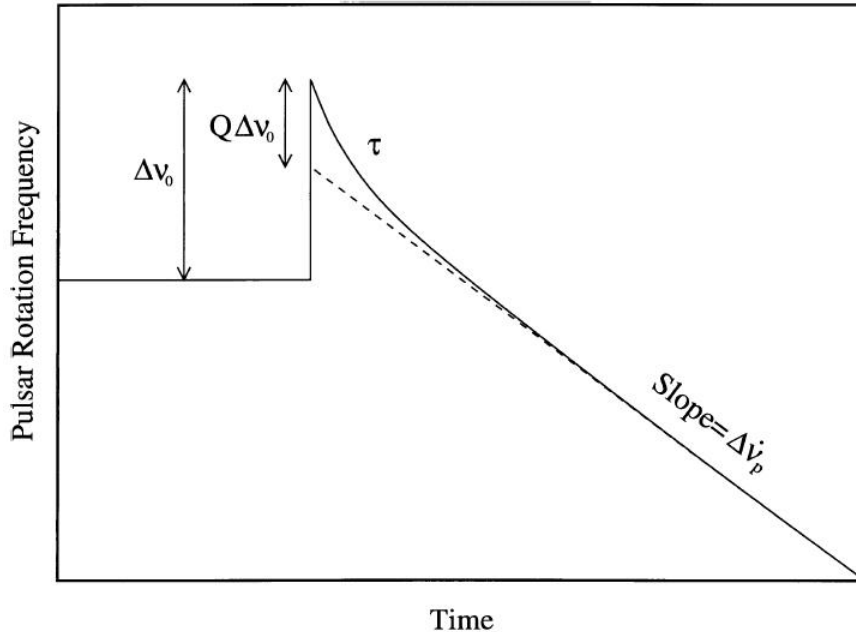


Glitches and precession

What is a glitch?



A sudden increase of rotation rate (limits are down to <12 sec in Vela).

ATNF catalogue gives >130 normal PSRs with glitches.

The most known: Crab and Vela

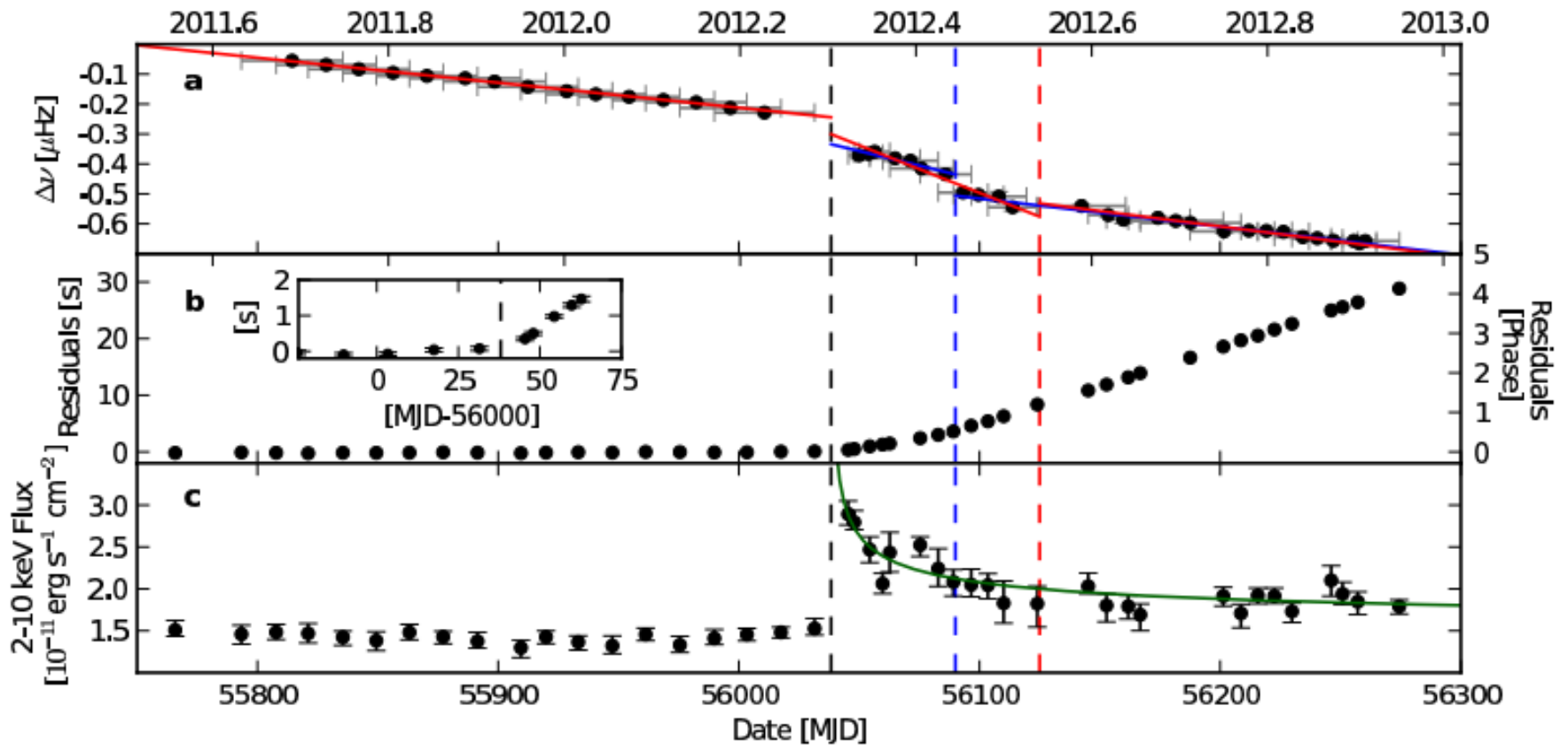
$$\Delta\Omega/\Omega \sim 10^{-9} - 10^{-6}$$

Spin-down rate can change after a glitch. Vela is spinning down faster after a glitch.

**Starquakes or/and vortex lines unpinning -
new configuration or transfer of angular momentum**

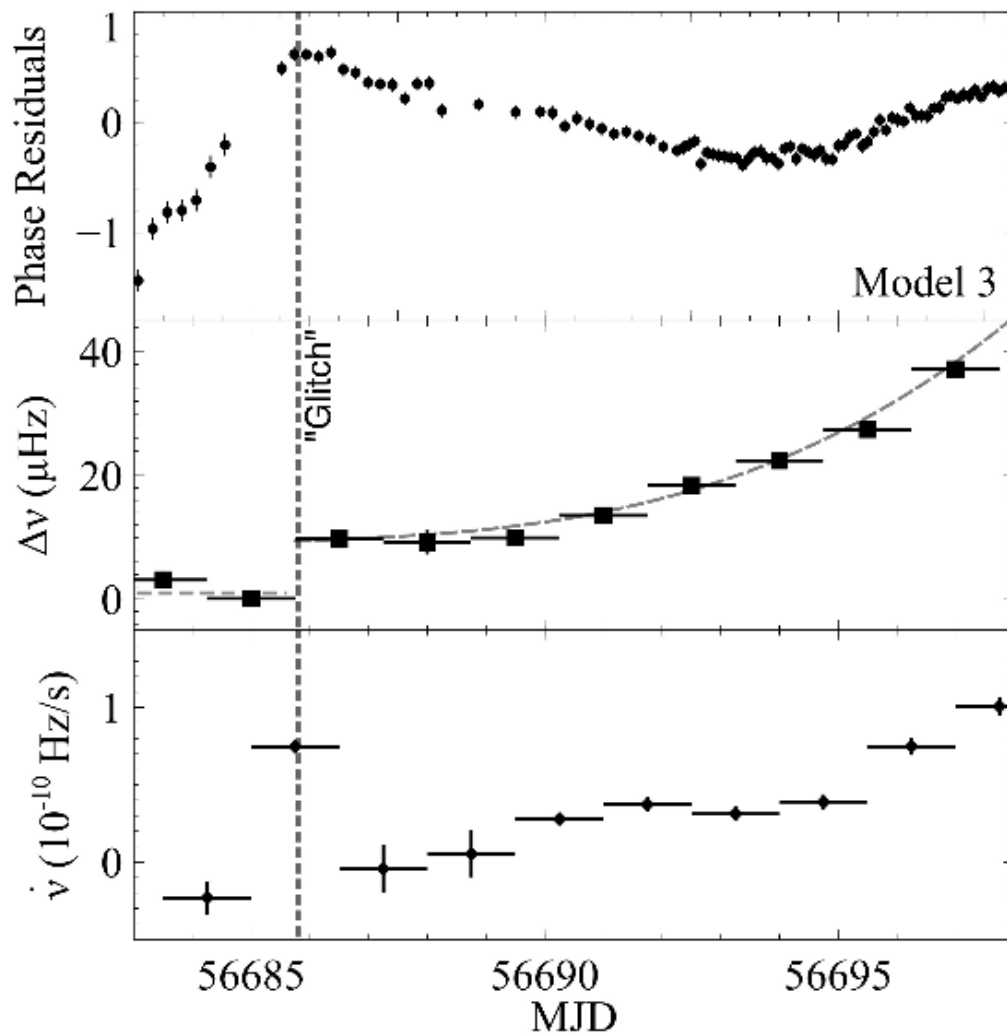
Glitches are important because they probe internal structure of a NS.

Anti-glitch of a magnetar



AXP 1E 2259+586

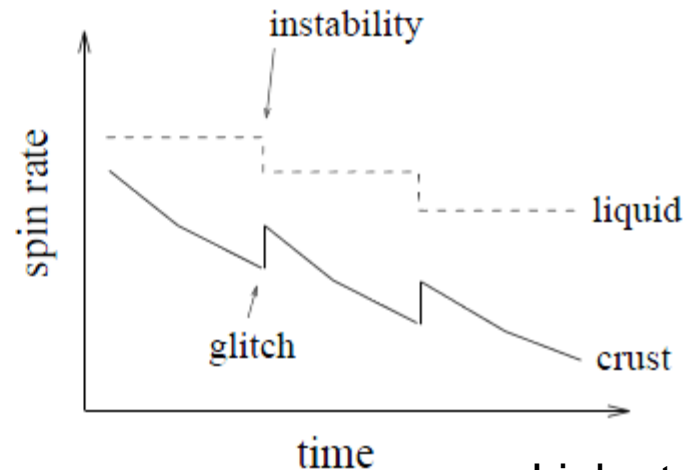
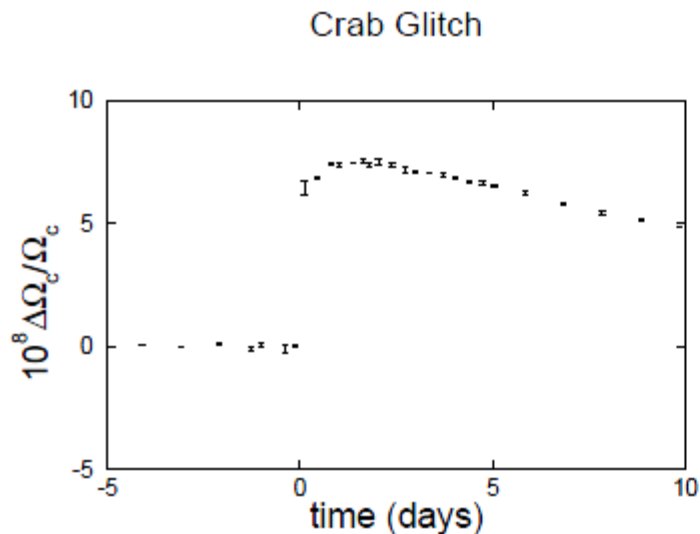
Glitches in accreting X-ray binaries



ULX M82 X-2
 $P_{\text{spin}} = 1.4$ sec

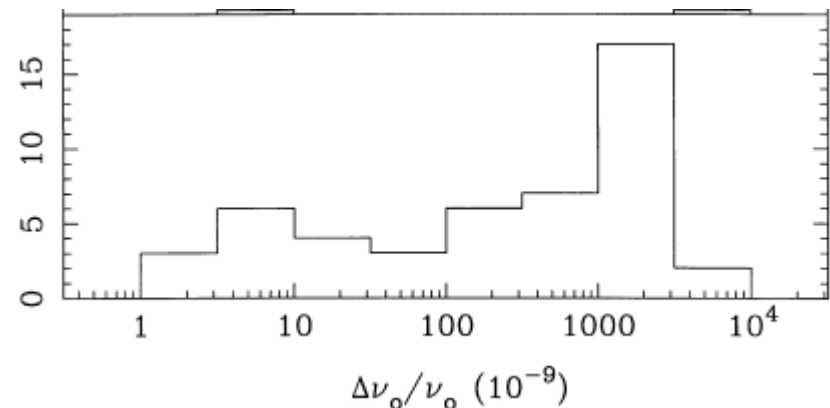
$$\Delta\nu/\nu \gtrsim 10^{-5}$$

Crab glitch and the general idea



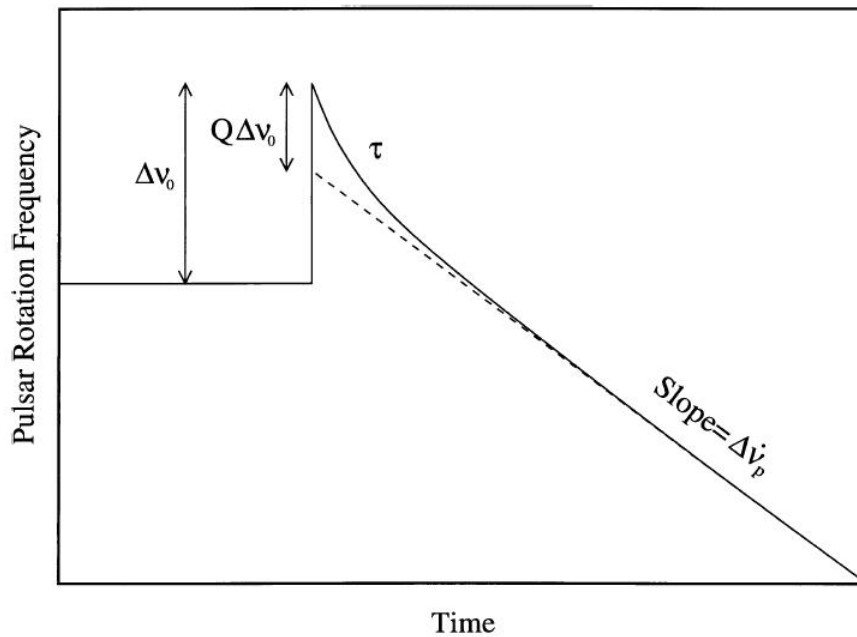
Link et al. (2000)

While the crust we see (and all coupled to it) is slowing down, some component of a star is not. Then suddenly (<12 sec) an additional momentum stored in such a “reservoir” is released and given to the crust. The crust spins-up, up the internal reservoir – down.



Lyne et al. (2000)

Glitches

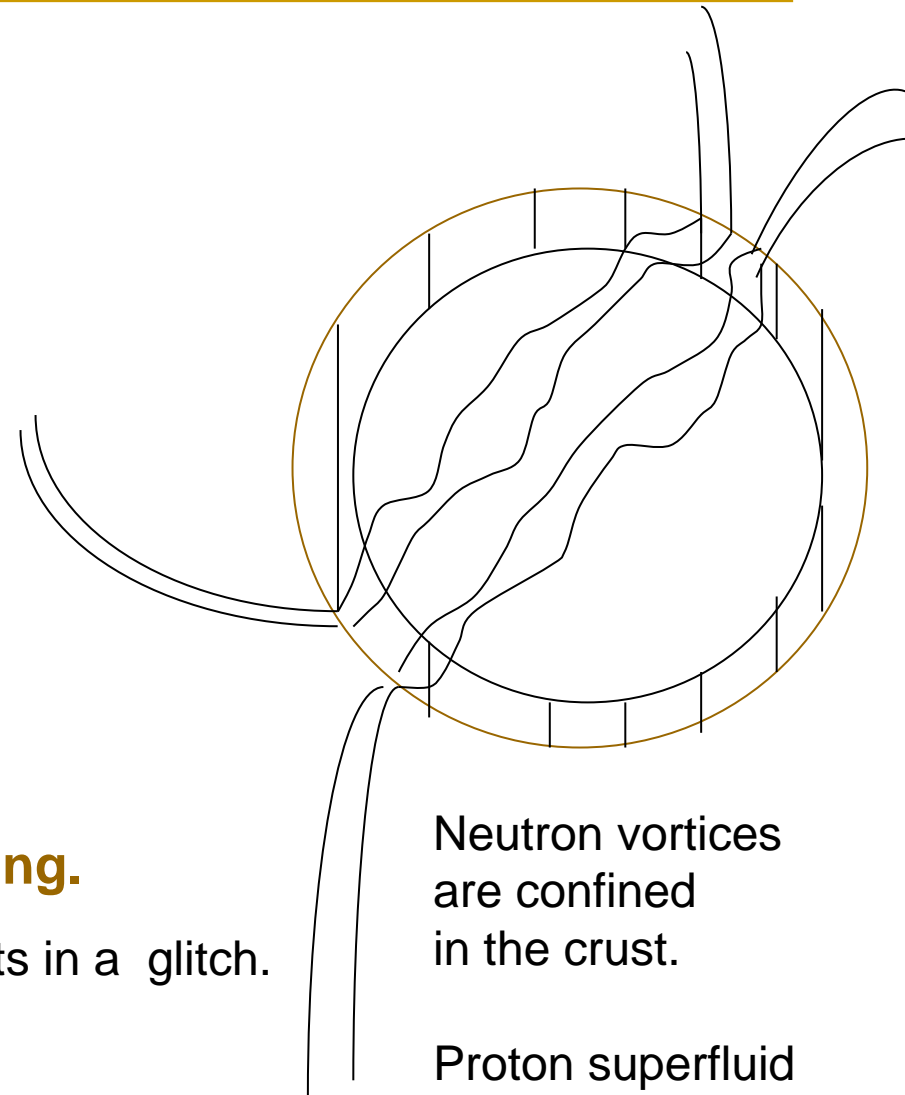


Starquakes or vortex lines unpinning.

