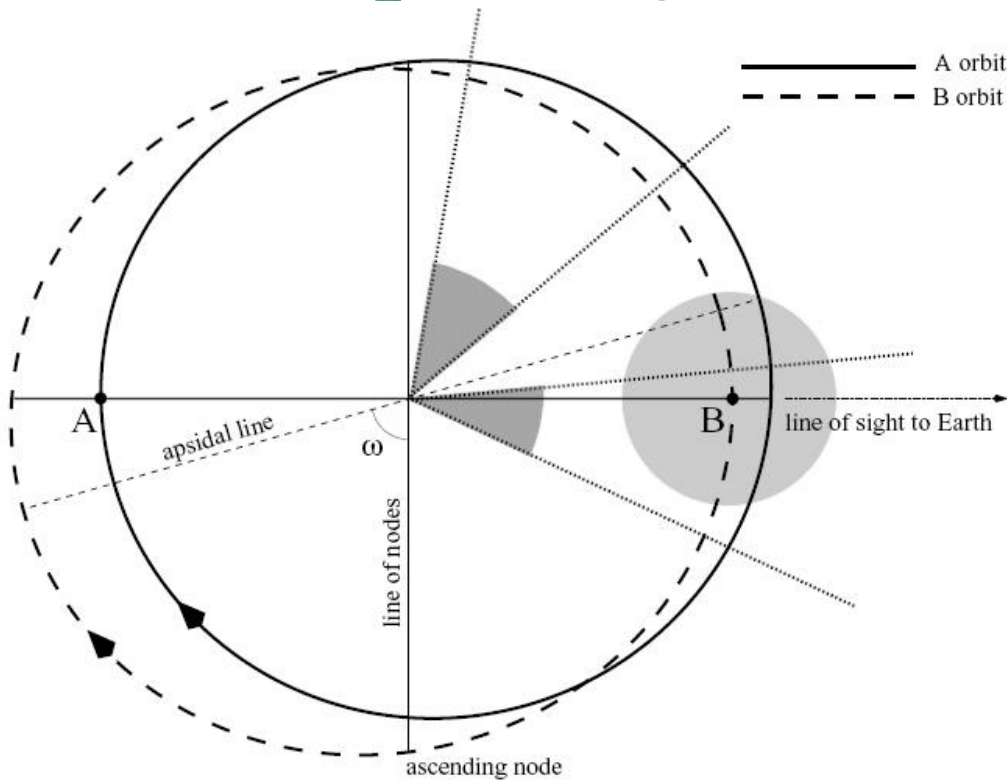


\dot{P}_b depends on the Shklovskii effect. So, if distance is not certain, it is difficult to have a good measurement of this parameter.

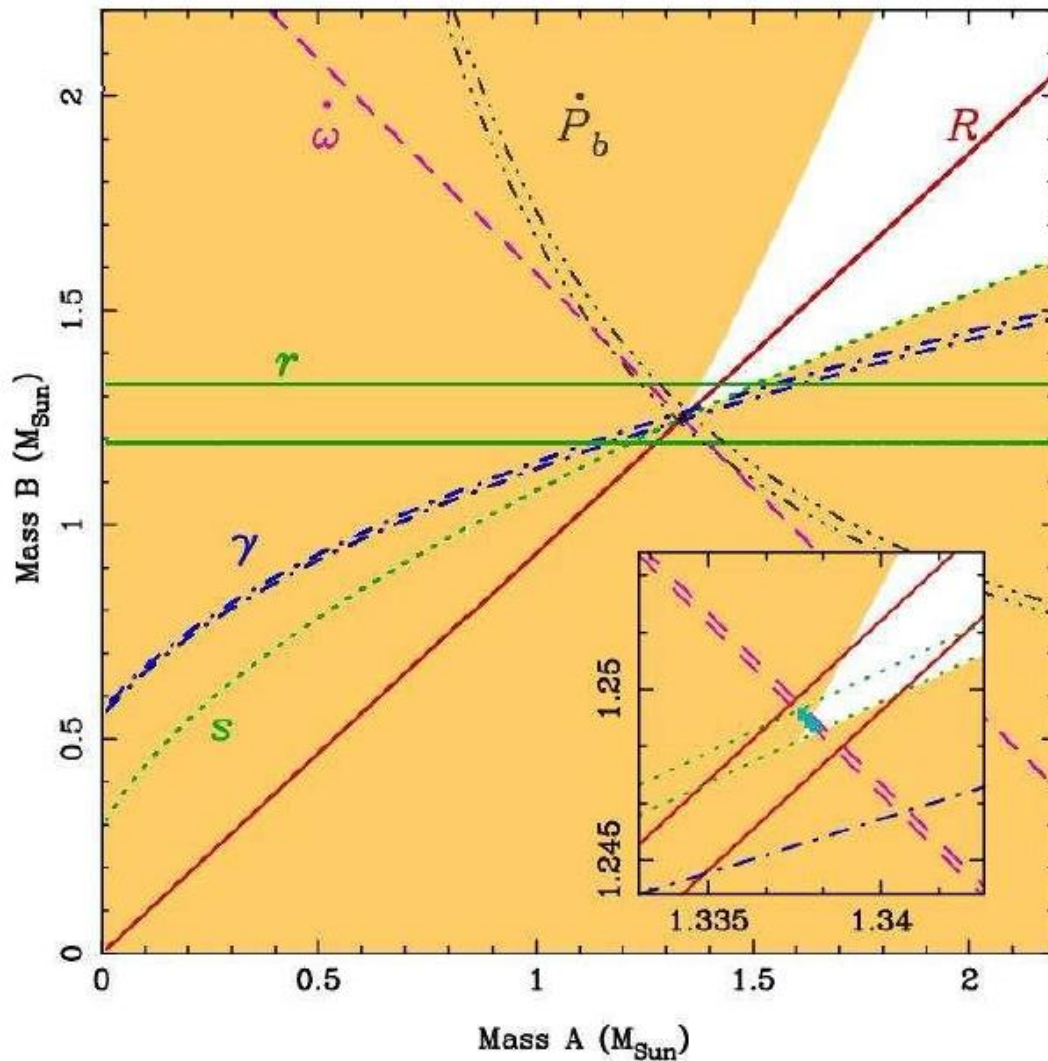
It is possible to invert the problem. Assuming that GR is correct, one can improve the distance estimate for the given source.

PSR B1534+12.

Double pulsar J0737-3039



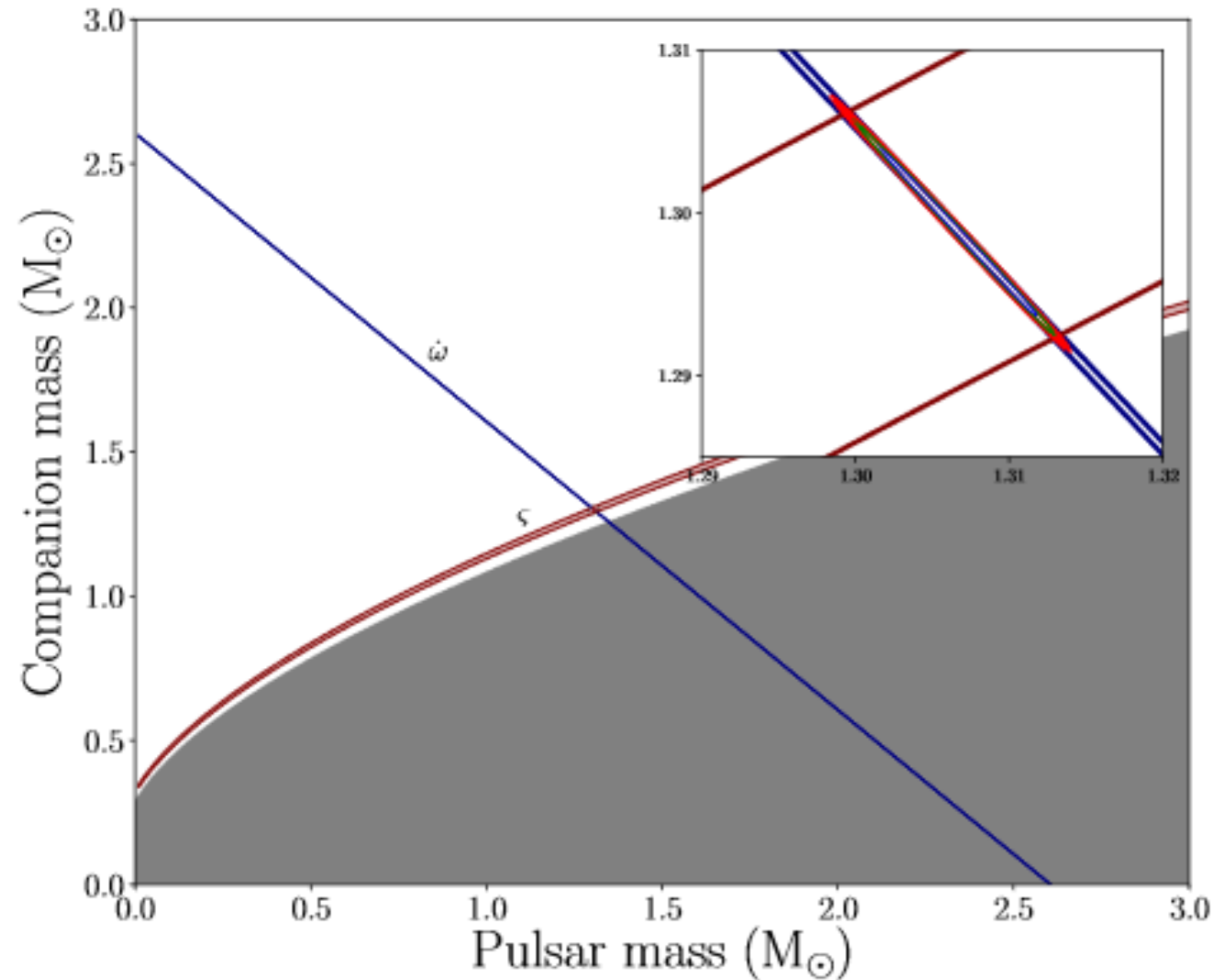
Masses for PSR J0737-3039



The most precise values.

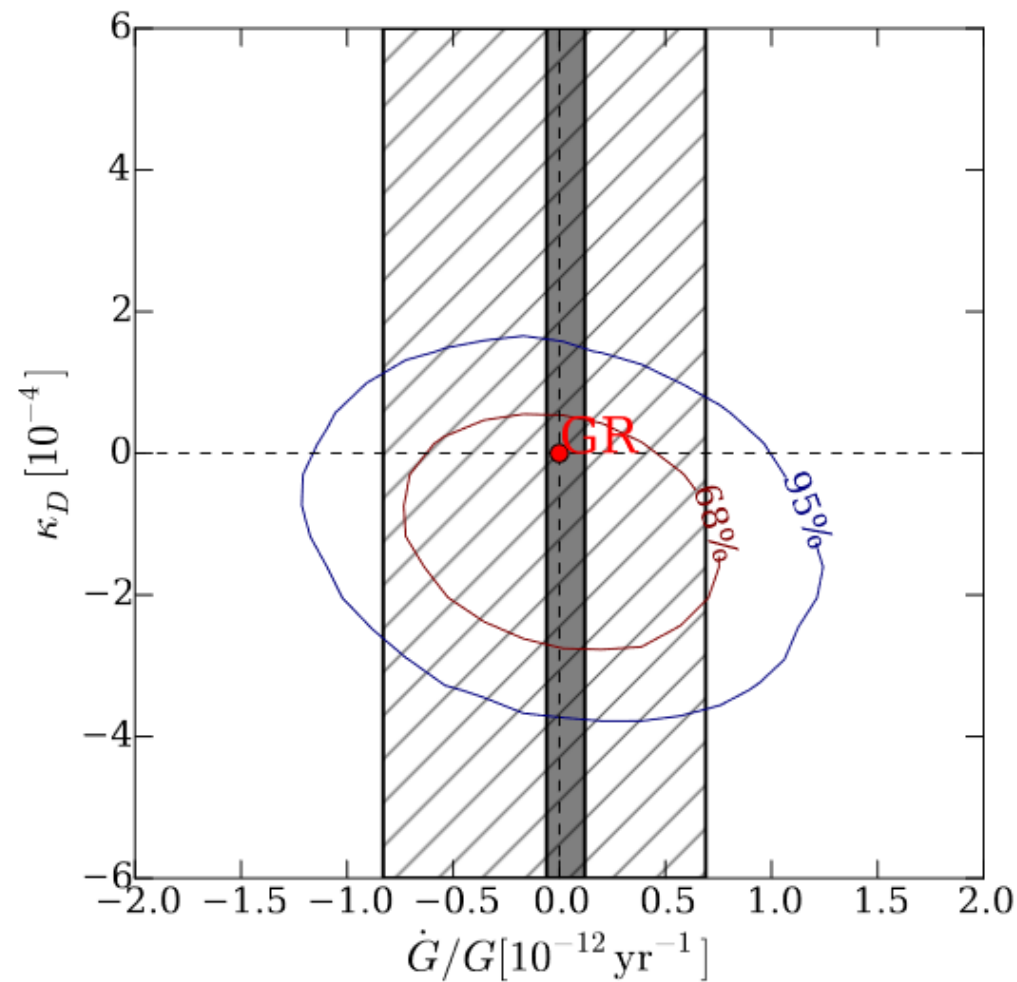
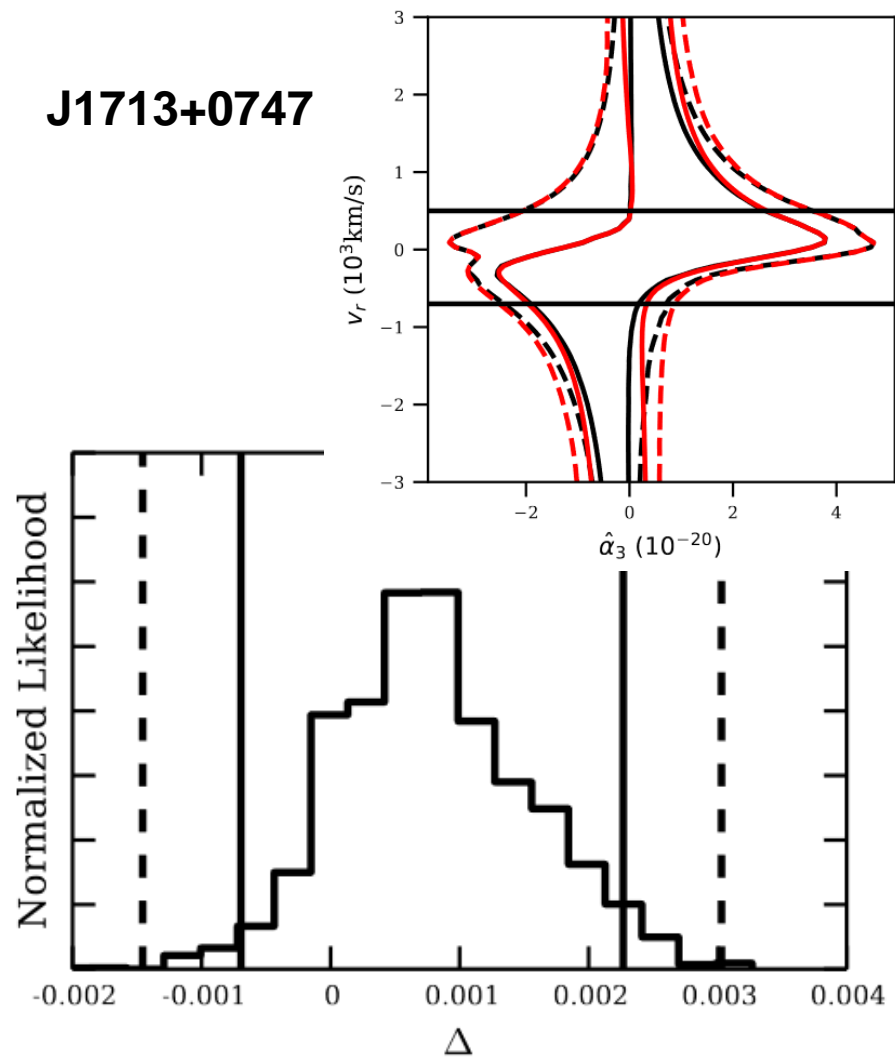
New mass estimates
have uncertainties <0.001

DNS J1829+2456 mass measurements

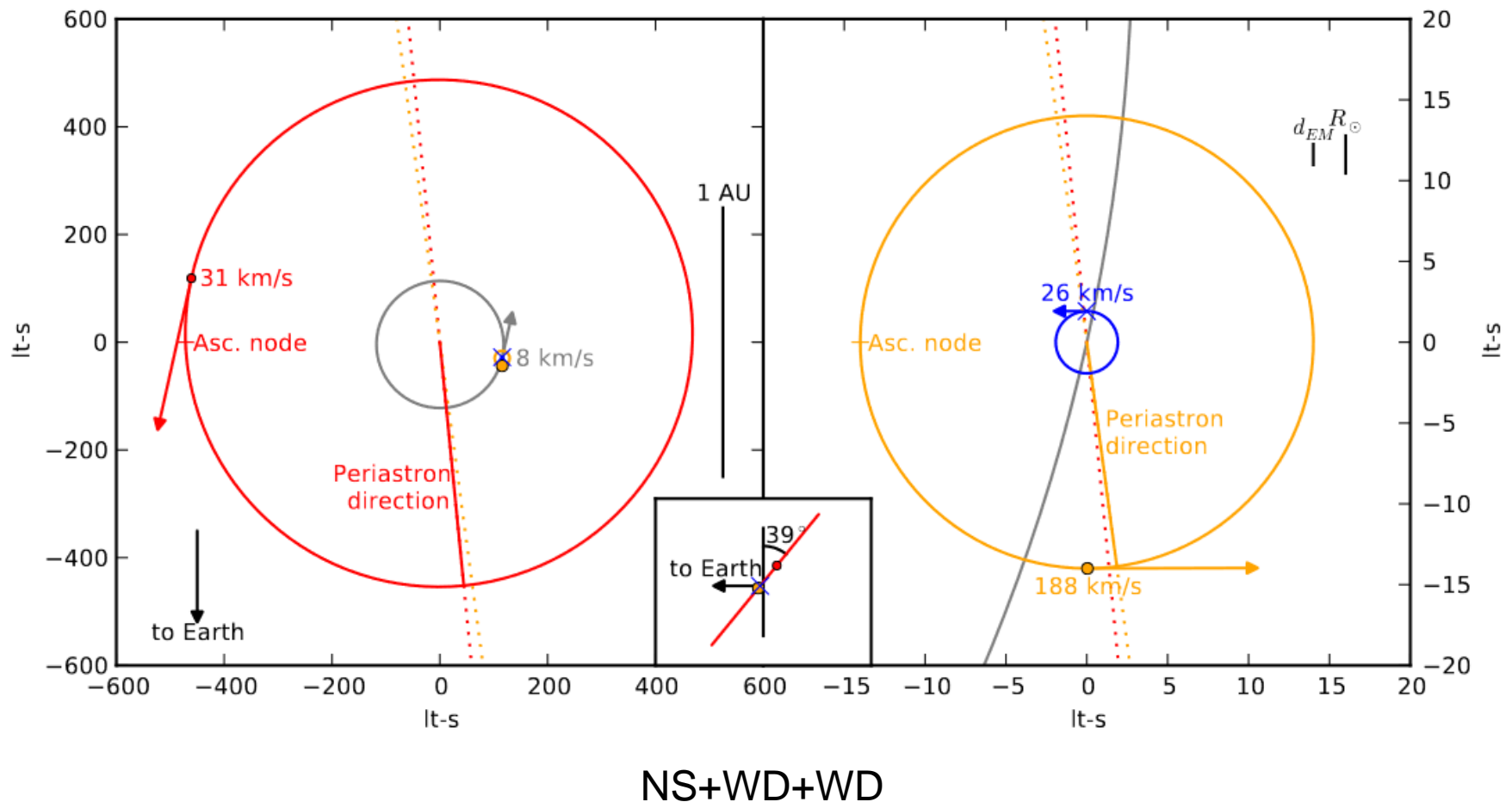


Tests of theories of gravity

J1713+0747



Testing strong equivalence principle with triple pulsar PSR J0337+1715



NS+WD binaries

Some examples

PSR J0437-4715. WD companion [[0801.2589](#), [0808.1594](#)].
The closest millisecond PSR. $M_{\text{NS}}=1.76\pm0.2$ solar.

The case of PSR J0751+1807.

Initially, it was announced that it has a mass ~ 2.1 solar [astro-ph/0508050].

However, then in 2007 at a conference the authors announced that the result was incorrect. Actually, the initial value was 2.1 ± 0.2 (1 sigma error).

New result: 1.26 ± 0.14 solar

[Nice et al. 2008, Proc. of the conf. “40 Years of pulsars”]

It is expected that most massive NSs get their additional “kilos” due to accretion from WD companions [[astro-ph/0412327](#)].

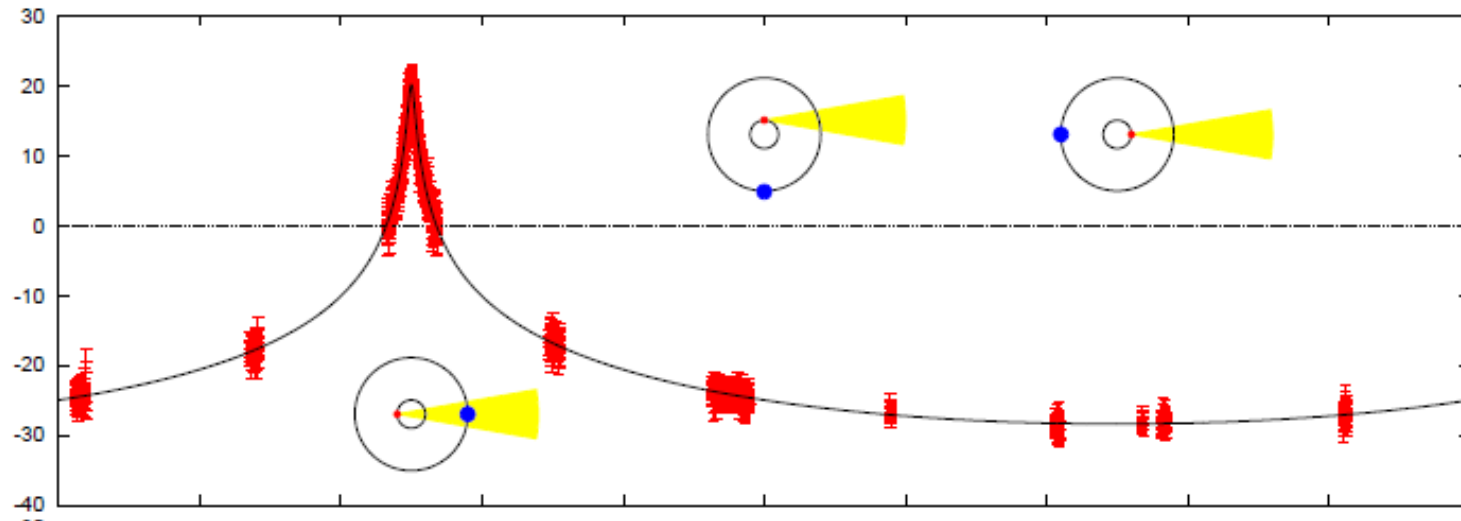
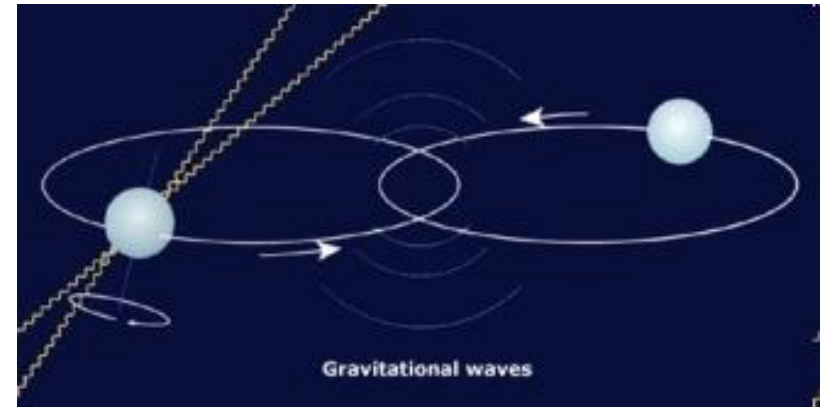
Very massive neutron star

Binary system: pulsar + white dwarf
PSR 1614-2230

Mass ~ 2 solar

About the WD see 1106.5497.

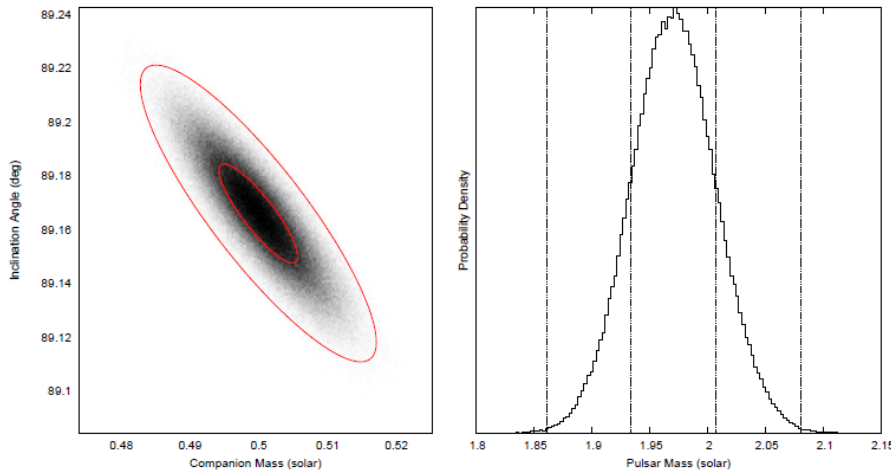
The object was identified in optics.



arXiv: 1010.5788

About formation of this objects see 1103.4996

Why is it so important?

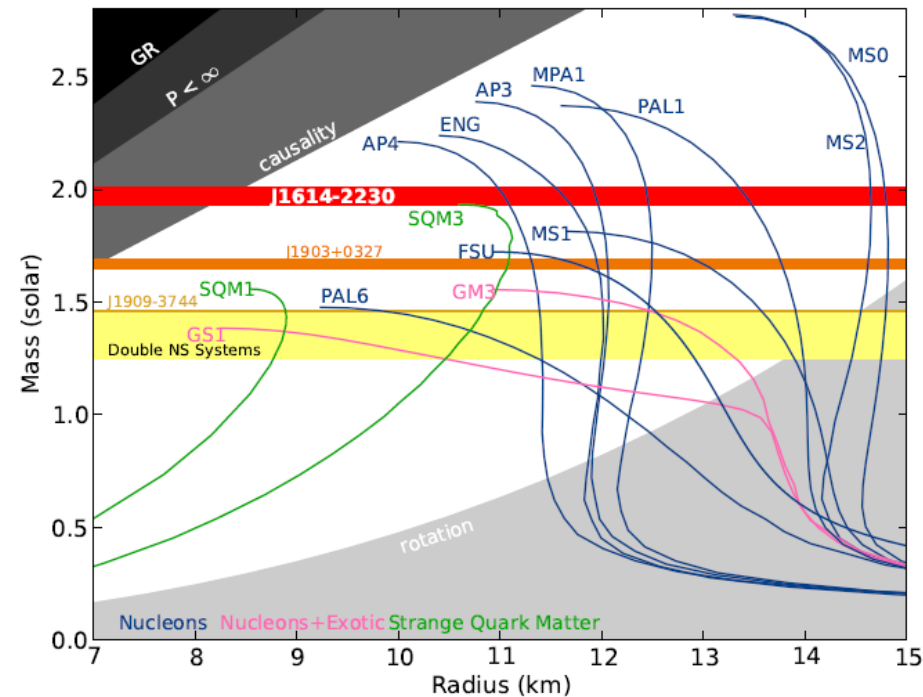


Collapse happens earlier for softer EoSs, see however, 1111.6929 about quark and hybrid stars to explain these data.

Interestingly, it was suggested that just <0.1 solar masses was accreted (1210.8331)

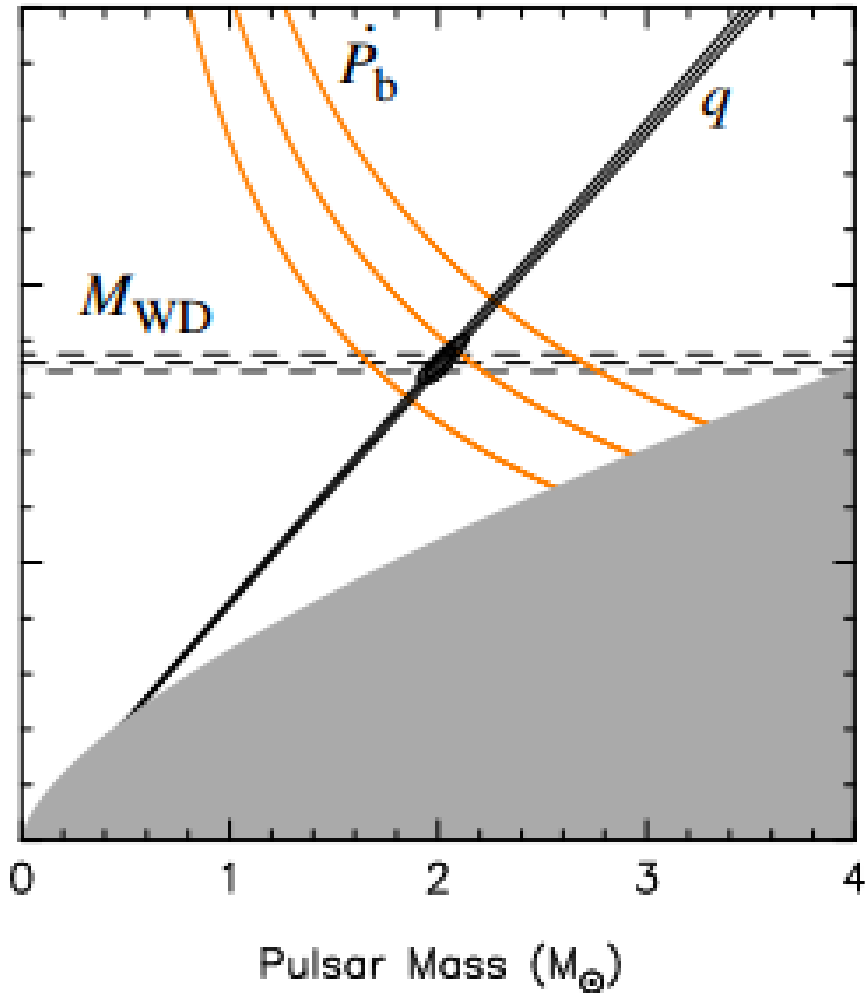
In the future specific X-ray sources (eclipsing msec PSR like SWIFT J1749.4–2807) can show Shapiro delay and help to obtain masses for a different kind of systems, see 1005.3527 , 1005.3479 .

The maximum mass is a crucial property of a given EoS



arXiv: 1010.5788

2.01 solar masses NS



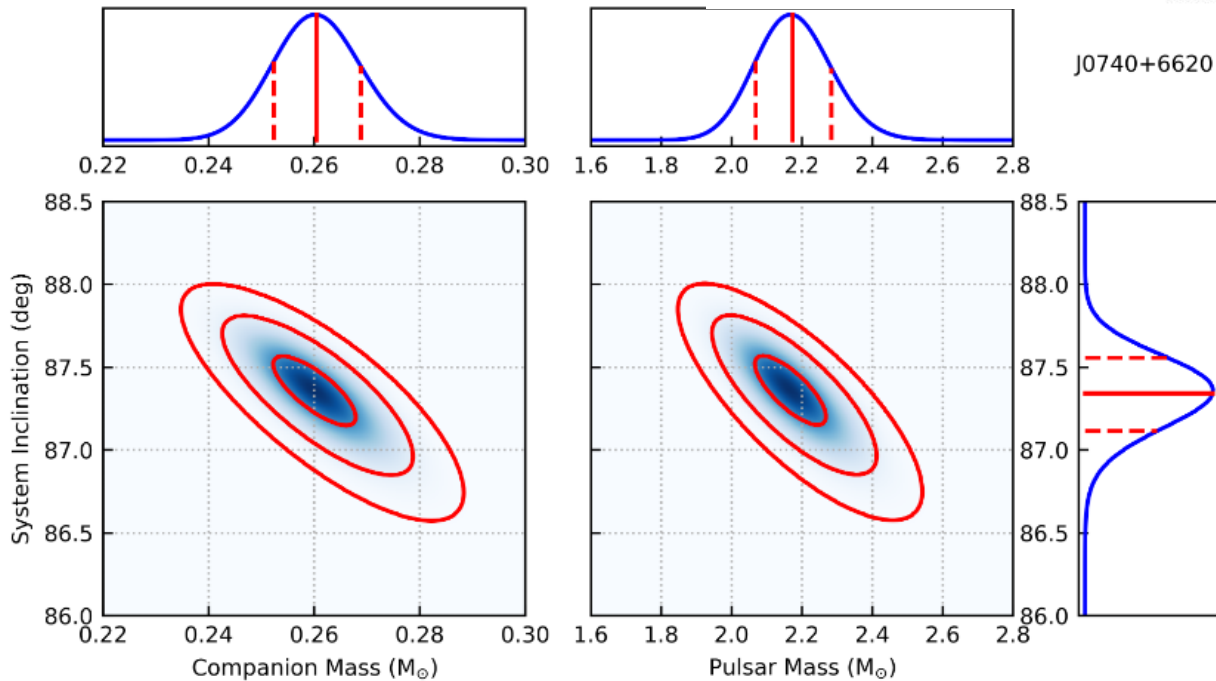
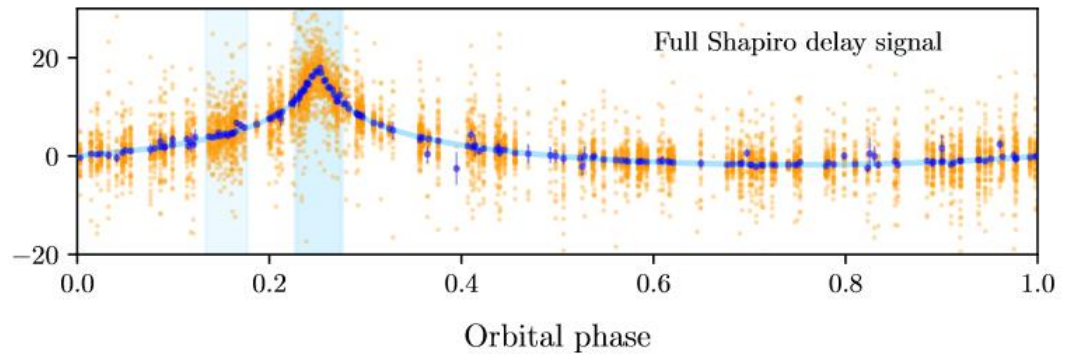
PSR J0348+0432
39 ms, 2.46 h orbit
WD companion

The NS mass is estimated to be:
1.97 – 2.05 solar mass at 68.27%
1.90 – 2.18 solar mass at 99.73%
confidence level.

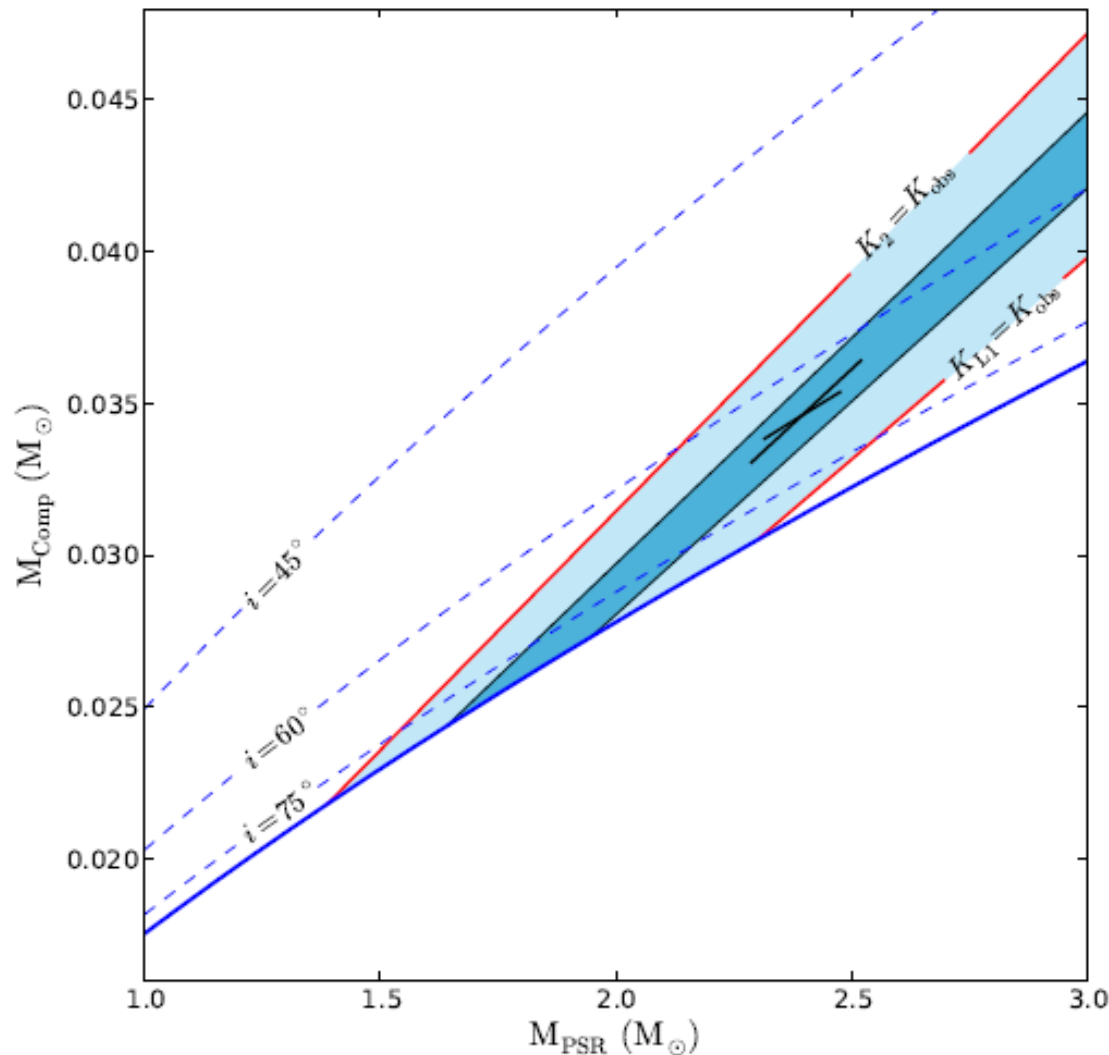
System is perfect for probing
theories of gravity as it is very compact.

2.14 solar mass NS

J0740+6620
2.14 solar masses



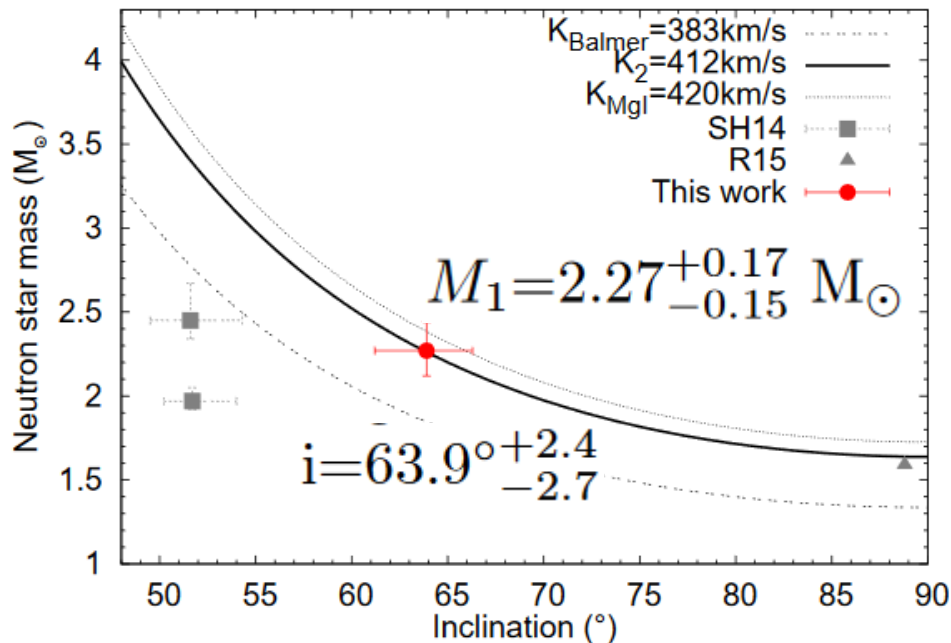
The most extreme (but unclear) example



BLACK WIDOW PULSAR
PSR B1957+20

2.4 \pm 0.12 solar masses

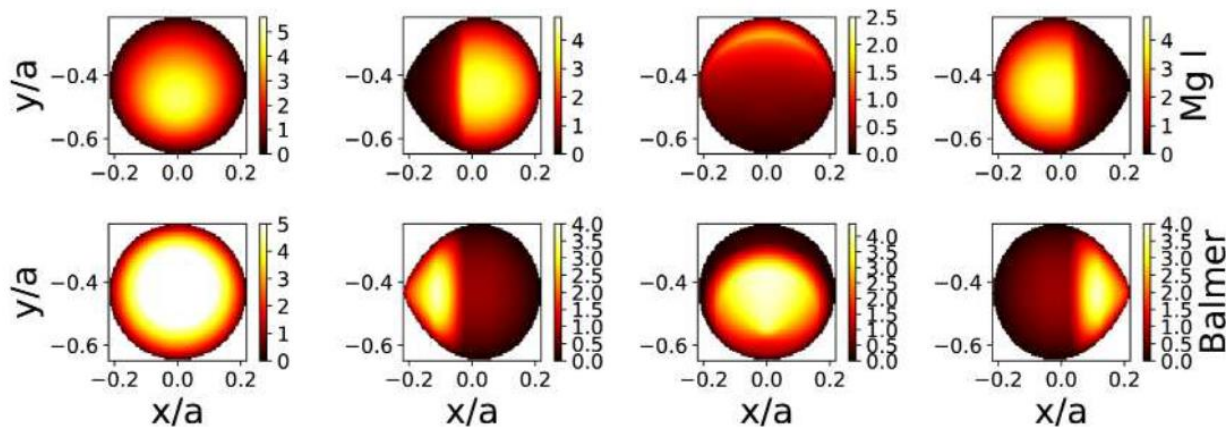
A massive NS in PSR J2215+5135



Different lines provide different velocity as they are emitted from different sides of the companion.

Different sides of the companion move with different velocity.

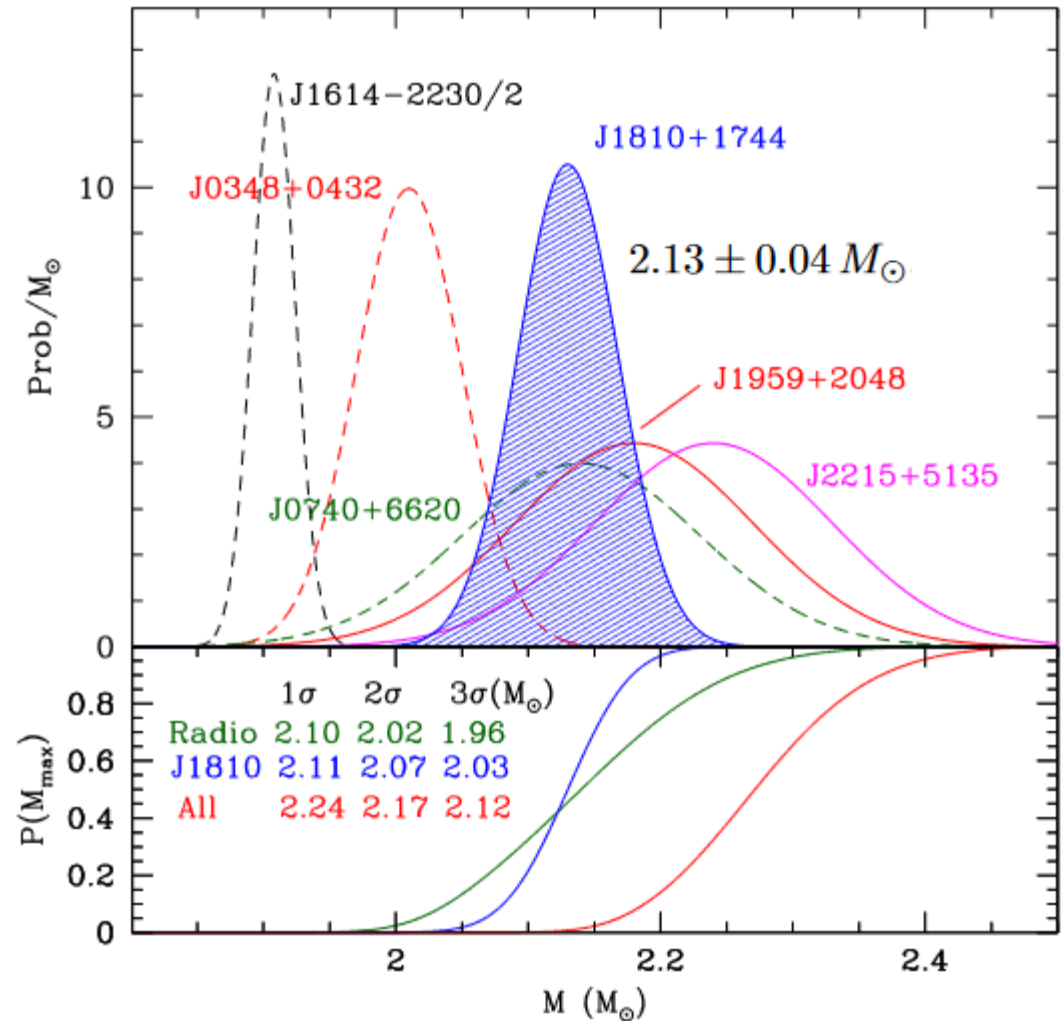
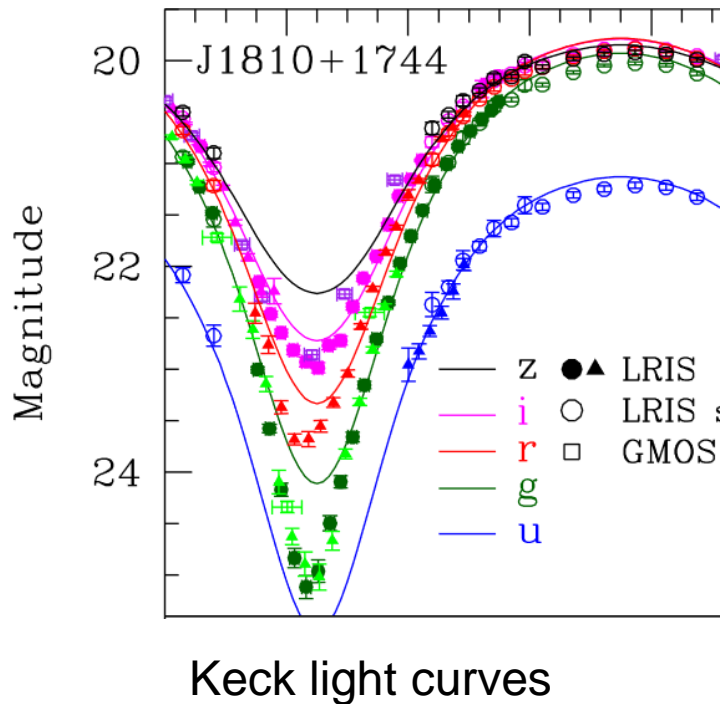
Thus, a correct model provides new mass determination.



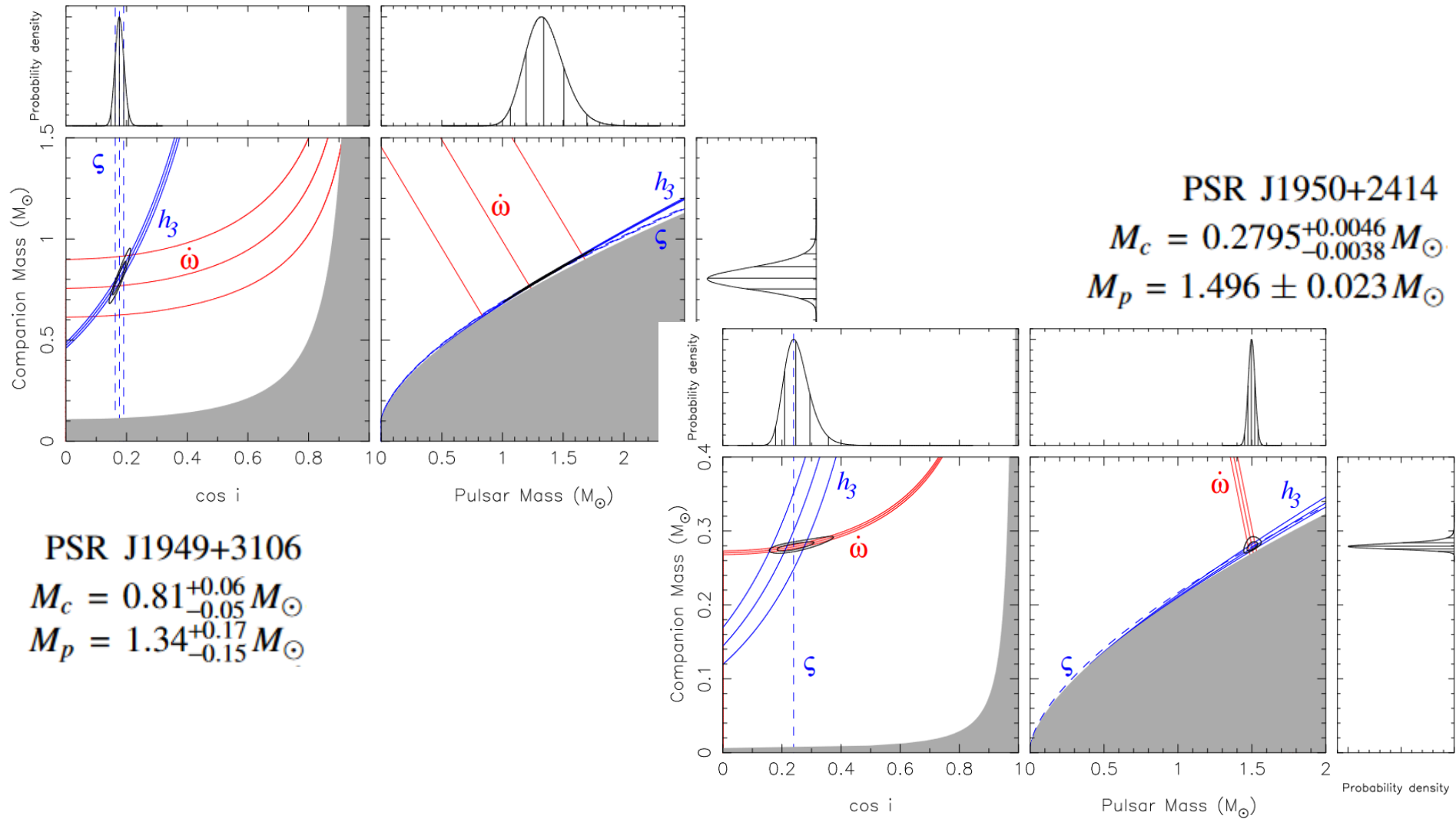
High mass of PSR J1810+1744

Black widow – like system

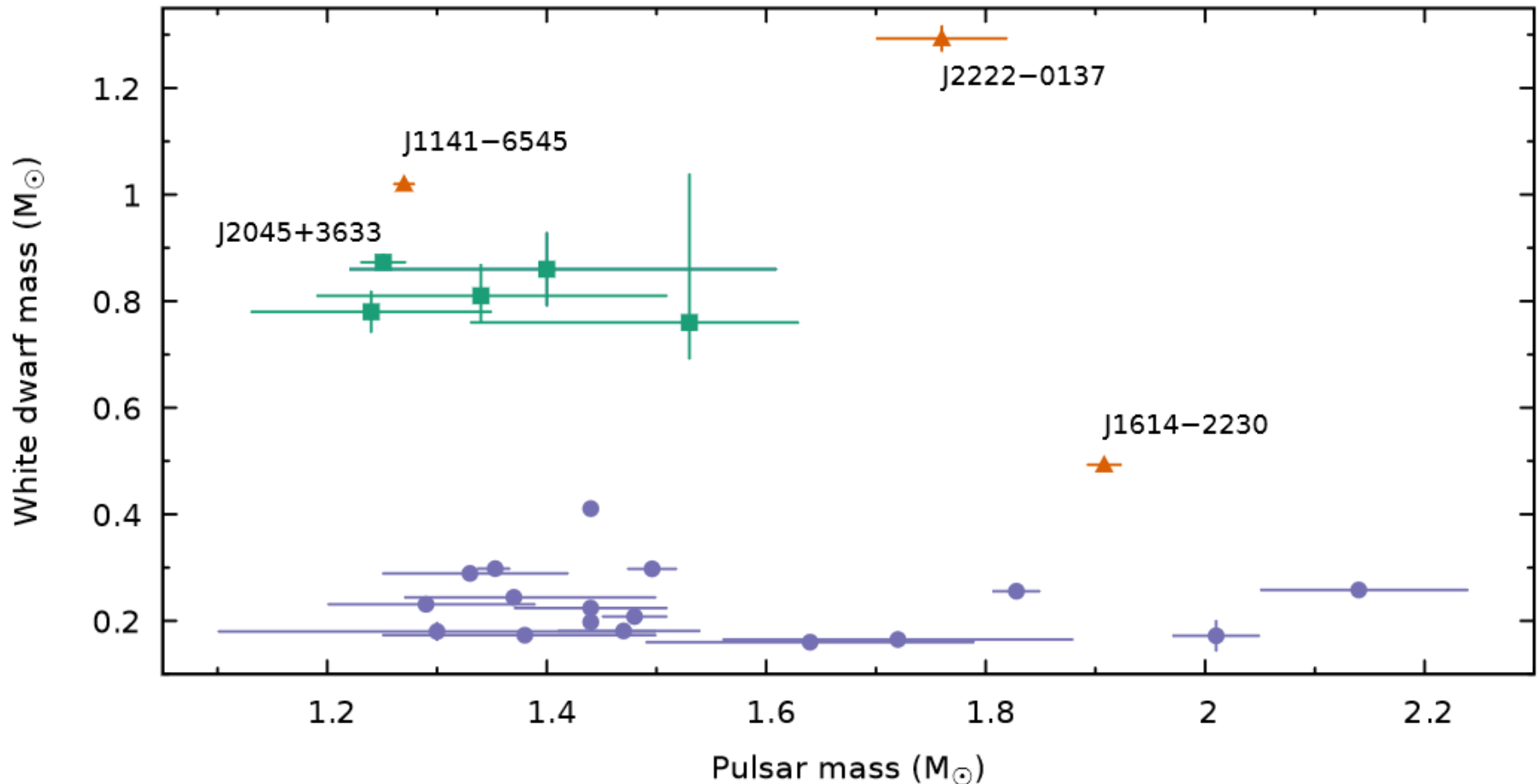
Detailed studies of companion
are necessary to measure mass.



Two new measurements

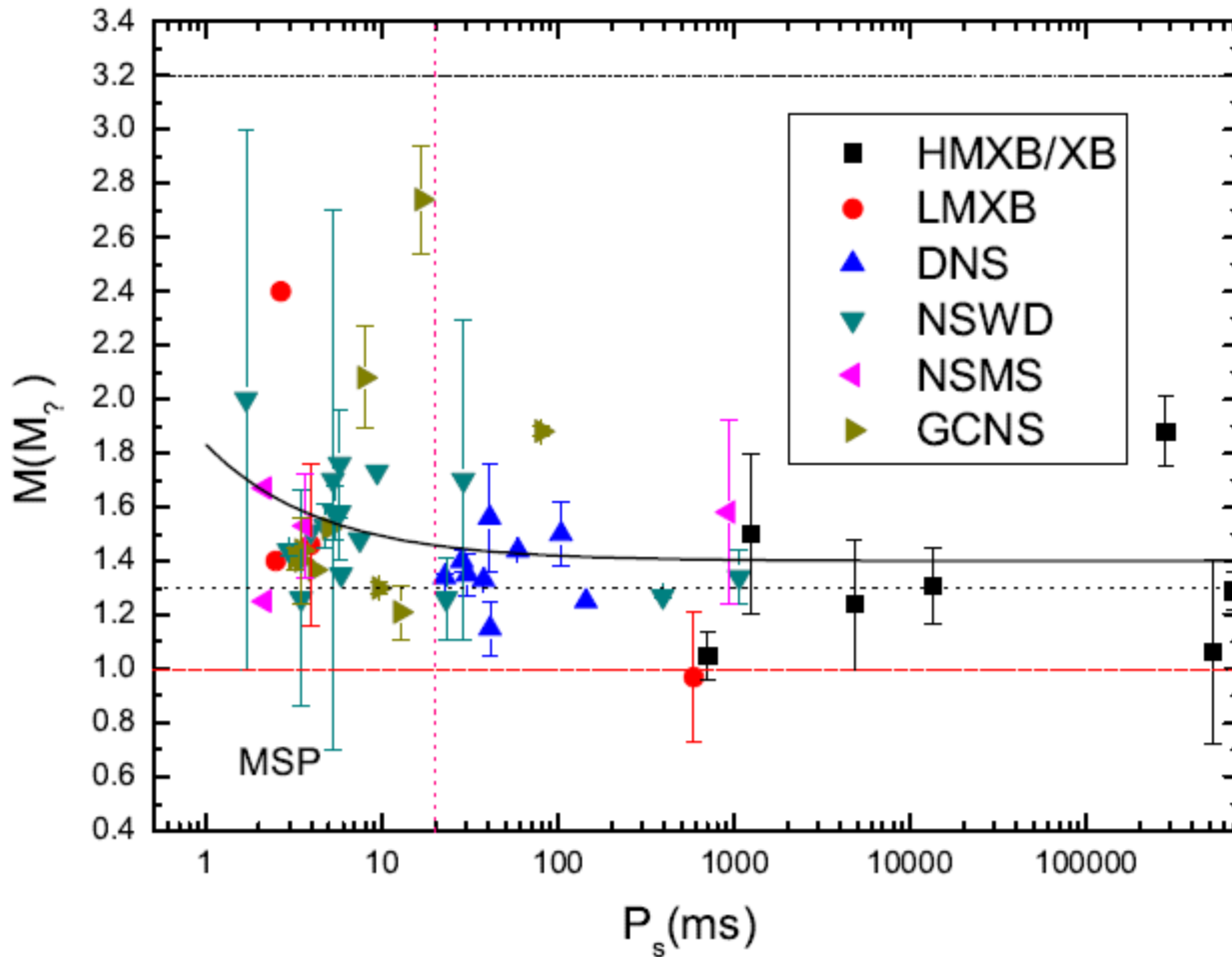


PSR-WD masses



Light helium white dwarf companions are shown as purple circles, and the systems with massive white dwarf (CO WD) companions are shown as green squares. Triangles – non-recycled PSRs (WD formed first).

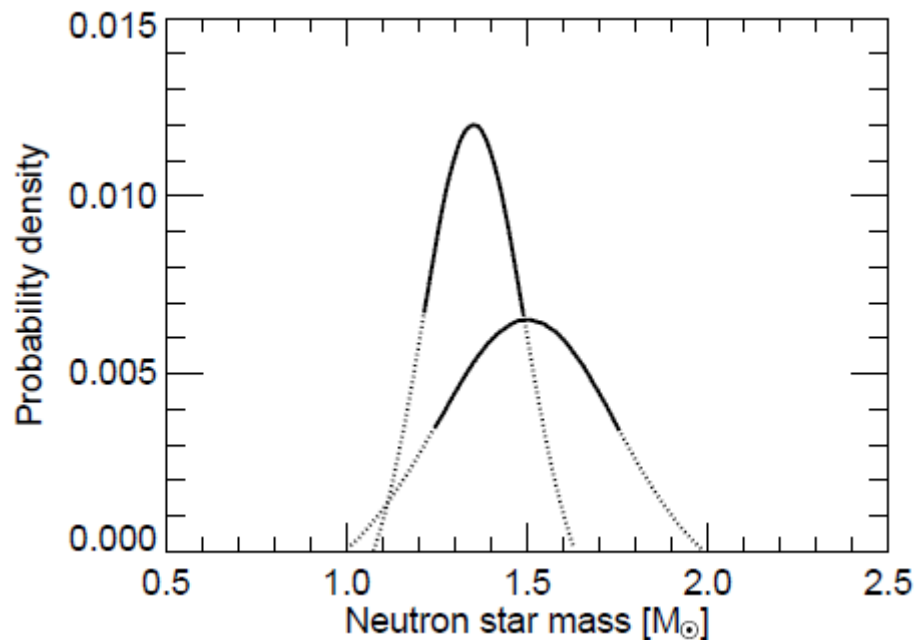
How much do PSRs accrete?



$$M = 1.4 + 0.43(P/\text{ms})^{-2/3}$$

Millisecond pulsars are
~0.2 solar masses more
massive than the rest ones.

DNS and NS+WD binaries



1.35 \pm 0.13 and 1.5 \pm 0.25

Cut-off at ~ 2.1 solar masses
can be mainly due to evolution
in a binary, not due to nuclear
physics (see 1309.6635)

Neutron stars in binaries

Study of close binary systems gives an opportunity to obtain mass estimate for progenitors of NSs (see for example, Ergma, van den Heuvel 1998 A&A 331, L29).

For example, an interesting estimate was obtained for GX 301-2.

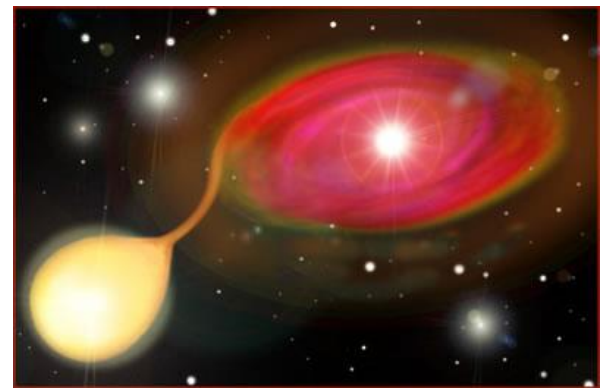
The progenitor mass is >50 solar masses.

On the other hand, for several other systems with both NSs and BHs progenitor masses are smaller: from 20 up to 50.

Finally, for the BH binary LMC X-3 the progenitor mass is estimated as >60 solar.

So, the situation is tricky.

Most probably, in some range of masses, at least in binary systems, stars can produce both types of compact objects: NSs and BHs.



Mass determination in binaries: mass function

$$f_v(m) \frac{m_x^3 \sin^3 i}{(m_x + m_v)^2} = 1,038 \cdot 10^{-7} K_v^3 P (1 - e^2)^{3/2},$$

m_x, m_v - masses of a compact object and of a normal star (in solar units),
 K_v – observed semi-amplitude of line of sight velocity of the normal star (in km/s),
 P – orbital period (in days), e – orbital eccentricity, i – orbital inclination
(the angle between the orbital plane and line of sight).

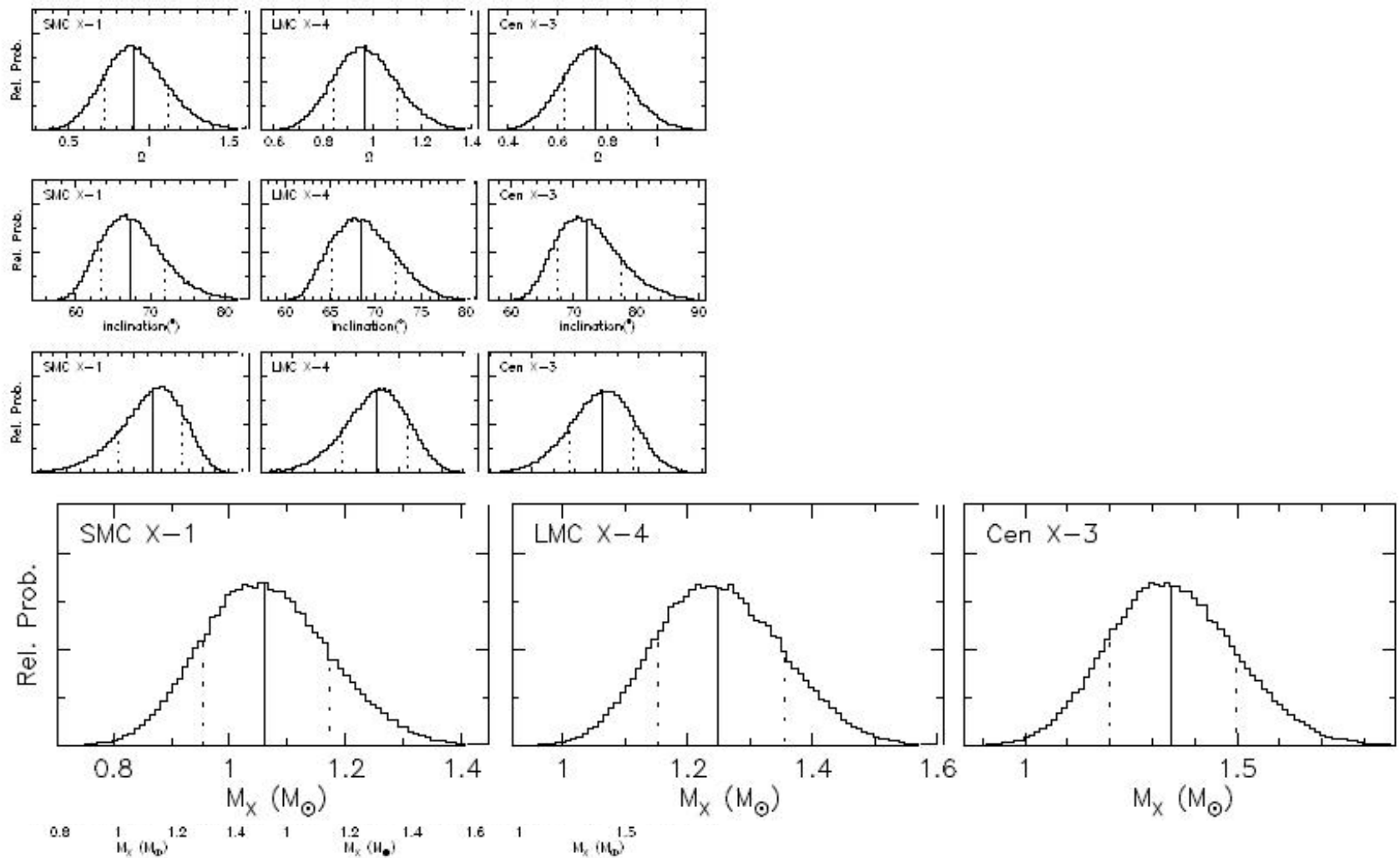
One can see that the mass function is the lower limit for the mass of a compact star.