Unpinning of superfluid vortex lines results in a glitch.

Vortex density is about $10^4 \text{ cm}^{-2} \text{ P}^{-1}$

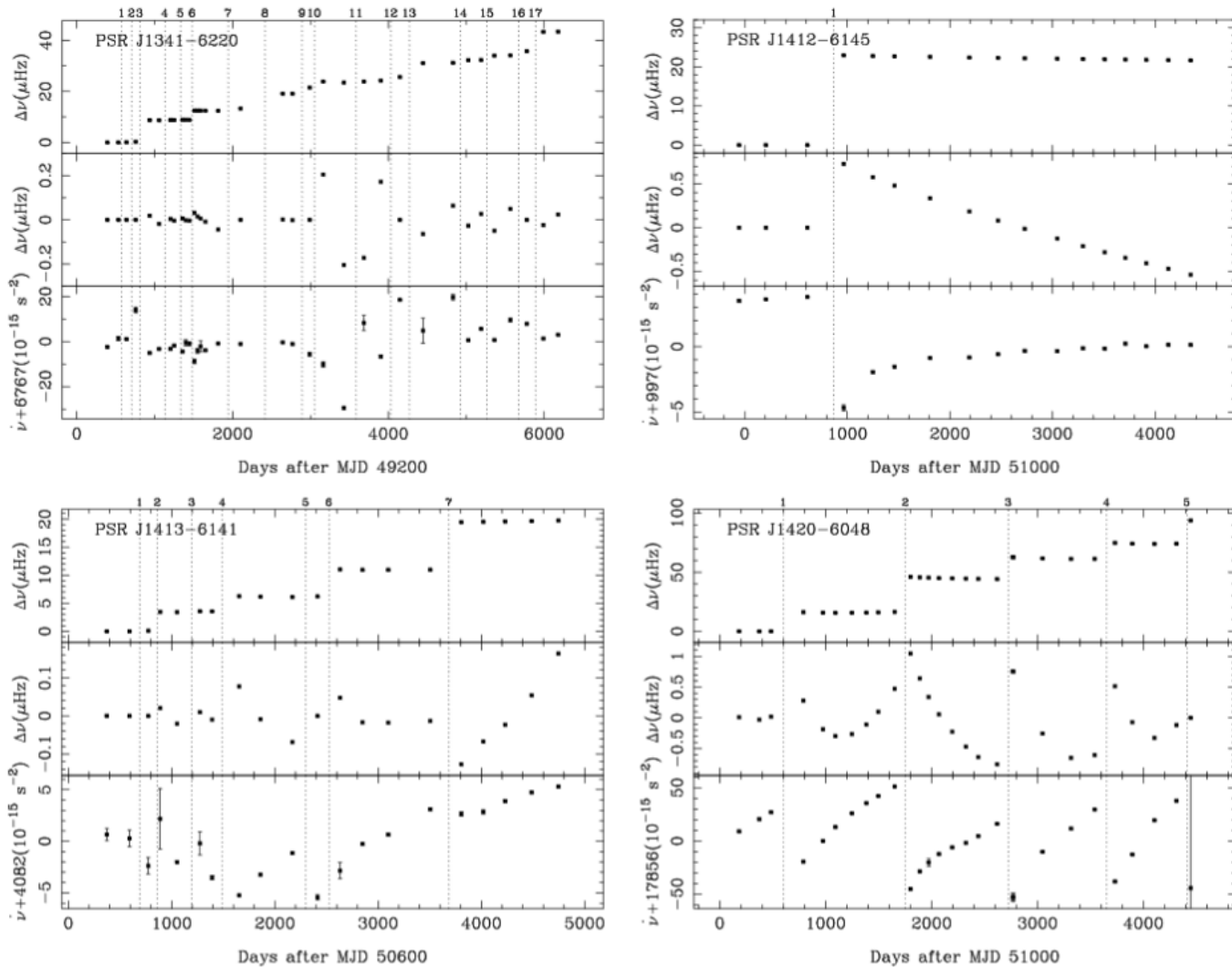
Flux lines density is $5 \cdot 10^{18} \text{ B}_{12} \text{ cm}^{-2}$



Neutron vortices
are confined
in the crust.

Proton superfluid
is strongly coupled
to the crust.

Glitch discovery and observations



The largest glitch of the Crab pulsar

2017 November 8

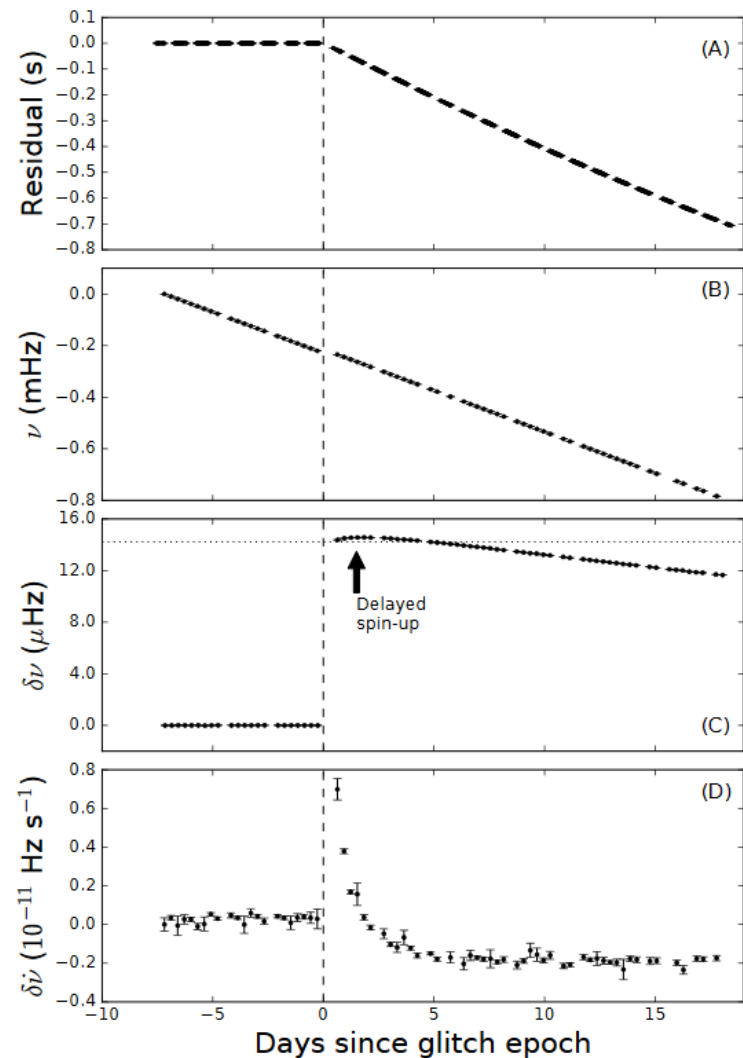
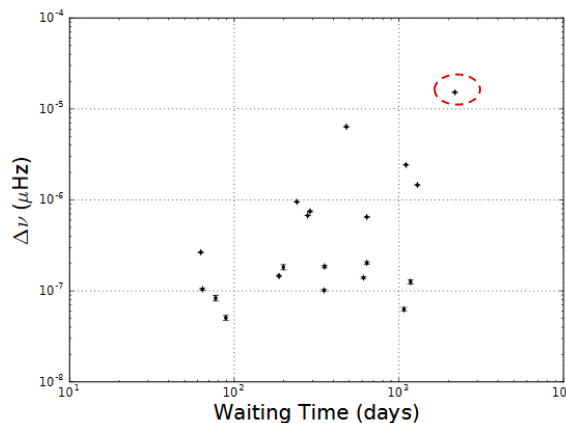
$$\Delta\nu = 1.530 \times 10^{-5} \text{ Hz}$$

$$\Delta\nu/\nu = 0.516 \times 10^{-6}$$

$$\Delta\dot{\nu}/\dot{\nu} = 7 \times 10^{-3}$$

The glitch occurred after the longest period of glitch inactivity – 6 years, – since beginning of daily monitoring in 1984.

No changes in the shape of the pulse profile, no changes in the X-ray flux.



1805.05110

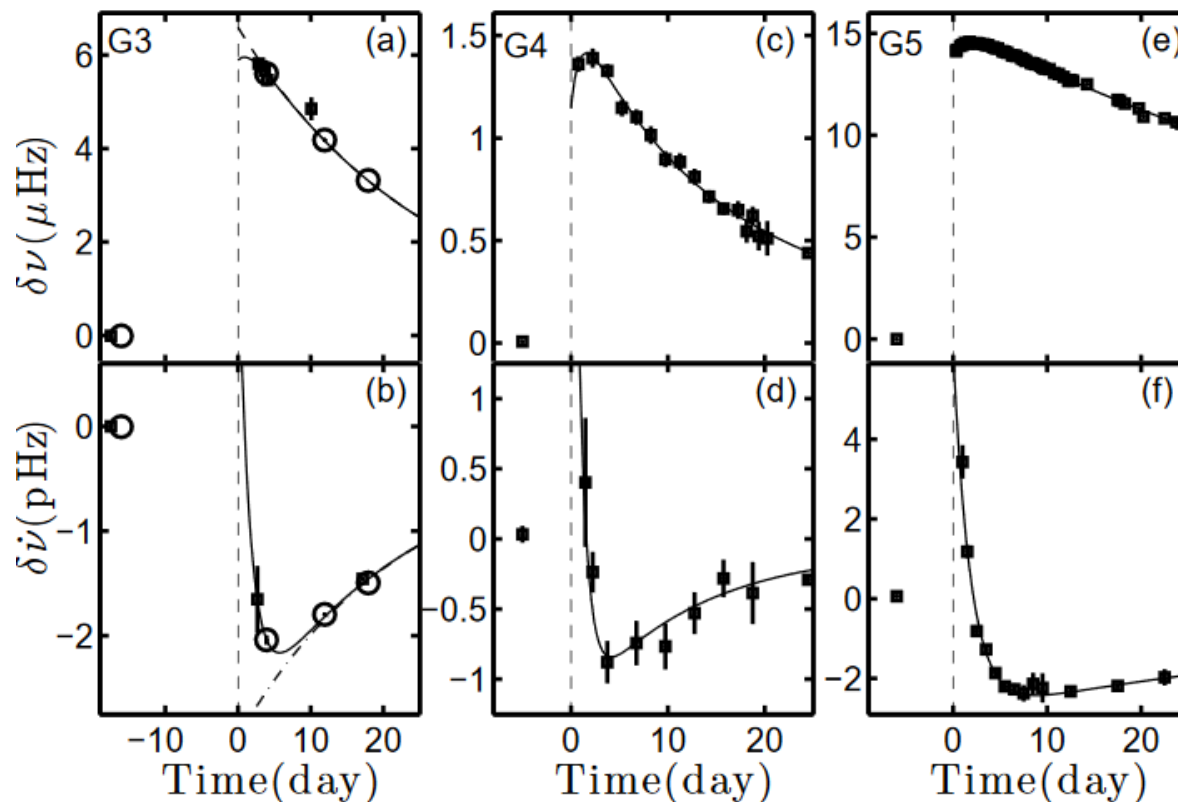
(See theoretical discussion in 1806.10168)

Delayed spin-up in Crab's glitches

Residuals: $\delta\nu = \Delta\nu_p + \Delta\dot{\nu}_p\Delta t + \sum_{i=1,2} \Delta\nu_{di} \exp(-\Delta t/\tau_i)$

$$\delta\dot{\nu} = \Delta\dot{\nu}_p + \sum_{i=1,2} \Delta\dot{\nu}_{di} \exp(-\Delta t/\tau_i)$$

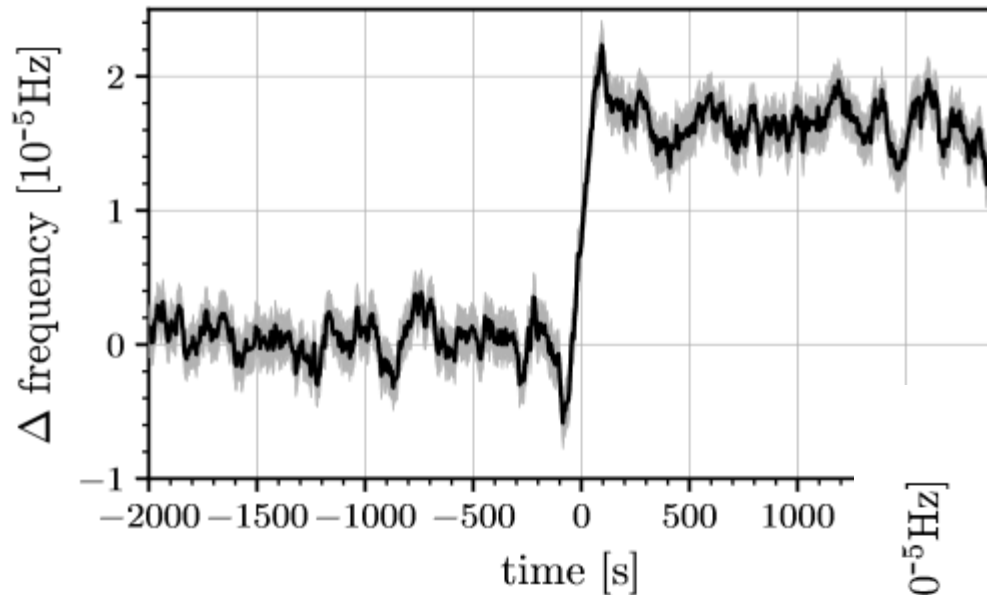
In delayed spin-up
 $\Delta\nu_{di}$ are negative,
 as well as $\Delta\dot{\nu}_{di}$



Three glitches with
 delayed spin-up.

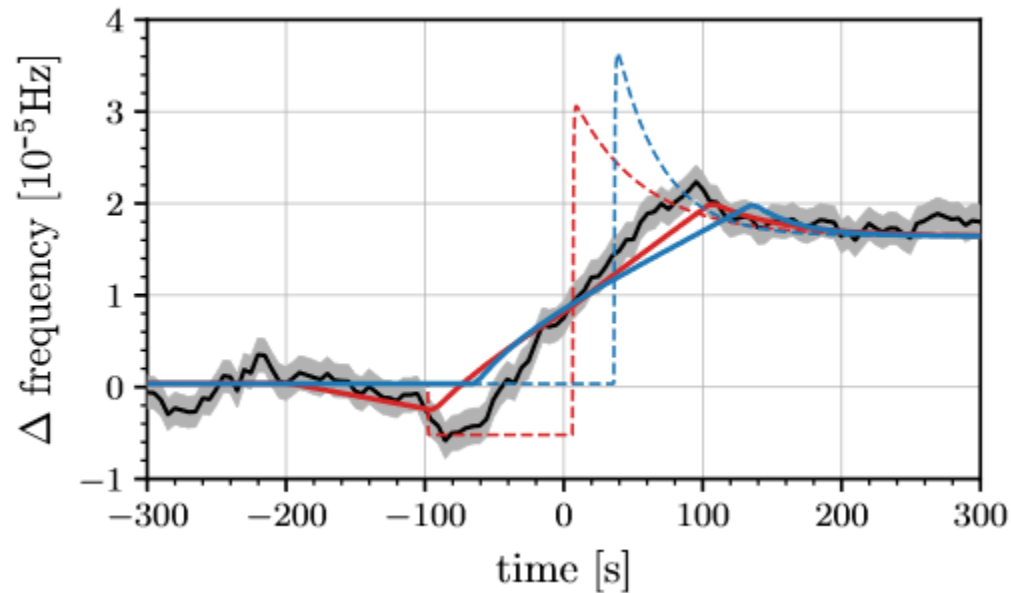
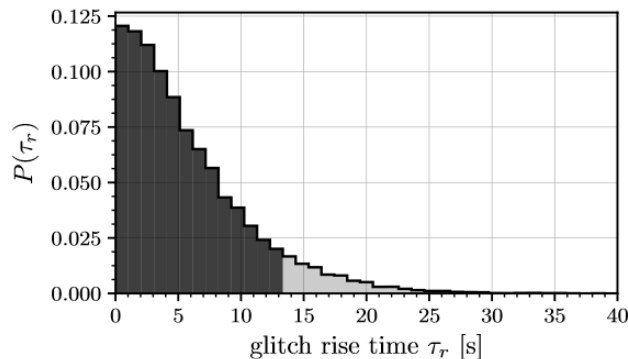
Dot-dashed (panel b) –
 is the fit without delayed
 spin-up.