The mass of a compact object can be calculated as:

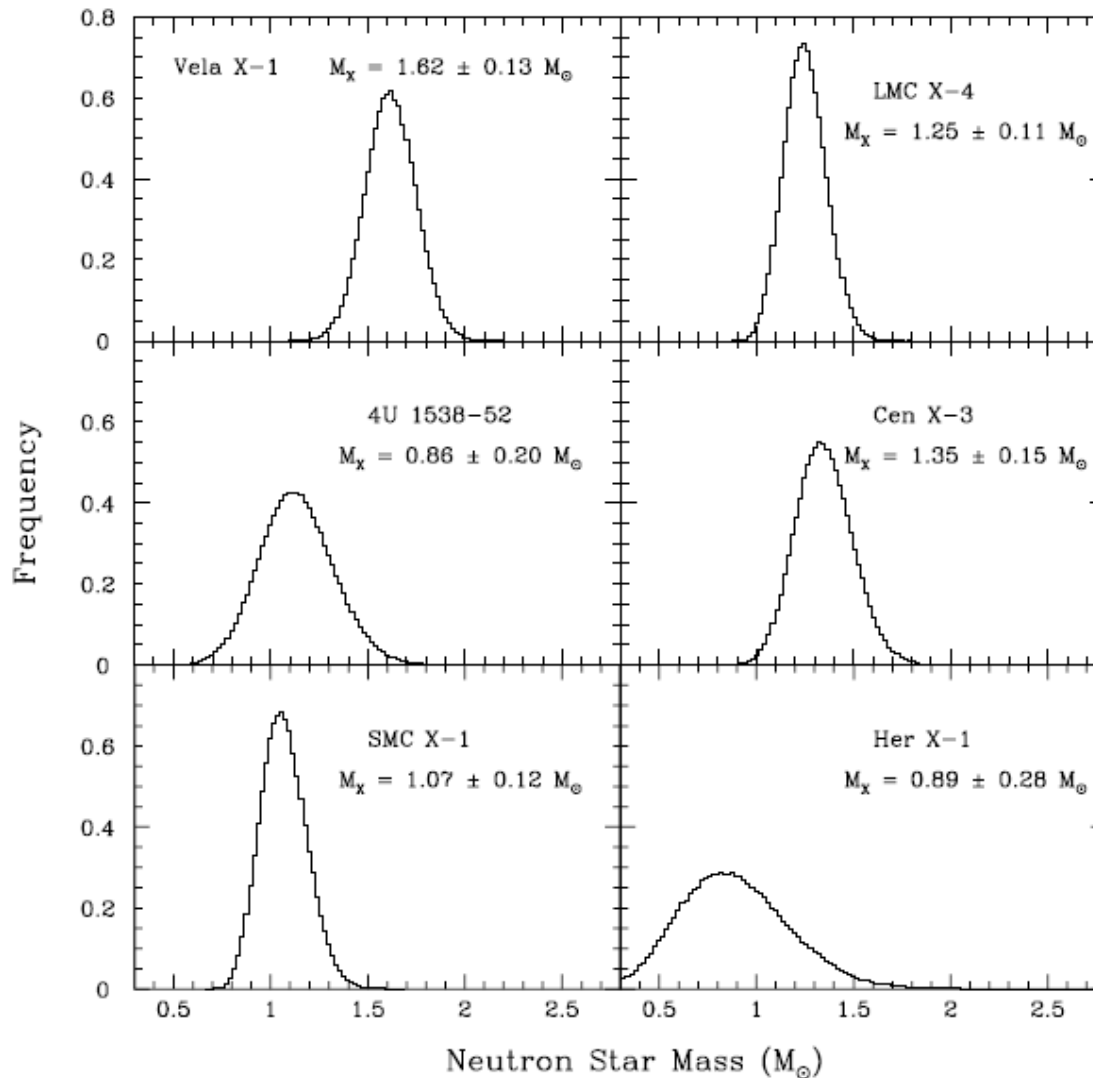
$$m_x = f_v(m) \left(1 + \frac{m_v}{m_x}\right)^2 \frac{1}{\sin^3 i}.$$

So, to derive the mass it is necessary to know (besides the line of sight velocity) independently two more parameters: mass ratio $q = m_x/m_v$, and orbital inclination i .

Some mass estimates

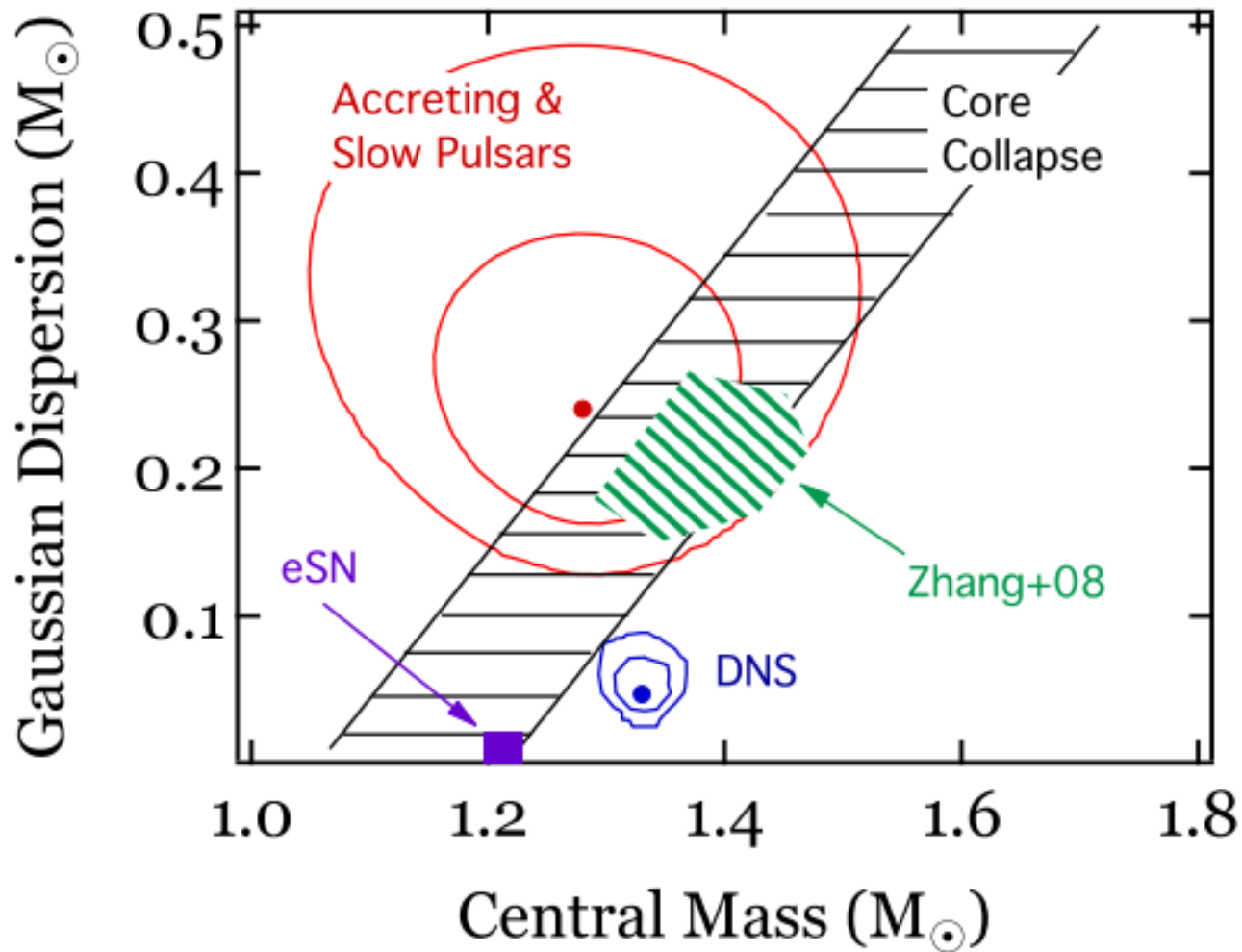


More measurements



Six X-ray binary systems.
All are eclipsing pulsars.

Altogether

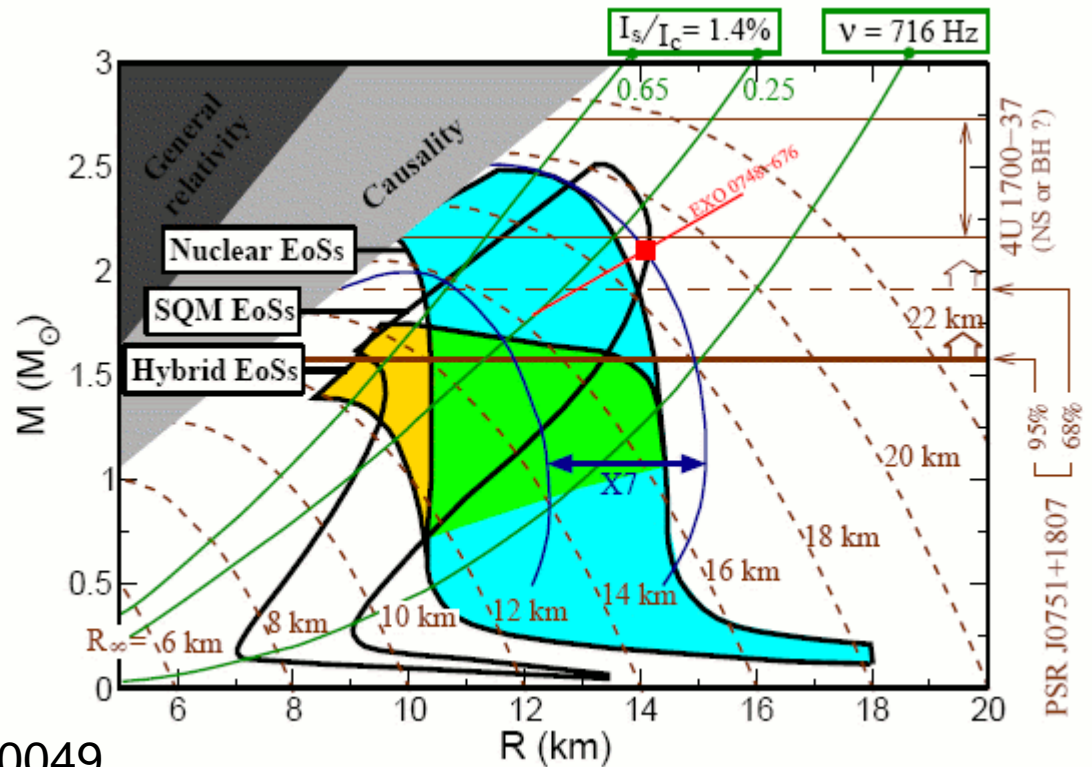


Mass-radius diagram and constraints

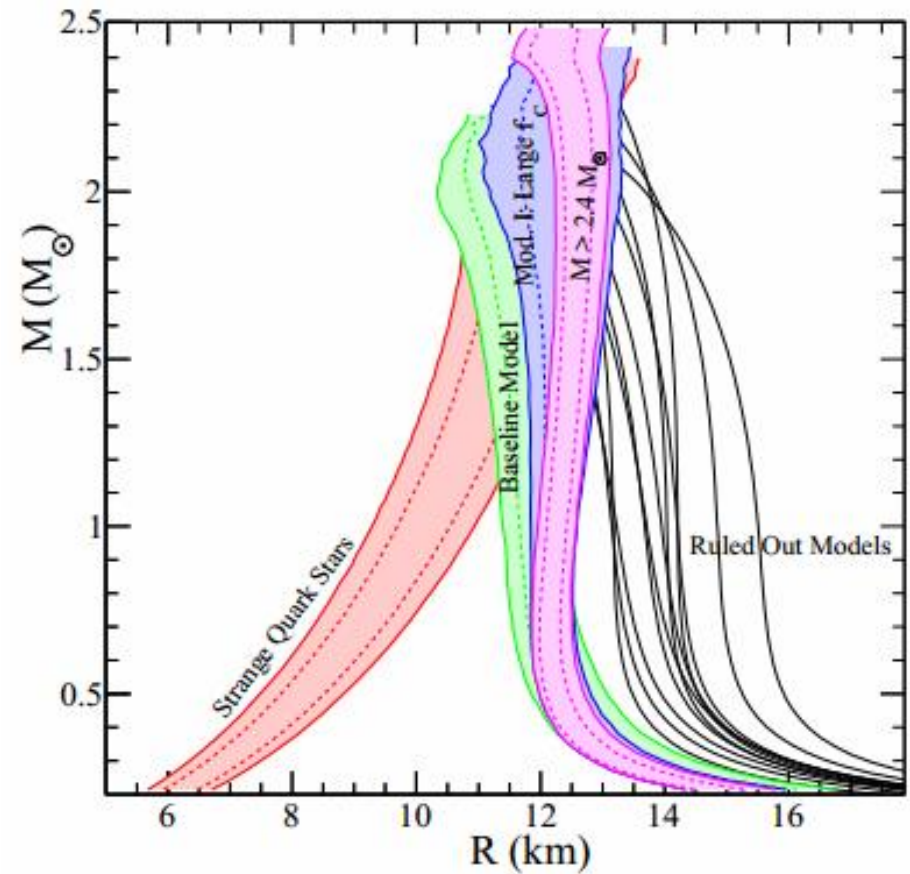
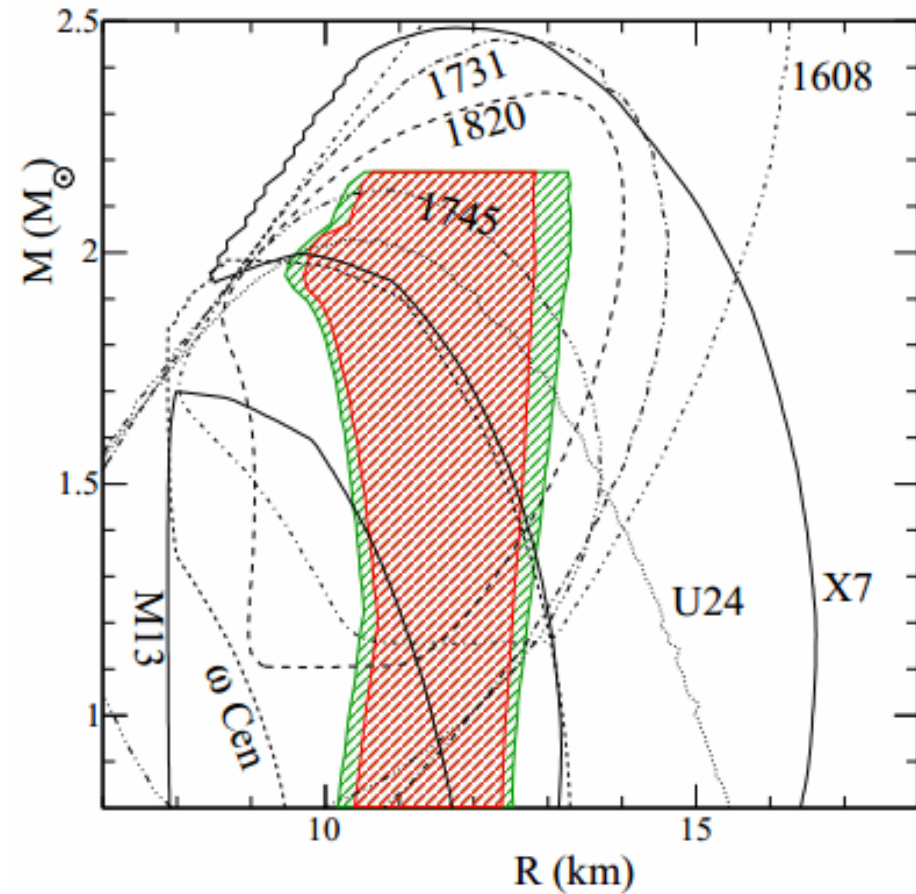
Unfortunately, there are no good data on independent measurements of masses and radii of NSs.

Still, it is possible to put important constraints. Most of recent observations favour stiff EoS.

Useful analytical estimates for EoS can be found in 1310.0049.



Observations vs. data



1205.6871

Some newer results by the same group are presented in 1305.3242

Mass and radius for a pulsar!

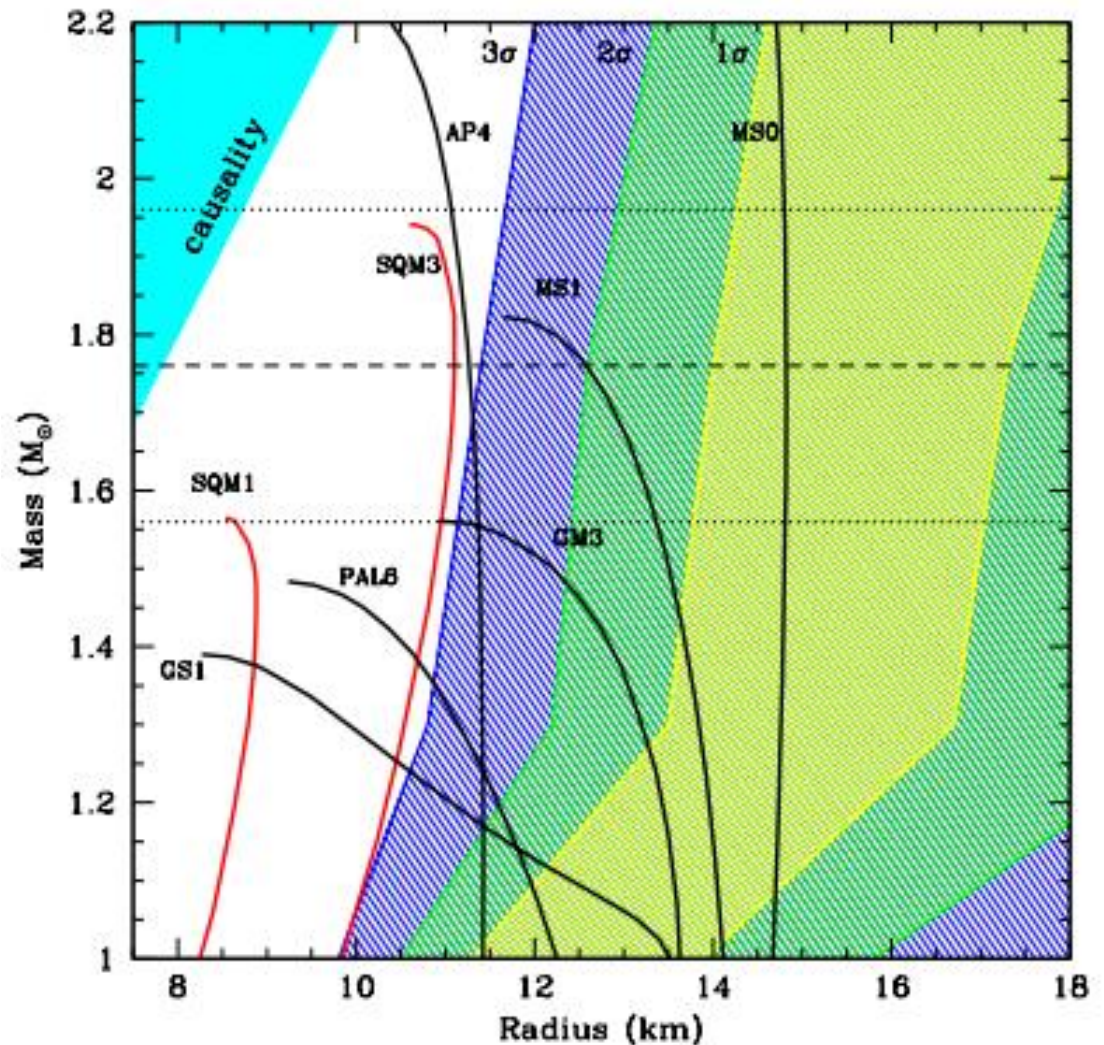
PSR J0437–4715 NS+WD

The nearest known mPSR
155-158 pc

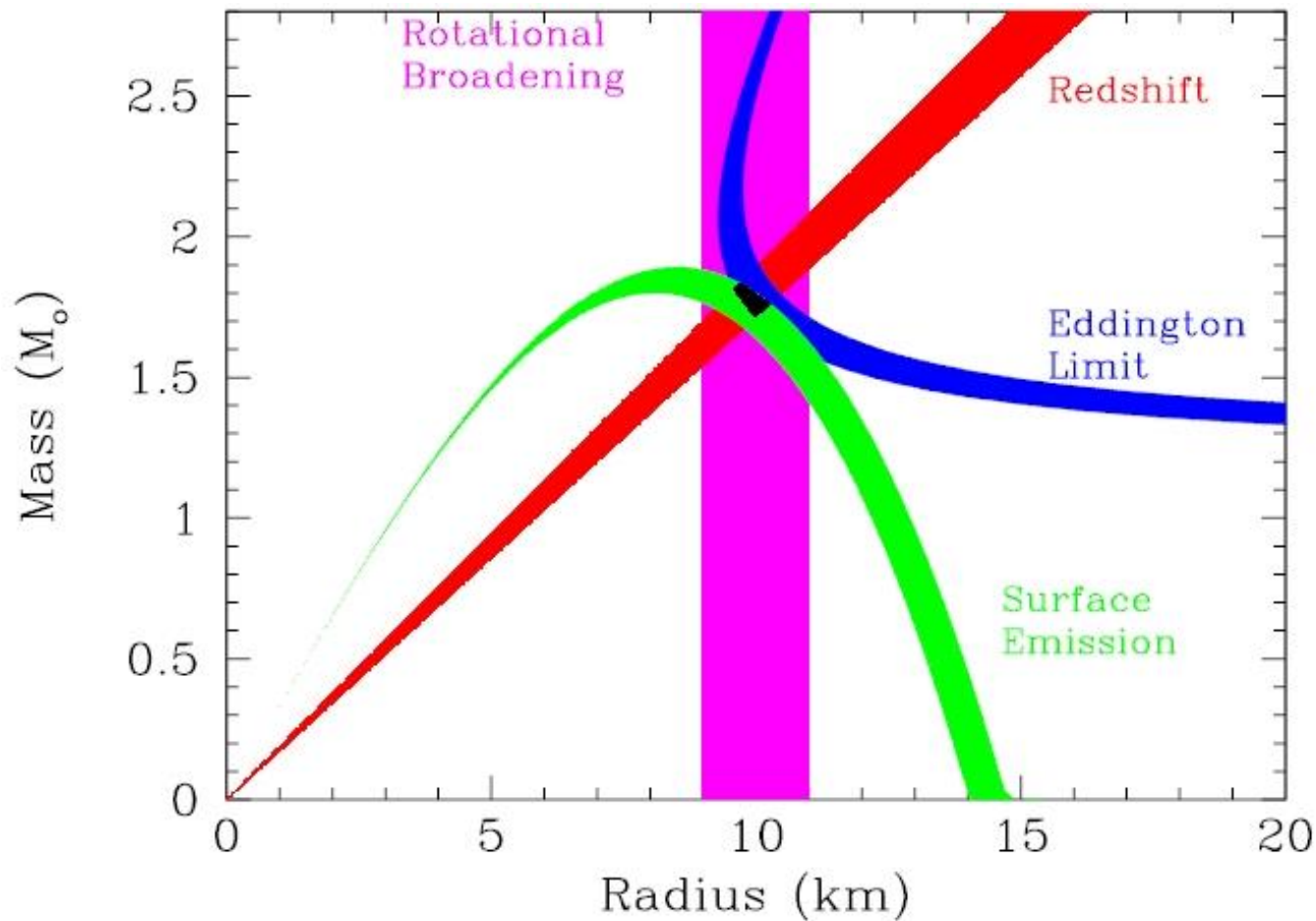
XMM-Newton observations
showed thermal emission.

H-atmosphere model fits.

Hot caps are non-antipodal.

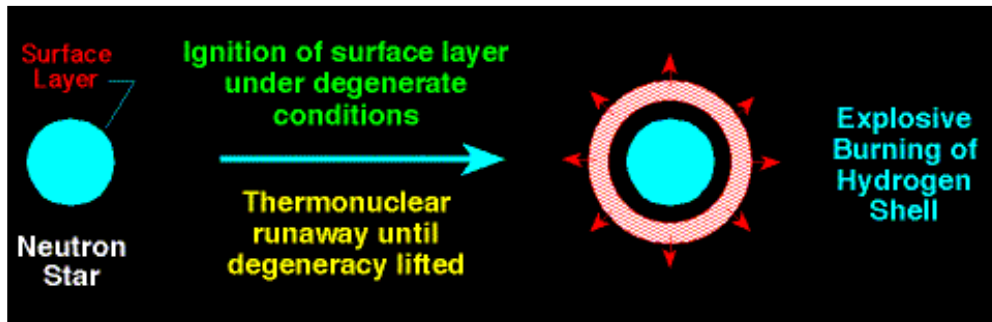


Combination of different methods



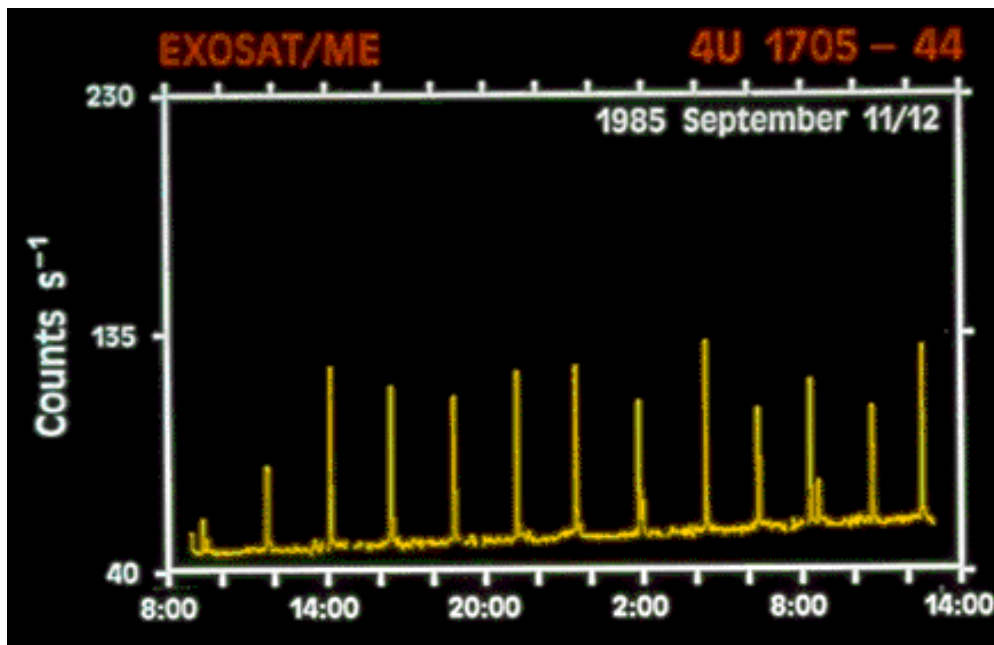
EXO 0748-676

Radius determination in bursters



Explosion with a \sim Eddington luminosity.

Modeling of the burst spectrum and its evolution.



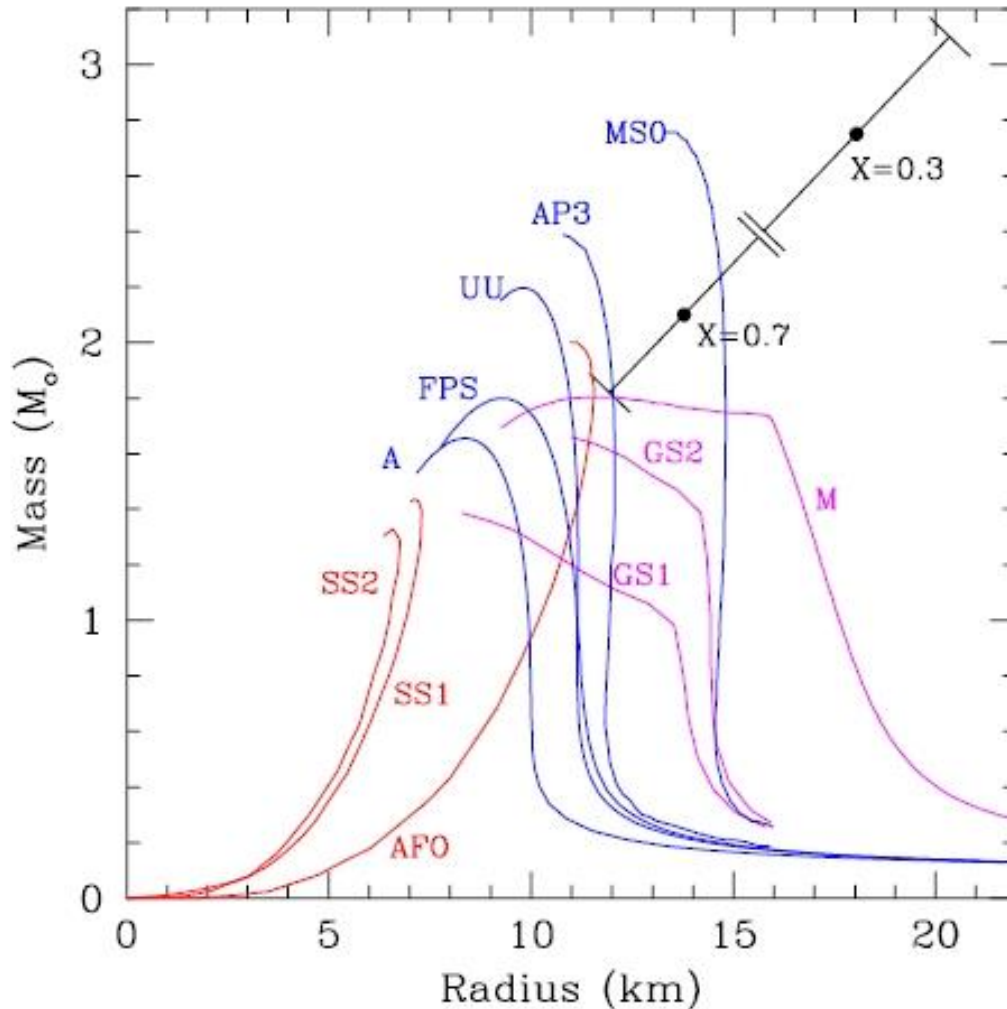
See, for example,
Joss, Rappaport 1984,
Haberl, Titarchuk 1995

More measurements

Continuously new measurements, critics and discussion appears

- 1104.2602 Systematic Uncertainties in the Spectroscopic Measurements of Neutron-Star Masses and Radii from Thermonuclear X-ray Bursts. II. Eddington Limit
 - 1104.5027 The Mass and Radius of the Neutron Star in the Bulge Low-Mass X-ray Binary KS 1731-260
 - 1103.5767 Systematic Uncertainties in the Spectroscopic Measurements of Neutron-Star Masses and Radii from Thermonuclear X-ray Bursts. I. Apparent Radii
 - 1105.1525 Mass and radius estimation for the neutron star in X-ray burster 4U 1820-30
 - 1105.2030 New Method for Determining the Mass and Radius of Neutron Stars
 - 1106.3131 Constraints on the Mass and Radius of the Neutron Star XTE J1807-294
 - 1111.0347 Constraints on neutron star mass and radius in GS 1826-24 from sub-Eddington X-ray bursts
 - 1201.1680 On the consistency of neutron-star radius measurements from thermonuclear bursts
 - 1204.3627 Constraints on the mass and radius of the accreting neutron star in the Rapid Burster
 - 1301.0831 The mass and the radius of the neutron star in the transient low mass X-ray binary SAX J1748.9-2021
-

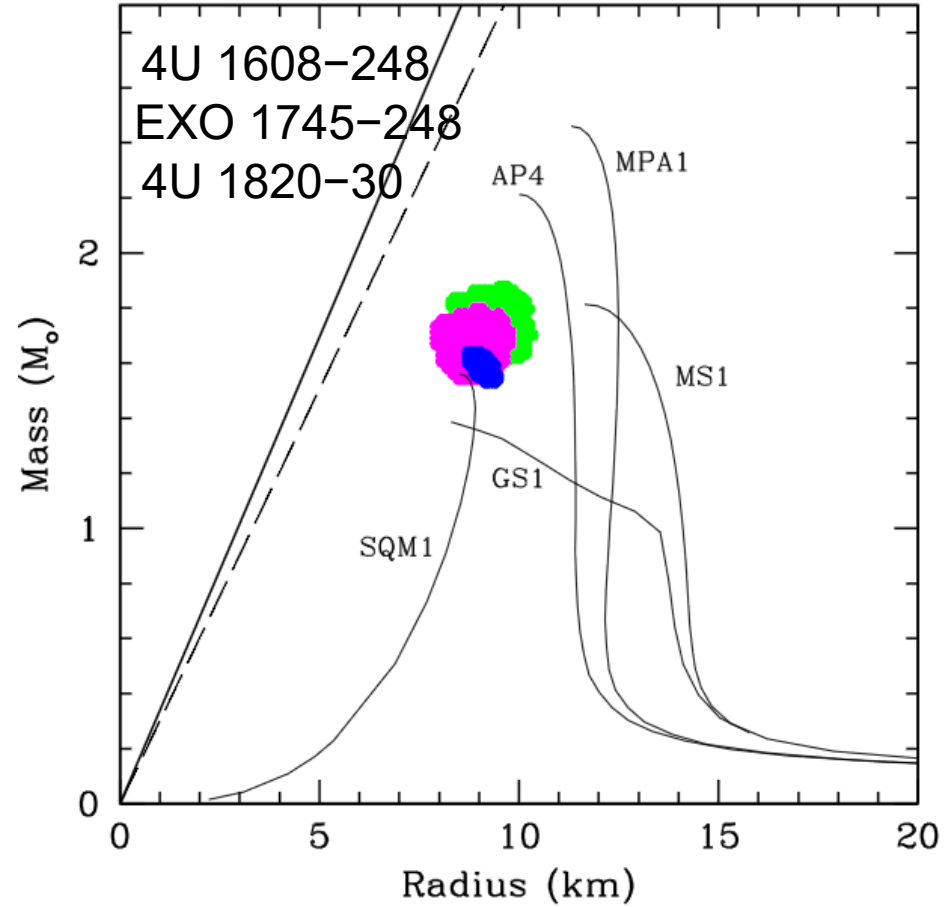
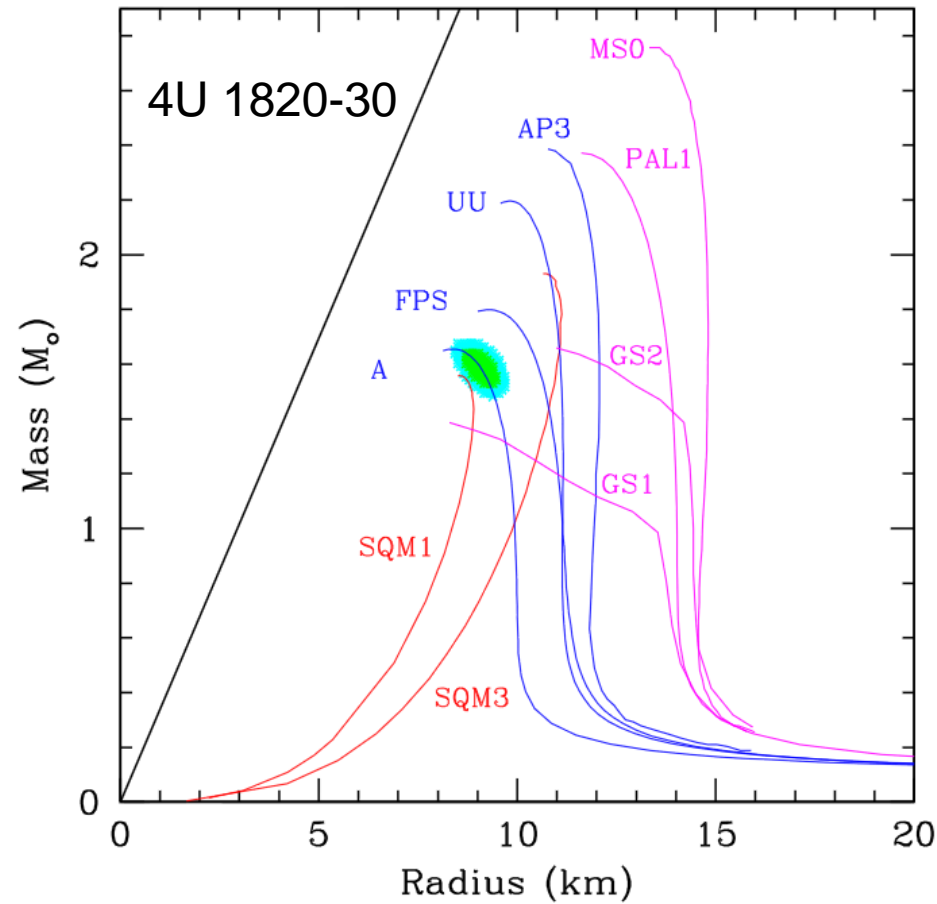
Limits on the EoS from EXO 0748-676



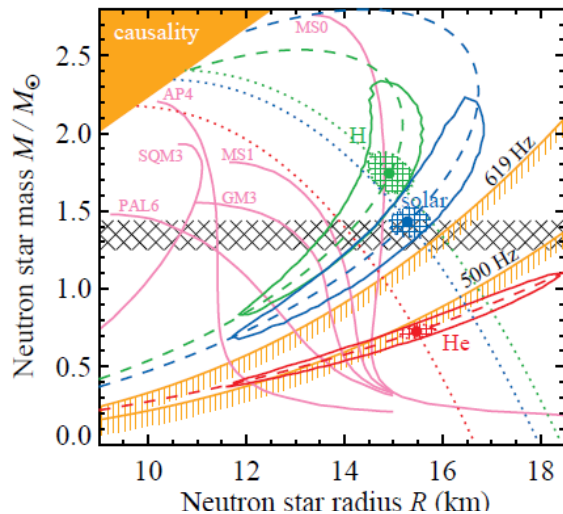
Stiff EoS are better.
Many EoS for strange matter are rejected.
But not all! (see discussion in Nature).

X- hydrogen fraction
in the accreted material

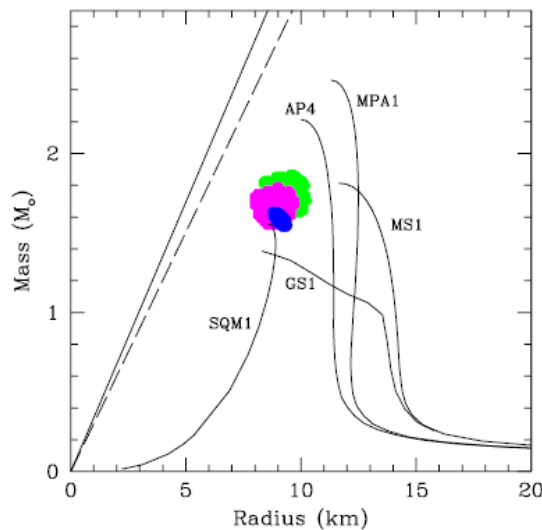
Some optimistic estimates



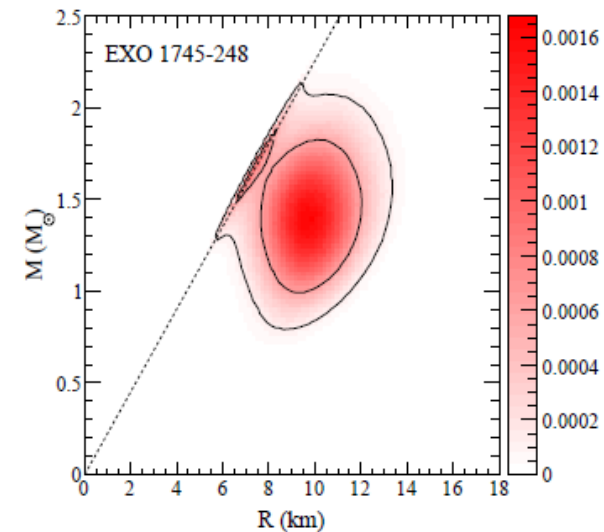
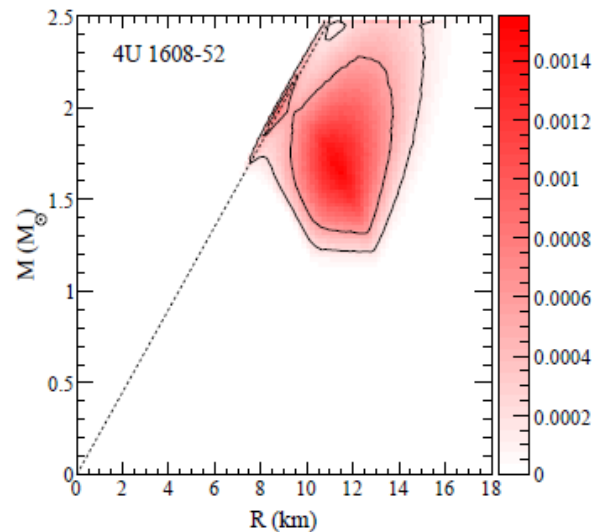
Pessimistic estimates



1004.4871



1005.0811

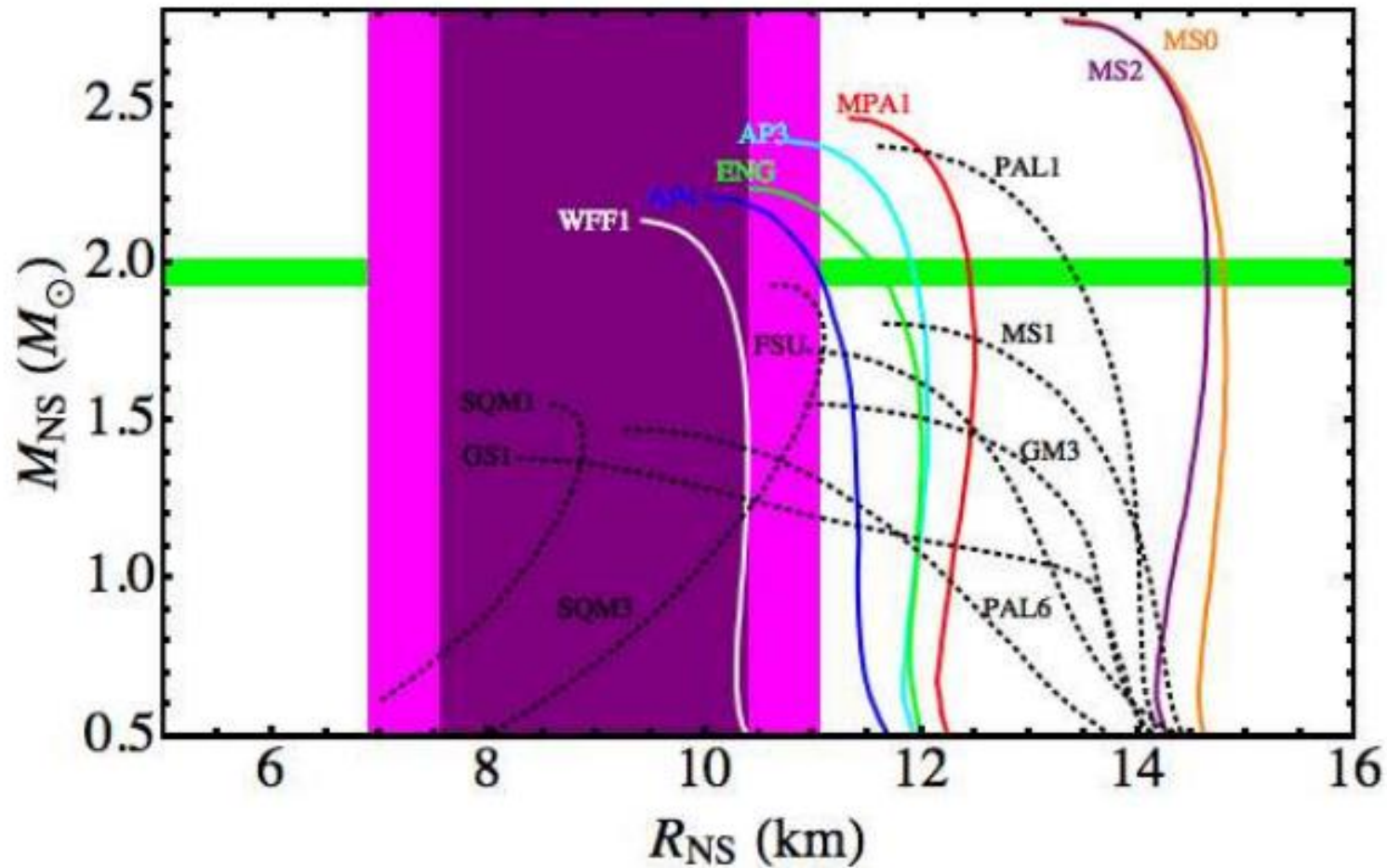


It seems that Ozel et al. underestimate different uncertainties and make additional assumptions.

1002.3153

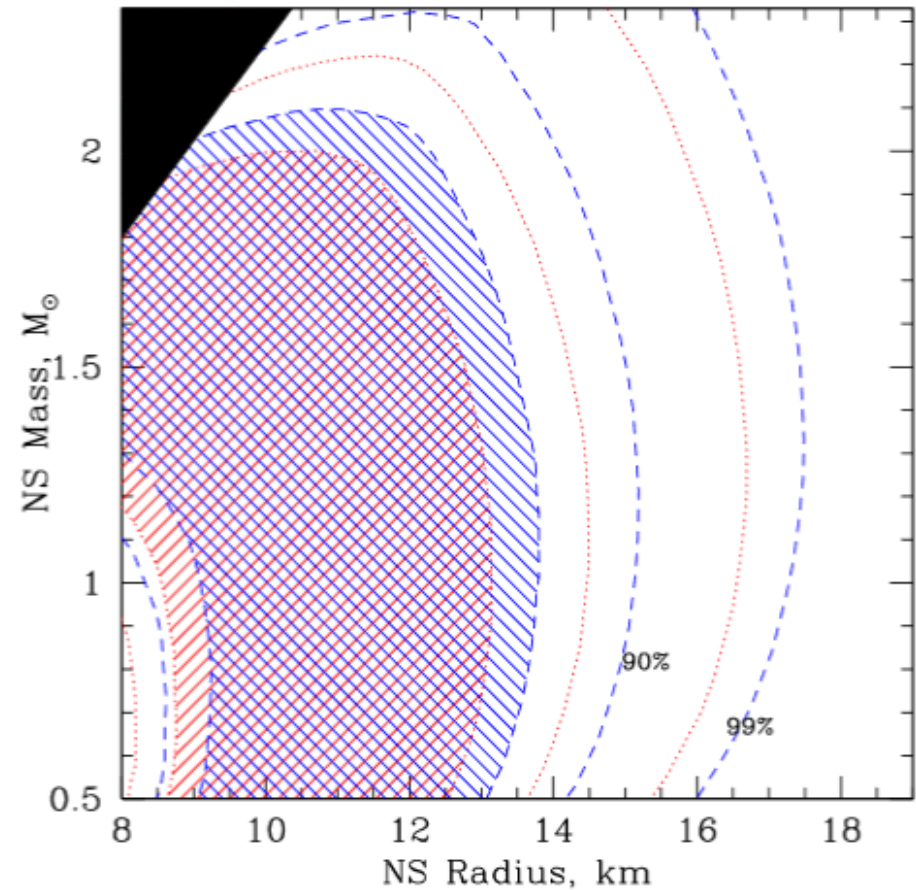
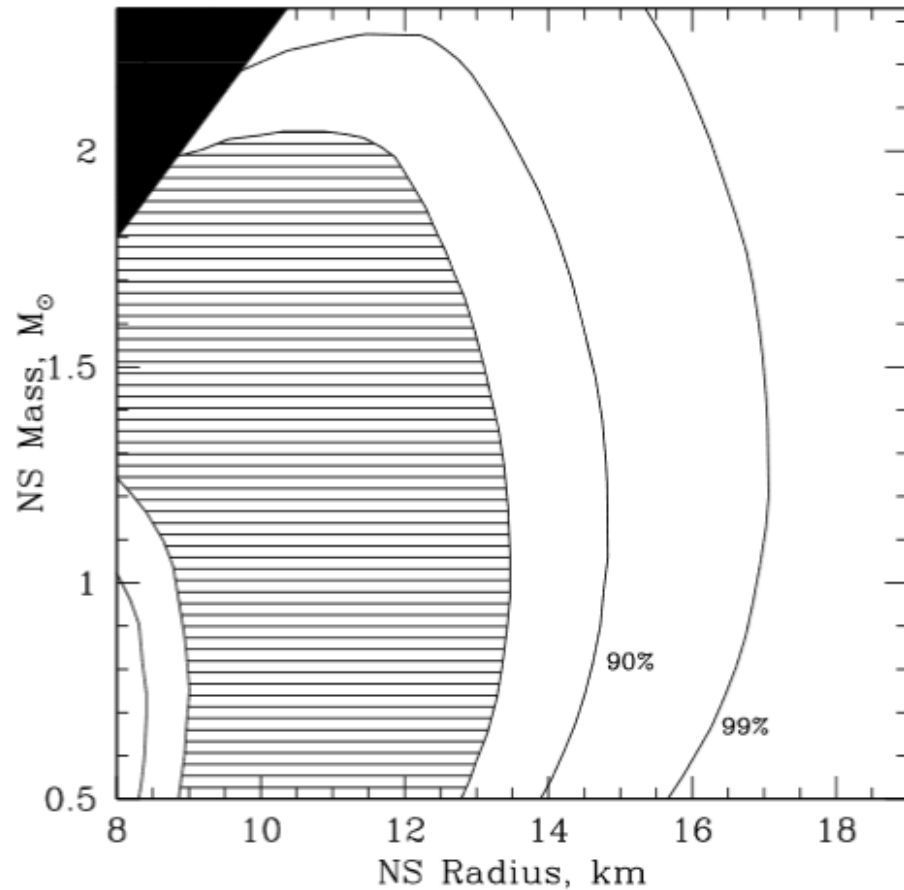
Radii measurements for qLMXBs in GCs

5 sources



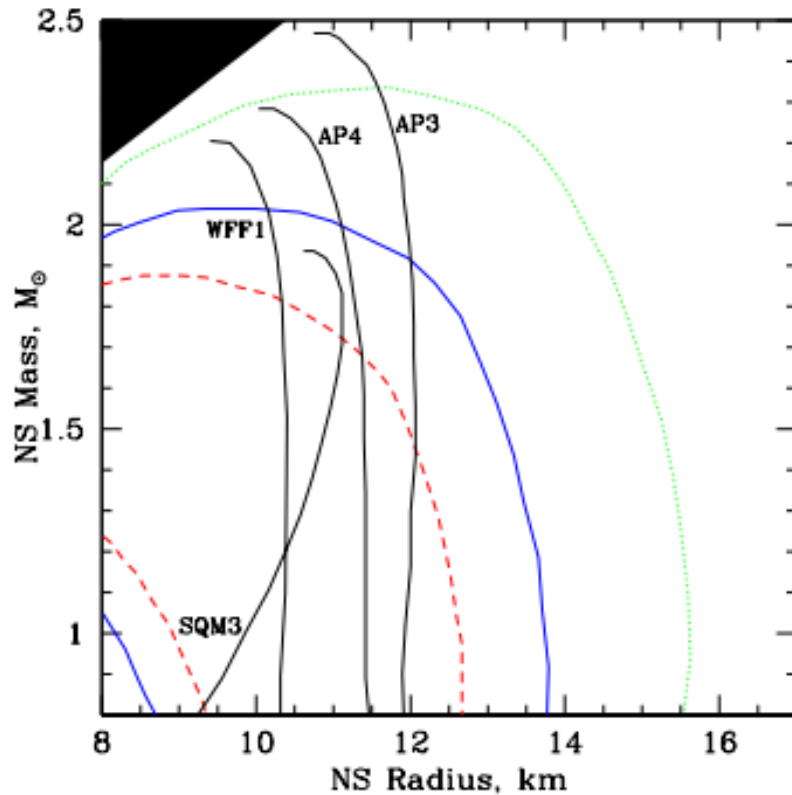
1302.0023, see new results in 1905.01081

Distance uncertainty

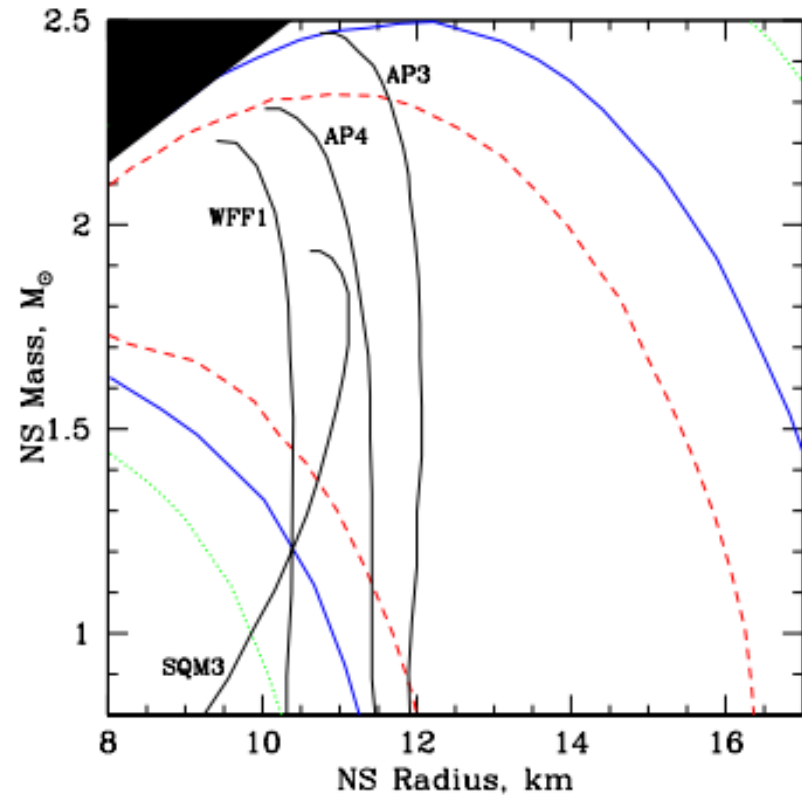


Atmospheric uncertainties

qLMXB in M13

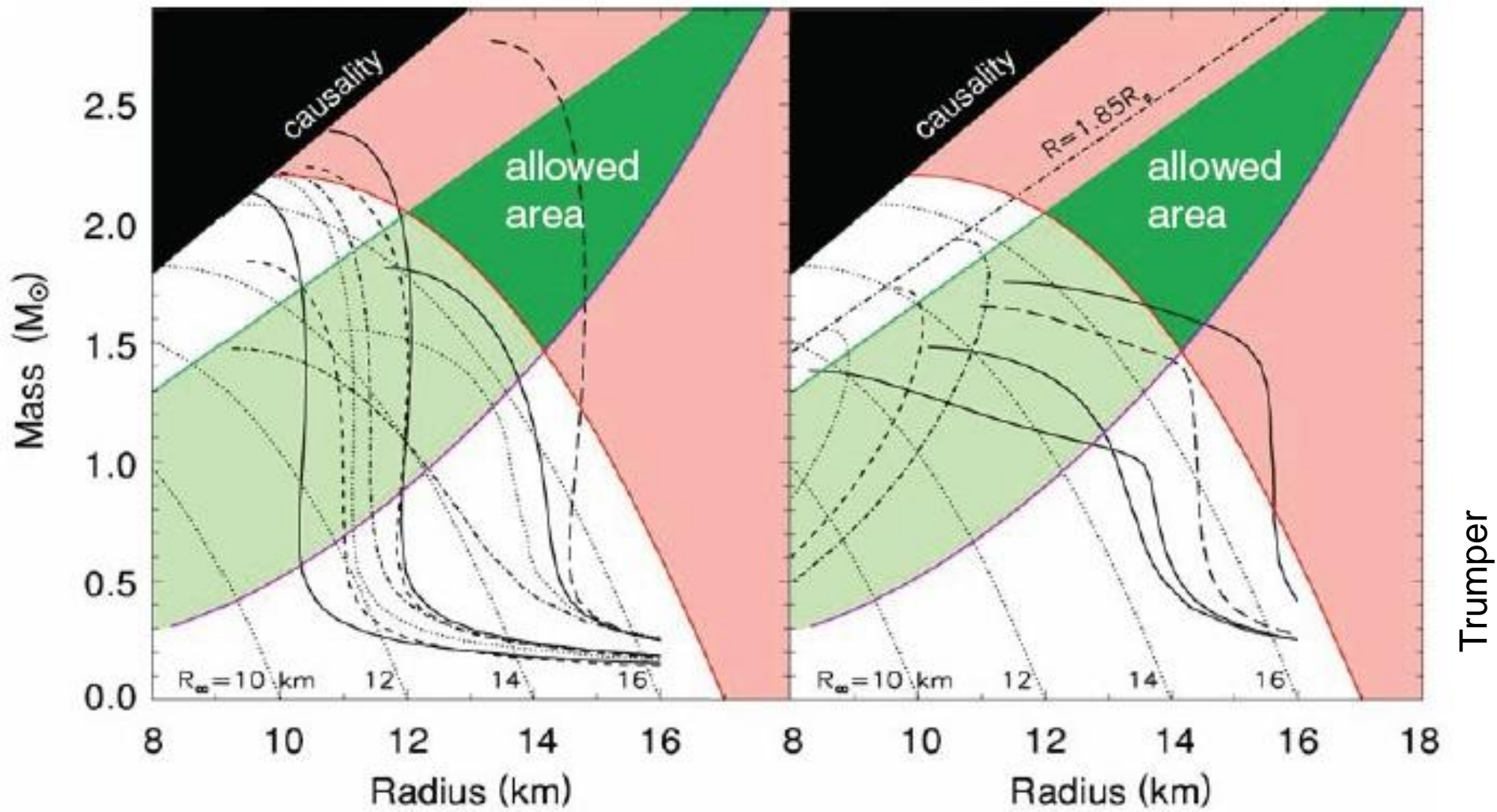


Hydrogene



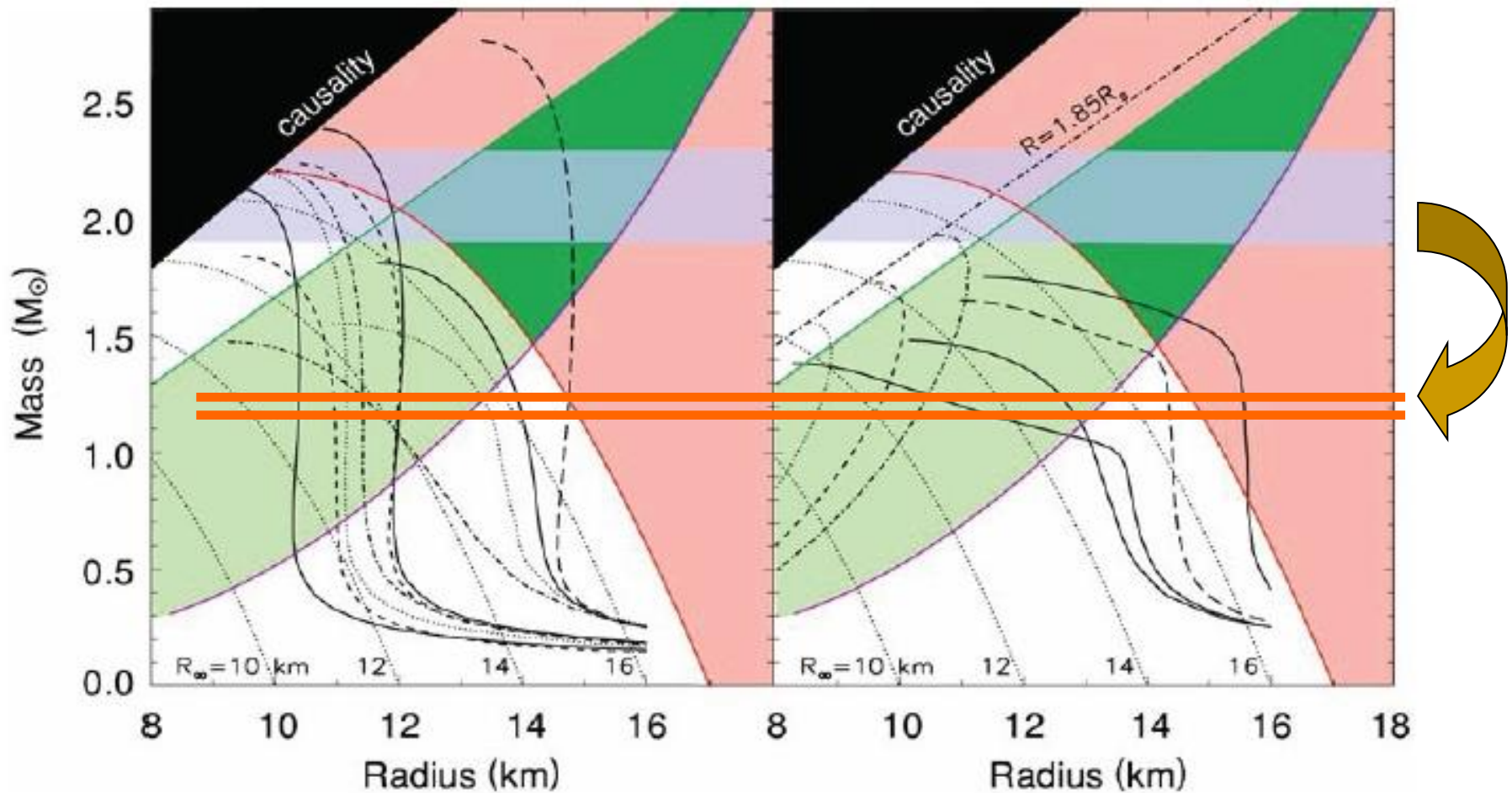
Helium

Limits from RX J1856



About M7 for constraints on the EoS see 1111.0447

PSR 0751+1807



Massive NS: 2.1 ± 0.3 solar masses – later shown to be wrong (!)
[see Nice et al. 2008]

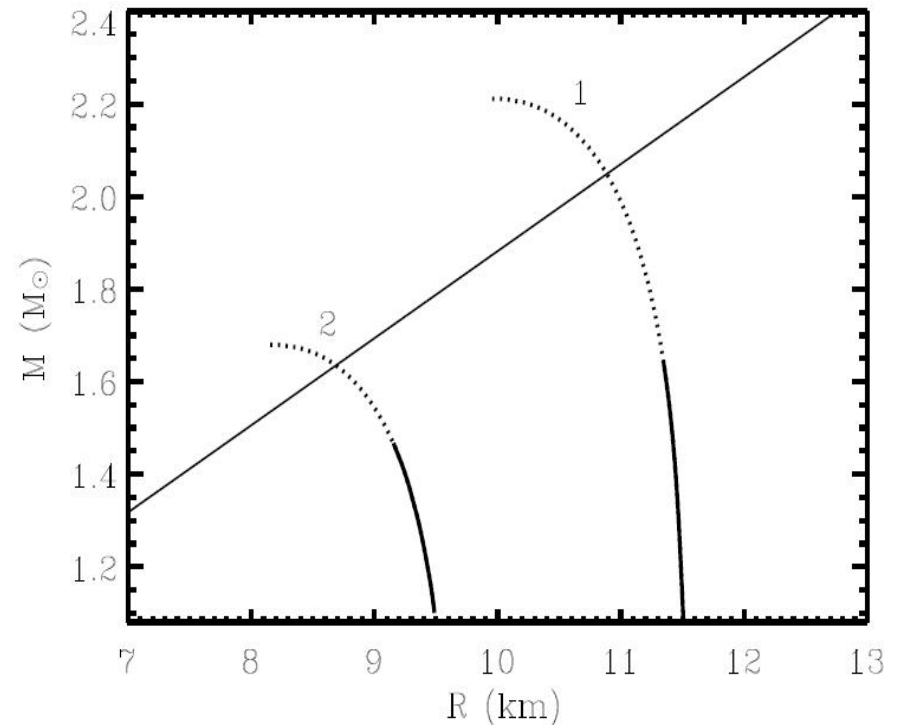
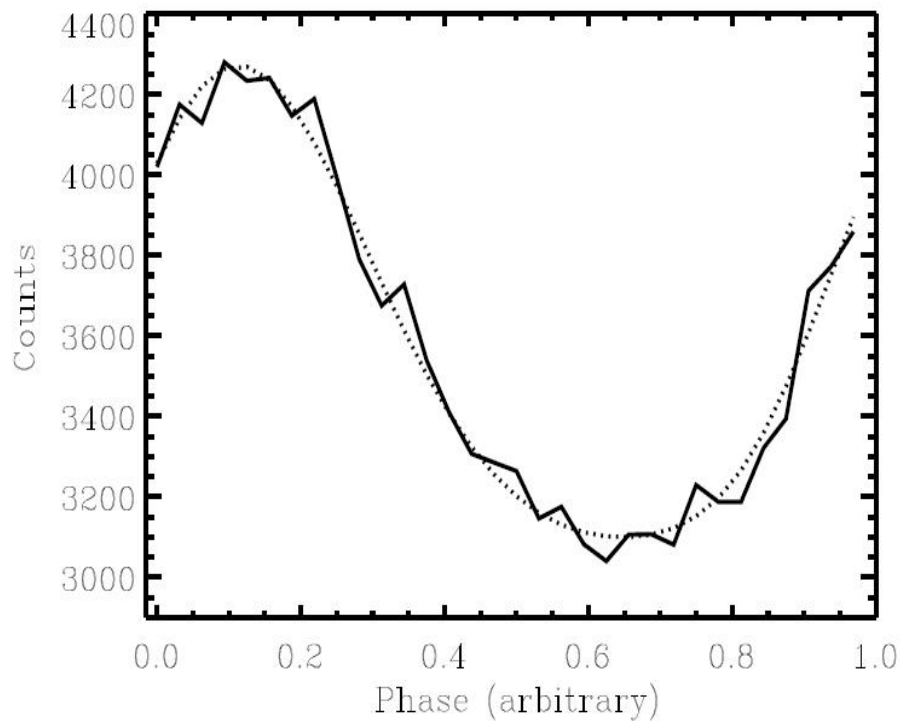
Trumper

Burst oscillations

Fitting light curves of X-ray bursts.

Oscillations due to rotation of a NS.

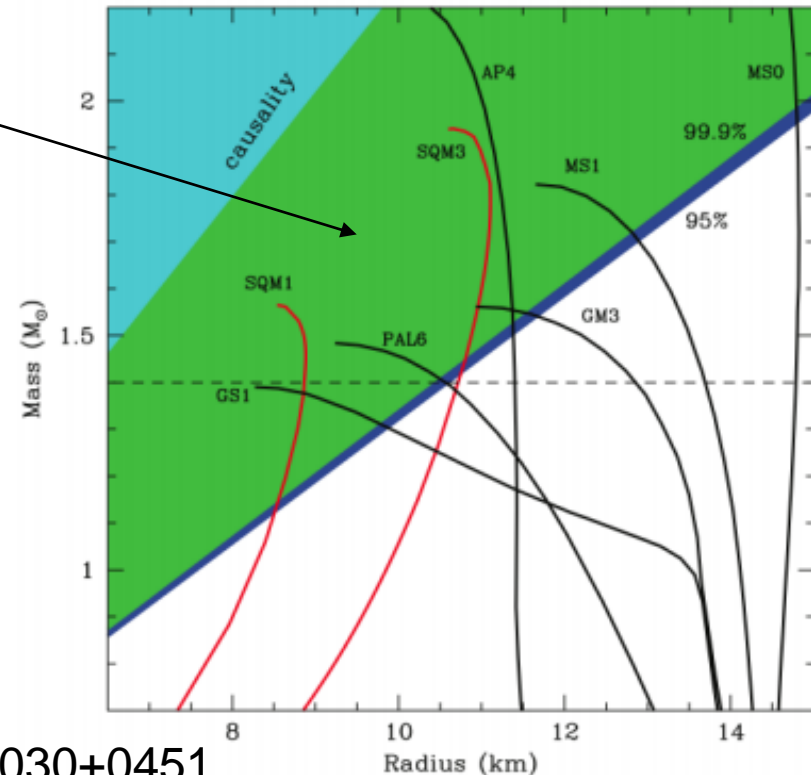
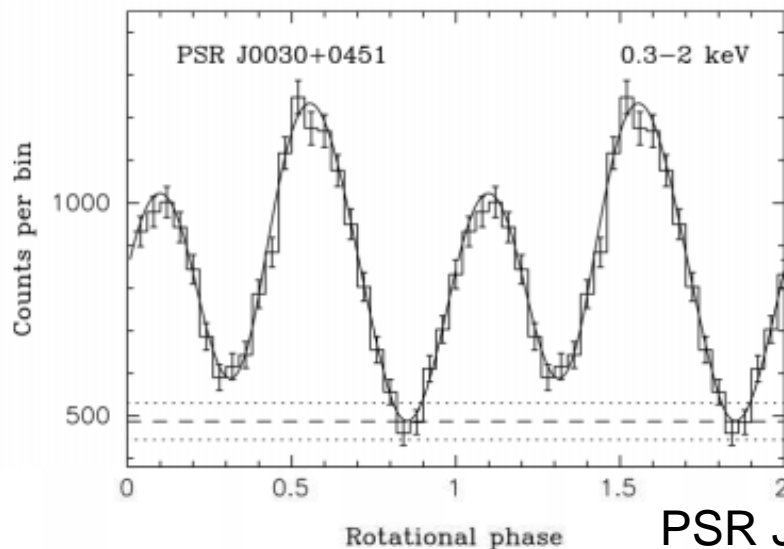
$Rc^2/GM > 4.2$ for the neutron star in XTE J1814-338.