Fastest Vela glitch (and fastest ever!)



12 December 2016

Glitch rise time <12 sec



1907.01124, see theoretical discussion in 2003.08724

Phenomenology and the Vela pulsar

$$\Delta J_i = I_c \Delta \Omega_i,$$

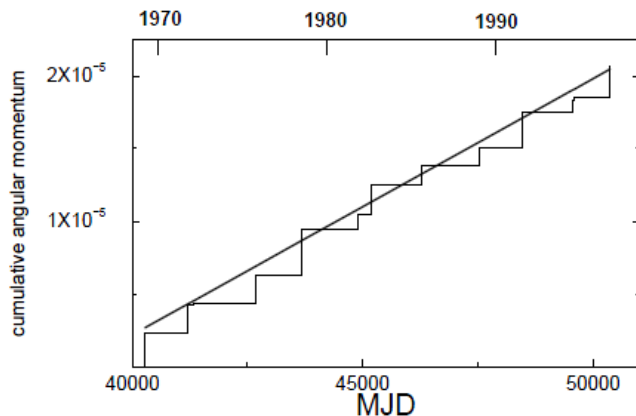
Glitches are driven by the portion of the liquid interior that is differentially rotating with respect to the crust.

$$J(t) = I_c \bar{\Omega} \sum_i \frac{\Delta \Omega_i}{\bar{\Omega}},$$

I_c – crust + everything coupled with (i.e., nearly all the star, except superfluid neutrons).

The average rate of angular momentum transfer associated with glitches is $I_c \bar{\Omega} A$,

$$A = (6.44 \pm 0.19) \times 10^{-7} \text{ yr}^{-1}. \quad \text{- Pulsar activity parameter}$$



Vela glitches are not random, they appear every ~840 days.

A – the slope of the straight line in the figure.

(A more sophisticated approach can be found in 2012.01539)

(Values are for the Vela PSR)

In Vela glitches can be related also to the outer core 1806.10168, 2001.09668

Role of the core

Only neutrons in the core are considered.

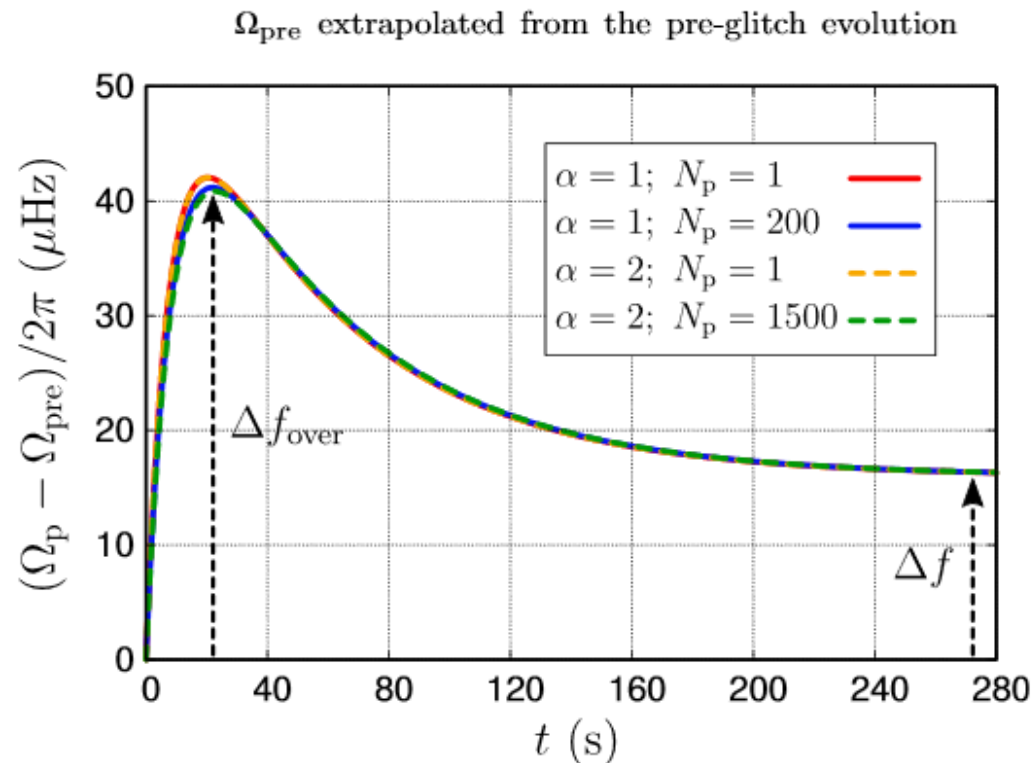
Three components: $I_n^{\text{pin}} + I_n^{\text{f}} + I_p = I$

- pinned superfluid neutrons in the outer core;
- free superfluid neutrons in the inner core;
- the rest of the star.

$$\dot{\Omega}_p = -\frac{I_n^{\text{f}}}{I_p} \dot{\Omega}_n^{\text{f}} - \frac{I_n^{\text{pin}}}{I_p} \dot{\Omega}_n^{\text{pin}} + \frac{\Gamma_{\text{ext}}}{I_p}, \quad \Gamma_{\text{ext}} = I \dot{\Omega}_{\infty}$$

$$\dot{\Omega}_n^{\text{f}} = 2 \mathcal{B}_f \Omega_n^{\text{f}} (\Omega_p - \Omega_n^{\text{f}}),$$

$$\dot{\Omega}_n^{\text{pin}} = 2 \mathcal{B}_{\text{pin}} \Omega_n^{\text{pin}} (\Omega_p - \Omega_n^{\text{pin}}),$$



General features of the glitch mechanism

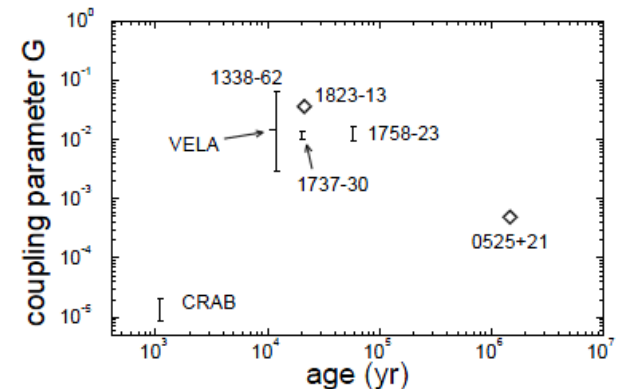
Glitches appear because some fraction (unobserved directly) rotates faster than the observed part (crust plus charged parts), which is decelerated (i.e., which is spinning-down).

$\dot{J}_{\text{res}} \leq I_{\text{res}} |\dot{\Omega}|$, The angular momentum is “collected” by the reservoir, related to differentially rotating part of a star (SF neutrons)

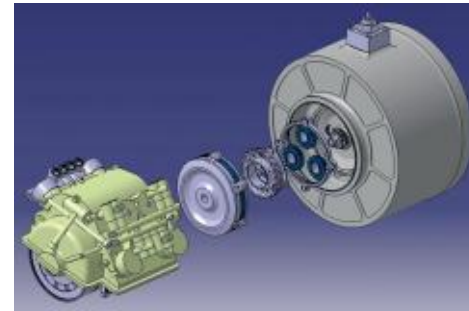
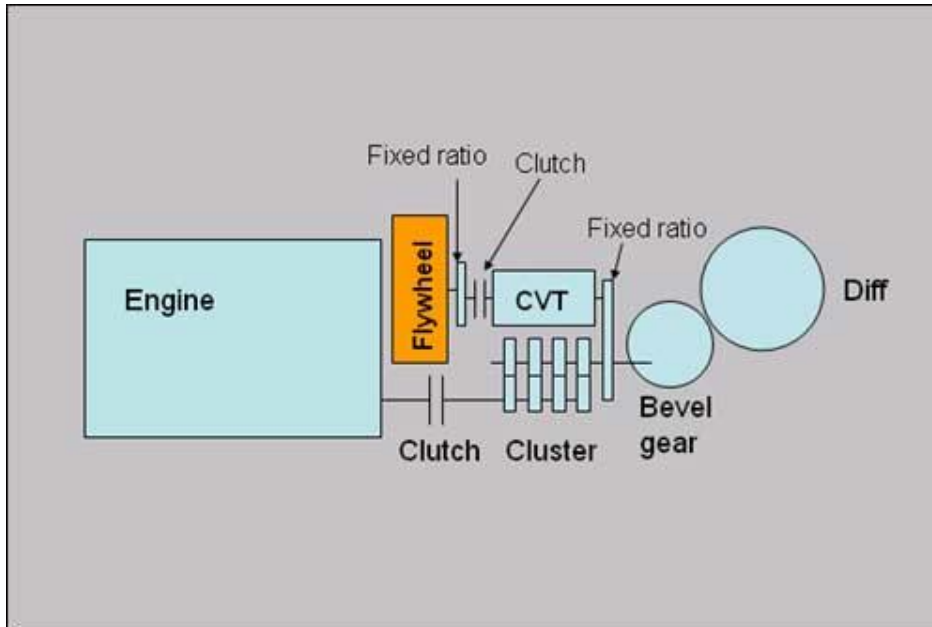
$\frac{I_{\text{res}}}{I_c} \geq \frac{\bar{\Omega}}{|\dot{\Omega}|} A \equiv G$, G – the coupling parameter. It can be slightly different in different sources.

$\frac{I_{\text{res}}}{I_c} \geq G_{\text{Vela}} = 1.4\%$. Glitch statistics for Vela provide an estimate for G.

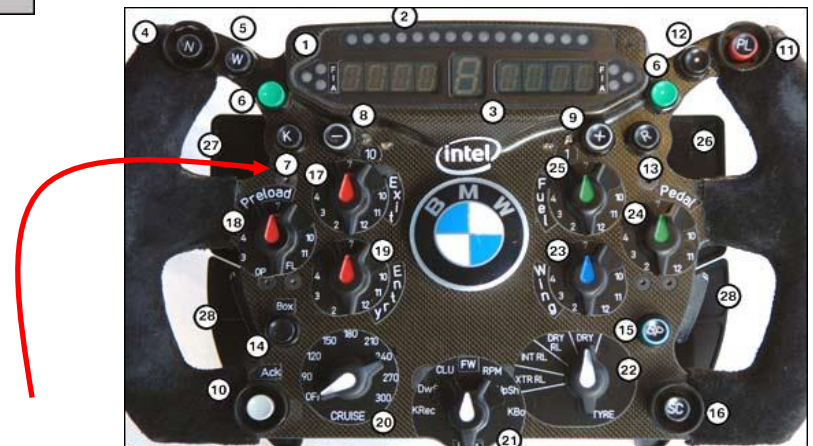
Superfluid is a good candidate to form a “reservoir” because relaxation time after a glitch is very long (~months) which points to very low viscosity.



KERS



Williams-F1 used mechanical KERS. Energy is stored in a flywheel.



Critical velocity difference

In most popular models glitches appear when the difference in angular velocity between the crust and the superfluid reaches some critical value.

$$I_{\text{super}}/I_c \sim 10^{-2}$$

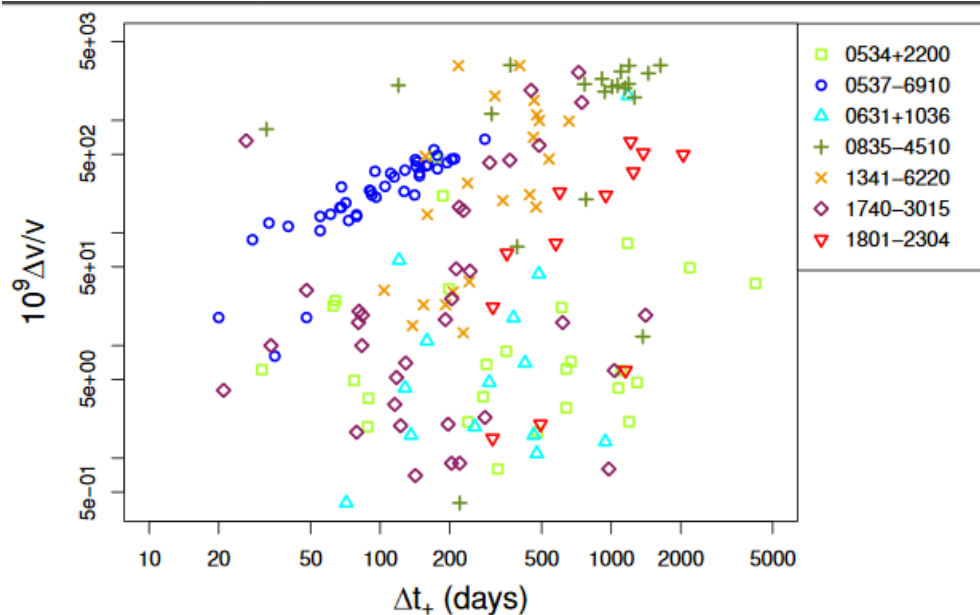
$$\Delta\Omega/\Omega \sim 10^{-6}$$

$\Delta\Omega$ – is for the crust (we see it!)

$$\Delta\Omega I_c = \Delta\Omega_{\text{super}} I_{\text{super}}$$

$$\Delta\Omega_{\text{super}} = \Delta\Omega I_c / I_{\text{super}} = \Omega 10^{-6} 10^2 = 10^{-4} \Omega$$