Pulse profile constraints

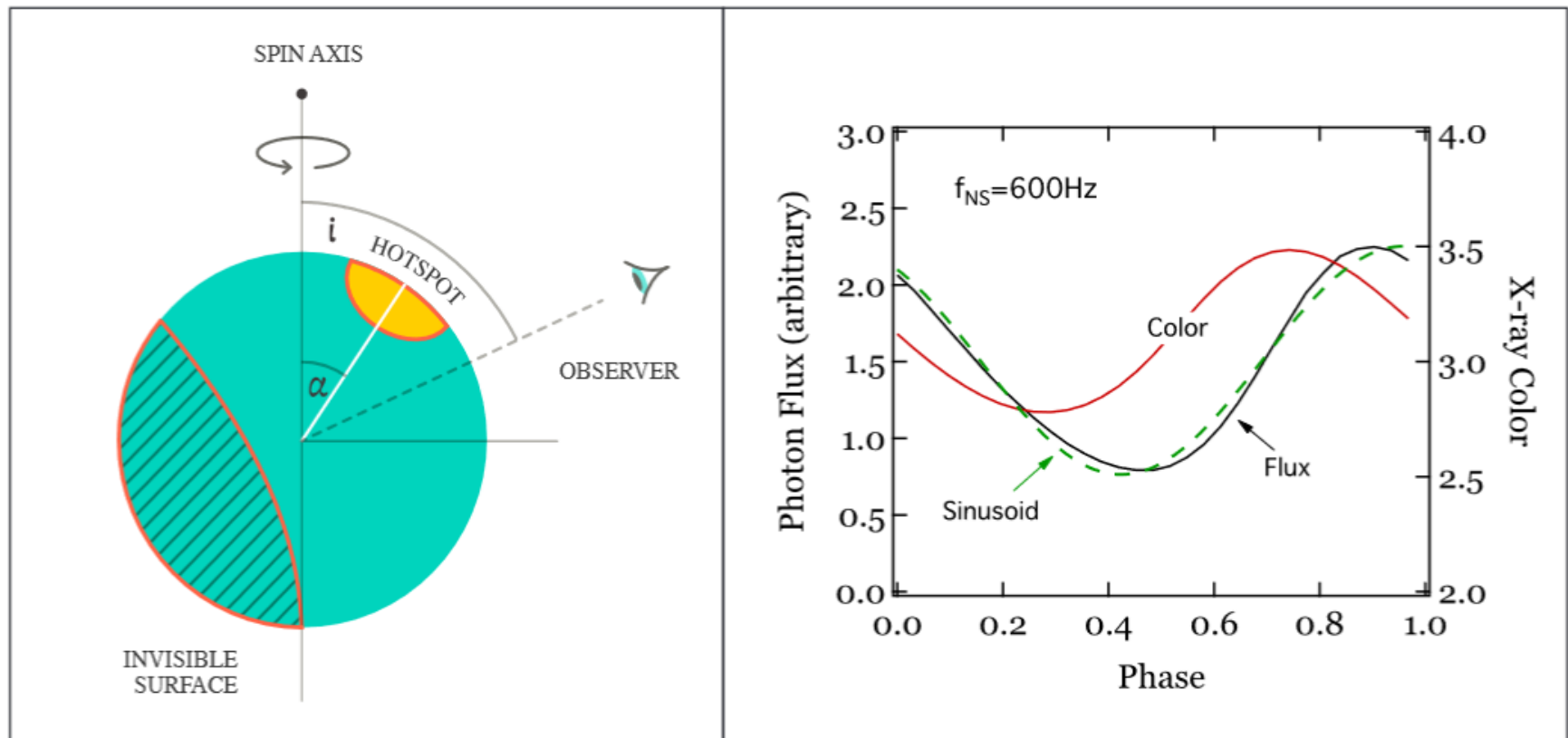
The idea is that: sharp pulses are possible only in the case of a large star.

Green – excluded region



Based on Bogdanov, Grindlay 2009

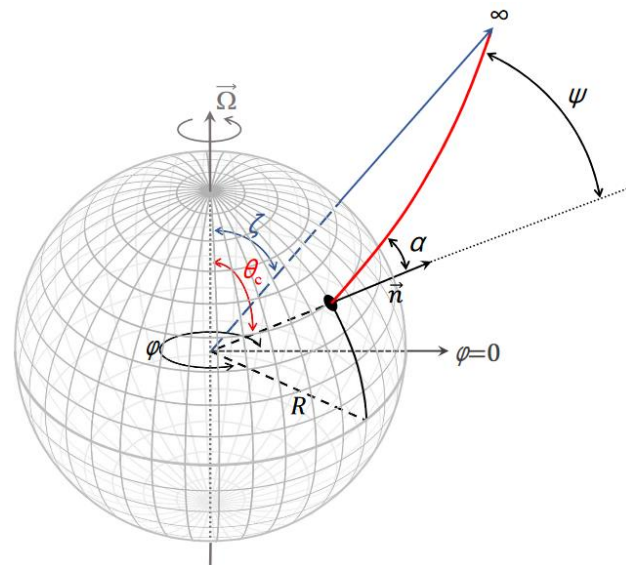
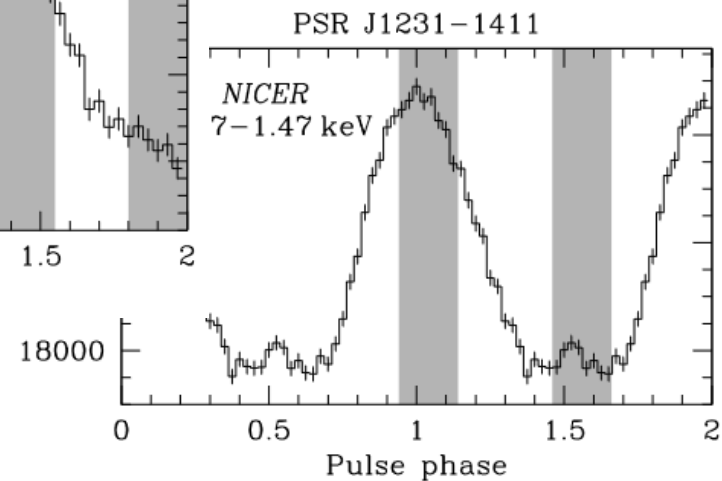
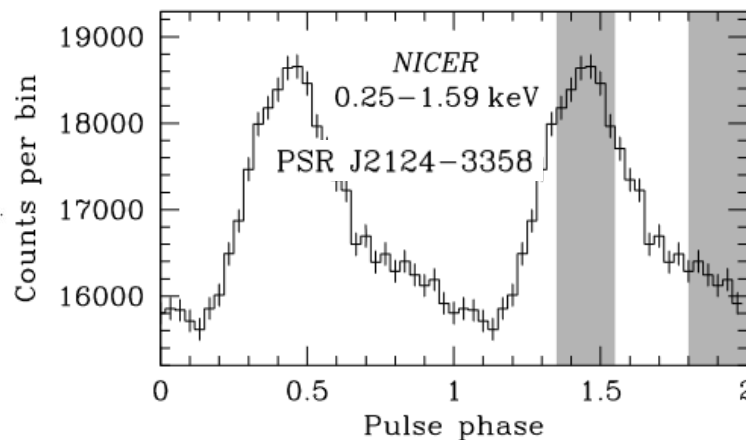
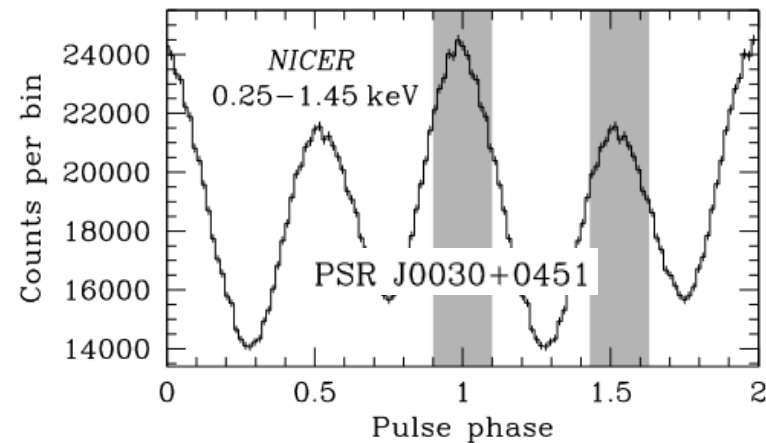
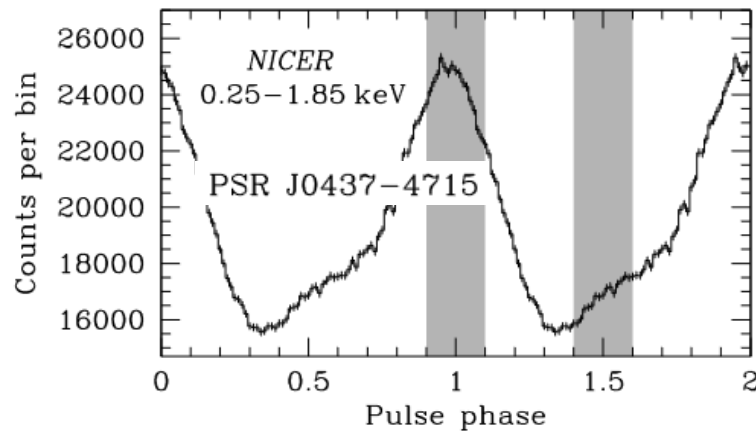
Hot spots and pulse profiles



As the neutron star rotates, emission from a surface hotspot generates a pulsation. The figure shows observer inclination i , and hotspot inclination α . The invisible surface is smaller than a hemisphere due to relativistic light-bending.

NICER's mPSRs

Four near-by
millisecond
radio pulsars:
J0437-4715
J0030+0451
J1231-1411
J2124-3358



Results from NICER. PSR J0030+0451

$$1.34^{+0.15}_{-0.16} M_{\odot}$$

$$12.71^{+1.14}_{-1.19} \text{ km}$$

For the ST-PST model

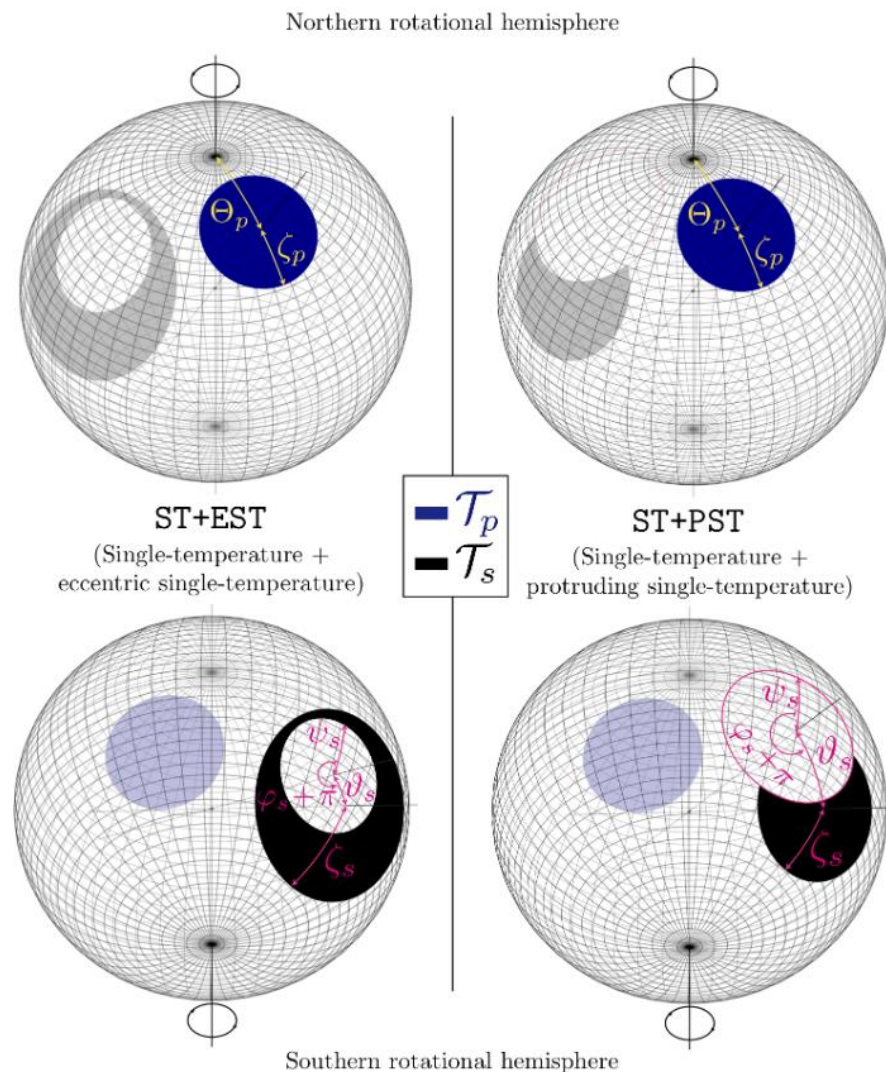
Single temperature+Protruding single temp.
No antipodal symmetry.

But several other tried models
are not ruled out.

For example, in the ST-CST model

$$M = 1.44^{+0.18}_{-0.19} M_{\odot}$$

$$R_{\text{eq}} = 13.89^{+1.22}_{-1.39} \text{ km}$$

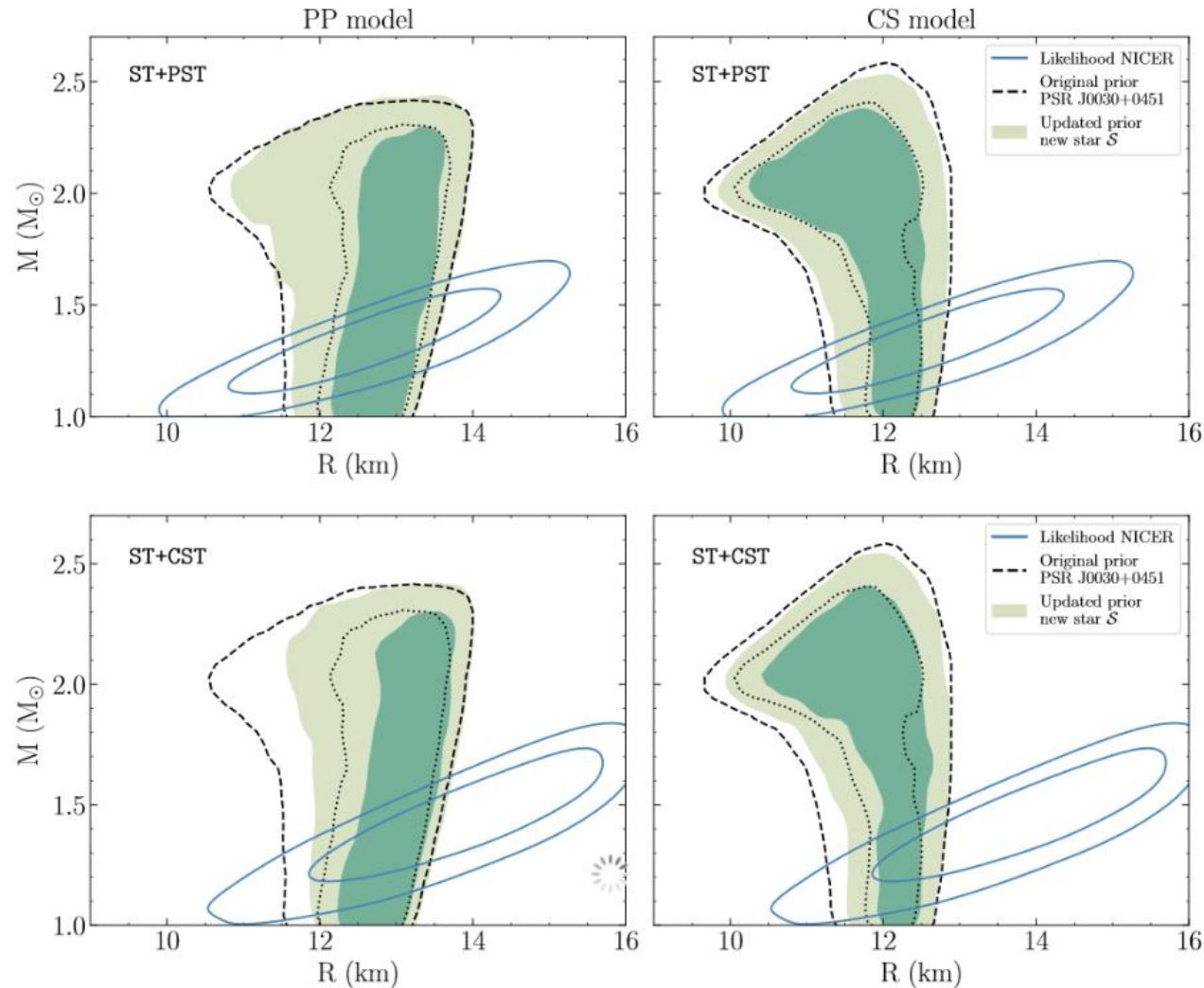


Results from NICER. PSR J0030+0451

Two types of EoS models

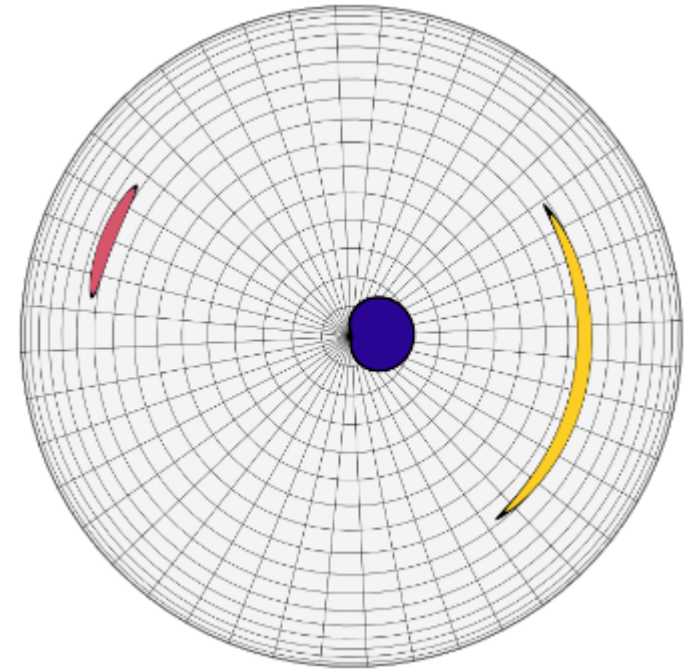
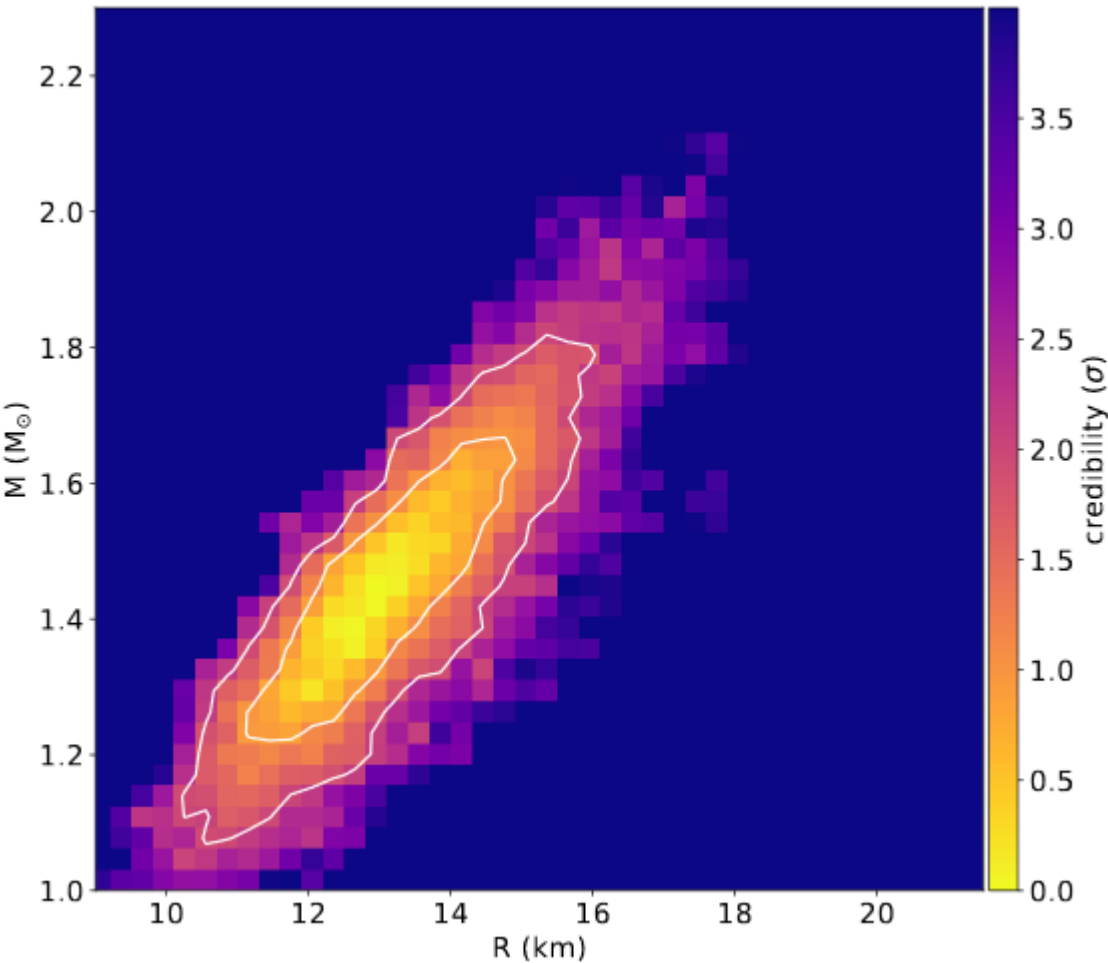
are considered:

- PP (piecewise-polytropic);
- CS (speed of sound).



Results from NICER. PSR J0030+0451

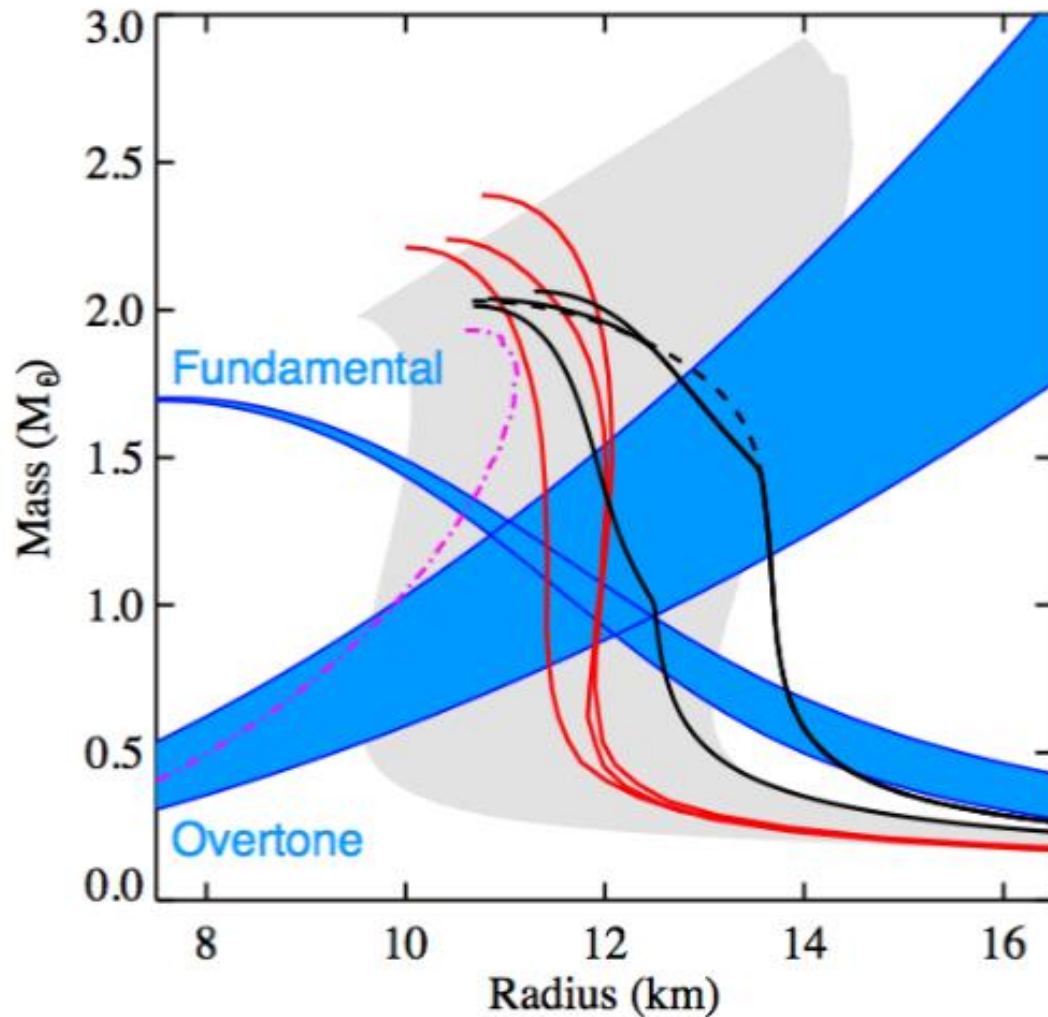
Three oval spots model.
Non-trivial field structure.



$$R_e = 13.02^{+1.24}_{-1.06} \text{ km}$$

$$M = 1.44^{+0.15}_{-0.14} M_\odot$$

Astroseismology

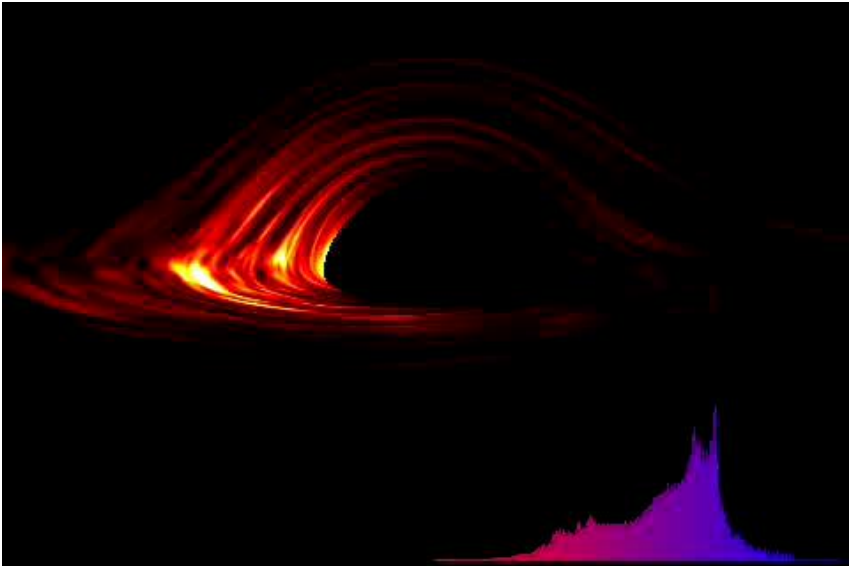


M – R diagram showing the seismological constraints for the soft gamma-ray repeater SGR 1806–20 using the relativistic torsional crust oscillation model of Samuelsson and Andersson (2007), in which the 29 Hz QPO is identified as the fundamental and the 625 Hz QPO as the first radial overtone. The neutron star lies in the box where the constraints from the two frequency bands overlap.

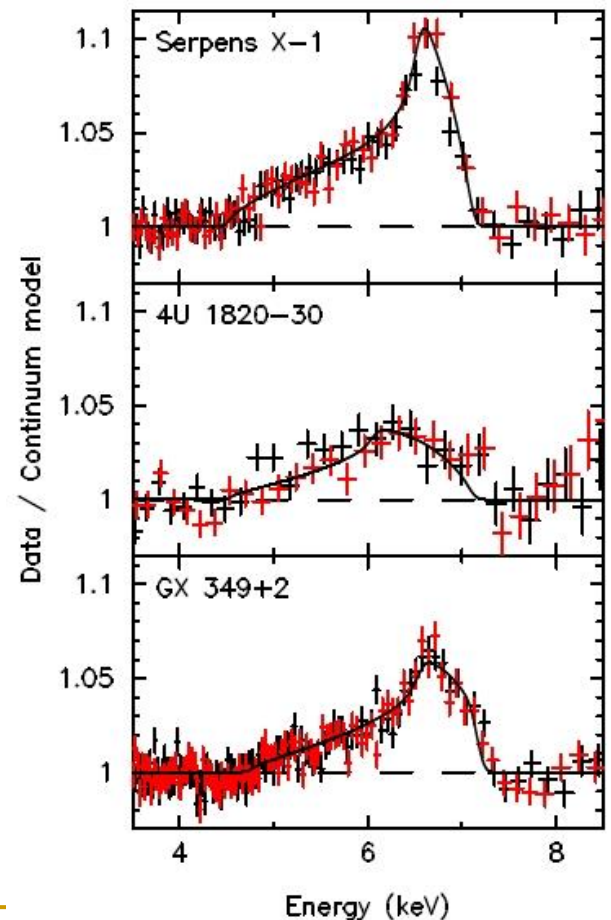
This is a simplified model. More detailed are in progress.

Fe K lines from accretion discs

Measurements of the inner disc radius provide upper limits on the NS radius.



Ser X-1 $<15.9 \pm 1$
4U 1820-30 $<13.8^{+2.9}_{-1.4}$
GX 349+2 $<16.5 \pm 0.8$
(all estimates for 1.4 solar mass NS)
[Cackett et al. arXiv: 0708.3615]

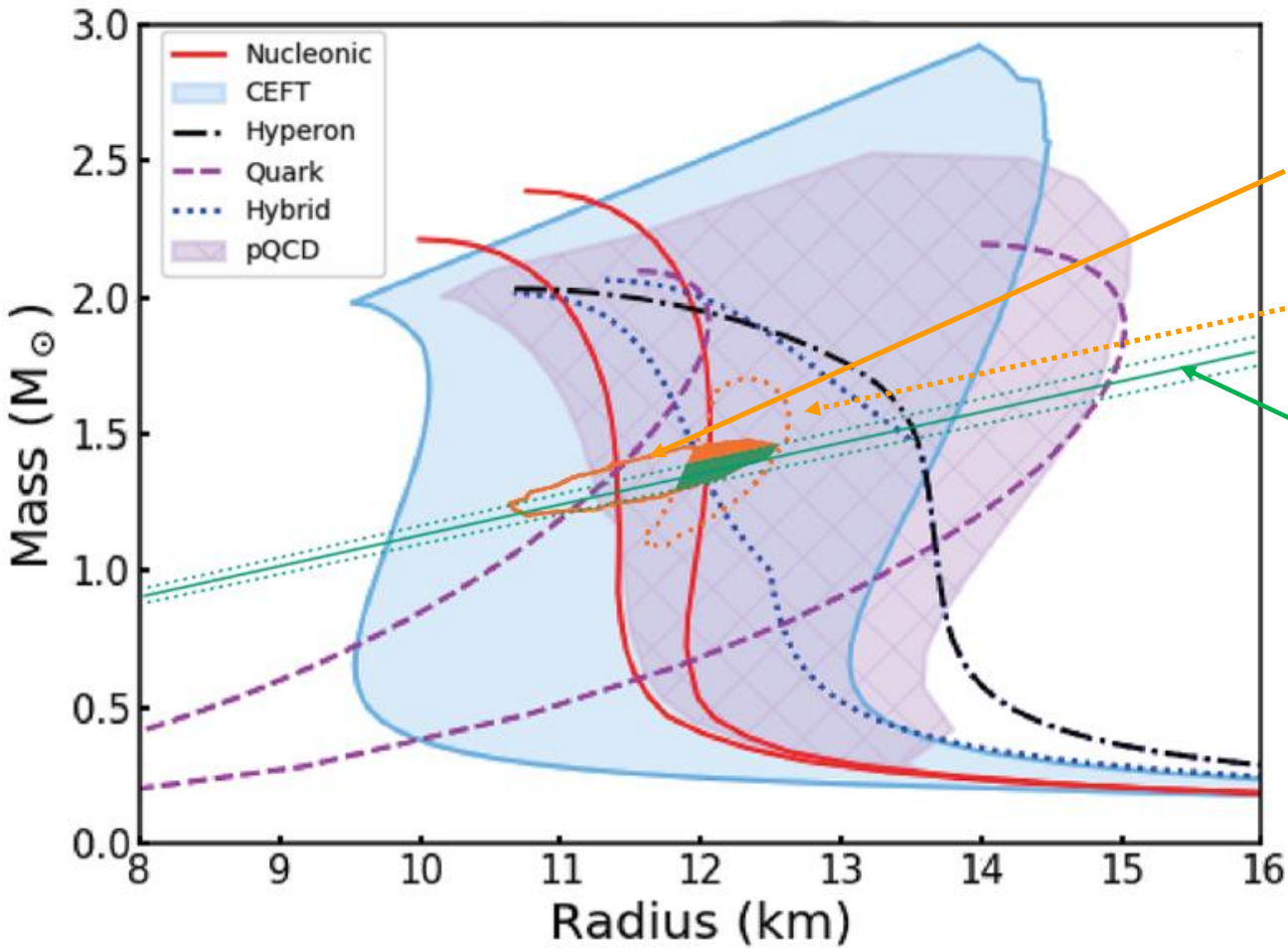


Suzaku observations

See also Papitto et al. arXiv: 0812.1149,
a review in Cackett et al. 0908.1098, and theory in 1109.2068.

Occultation of the disc by the NS

Results of modeling are shown. Large area detectors are necessary.



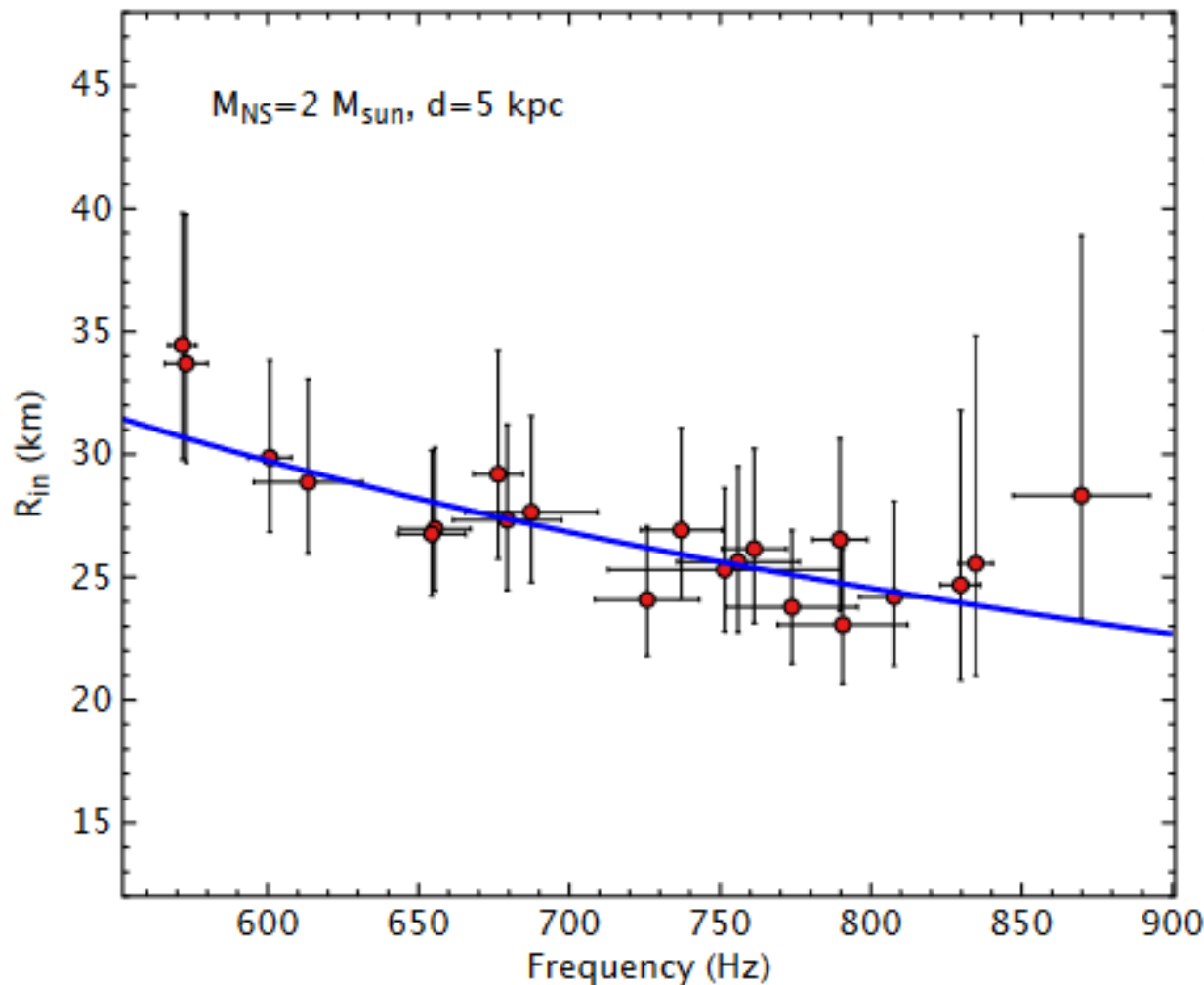
Pulse profile measurements

Atmospheric modeling

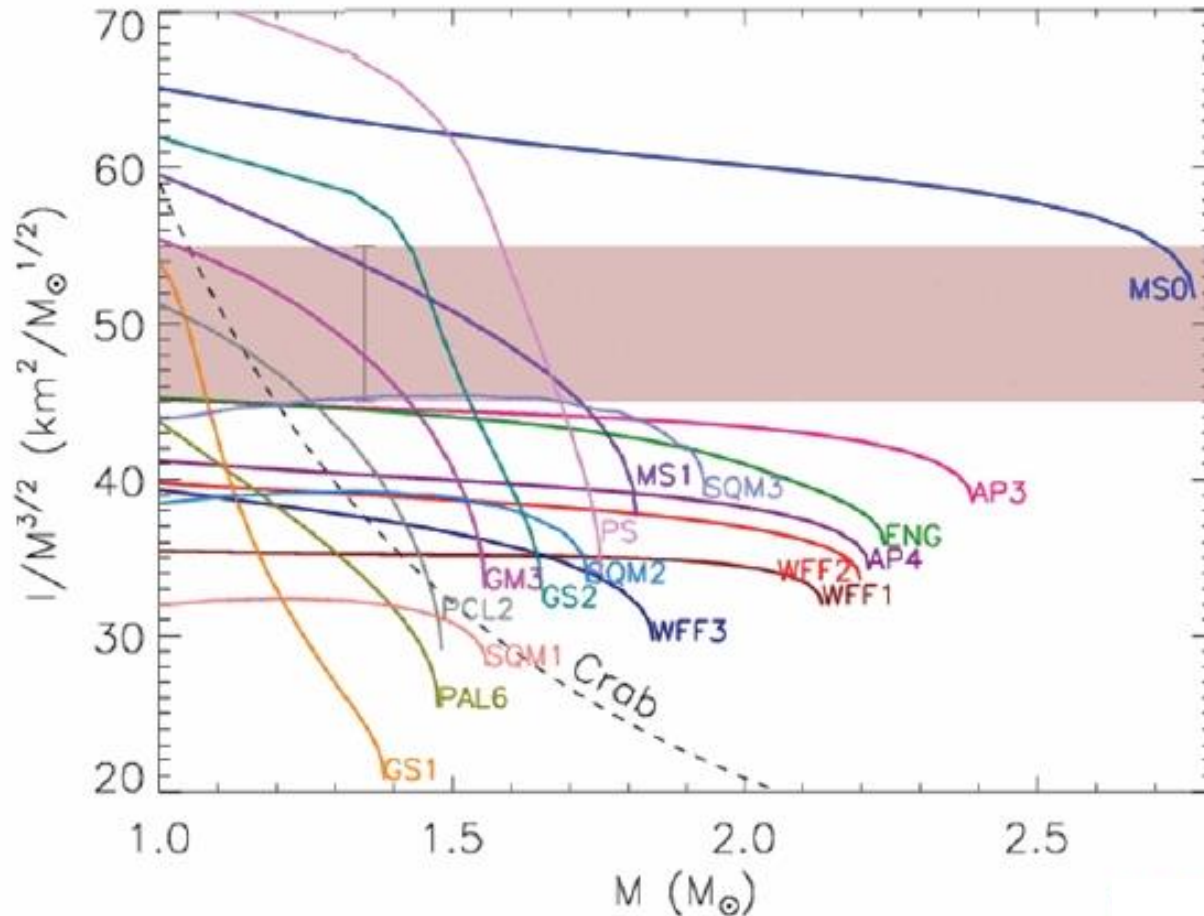
Kalpha line with shading

The authors tested a new method to measure R/M by observations of variations of the Kalpha line profile due to occultations of part of a disc by a NS. Not effective, yet.

Fits from QPOs



Limits on the moment of inertia



Spin-orbital interaction

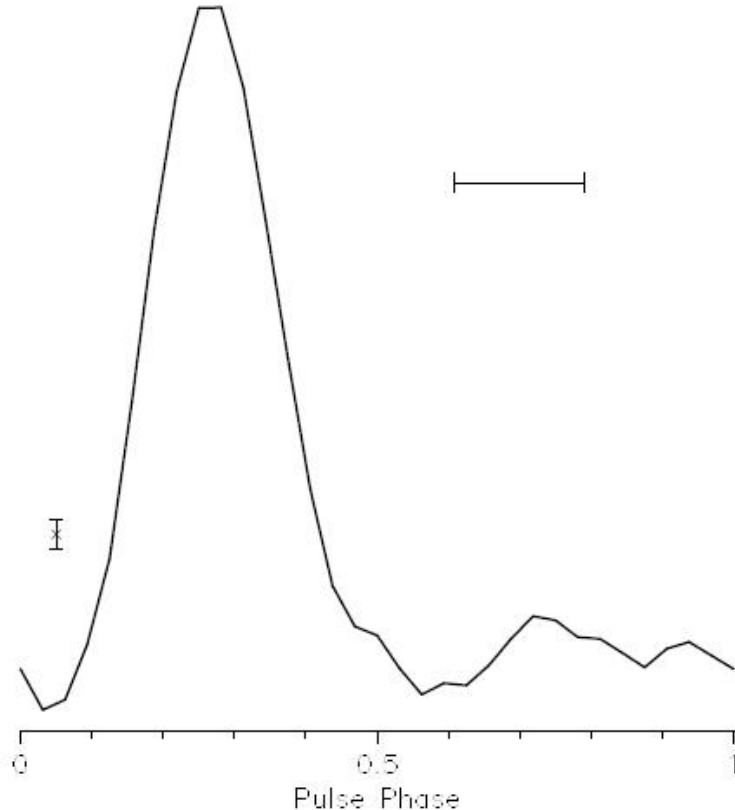
PSR J0737-3039
(see Lattimer, Schutz
astro-ph/0411470)

The band refers to a
hypothetical 10% error.
This limit, hopefully,
can be reached in
several years of observ.

See a more detailed
discussion in 1006.3758

Most rapidly rotating PSR

716-Hz eclipsing binary radio pulsar in the globular cluster Terzan 5

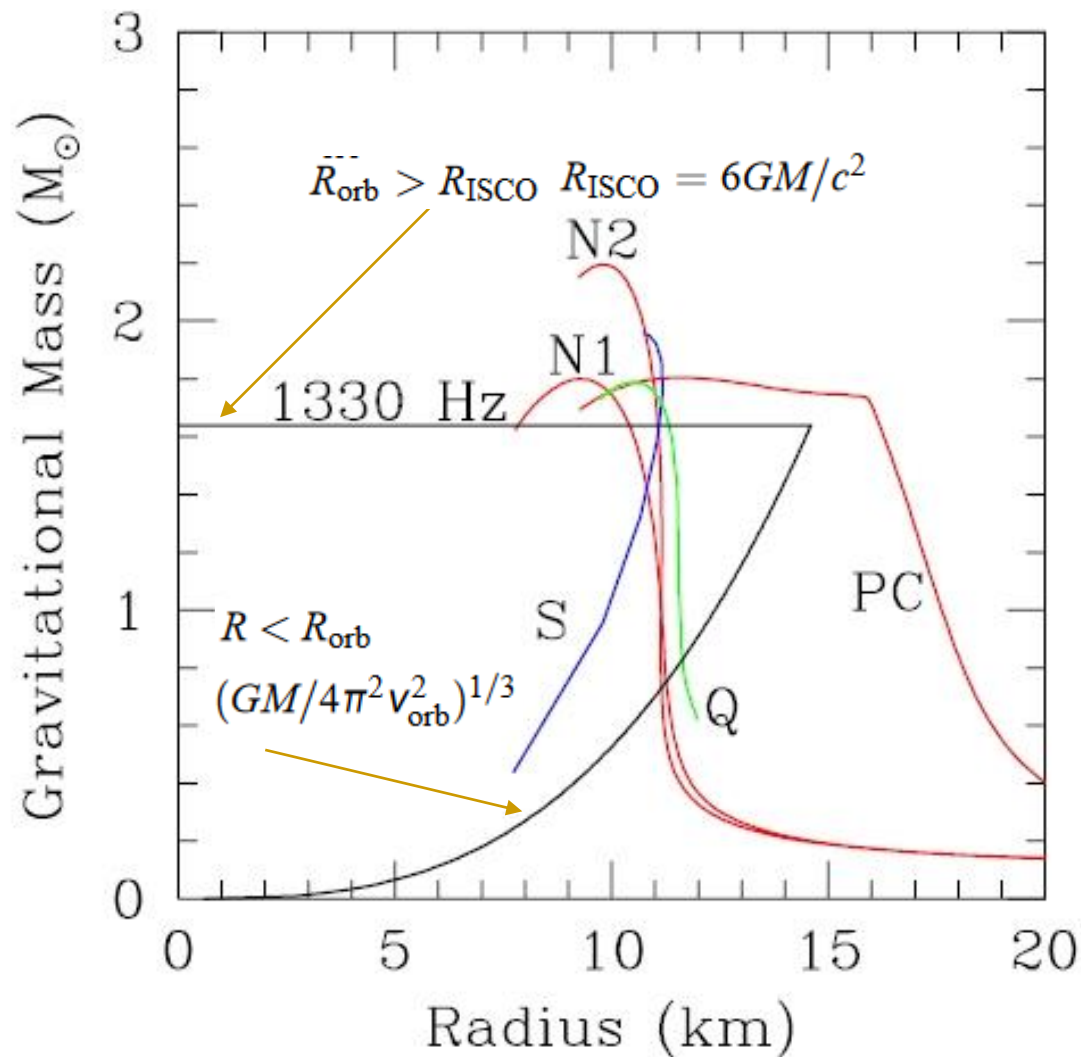


Previous record
(642-Hz pulsar B1937+21)
survived for more than 20 years.

**Interesting calculations
for rotating NS have been
performed recently by Krastev et al.
arXiv: 0709.3621**

Rotation starts to be important
from periods ~ 3 msec.

QPO and rapid rotation



XTE J1739-285

1122 Hz

[P. Kaaret](#) et al.
astro-ph/0611716

1330 Hz – one of the
highest QPO frequency

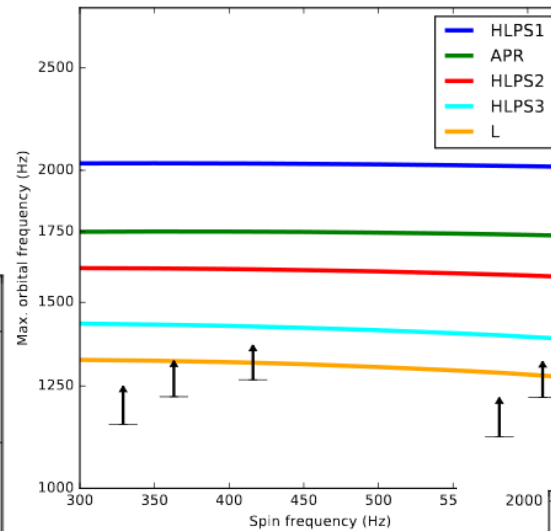
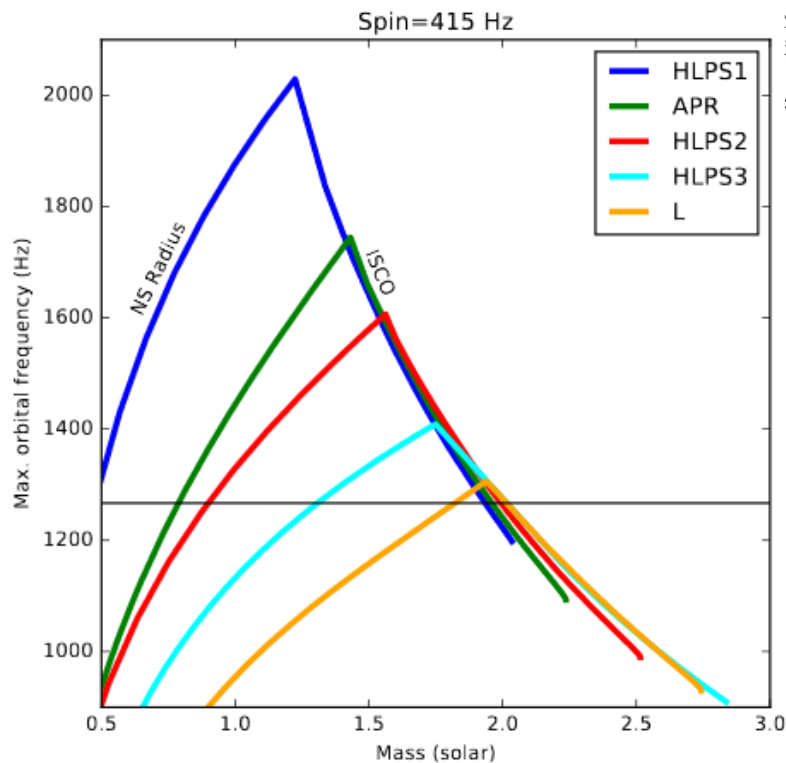
The line corresponds to
the interpretation, that
the frequency is that
of the last stable orbit,
 $6GM/c^2$

New measurements for 4U 0614+09

$$M \leq 2.2(v_{orb}/1000 \text{ Hz})^{-1}(1+0.75j)M_{\odot}$$

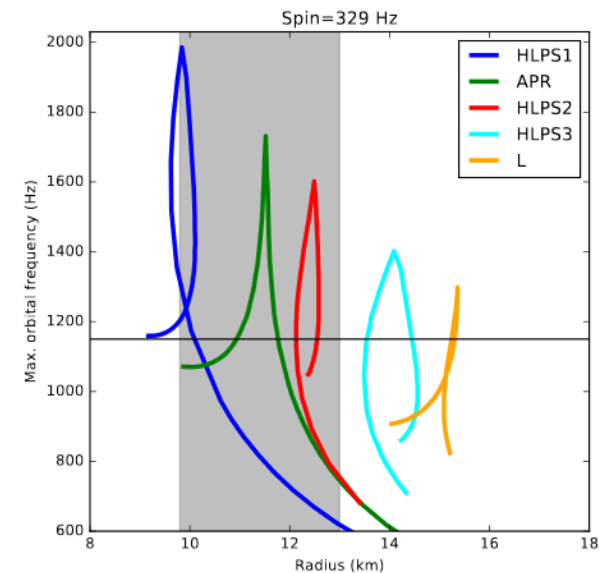
$$R \leq 19.5(v_{orb}/1000 \text{ Hz})^{-1}(1+0.2j)\text{km}$$

$$j \equiv cJ/GM^2$$

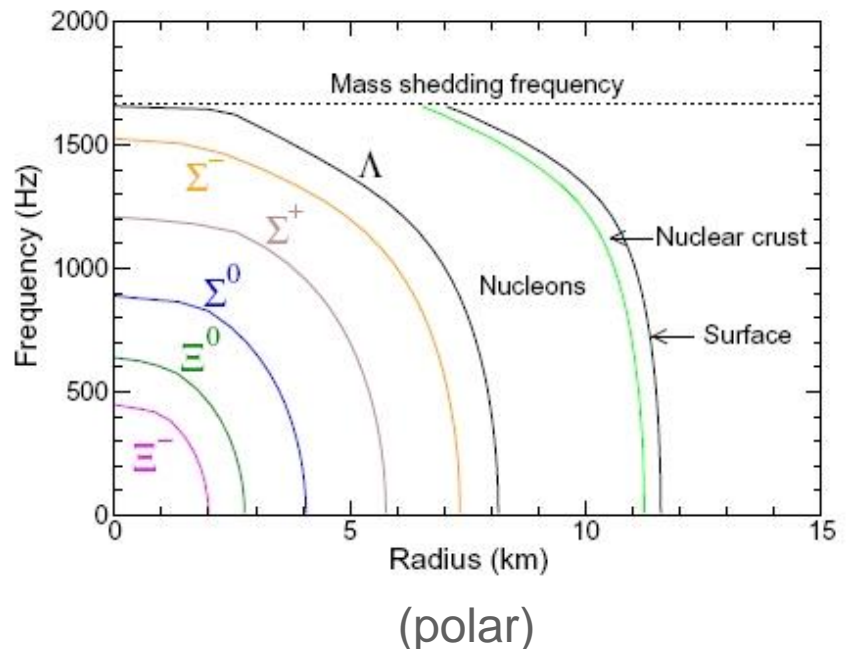
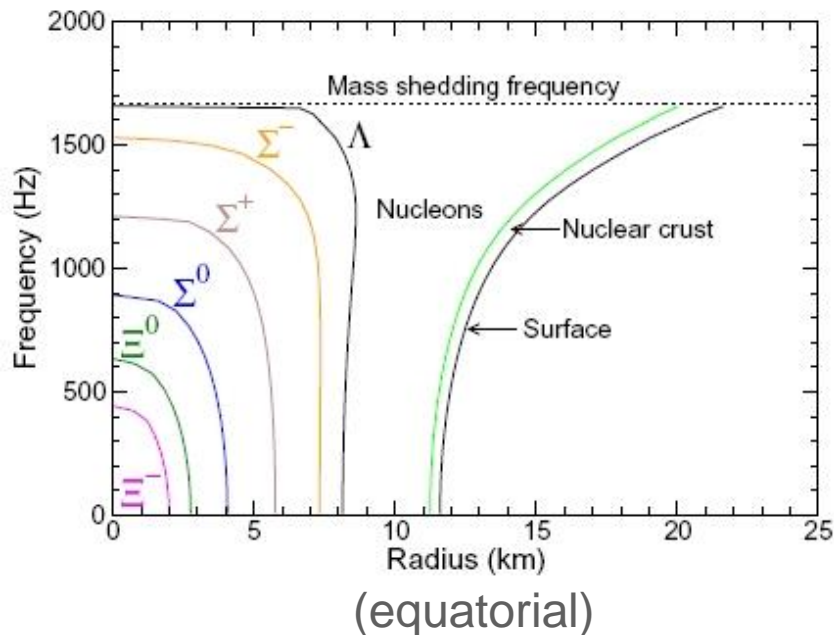


1267 Hz

$$r_{\min} = \begin{cases} 6GM/c^2, & r_{\text{ISCO}} > R_{\text{NS}} \\ R_{\text{NS}}, & r_{\text{ISCO}} < R_{\text{NS}} \end{cases}$$



Rotation and composition

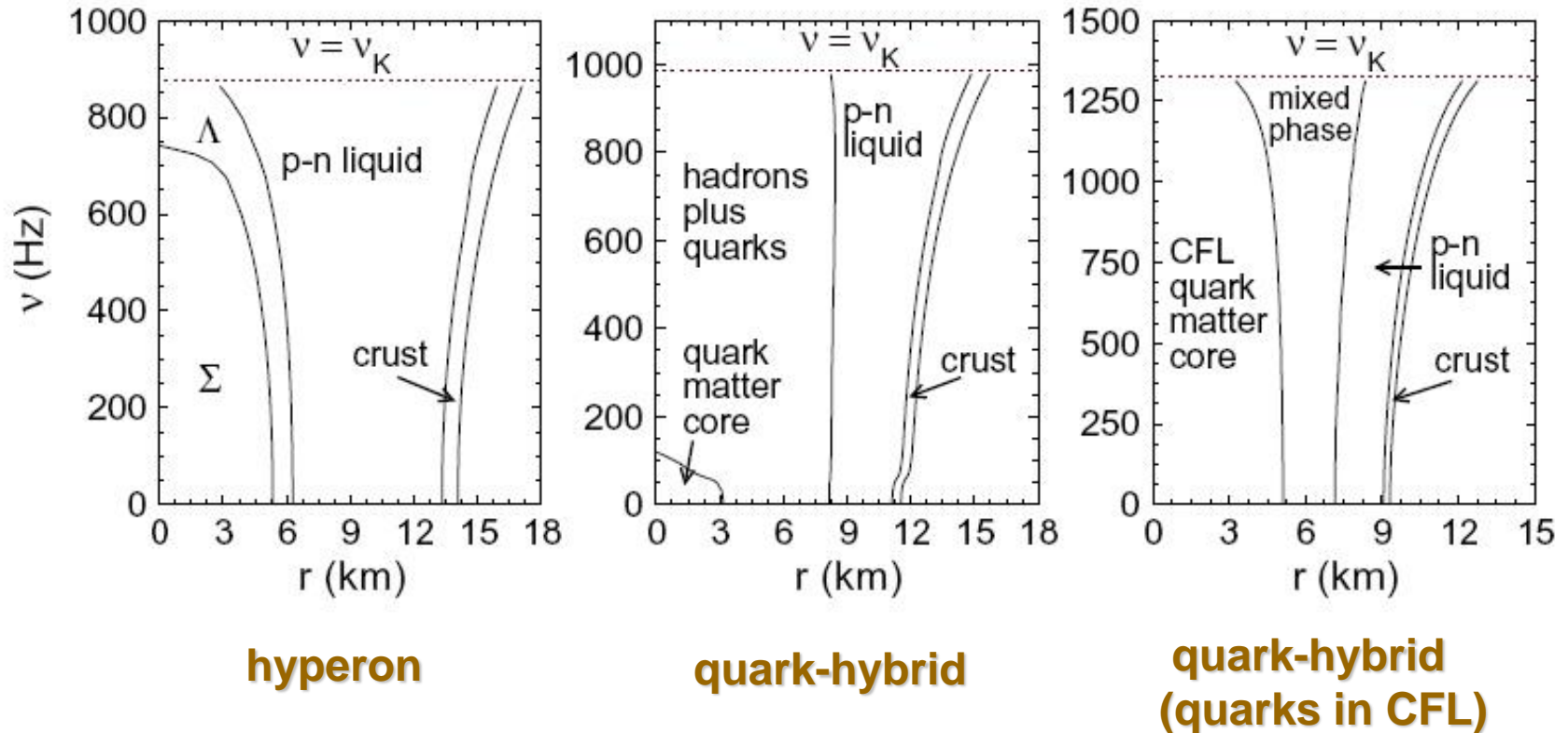


Computed for a particular model:
density dependent relativistic Brueckner-Hartree-Fock (DD-RBHF)

(Weber et al. arXiv: 0705.2708)

Detailed study of the influence of rotation onto structure and composition is
given in 1307.1103

Rotation and composition



(Weber et al. arXiv: 0705.2708)

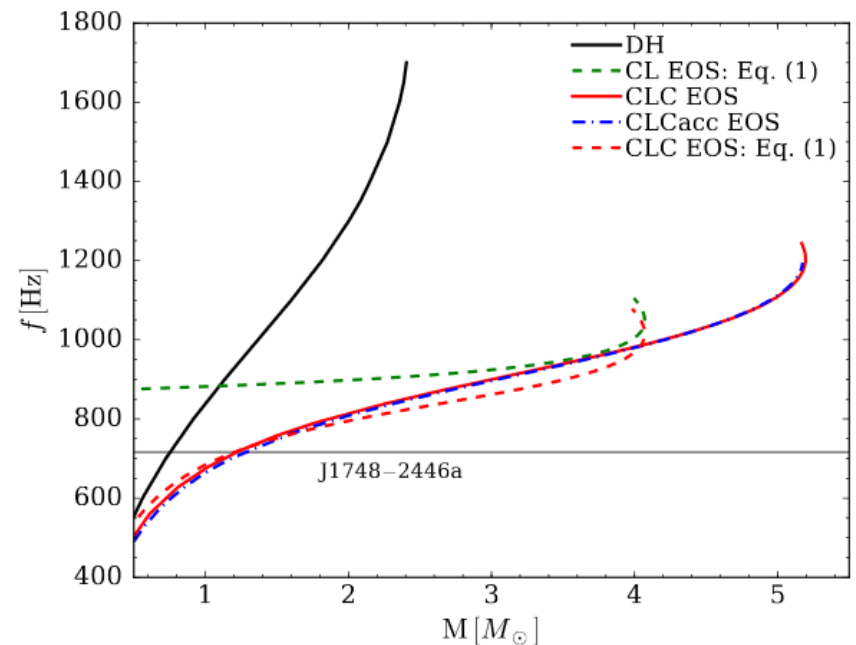
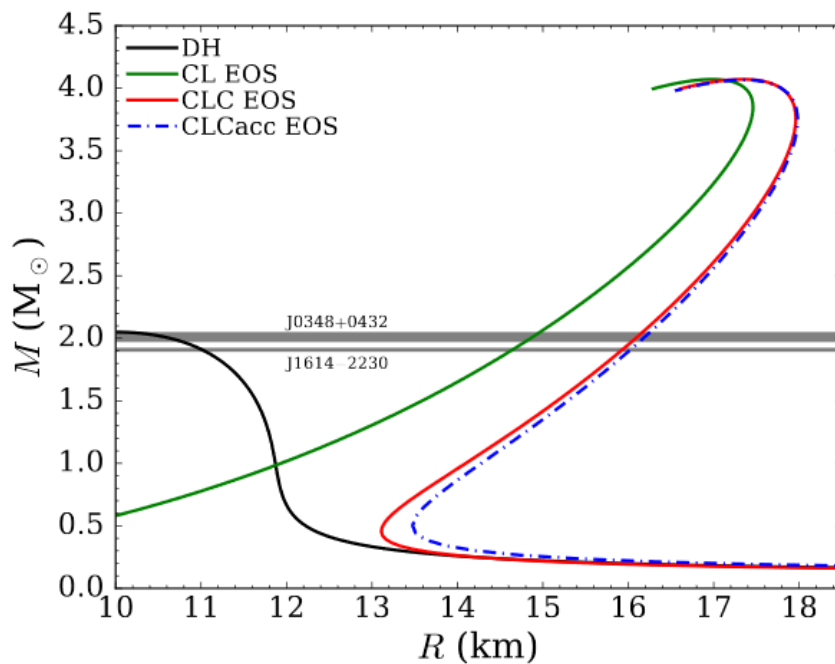
1.4 solar mass NS (when non-rotating)

Limiting rotation

$$f_{\max}^{\text{EOS}} = C_{\max} \left(\frac{M_{\max}^{\text{stat}}}{M_{\odot}} \right)^{1/2} \left(\frac{R_{M_{\max}}^{\text{stat}}}{10 \text{ km}} \right)^{-3/2}$$

$$C_{\max} = 1.22 \text{ kHz}$$

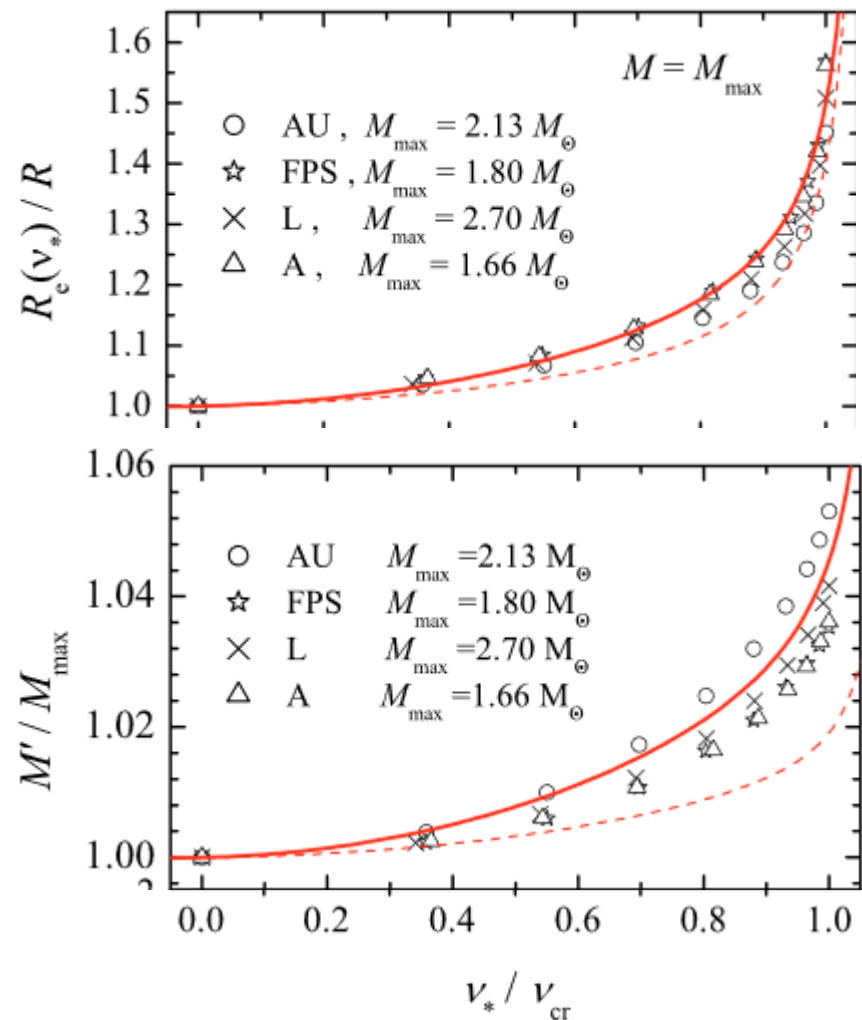
Without additional assumptions for realistic EoS it is expected that NS can rotate faster than $f=716 \text{ Hz}$ for masses close to the limiting value.



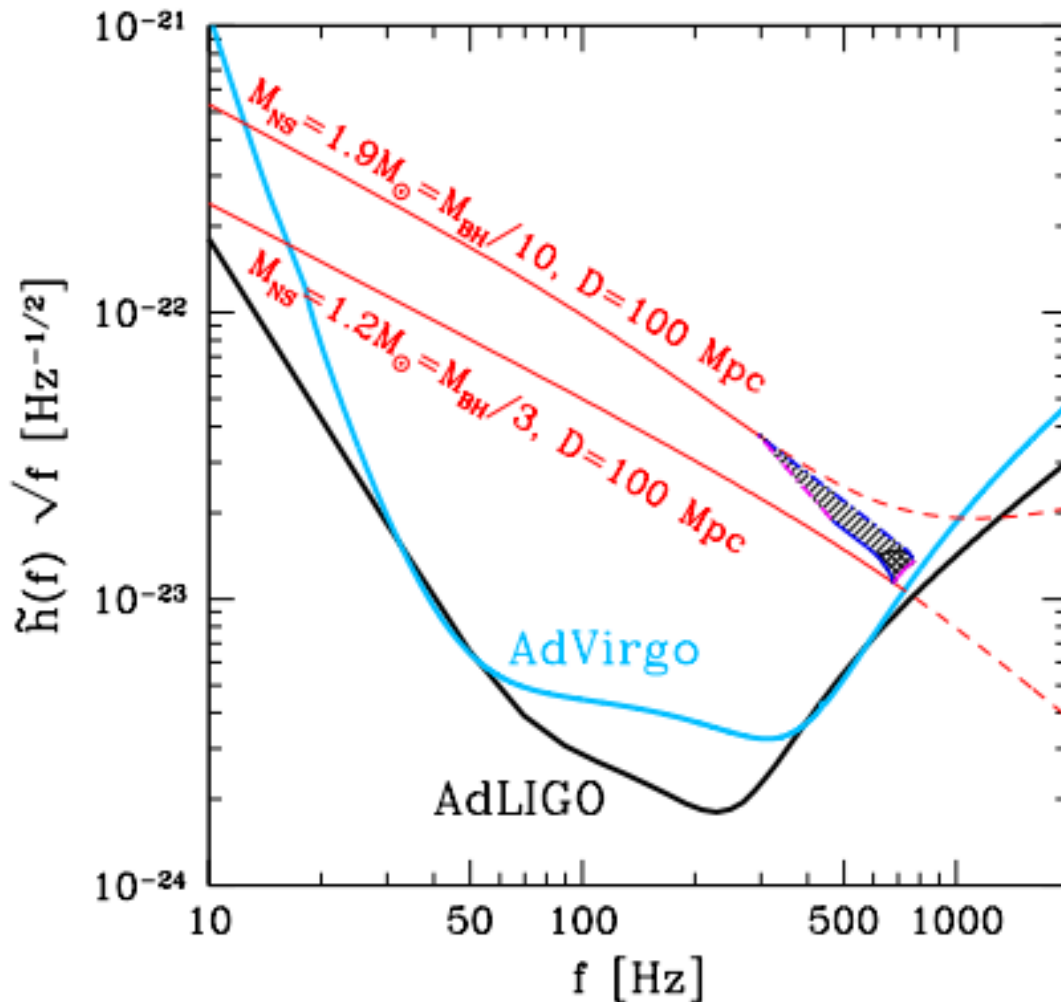
Parameters of extremely rotating NSs

$$\nu_{\text{cr}} = 1278 M_{1.4}^{1/2} \left(\frac{10 \text{ km}}{R} \right)^{3/2} \text{ Hz}$$

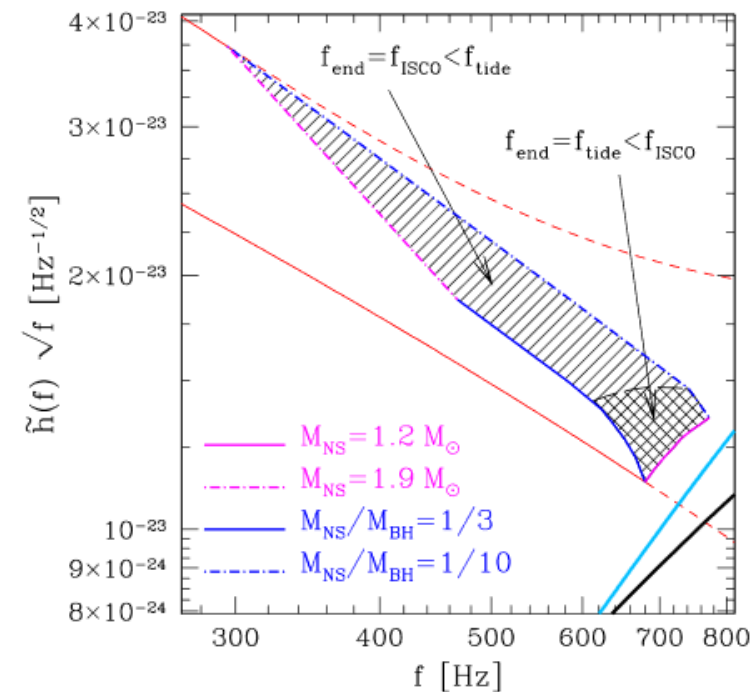
$$R_e = R \left[0.9766 + \frac{0.025}{1.07 - \bar{\nu}} + 0.07 M_{1.4}^{3/2} \bar{\nu}^2 \right]$$



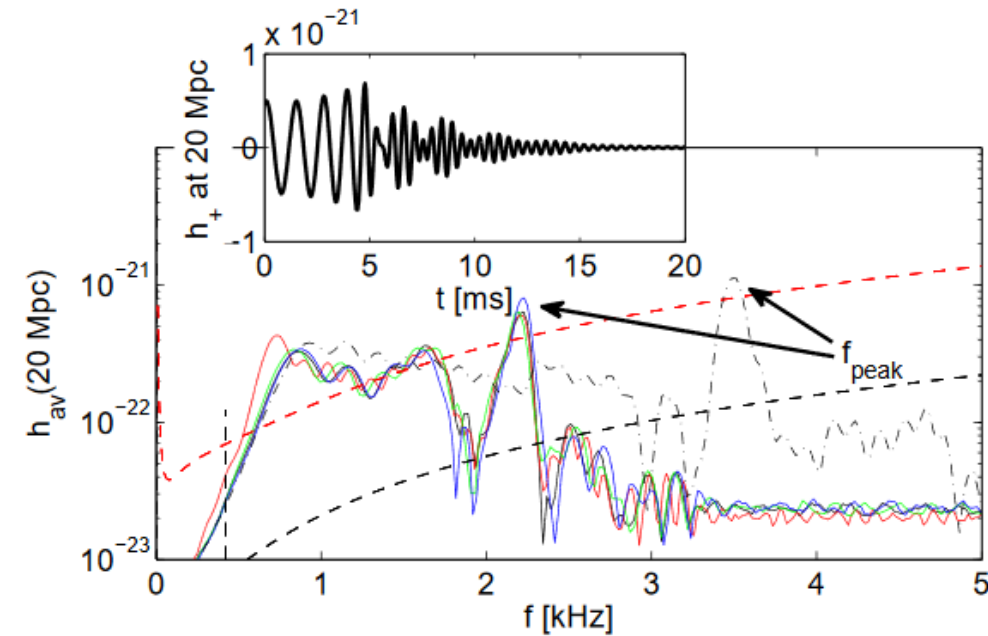
Limits on the EoS from GW observations



For stiff EoS
AdLIGO and AdVIRGO
can detect signatures
in the GW signal
during BH-NS mergers.