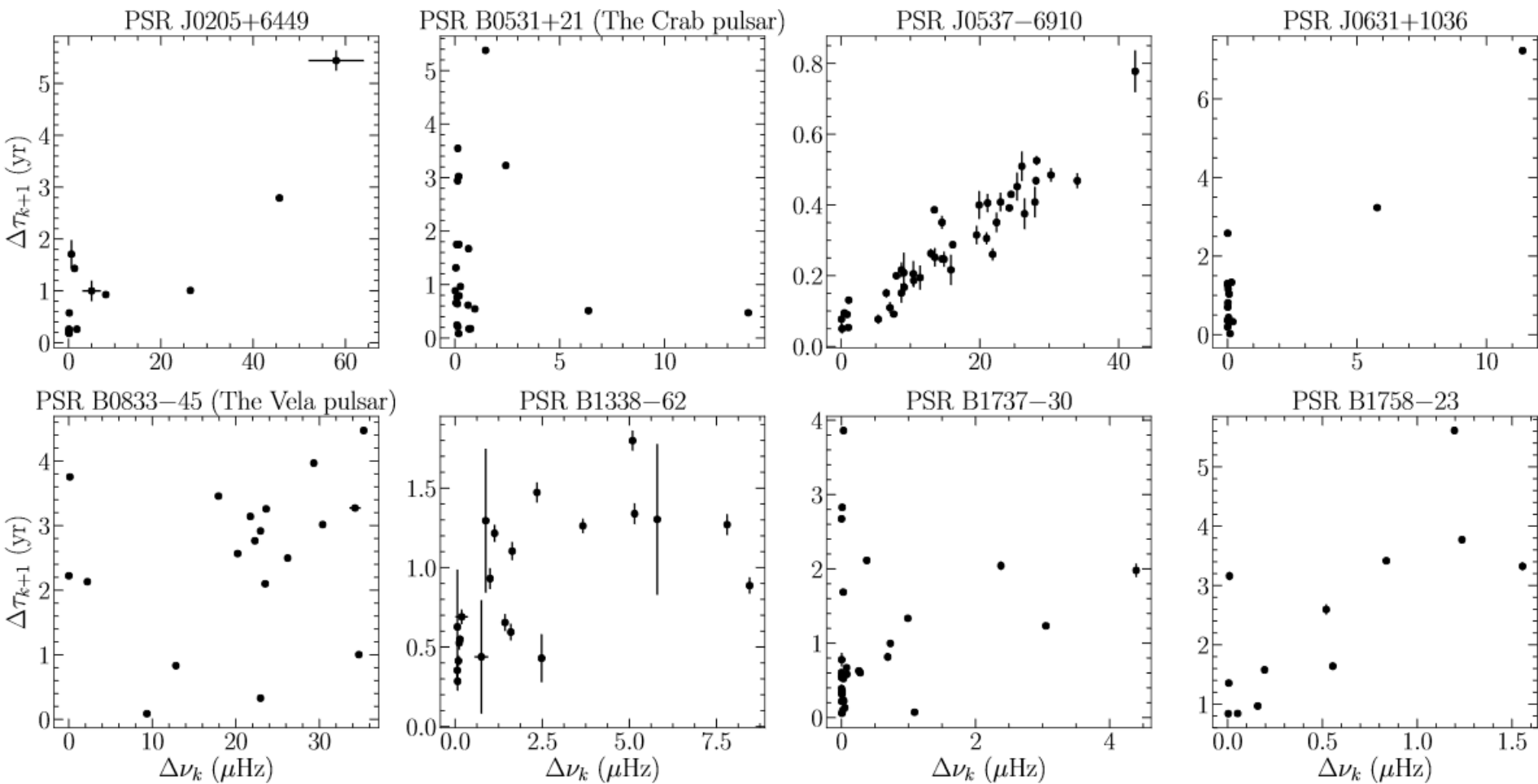
Glitch size – waiting time correlation



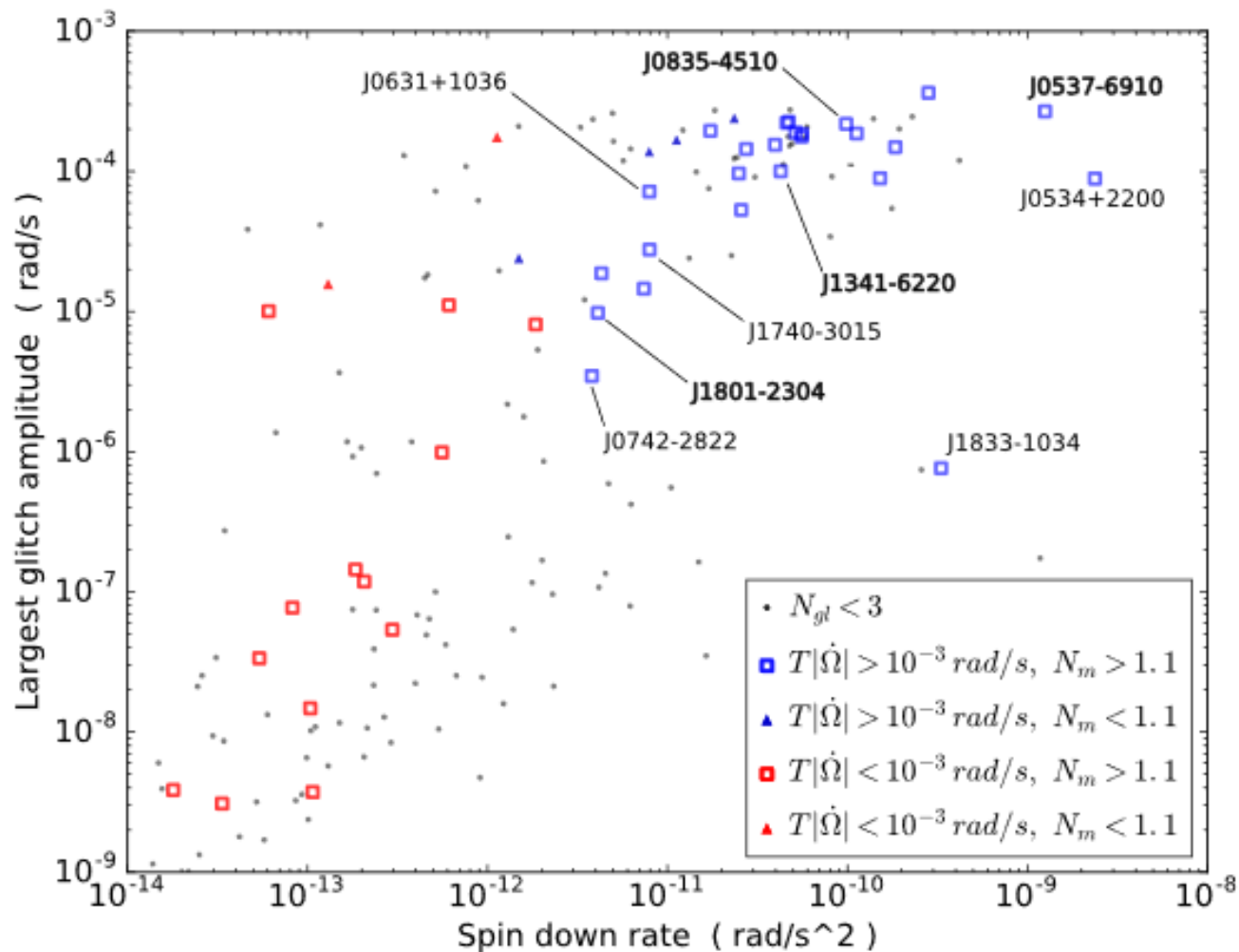
No correlation of a glitch size
with time since the previous glitch,
or with time before the next one.

Only for 0537 there is a correlation
(see 1907.09887).
It is observed in X-rays!

Glitch size vs. time to the next glitch

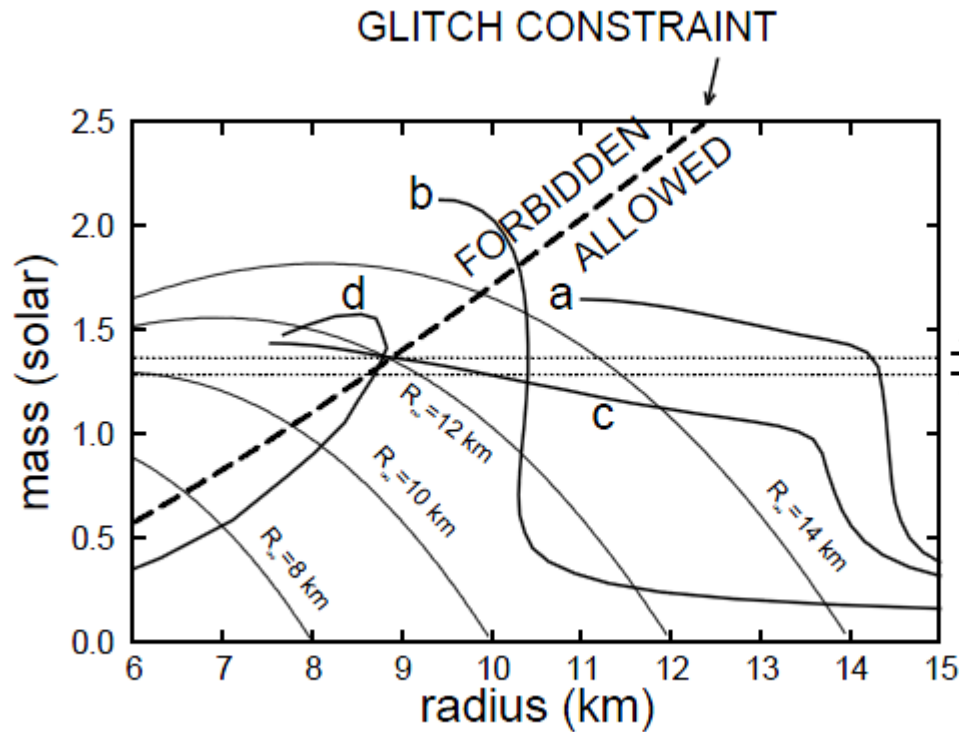


Glitch size – spin down rate correlation



1809.07834. About autocorrelations search see 1907.09143.

EoS and glitches



$$R = 3.6 + 3.9M/M_{\odot}.$$

$P_t = 0.65 \text{ MeV fm}^{-3}$
 $n_t = 0.075 \text{ fm}^{-3}$
 pressure and density
 on the core-crust boundary.

$$\Lambda \equiv (1 - 2GM/Rc^2)^{-1}$$

$$\frac{\Delta I}{I} \simeq \frac{28\pi}{3} \frac{P_t R^4}{GM^2} \left[1 + \frac{8P_t}{n_t m_n c^2} \frac{4.5 + (\Lambda - 1)^{-1}}{\Lambda - 1} \right]^{-1}$$

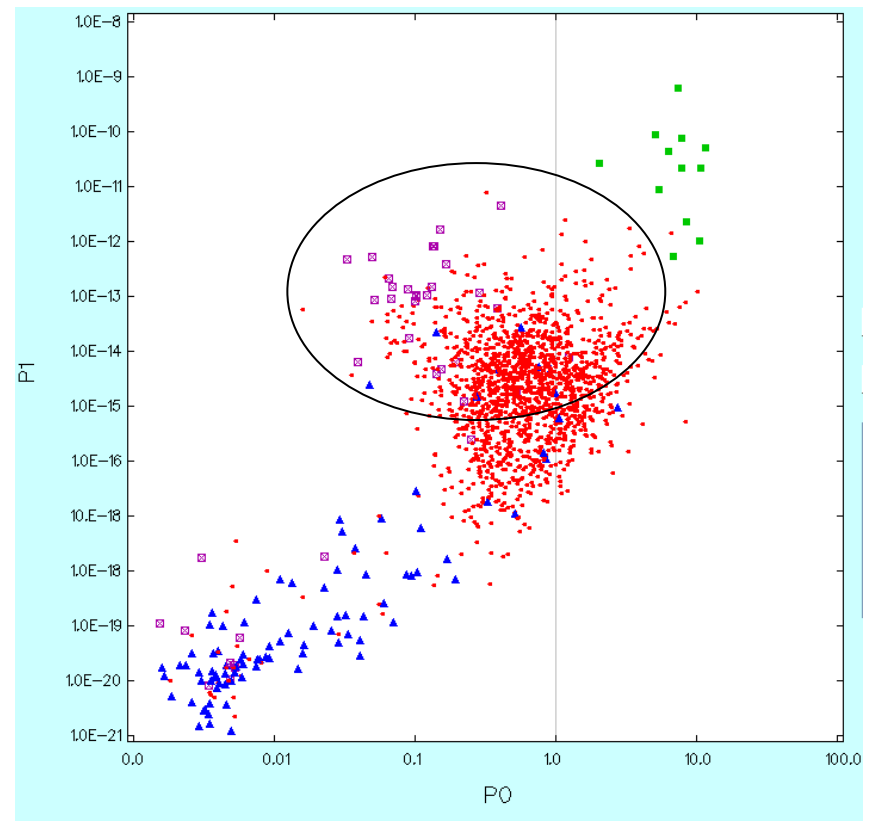
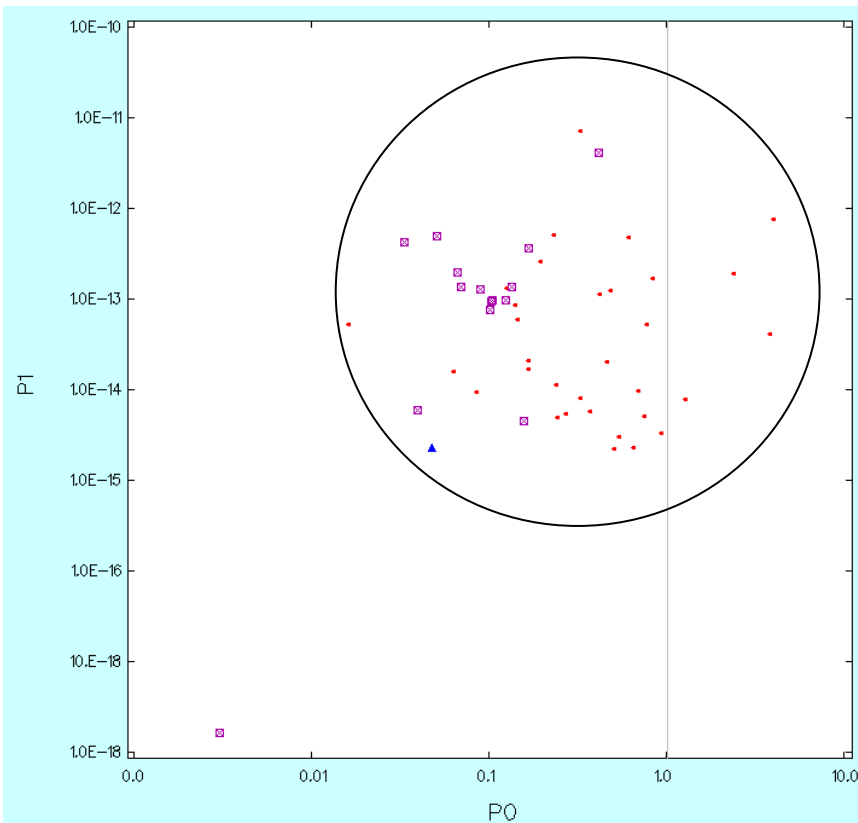
$$\Delta I / (I - \Delta I) \geq \Delta I / I_c \geq I_{\text{res}} / I_c \geq 0.014.$$

Link et al. 0001245

See some critics in 1207.0633 “Crust is not enough” and 1210.8177
 Further discussion – in 1404.2660, 1809.07834.

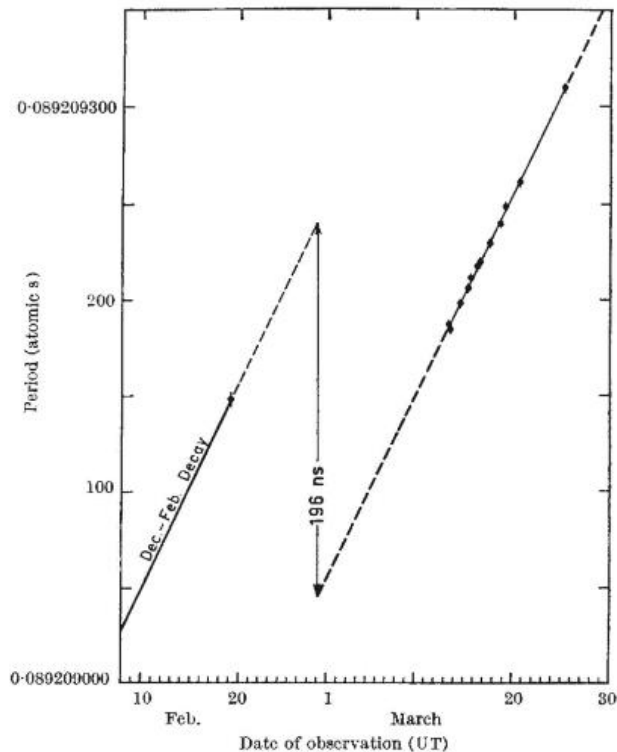
Which PSRs do glitch?

On average young pulsars with larger spin-down glitch more frequently

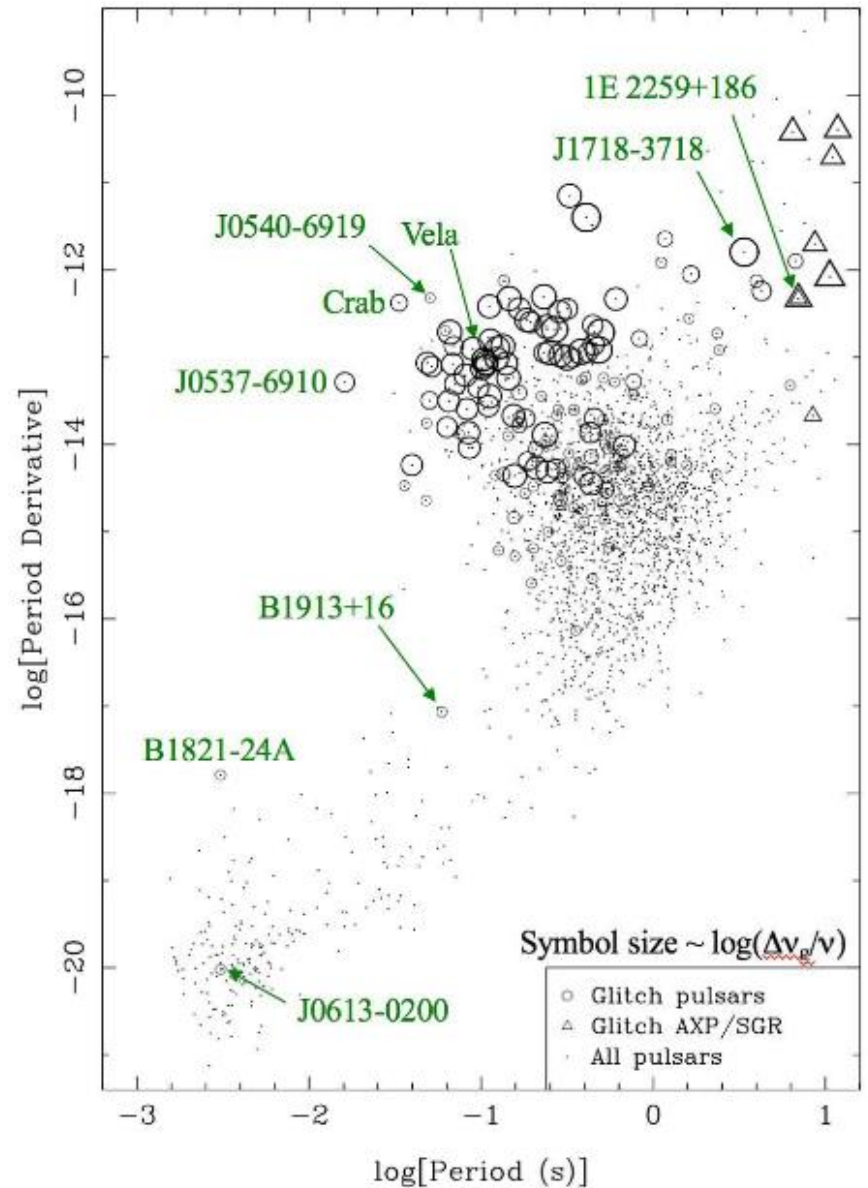


P-Pdot

>520 glitches
in >180 PSRs



Vela glitch in 1969

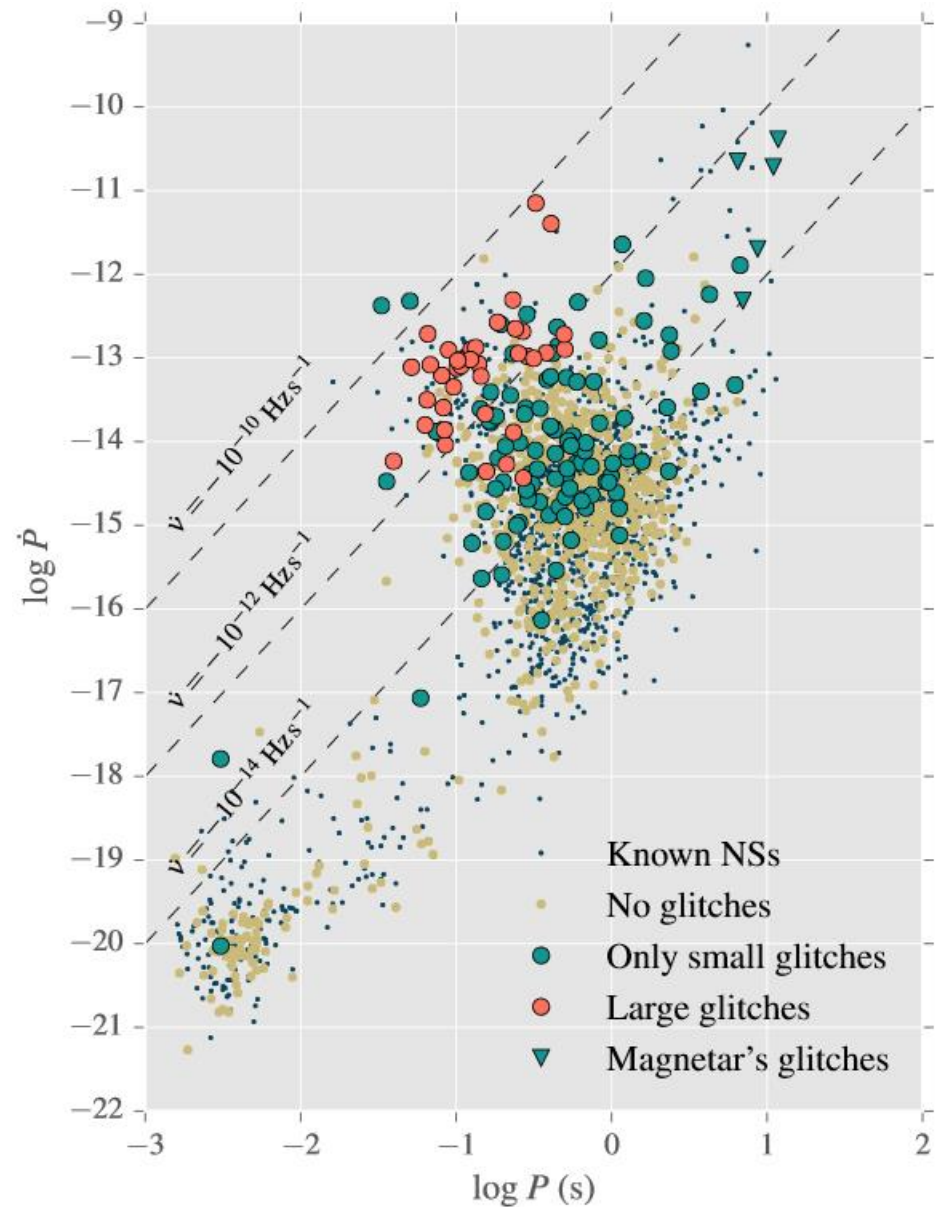
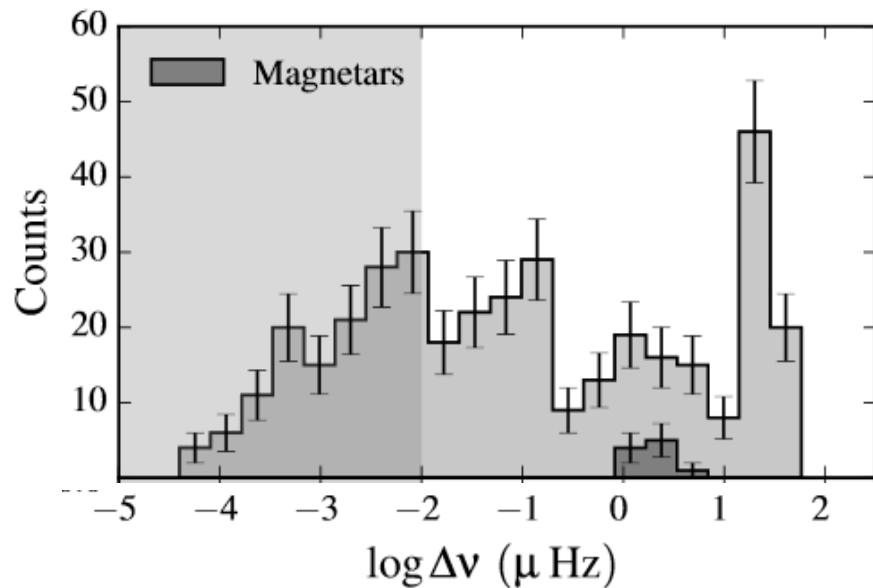


1801.04332

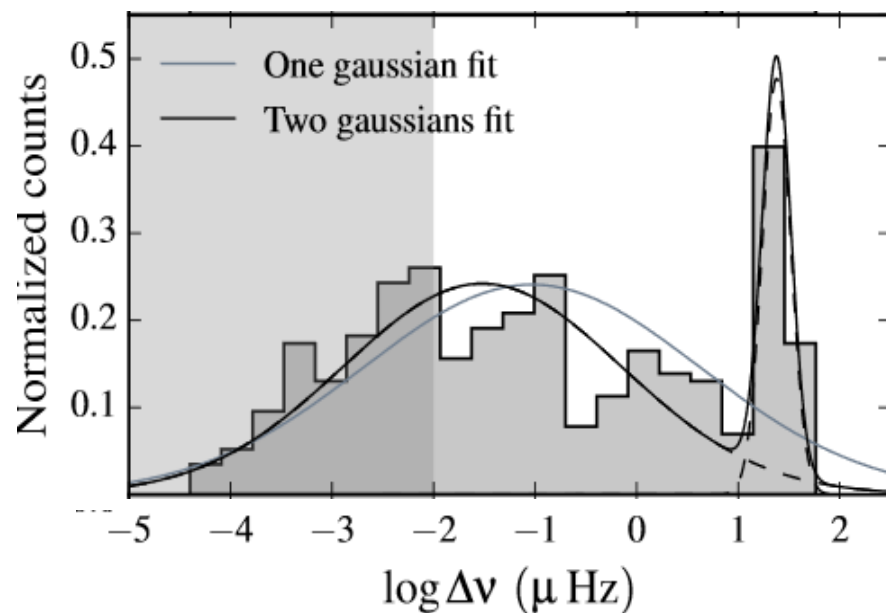
In 2020: ~600 glitches in ~200 PSRs

Statistics

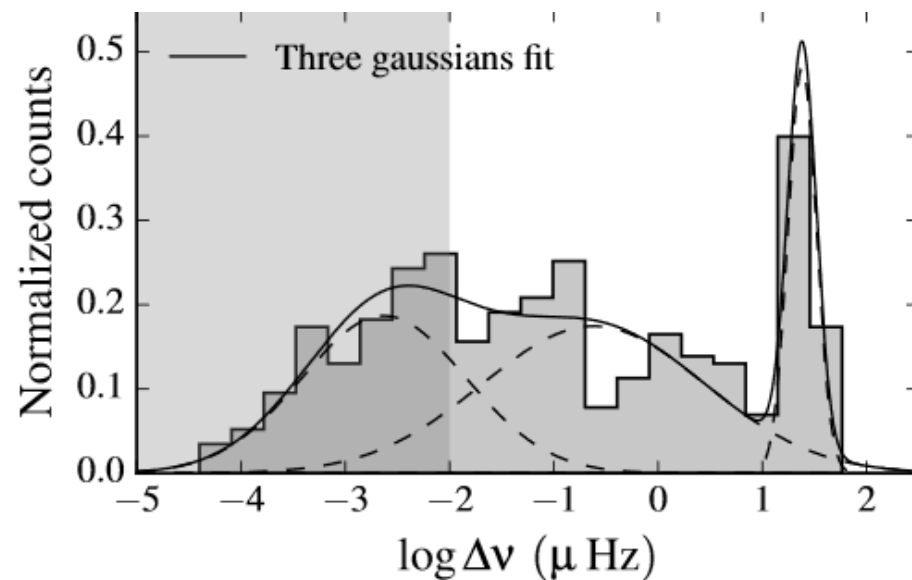
384 glitches in 141 NSs



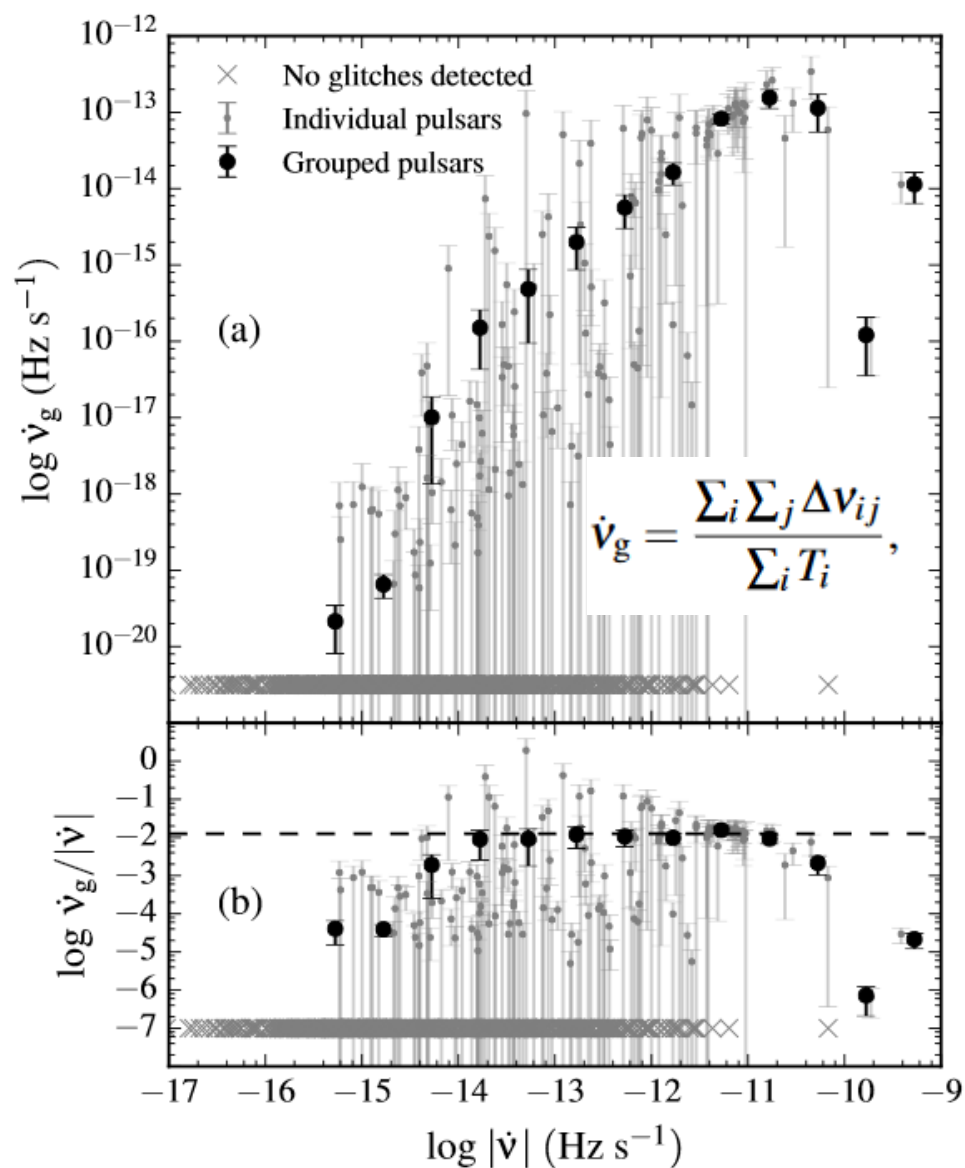
Glitches properties

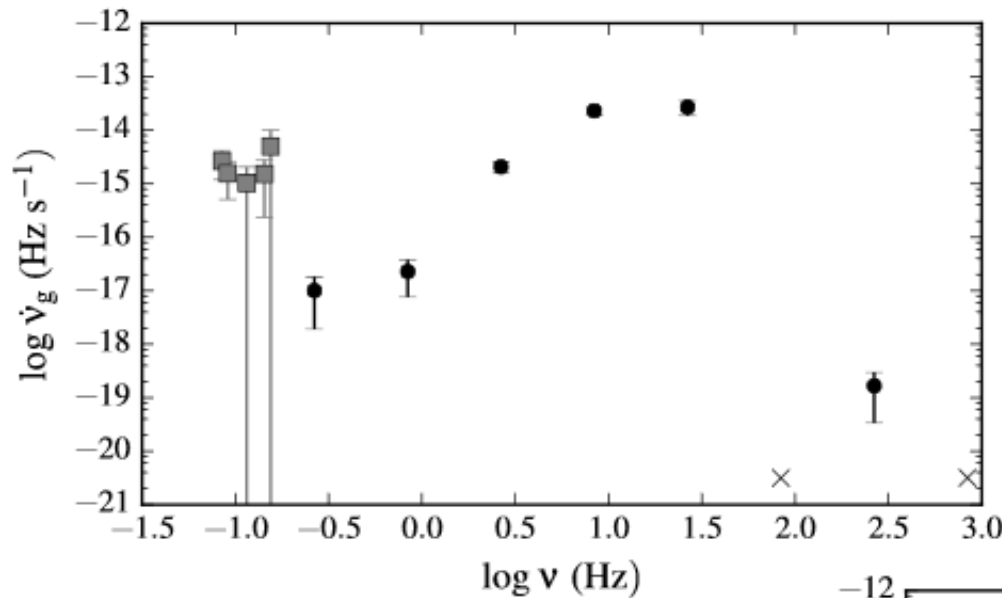


384 glitches in 141 NSs



# bin	$\log \dot{\nu} $ (Hz s ⁻¹)	$\sum T_i$ (yr)	N_ℓ	N_t	N_{pg}	N_p
1	-16.75	117	0	0	0	7
2	-16.25	430	0	0	0	25
3	-15.75	1233	0	0	0	70
4	-15.25	2478	0	3	3	139
5	-14.75	2675	0	11	8	142
6	-14.25	1973	0	25	16	105
7	-13.75	2083	0	35	20	113
8	-13.25	1706	1	29	18	105
9	-12.75	1312	3	26	14	81
10	-12.25	745	4	38	15	48
11	-11.75	493	8	74	15	33
12	-11.25	357	37	78	18	20
13	-10.75	66	13	19	5	5
14	-10.25	44	4	8	2	3
15	-9.75	16	0	2	1	1
16	-9.25	46	0	25	1	1