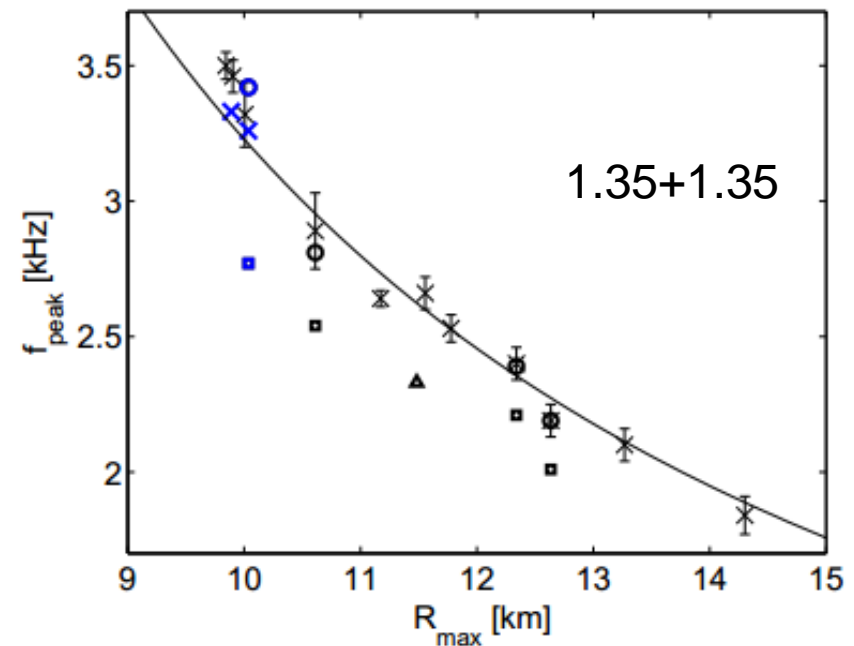


Another constraint



Orientation-averaged spectra of the GW signal for the Shen (solid) and the eosUU (black dashed-dotted) EoSs and the AdvLIGO (red dashed) and ET (black dashed) unity SNR sensitivities. The inset shows the GW amplitude with +polarization at a polar distance of 20 Mpc for the Shen EoS

Measuring NS-NS mergers one can better constraint the EoS.



GW170817: deformability Λ

Many papers are published based on detection of GW signal from GW170817:
1803.00549, 1804.08583, 1805.09371, 1805.11579, 1805.11581, 1901.04138.

$$\tilde{\Lambda} = \frac{16}{13} \frac{(12q + 1)\Lambda_1 + (12 + q)q^4\Lambda_2}{(1 + q)^5},$$

$$q = m_2/m_1 \leq 1$$

$$\Lambda_{1,2} = \frac{2}{3} k_2 \left(\frac{R_{1,2} c^2}{G m_{1,2}} \right)^5$$

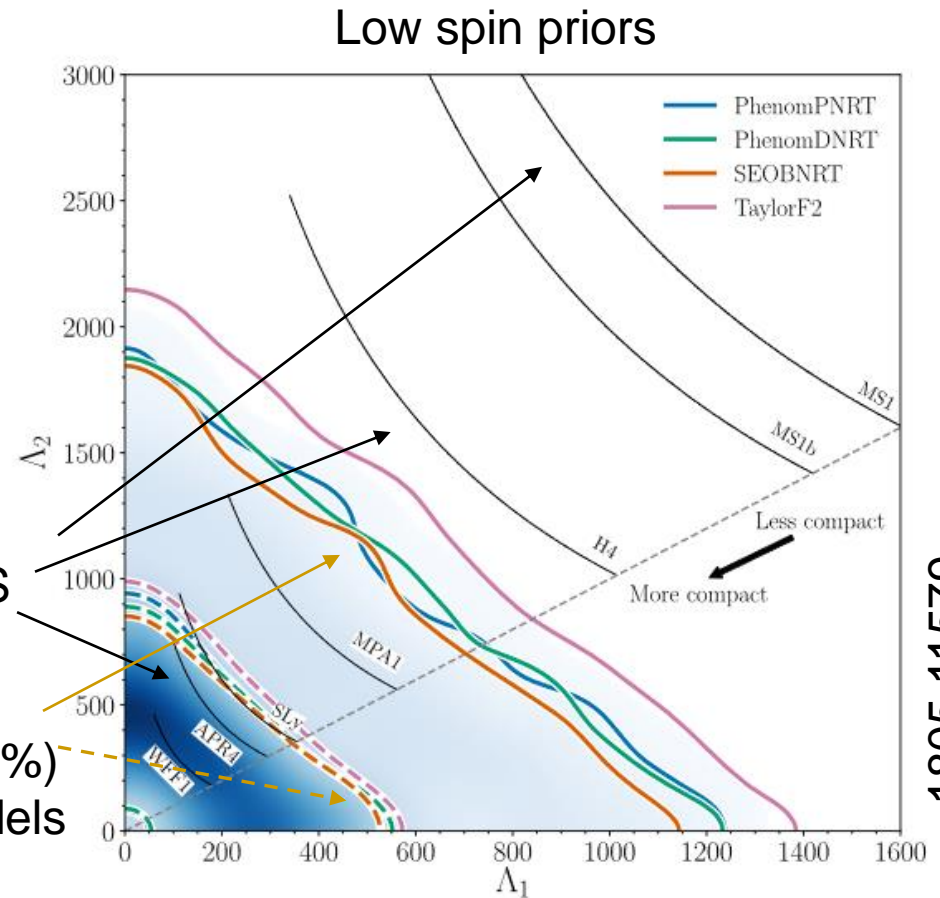
$$\beta = Gm/(\dot{R}c^2)$$

$$k_2 \sim \beta^{-1}$$

$$\Lambda \sim \beta^{-6}$$

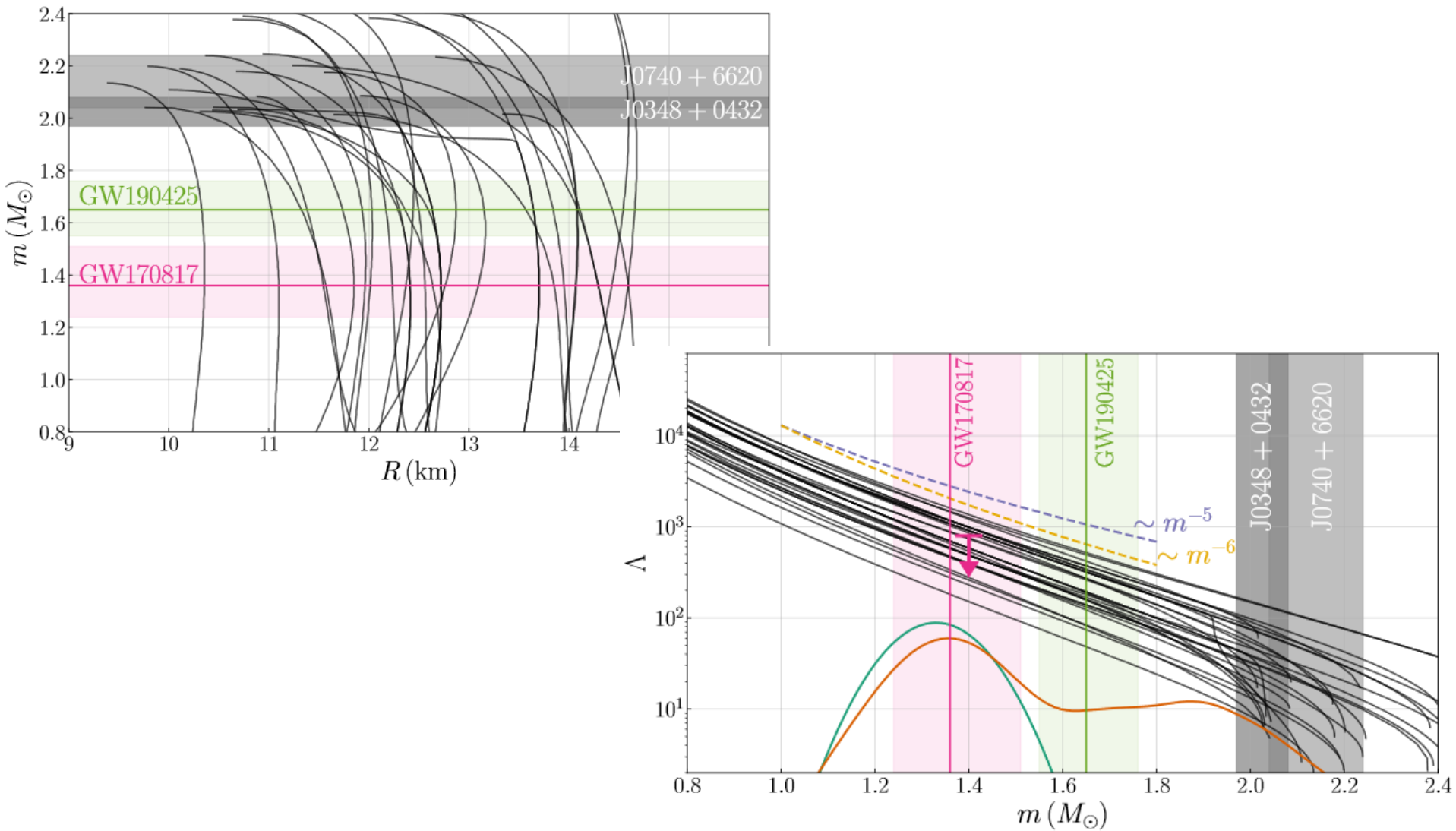
Solid – theoretical EoS

Colored – limits
(dashed 50%, solid 90%)
for four waveform models

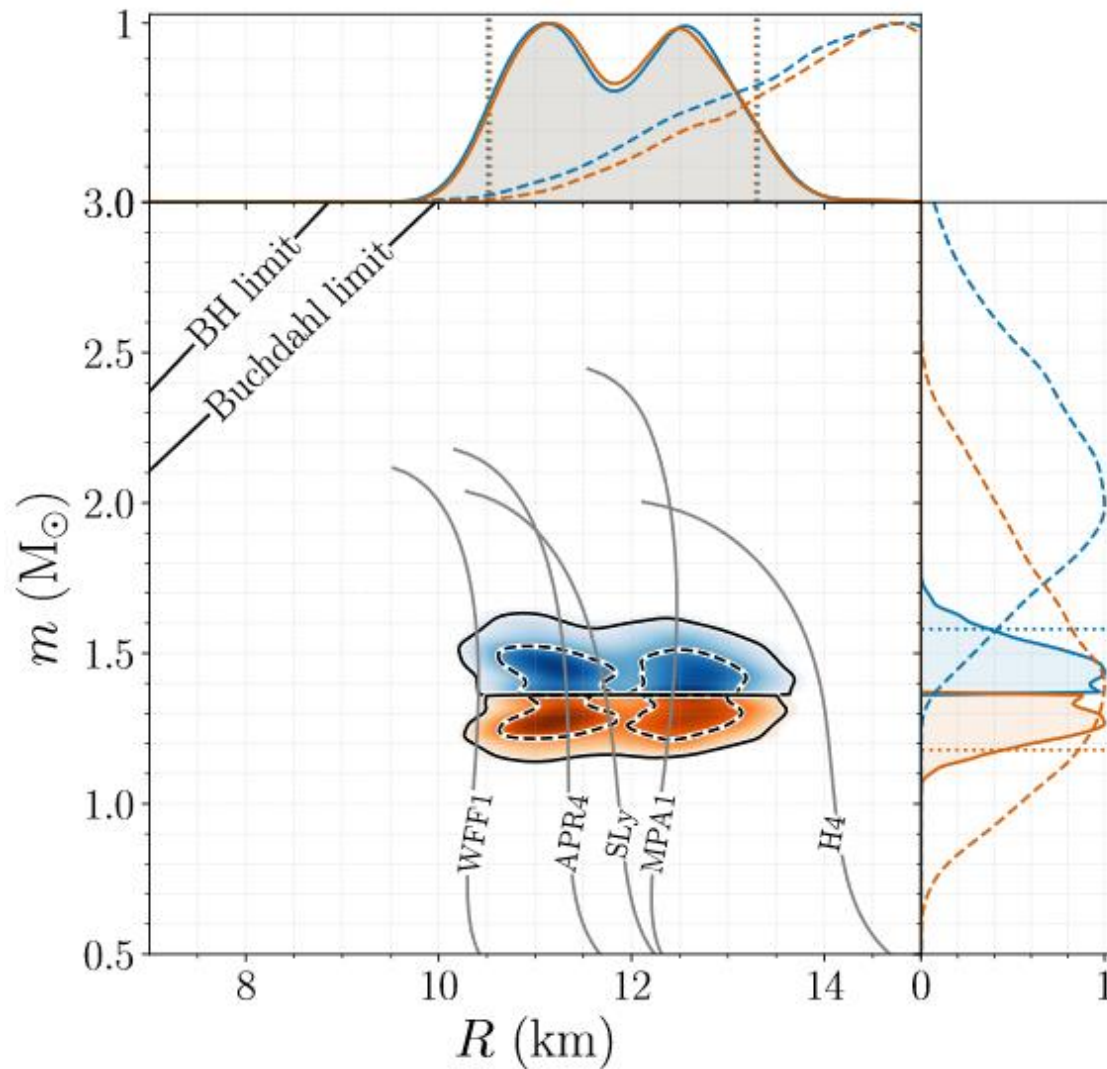


Collapse to a BH after ~1 sec? (1901.04138)

Parameters of NSs in known coalescences



GW170817: M-R

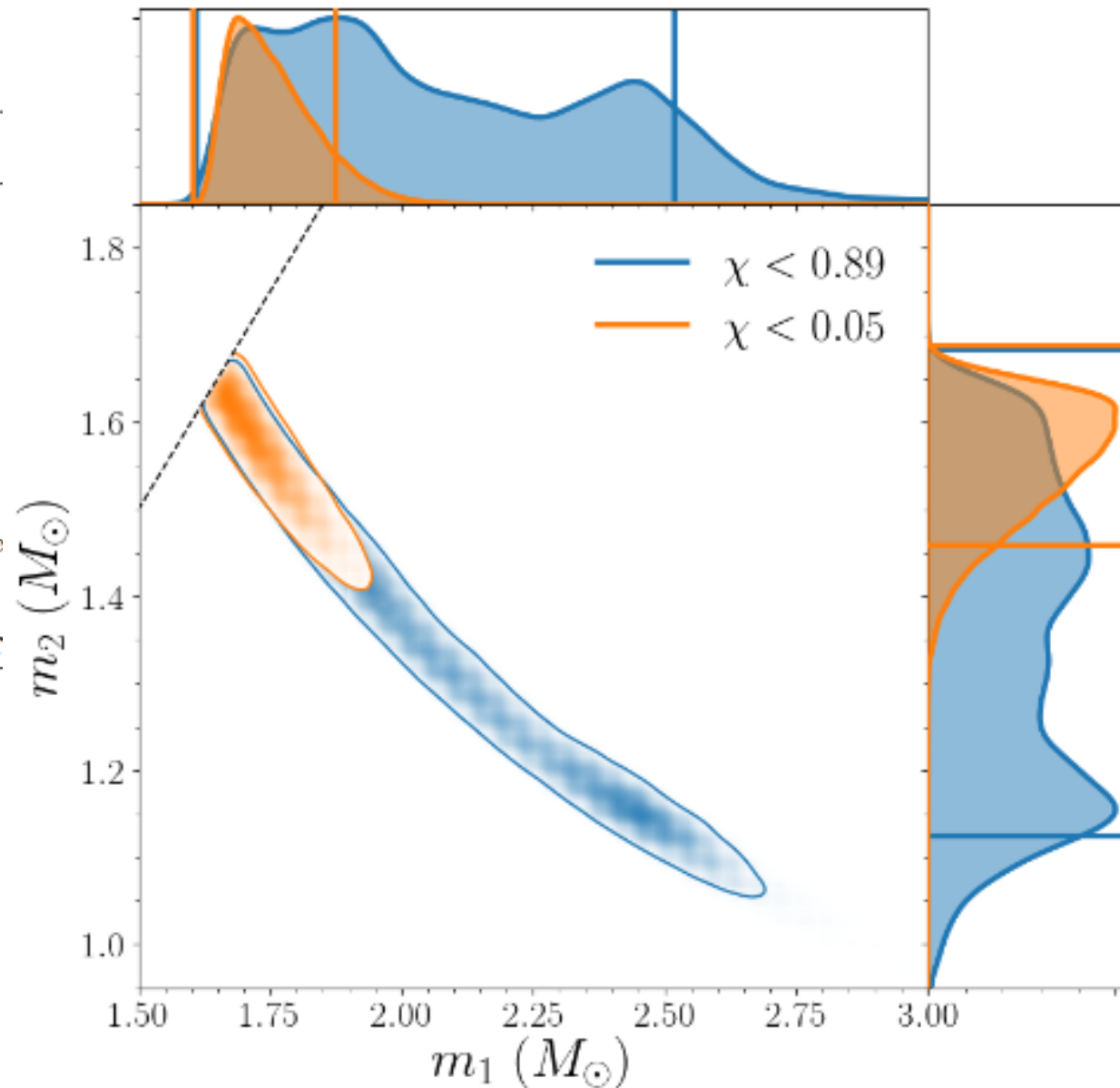


$$R_1 = 11.9^{+1.4}_{-1.4} \text{ km}$$

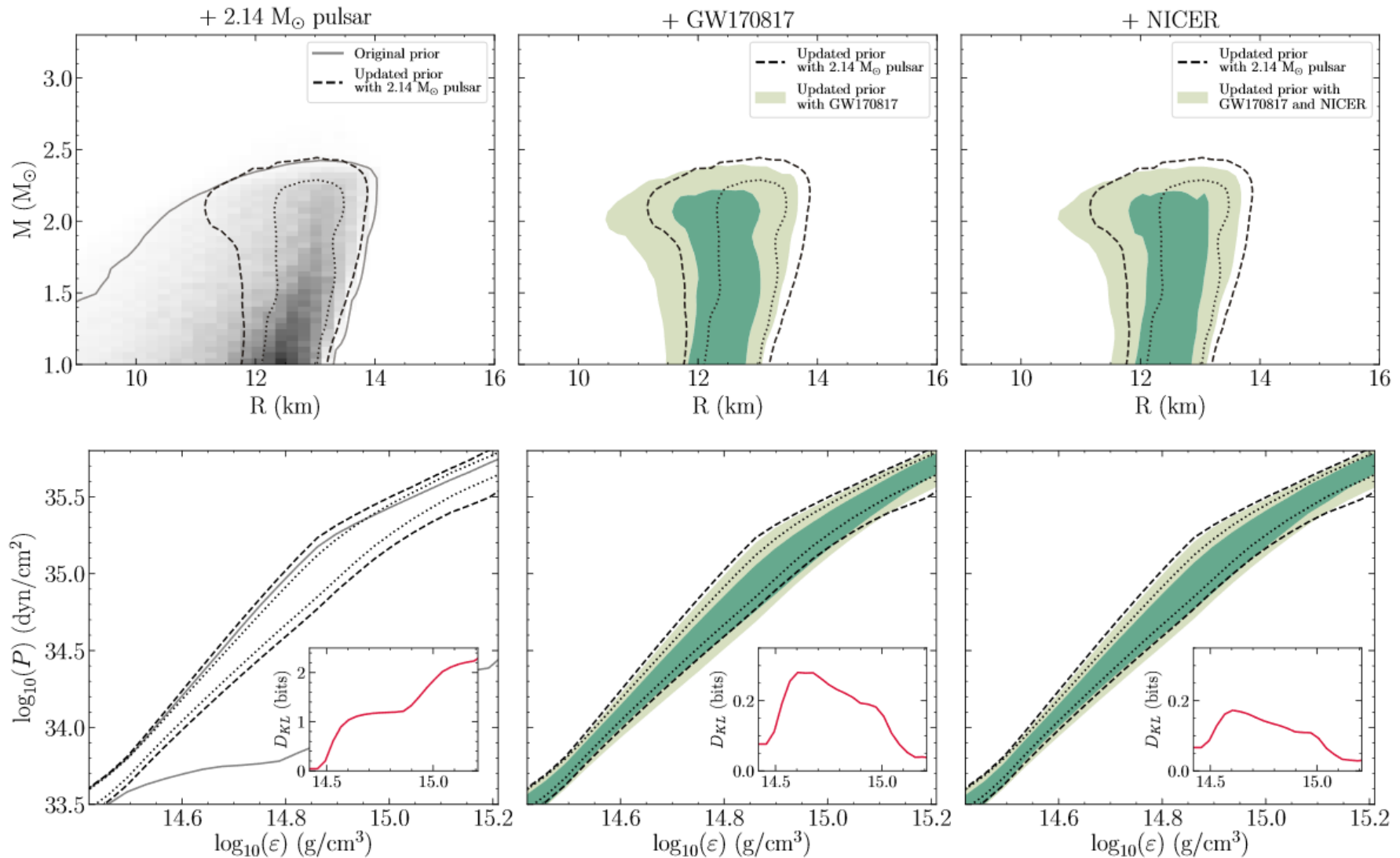
$$R_2 = 11.9^{+1.4}_{-1.4} \text{ km}$$

GW190425

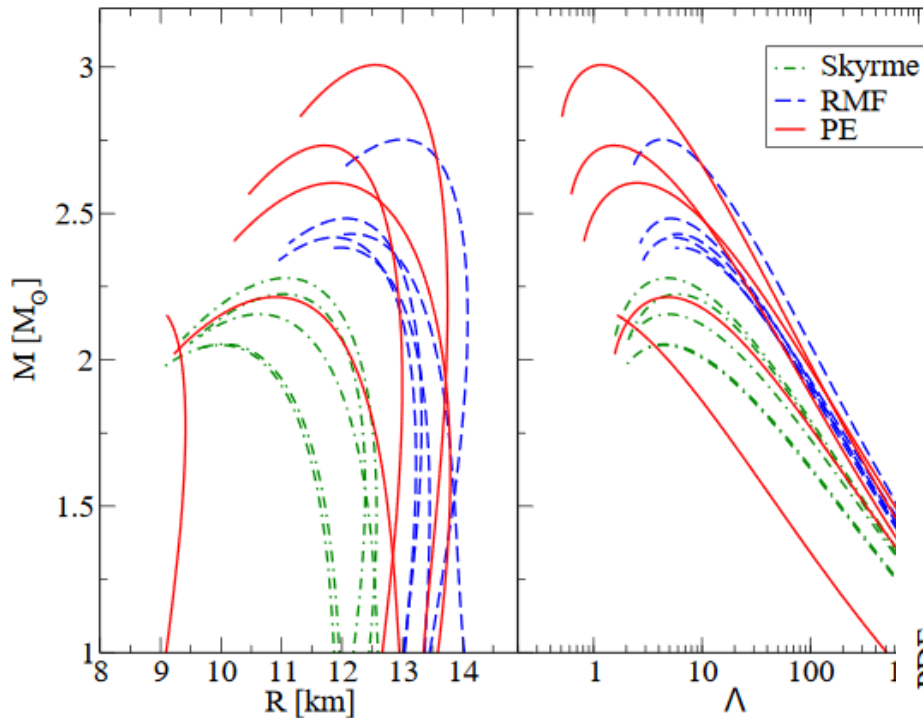
Primary mass m_1
Secondary mass m_2
Chirp mass \mathcal{M}
Detector-frame chirp mass
Mass ratio m_2/m_1
Total mass m_{tot}
Effective inspiral spin parameter χ_e
Luminosity distance D_L
Combined dimensionless tidal defor



LIGO+NICER=EoS ?



LIGO+NICER=EoS ?



$$Q_{ij} = -\lambda \mathcal{E}_{ij}.$$

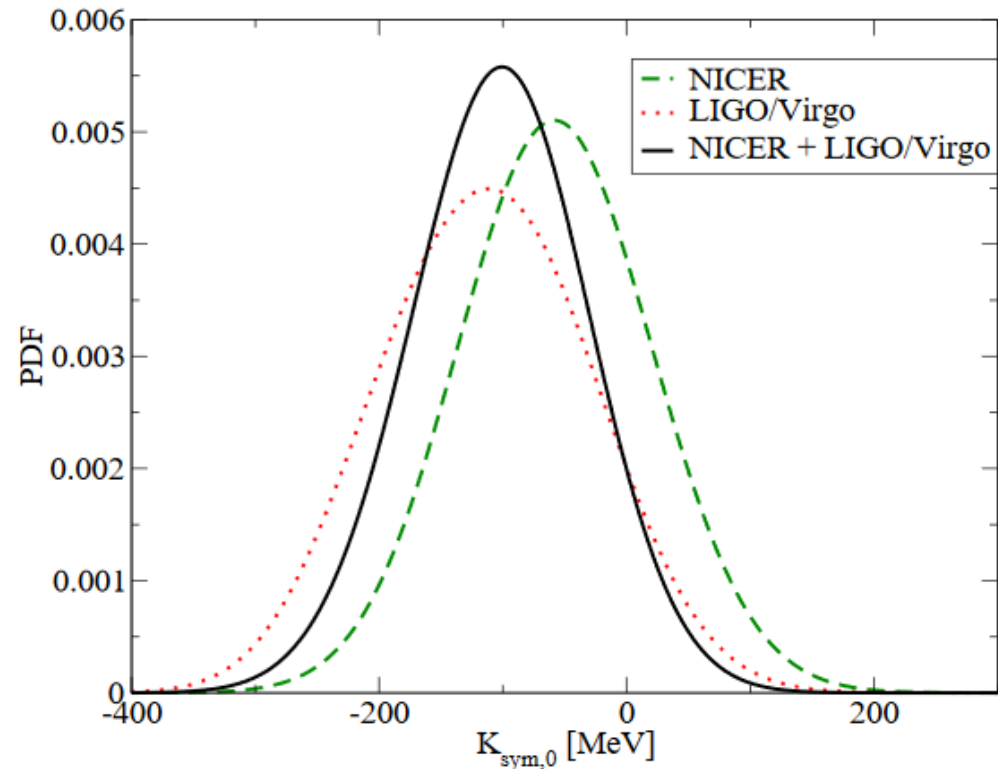
$$\Lambda \equiv \lambda / M^5.$$

$$\tilde{\Lambda} = \frac{16}{13} \frac{(1 + 12q)\Lambda_1 + (12 + q)q^4\Lambda_2}{(1 + q)^5}$$

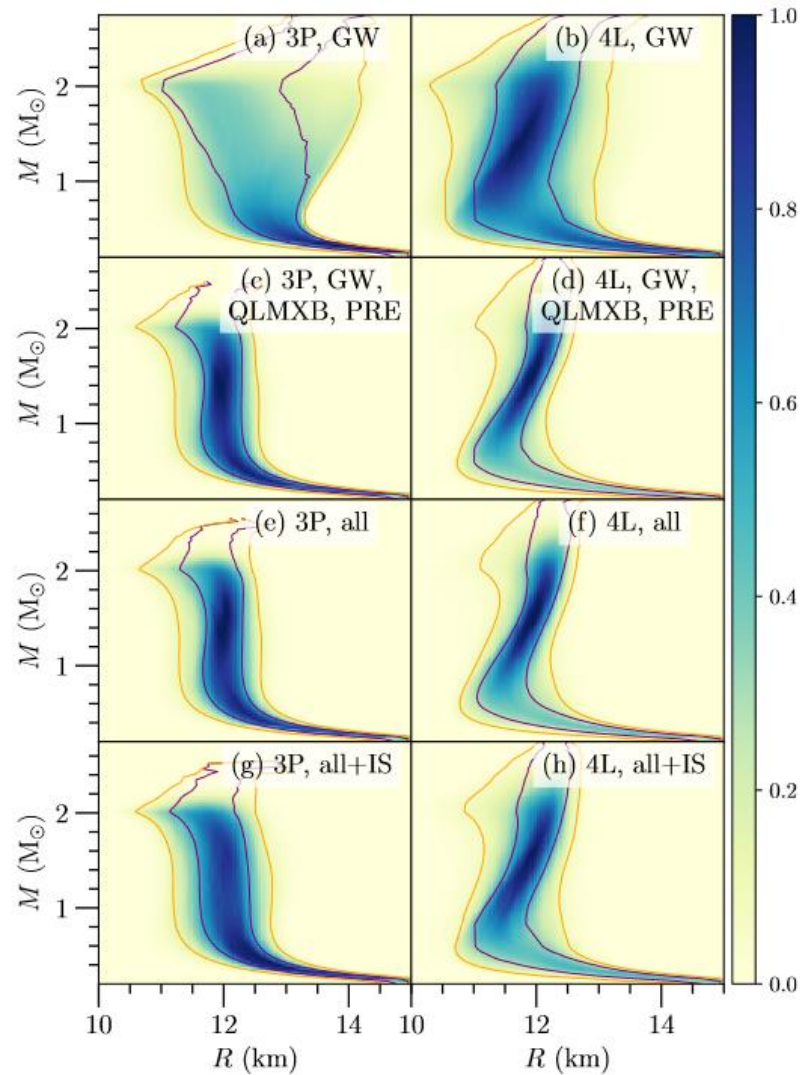
$$e(n, \delta) = e(n, 0) + S_2(n)\delta^2 + \mathcal{O}(\delta^4).$$

$$e(n, 0) = e_0 + \frac{K_0}{2}y^2 + \frac{Q_0}{6}y^3 + \mathcal{O}(y^4),$$

$$S_2(n) = J_0 + L_0y + \frac{K_{\text{sym},0}}{2}y^2 + \mathcal{O}(y^3),$$



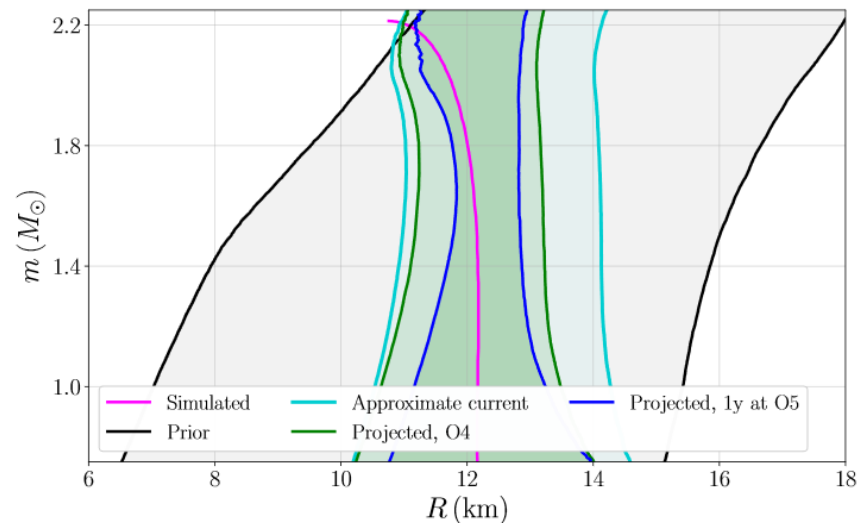
GW+binaries+EoS parametrization



It is important how to parametrize an EoS. Two columns correspond to different approaches to parametrize the EoS.