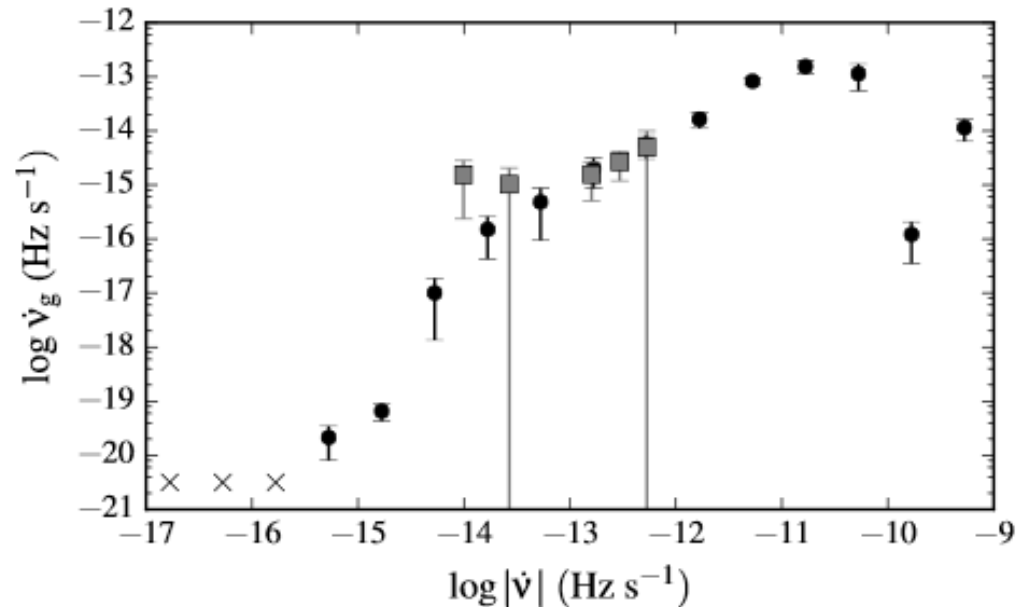




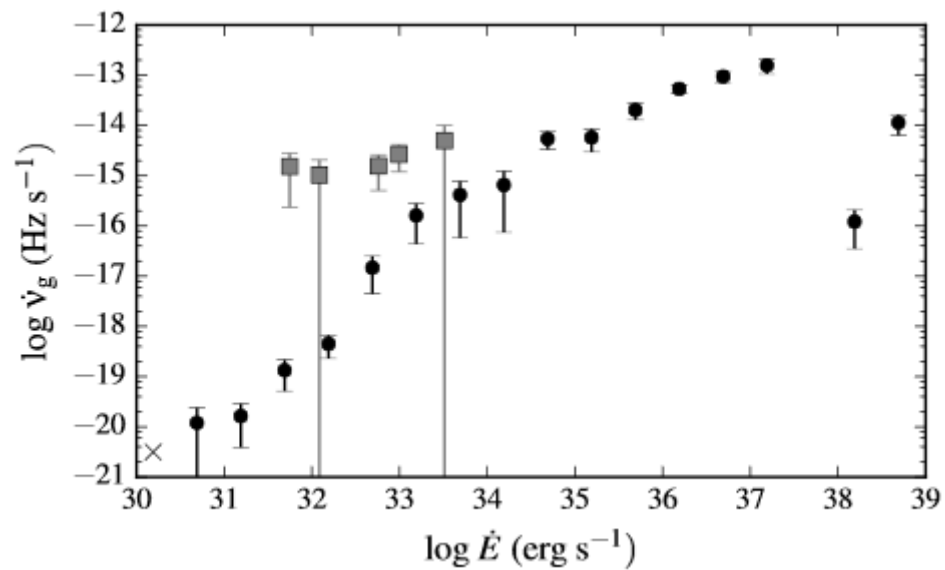
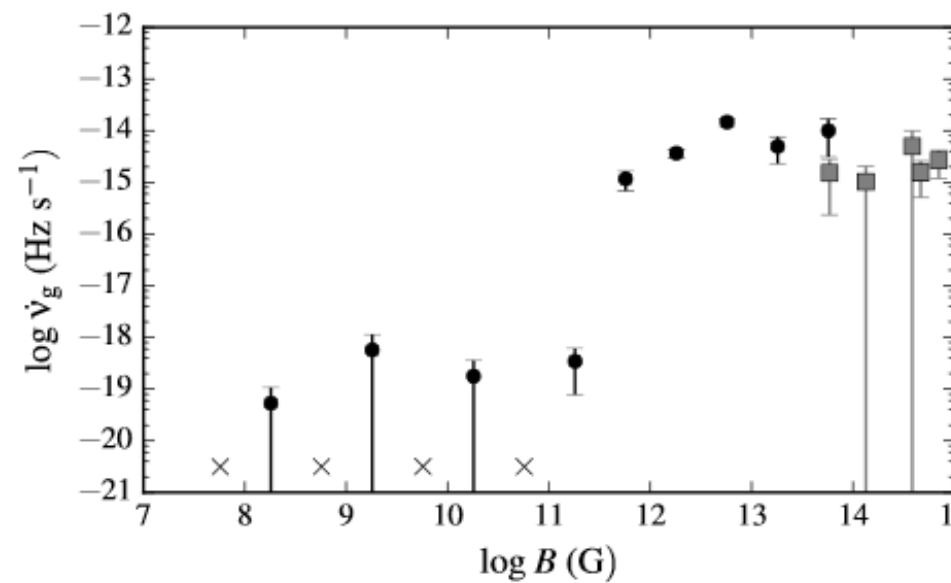
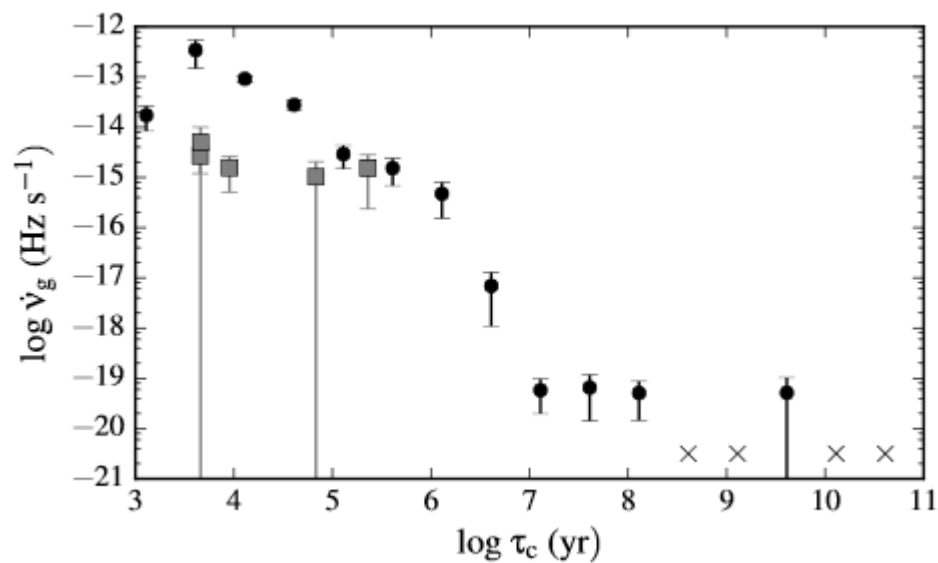
All PSRs in the survey are included, also those with no detected glitches.

$$\dot{v}_g = \frac{\sum_i \sum_j \Delta v_{ij}}{\sum_i T_i},$$

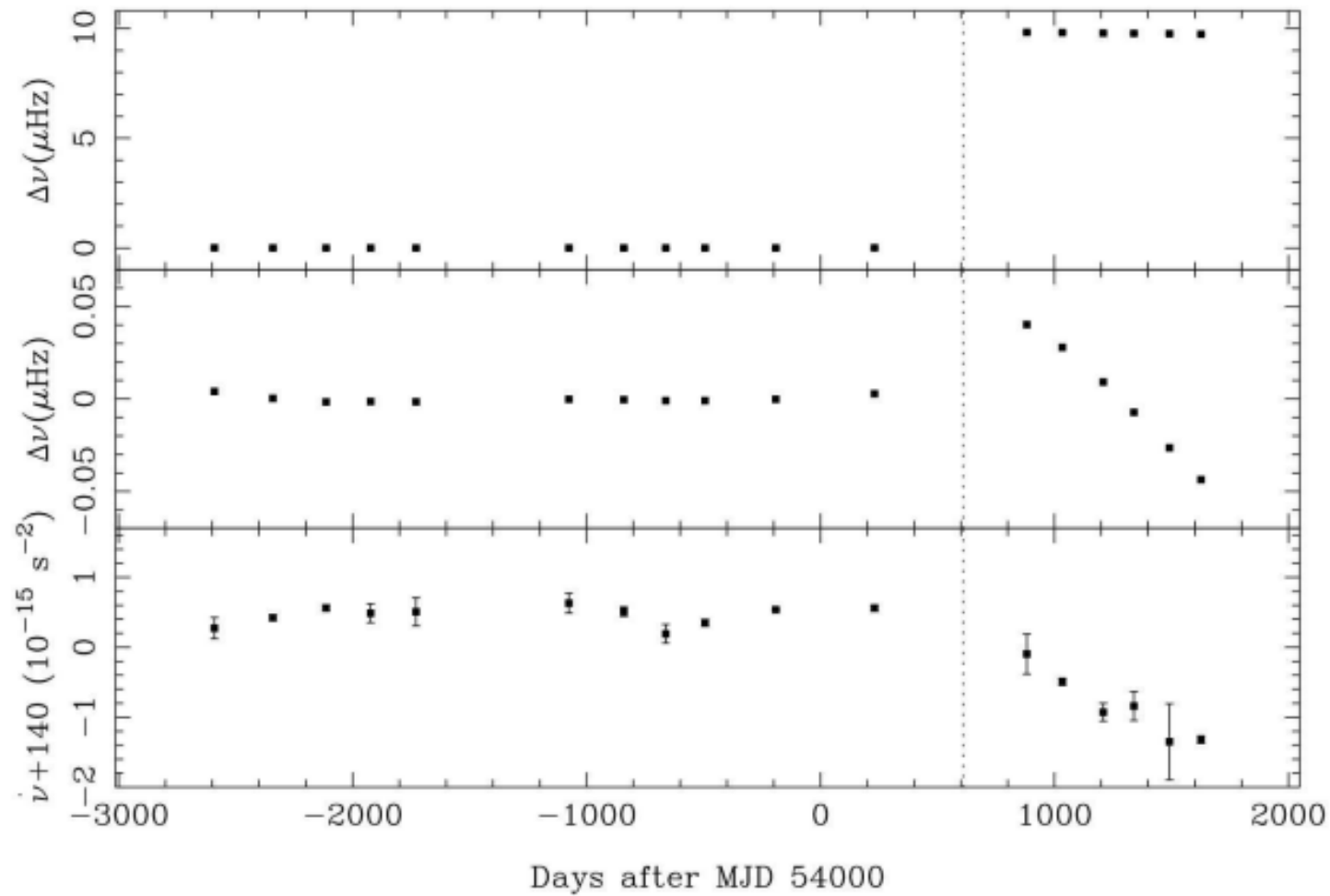
The double sum runs over every change in frequency Δv_{ij} due to the glitch j of the pulsar i , and T_i is the time over which pulsar i has been searched for glitches.



$$\dot{v}_g = \frac{\sum_i \sum_j \Delta v_{ij}}{\sum_i T_i},$$

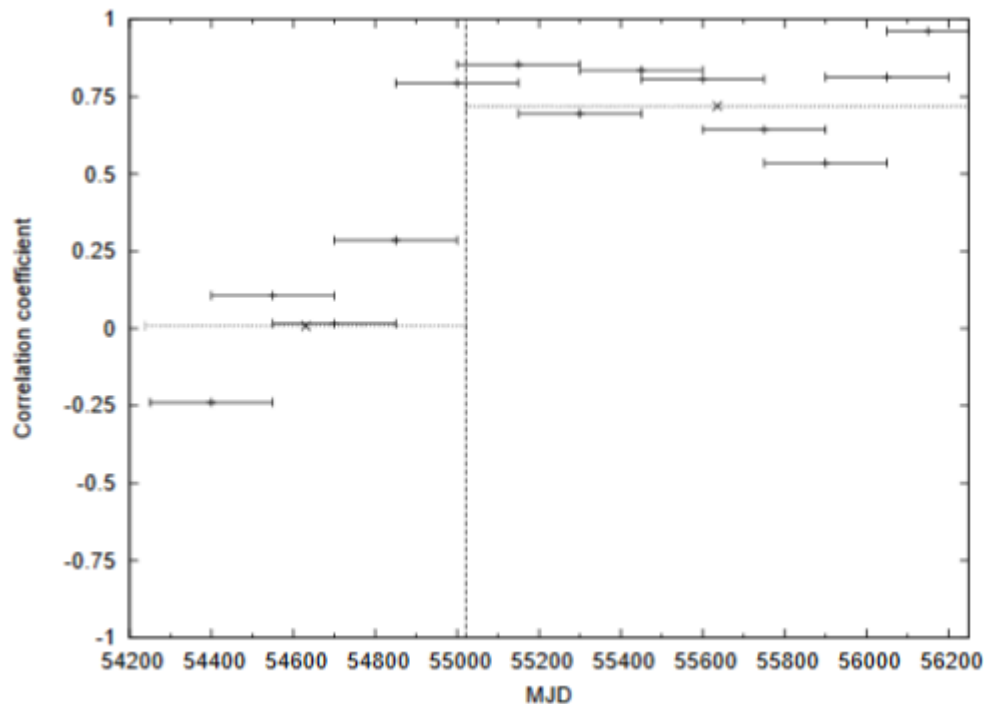


The largest glitch



$33 \cdot 10^{-6}$

Glitch and radio properties



PSR J0742-2822

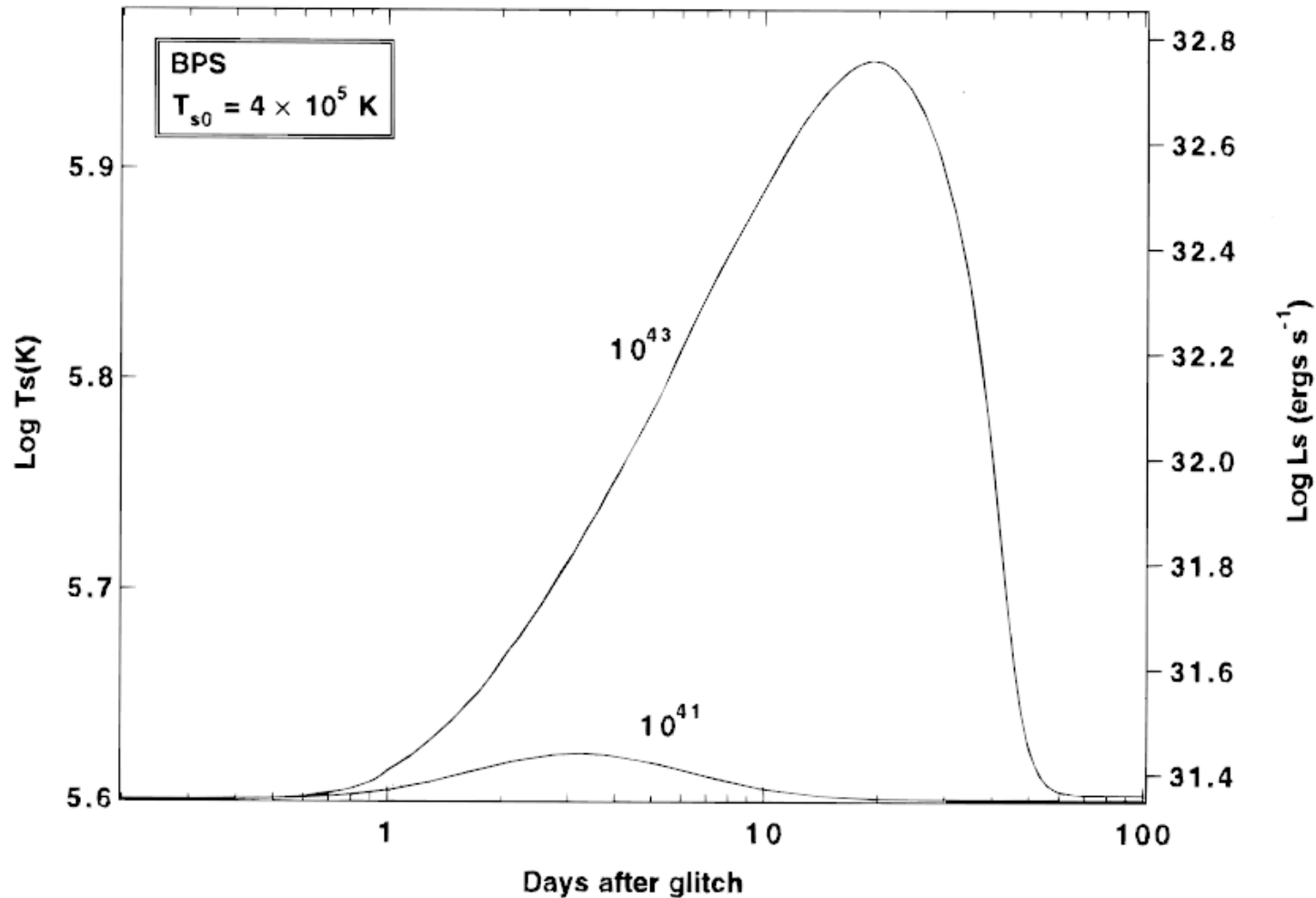
exhibits two distinct emission states that are identified by discrete changes in the observed pulse profile.

Correlation between frequency derivative and smoothed pulse shape parameter for overlapping 300-day intervals.

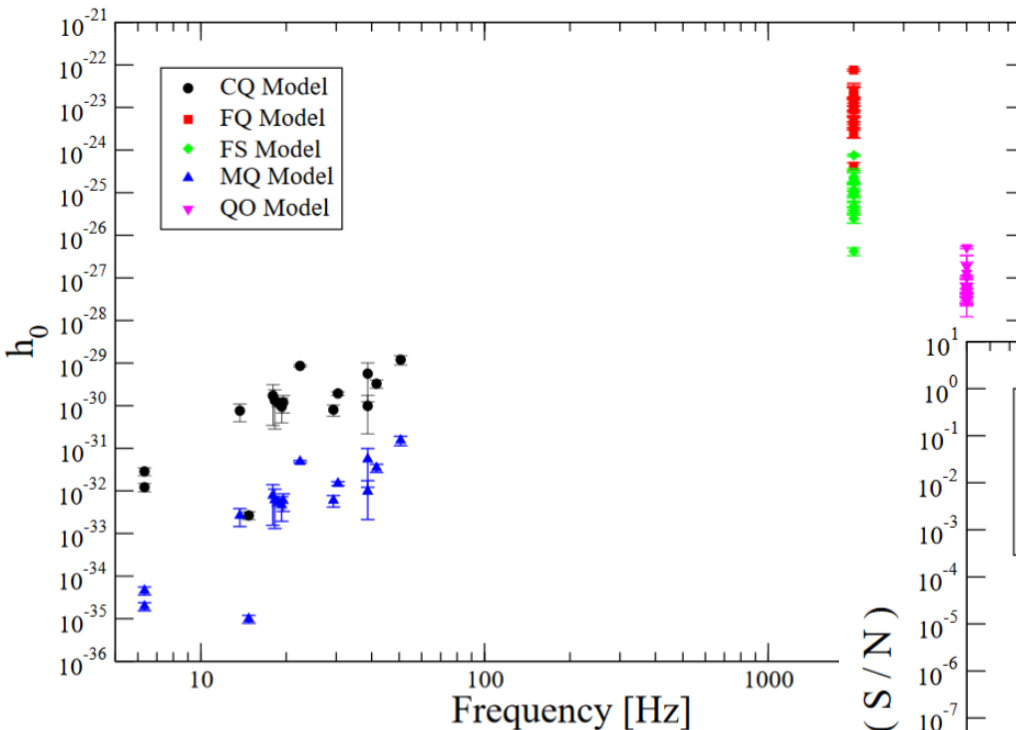
The vertical dashed line at MJD 55022 indicates the epoch of a glitch.

Also shown with dotted bars is the same correlation when computed for the entire pre and post-glitch epochs.

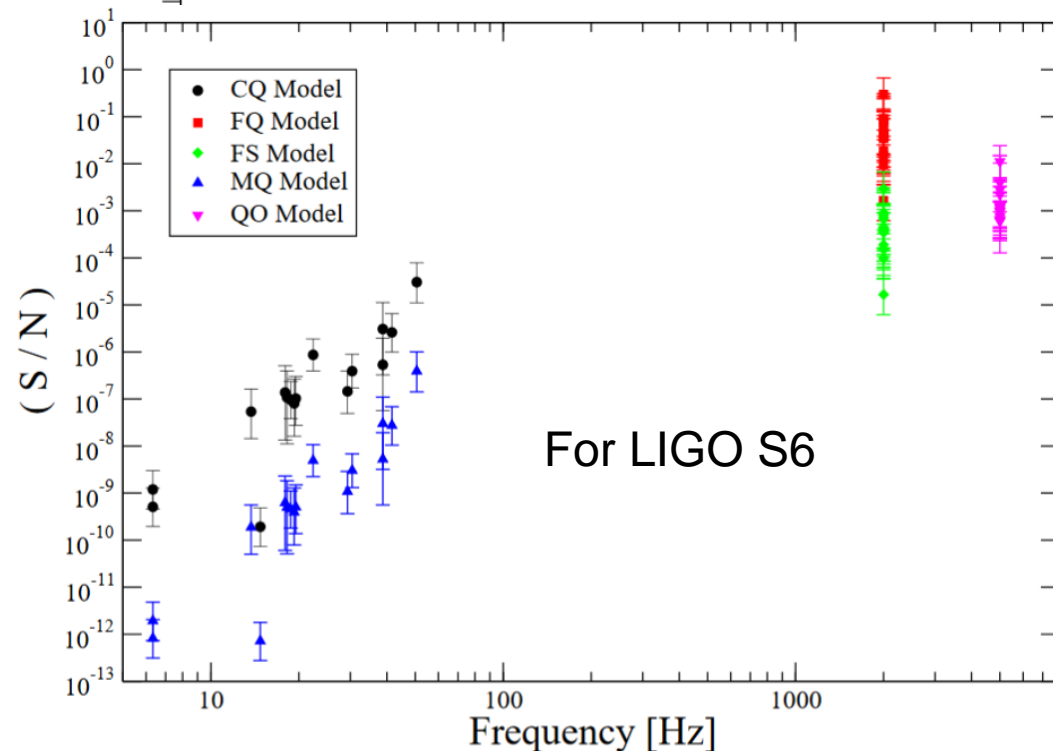
Thermal effect of a glitch



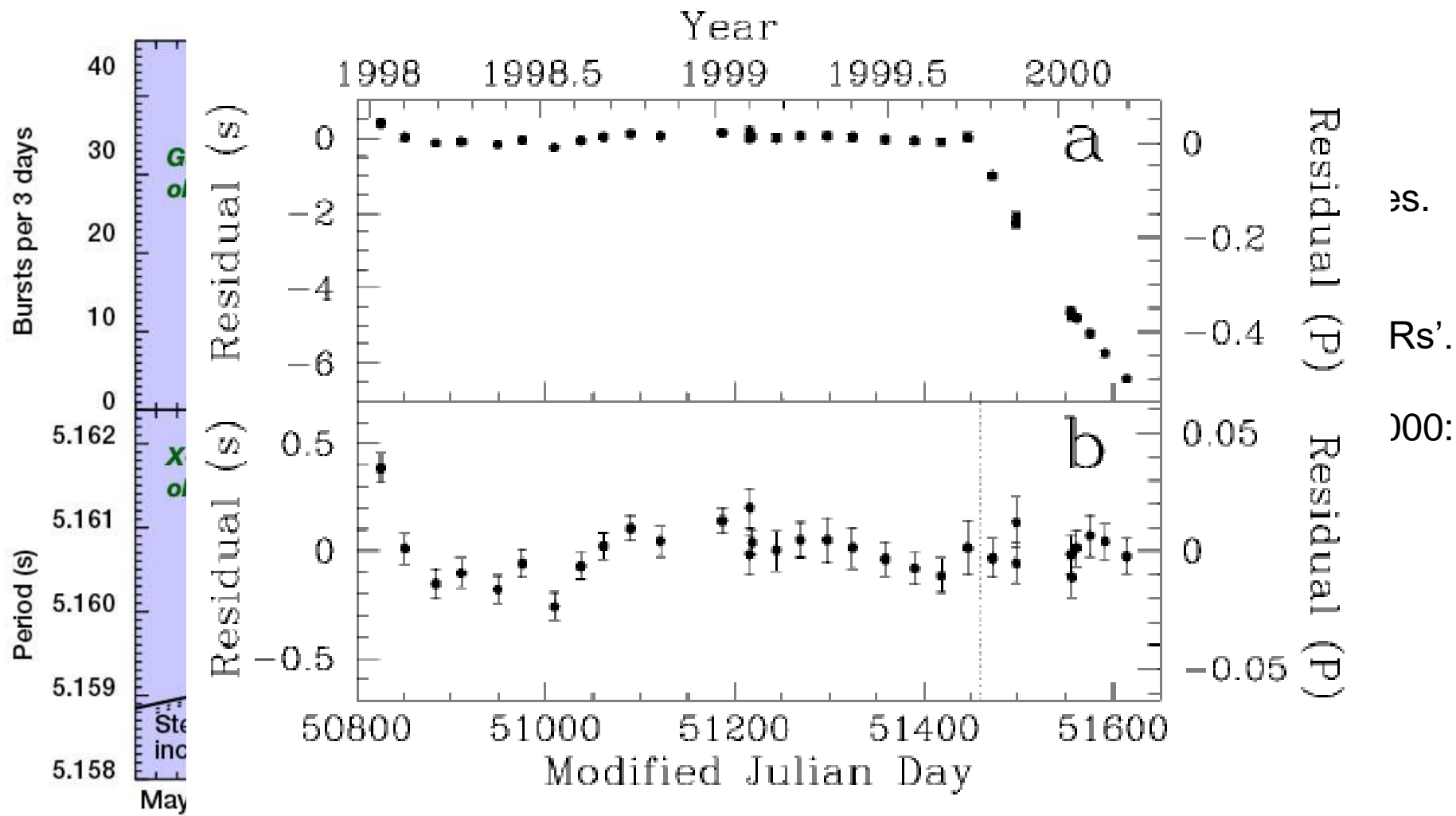
Gravitational waves from glitches



In some very optimistic models GW signals from PSRs glitches can be detected already with existing detectors (aLIGO, adVIRGO).



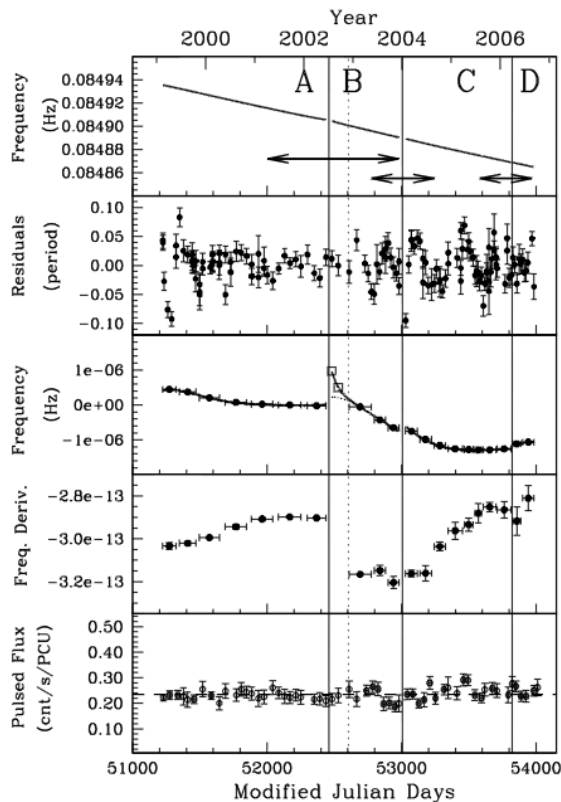
Glitches of magnetars



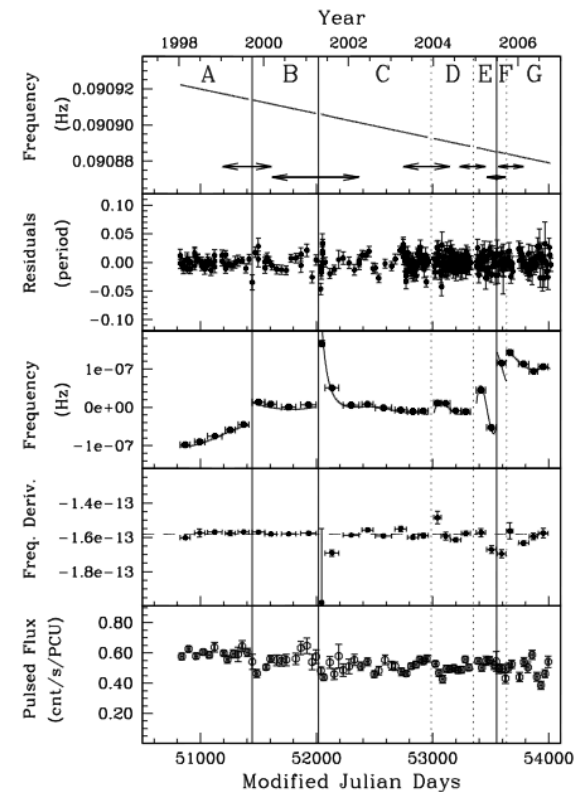
About modeling of magnetar bursts see 1203.4506:
glitches always are accompanied by energy release.

Glitches and bursts

Sometime magnetar glitches are related to bursts, sometime – not.



The pulsed flux was nearly constant during glitches.

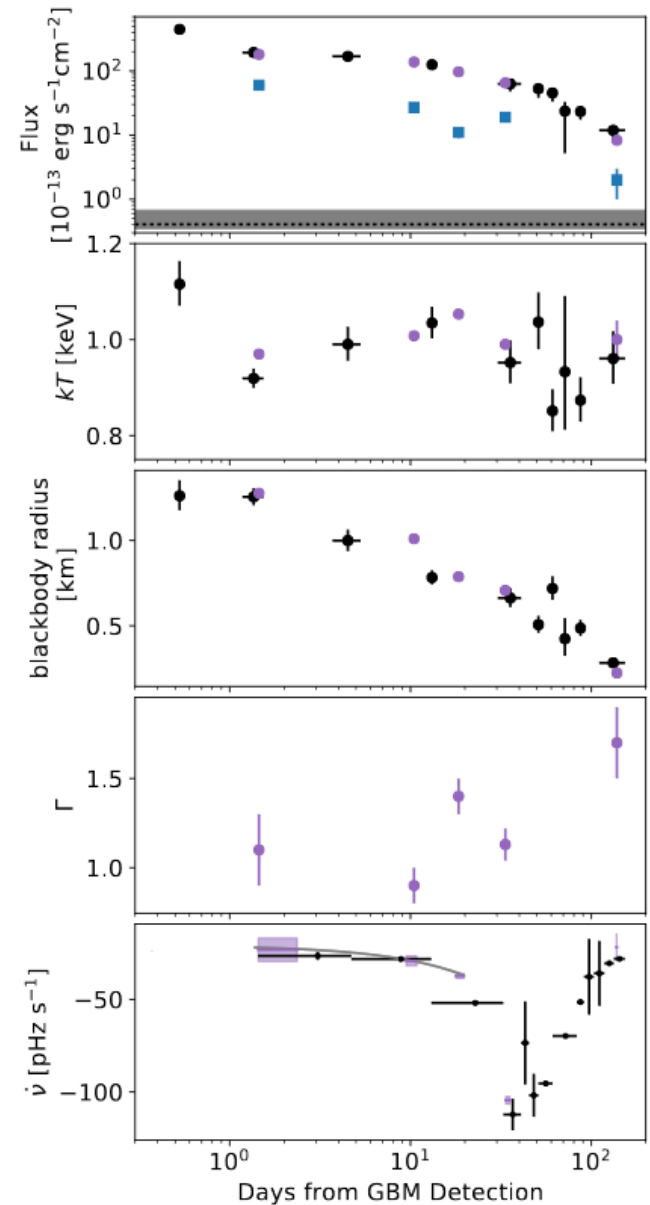
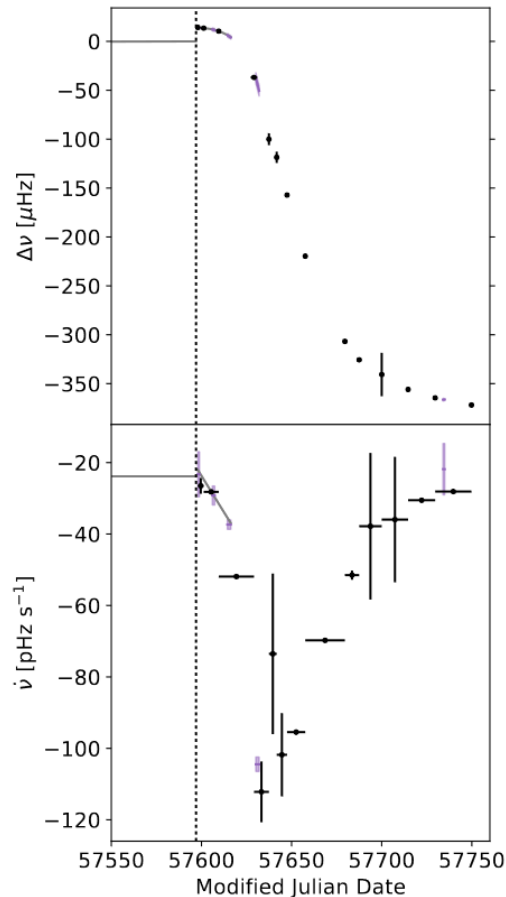


Glitch and bursts from PSR J1119–6127

Young
highly magnetized
radio pulsar.

Outburst with
many flares.

Glitch properties
confirm the model
of magnetospheric
perturbation and
energy release.
Spin behavior
correlates with
pulse profile
and spectral changes.



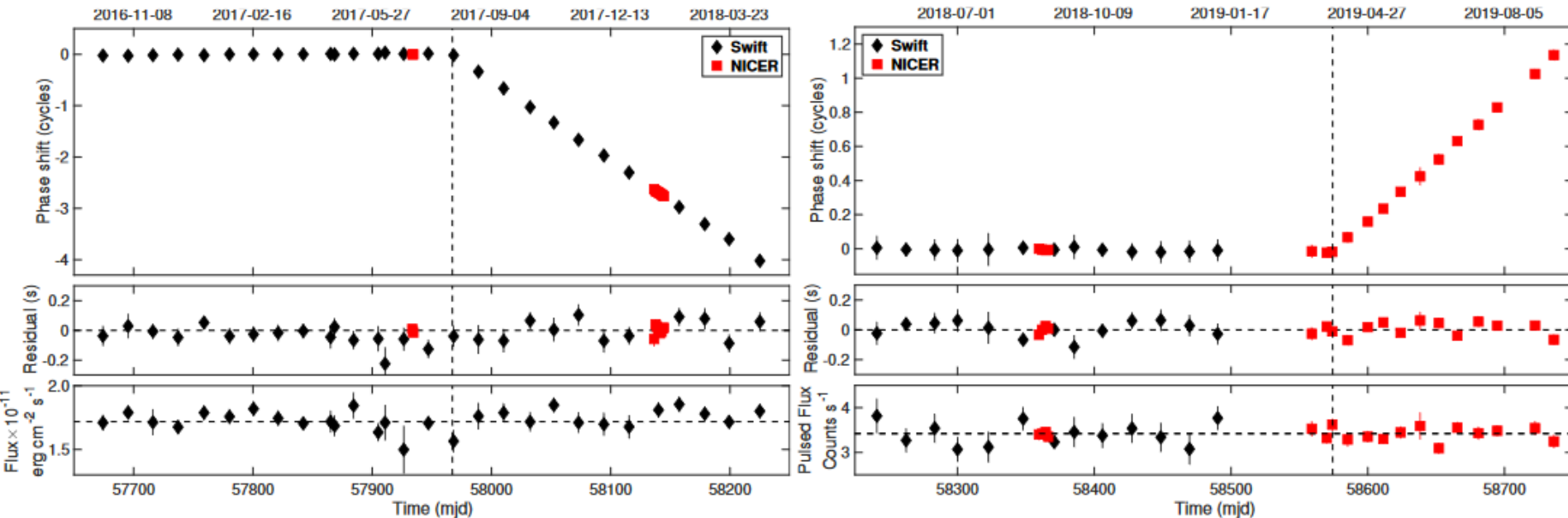
Quiet magnetar glitches and anti-glitches

1E 2259+586 – the magnetar that anti-glitched.