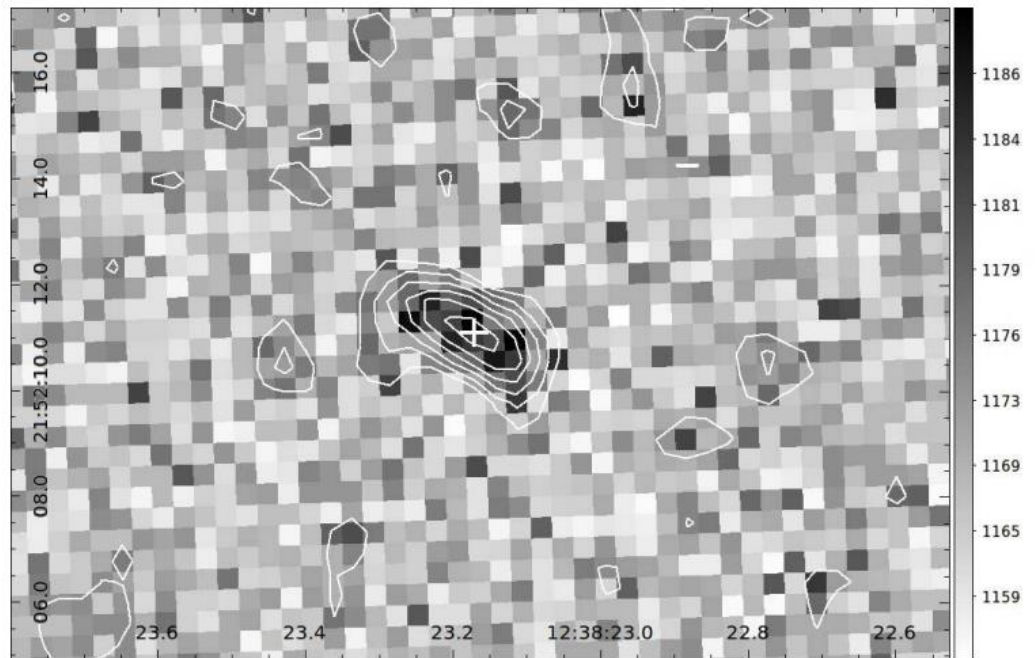
The last line included effect if systematic uncertainties (IS – intrinsic scattering)

PRE – photospheric radius expansion

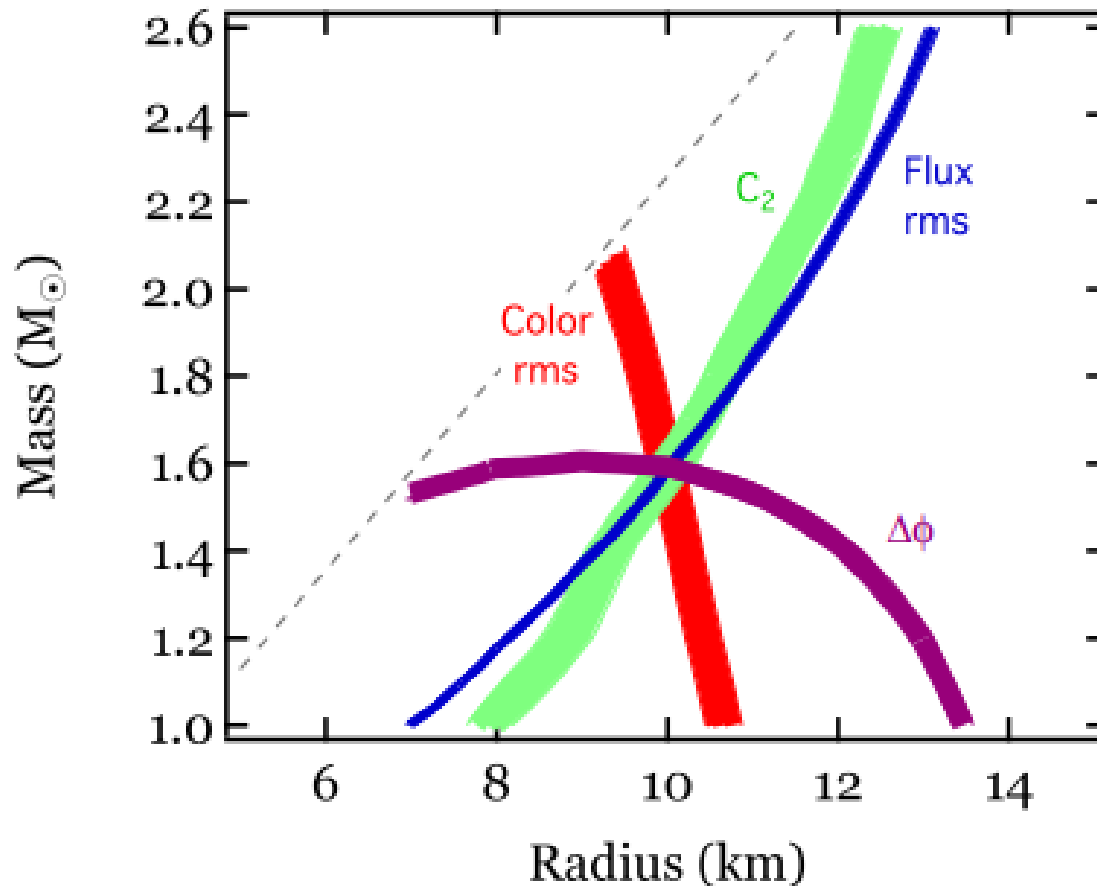


Microlensing and weak lensing

In the future (maybe already with Gaia)
it can possible to determine NS mass with lensing.
Different techniques can be discussed:
photometric (normal) microlensing (1009.0005),
astrometric microlensing, weak lensing (1209.2249).



Future X-ray measurements



Valid for future observations aboard NICER and LOFT space projects.

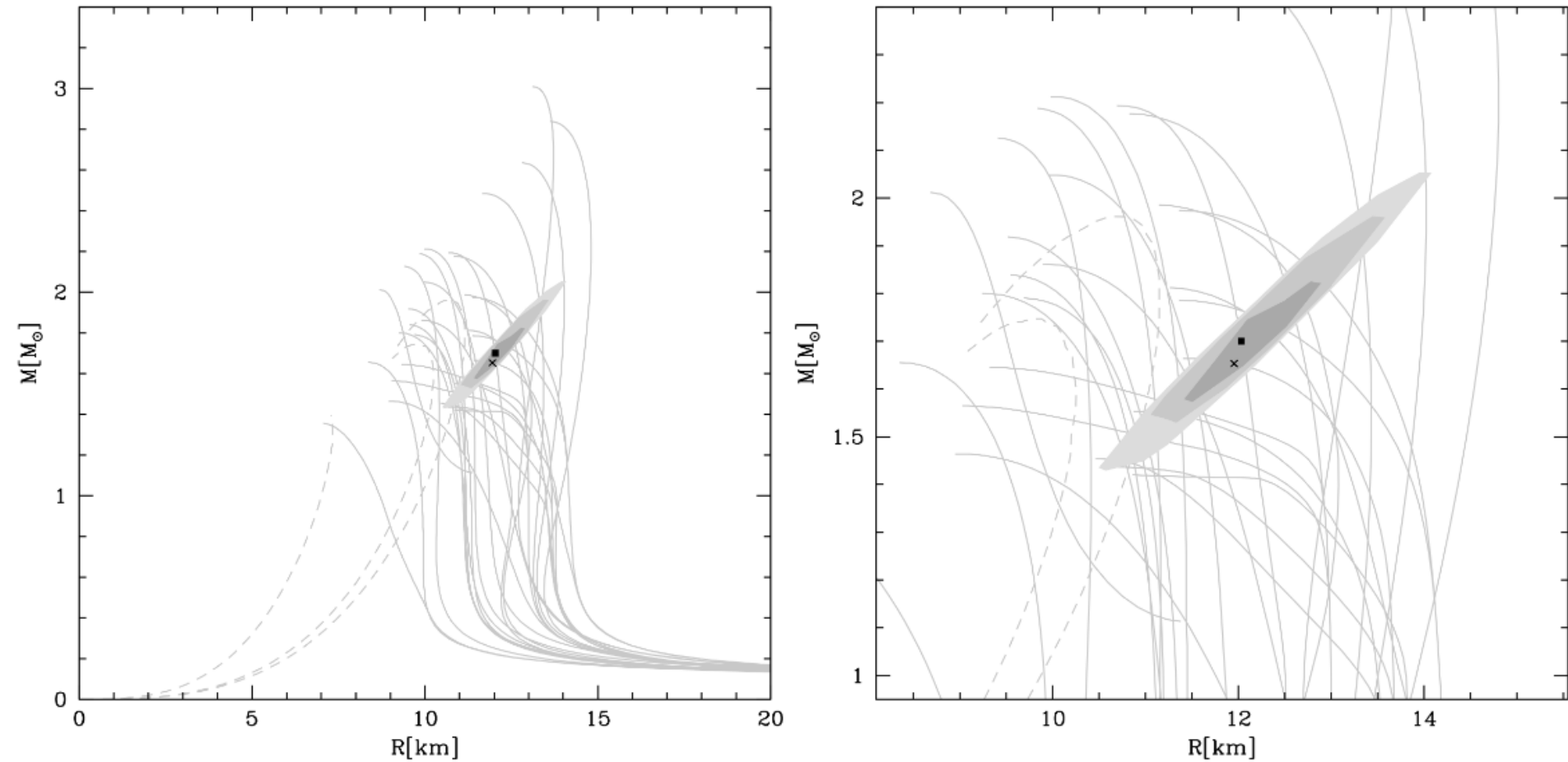
Data based on pulse profile.

The idea is to observe X-ray pulsars with spin periods ~few msec and to collect about 10^6 counts.

It allows to derive from the pulse profile a lot of info about a NS.

ATHENA

Using only spectra M and R can be determined within 3-10% and 2-8%, respectively.



1912.01608

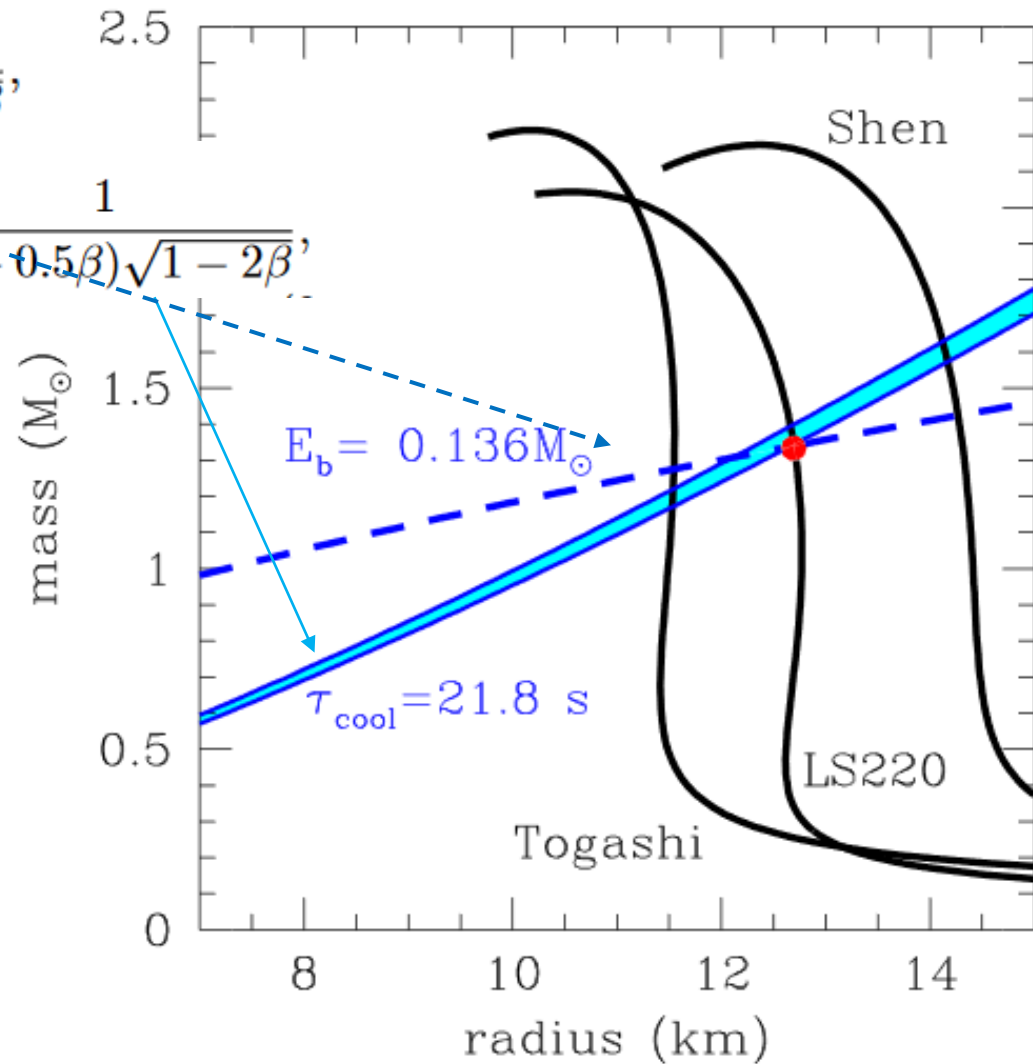
Exotics: neutrino signal

$$\frac{E_b}{m} = \frac{0.6\beta}{1 - 0.5\beta}, \quad \tau_{\text{cool}} \propto \frac{E_b}{r^2 \sqrt{1 - 2\beta}},$$

$$\tau_{\text{cool}} = \tau^* \left(\frac{m}{1.4M_\odot} \right)^2 \left(\frac{r}{10 \text{ km}} \right)^{-3} \frac{1}{(1 - 0.5\beta)\sqrt{1 - 2\beta}},$$

$$\beta = Gm/rc^2$$

Cooling timescale is longer for a model with a larger neutron star mass and a smaller neutron star radius.



References

- Observational Constraints on Neutron Star Masses and Radii 1604.03894
The review is about X-ray systems
 - Mass, radii and equation of state of neutron stars 1603.02698
The review about different kinds of measurements, including radio pulsars.
Recent lists of mass measurements for different NSs.
 - Measuring the neutron star equation of state using X-ray timing 1602.01081
The review about EoS and X-ray measurements
 - The masses and spins of neutron stars and stellar-mass black holes 1408.4145
The review covers several topics. Good brief description of radio pulsar mass measurements.
 - Properties of DNS systems. 1706.09438
The review covers all aspects of observations, formation and evolution.
 - Testing the equation of state of neutron stars with electromagnetic observations. 1806.02833
The review describes observational tests of the EoS.
-

NS+NS binaries

Secondary companion in double NS binaries can give a good estimate of the initial mass if we can neglect effects of evolution in a binary system.

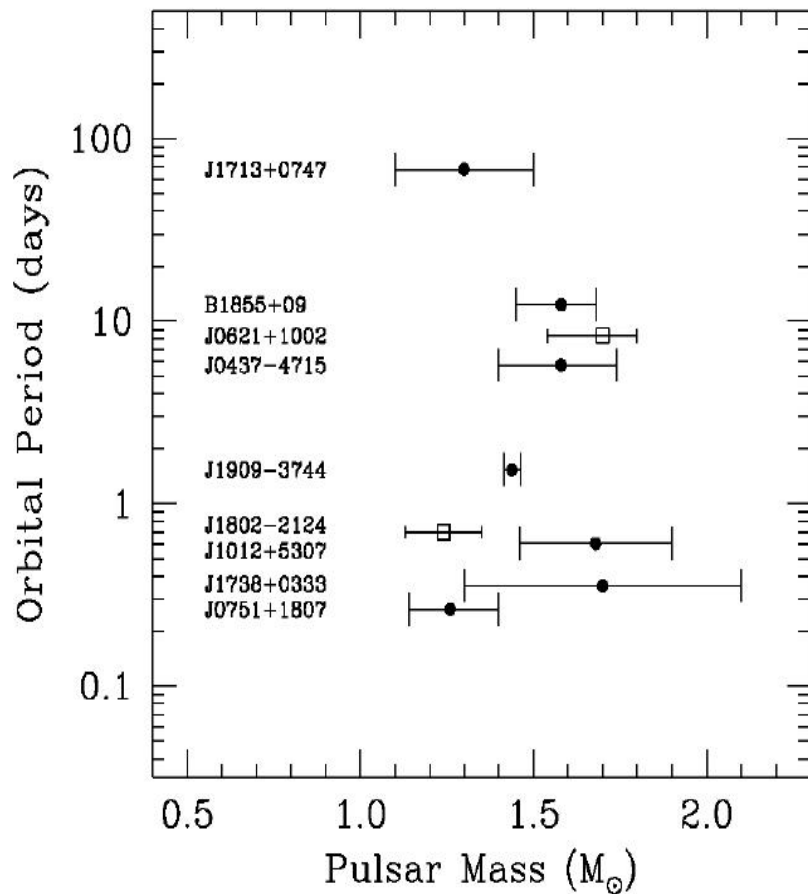
	Pulsar	Pulsar mass	Companion mass
GC →	B1913+16	1.44	1.39
	B2127+11C	1.36	1.35
	B1534+12	1.33	1.35
	J0737-3039	1.34	1.25
	J1756-2251	1.40	1.18
Non-recycled →	J1518+4904	<1.17	>1.55 → 0808.2292
	J1906+0746	1.25	1.37
	J1811-1736	1.56	1.12
	J1829+2456	1.2	1.4

Also there are candidates, for example
PSR J1753-2240
[arXiv:0811.2027](#)

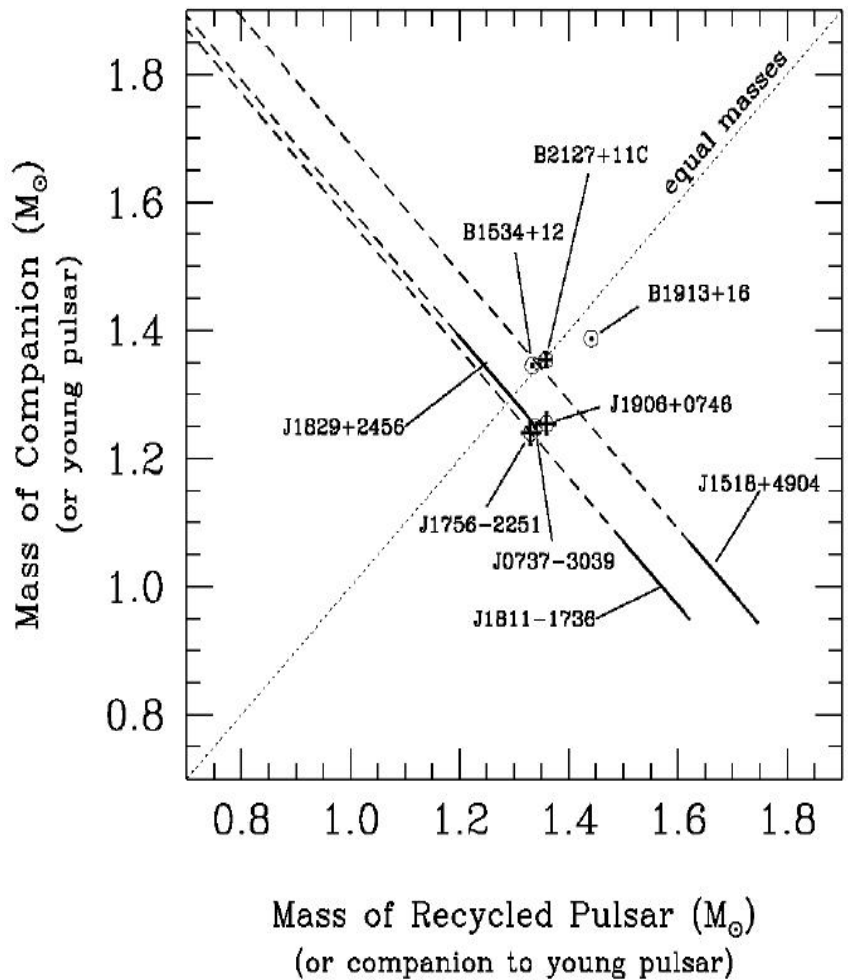
In NS-NS systems we can neglect all tidal effects etc.

See a review on formation and evolution of
DNS binaries in 1706.09438

Pulsar masses

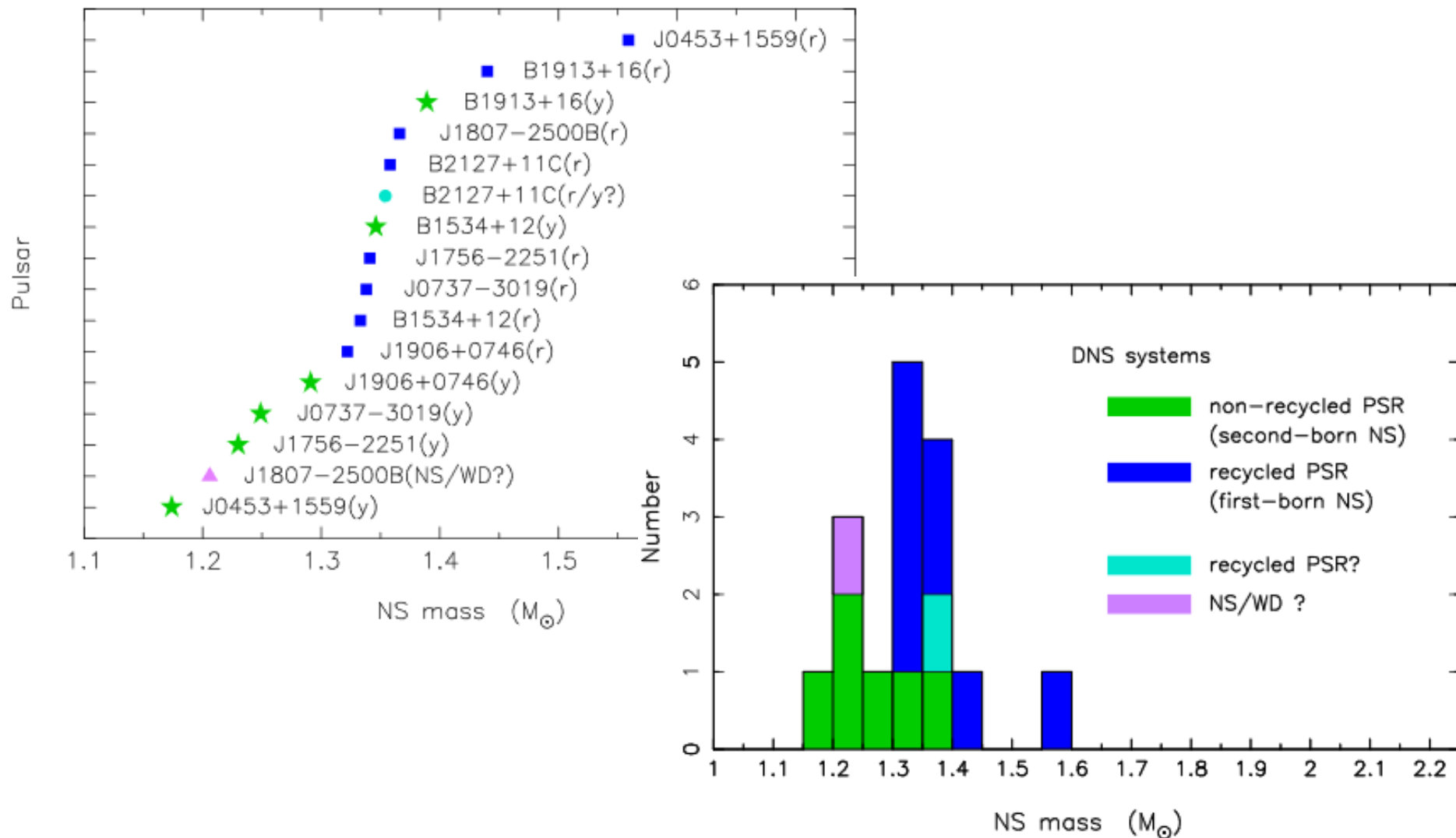


With WD companions



With NS companions

Mass distribution



PSR J1518+4904

[Janssen et al. arXiv: 0808.2292]

Surprising results !!!

Mass of the recycled pulsar is
<1.17 solar masses

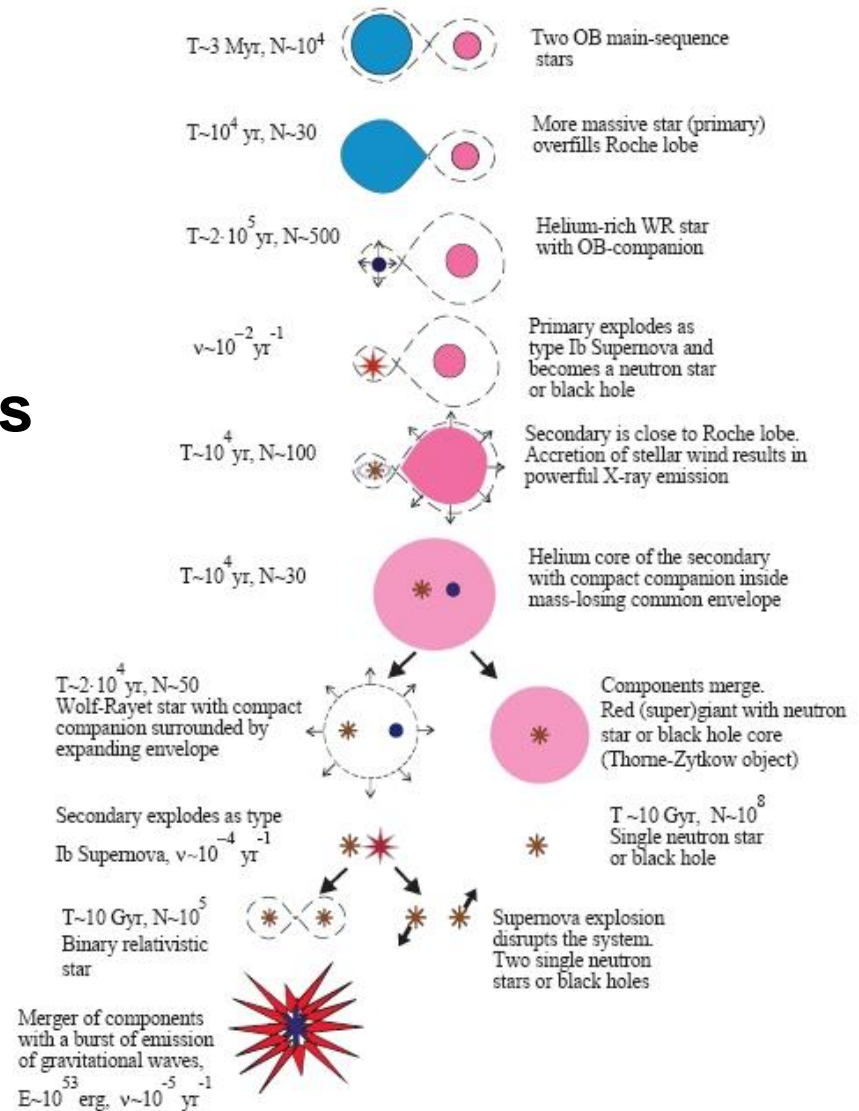
Mass of its component is
>1.55 solar masses

Central values are even more shocking:

$0.72^{+0.51}_{-0.58}$ and $2.00^{+0.58}_{-0.51}$

$V \sim 25$ km/s, $e \sim 0.25$

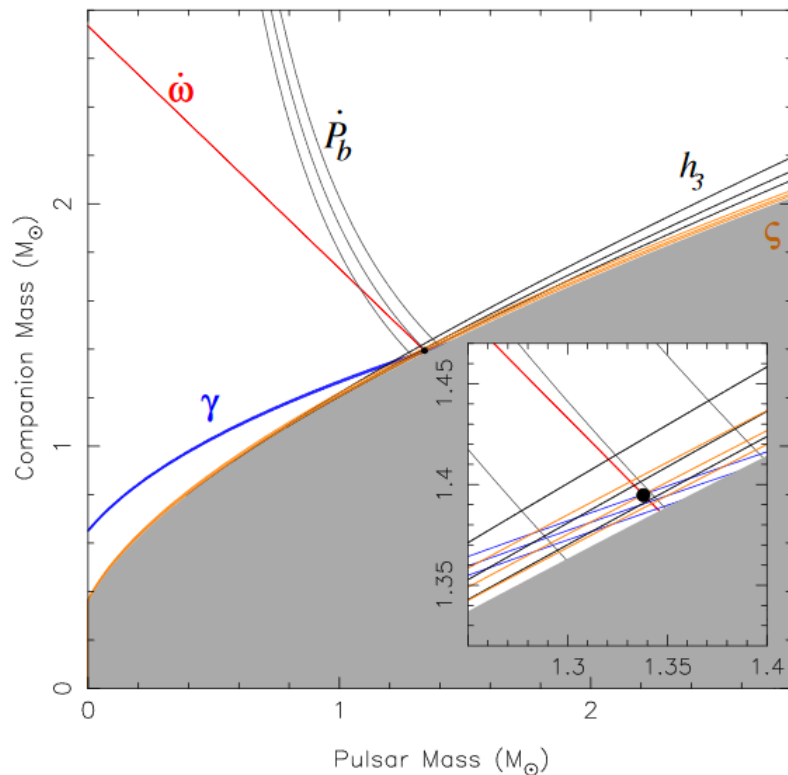
The second SN was e⁻-capture?



New measurements show less extreme values, see table 1 in 1603.02698:
<1.768 and >0.95 solar masses. Total mass is the same 2.7183 solar masses.

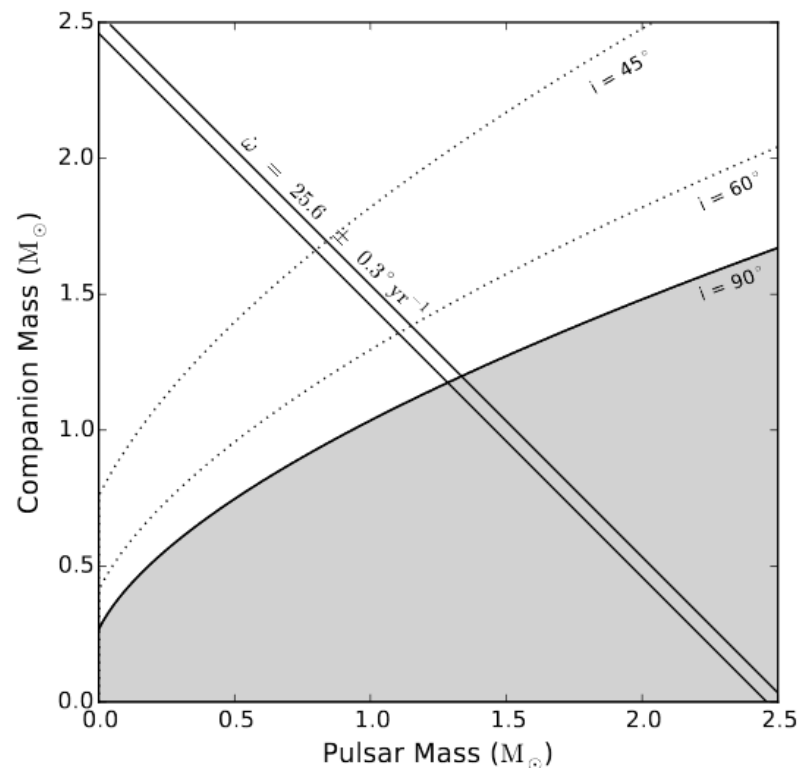
Recent discoveries with records

PSR J1757–1854
4.4 hours



$$\dot{\omega} \simeq 10.37^{\circ} \text{ yr}^{-1},$$

PSR J1946+2052
1.88 hours



$$\dot{\omega} = 25.6 \pm 0.3 \text{ deg yr}^{-1},$$

1711.07697

1802.01707

More measurements

PSR J1738+0333 NS+WD

arXiv: 1204.3948

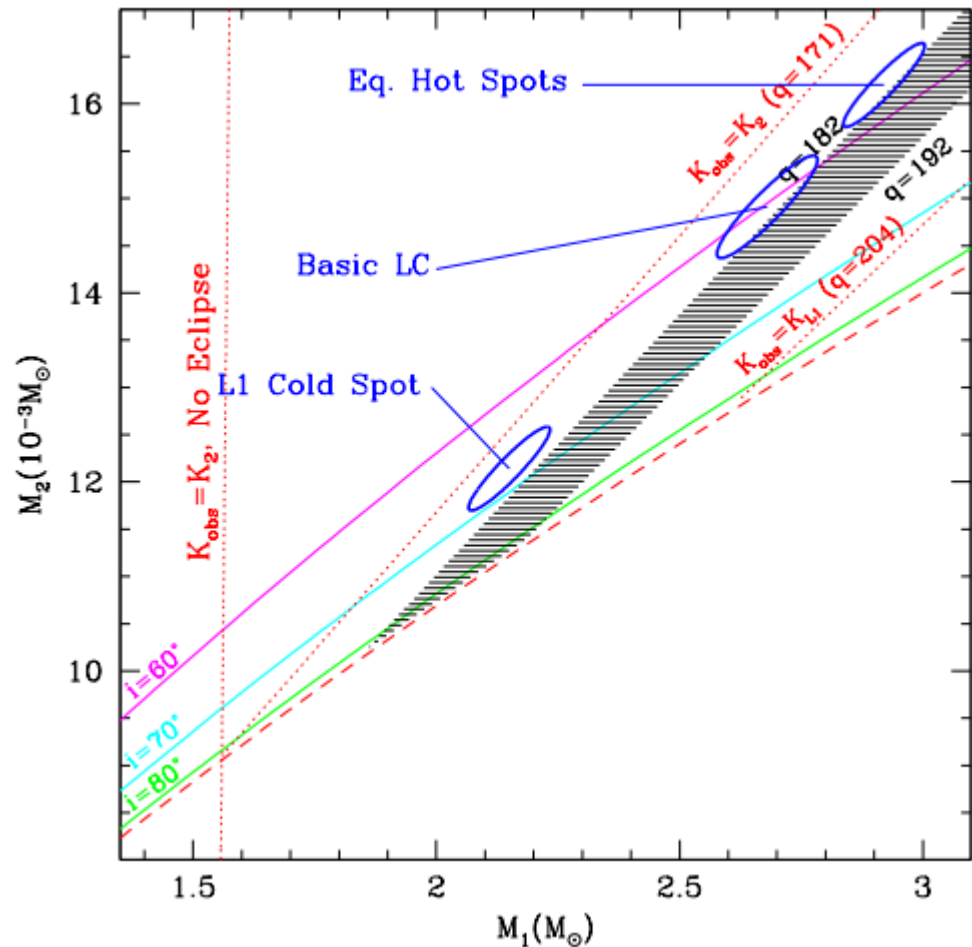
$$M_{\text{WD}} = 0.181 \pm 0.007 \text{--} 0.005 M_{\odot}$$

$$M_{\text{PSR}} = 1.47 \pm 0.07 \text{--} 0.06 M_{\odot}$$

PSR J1311–3430

arXiv: 1210.6884

MPSR > 2.1 at least!



Radius measurement

Fitting X-ray spectrum of a low-mass X-ray binary in quiescent state.

Mostly sources in globular clusters.

For 4 objects ~10% precision. But this is for fixed mass.

For U24 in NGC 6397 $R_{\text{NS}} = 8.9^{+0.9}_{-0.6}$ km for 1.4 solar masses.

For the radius observed from infinity: $11.9^{+2.2}_{-2.5}$ km