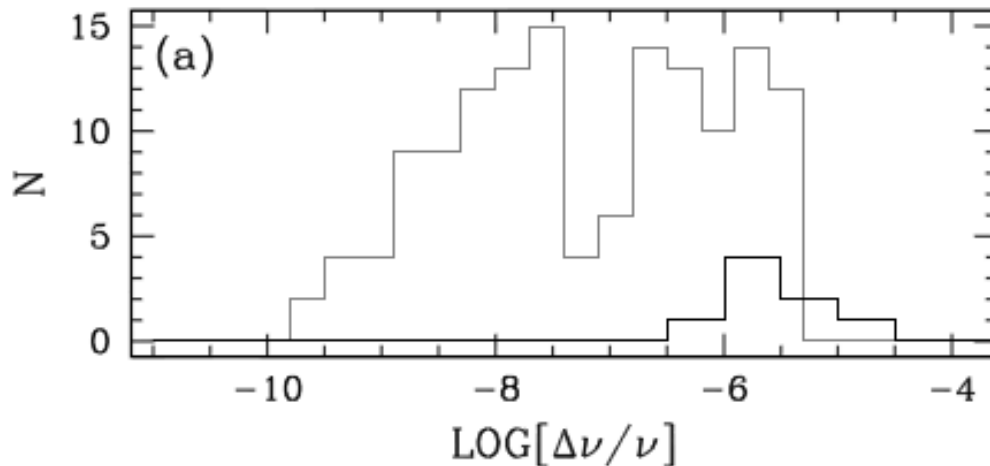
The new anti-glitch is similar to the original one.

But no changes in flux or/and pulse profile are observed.

A new glitch also was not accompanied with any changes in flux or/and profile.

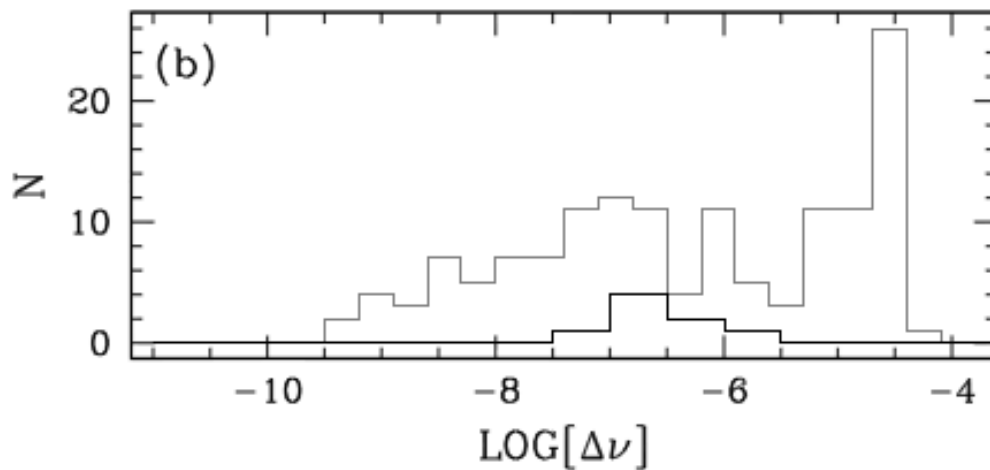


PSRs vs. magnetars



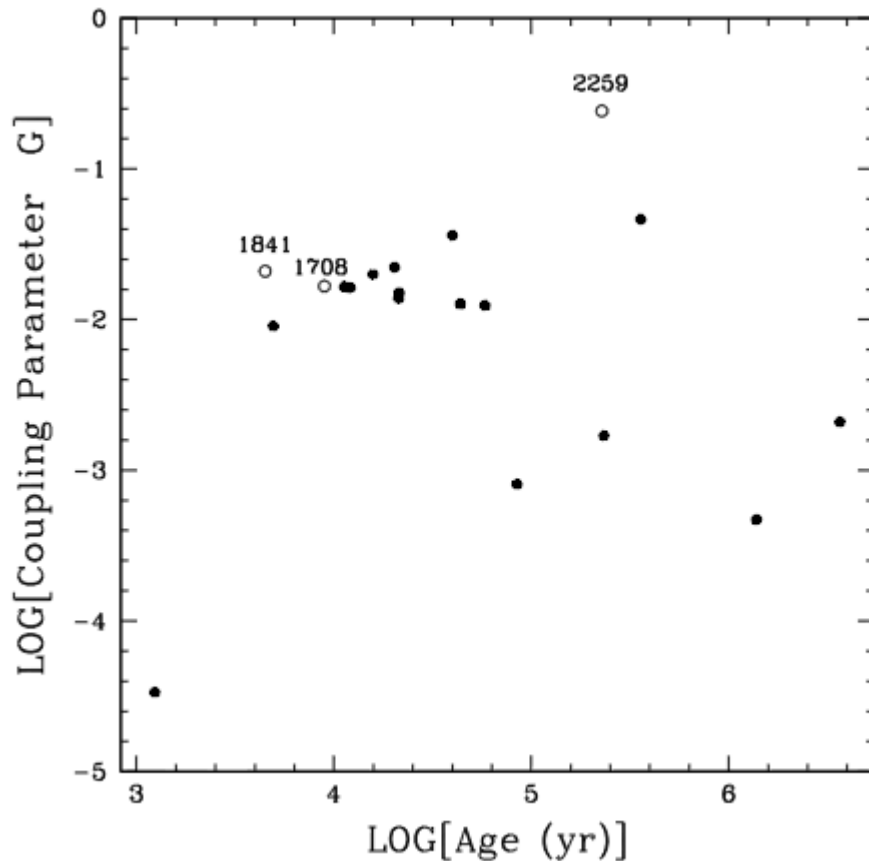
Nearly all known persistent AXPs now seem to glitch.

In terms of fractional frequency change, AXPs are among the most actively glitching neutron stars, with glitch amplitudes in general larger than in radio pulsars.



However, in terms of absolute glitch amplitude, AXP glitches are unremarkable.

Are PSRs and magnetar glitches similar?



$$\dot{J}_{\text{res}} \leq I_{\text{res}} |\dot{\Omega}|, \quad \frac{I_{\text{res}}}{I_c} \geq \frac{\bar{\Omega}}{|\dot{\Omega}|} A \equiv G,$$

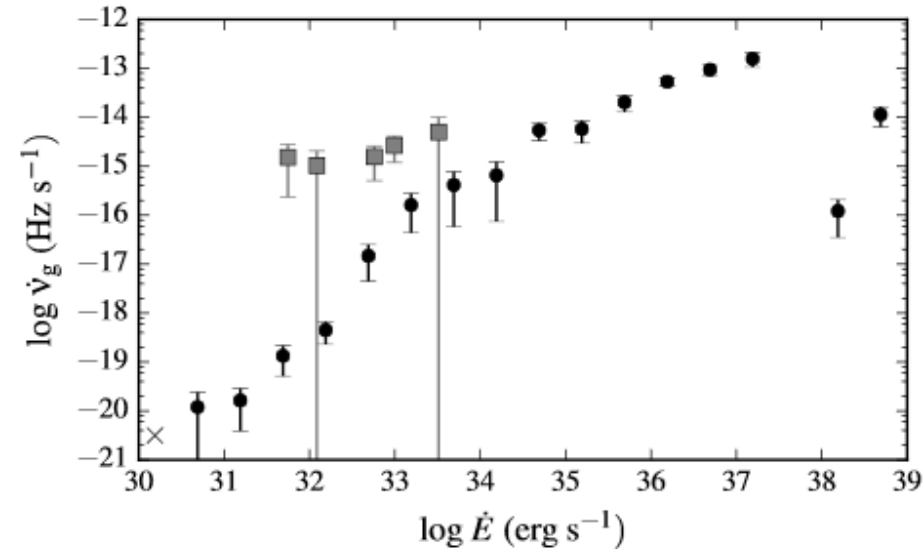
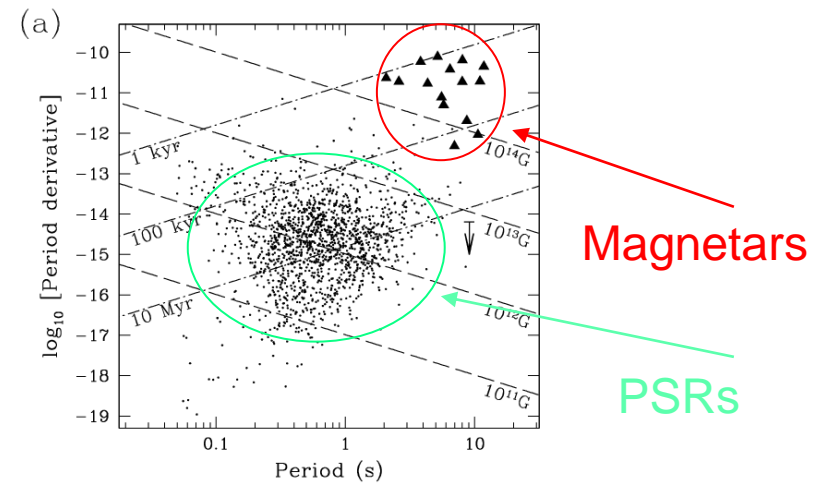
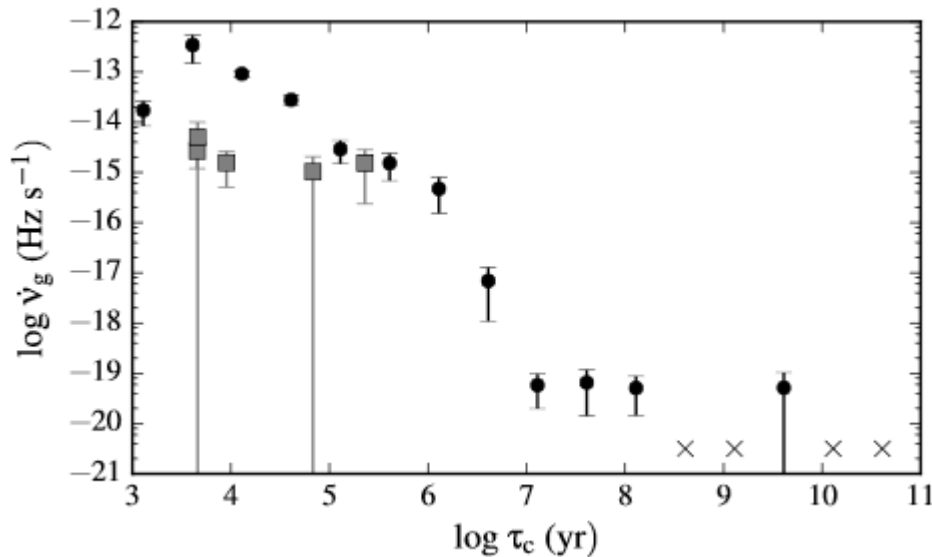
$$\frac{I_{\text{res}}}{I_c} \geq G_{\text{Vela}} = 1.4\%.$$

It seems that for some AXP glitches G is much larger than for PSRs. Dib et al. propose that it can be related to the role of core superfluid.

Many others proposed that glitches of magnetars can be related to magnetic field dissipation in the crust. As the field can be dynamically important there, its decay can result in crust cracking.

PSRs vs. Magnetars

Glitch activity of the magnetars with the smallest characteristic ages is lower than that of the rotation-powered pulsars with similar characteristic ages. However, their activity is larger than that of pulsars of equal spin-down power.



CCOs also glitch!

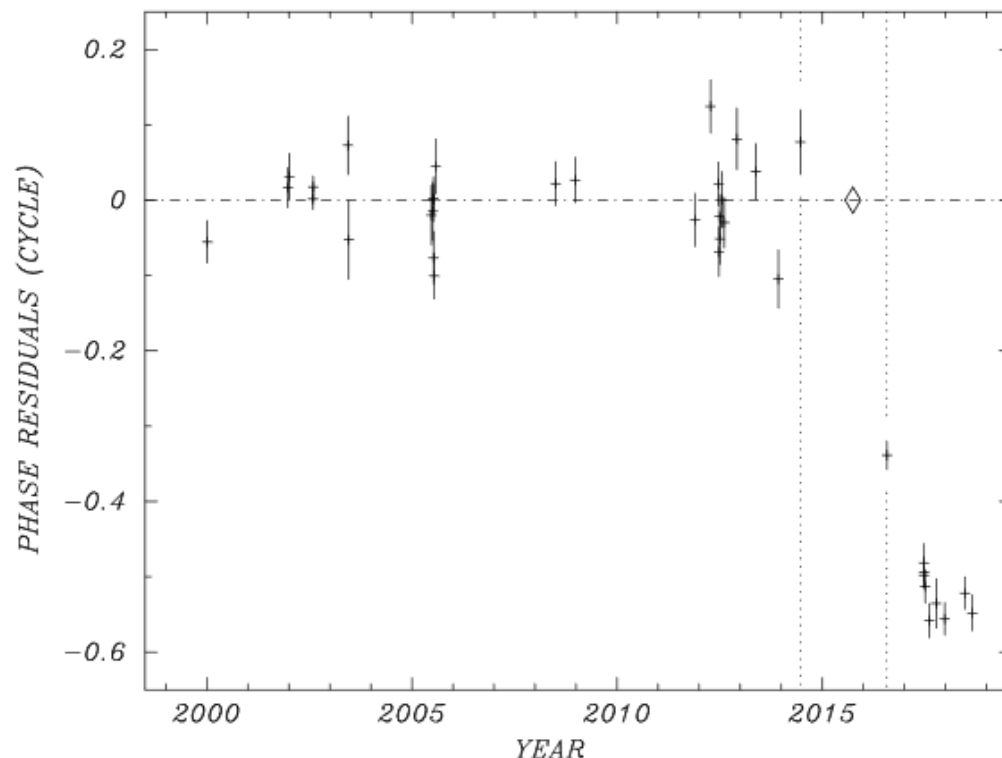
1E 1207.4–5209

0.424 s

$B_s \approx 9 \times 10^{10}$ G

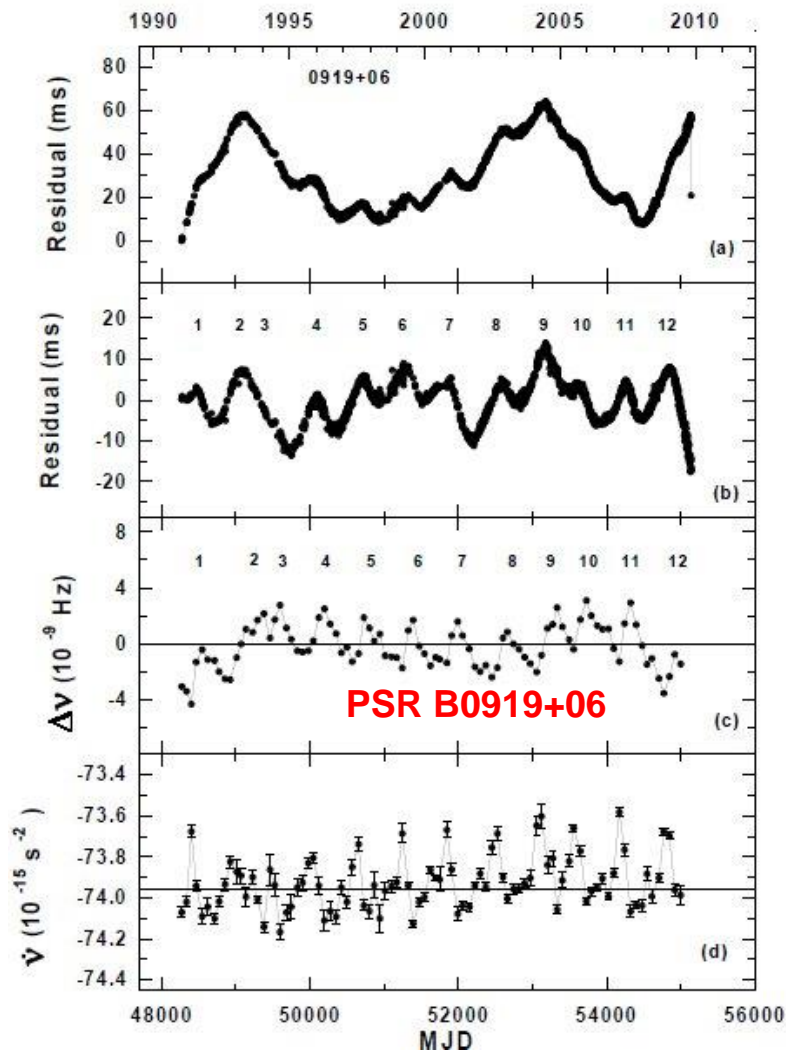
$\Delta f/f = (2.8 \pm 0.4) \times 10^{-9}$

Parameter	Pre-glitch ^a	Post-glitch ^b
Epoch	2012	2017
N_H (cm ⁻²) (fixed)	1.66×10^{21}	1.66×10^{21}
kT_1 (keV)	$0.0801^{+0.0046}_{-0.0042}$	$0.0742^{+0.0042}_{-0.0037}$
kT_2 (keV)	$0.2513^{+0.0022}_{-0.0021}$	$0.2509^{+0.0021}_{-0.0021}$
E_0 (keV)	$0.712^{+0.012}_{-0.011}$	$0.710^{+0.017}_{-0.015}$
σ_0 (keV) (fixed)	0.08	0.08
τ_0	0.26	0.22
E_1 (keV)	$1.4292^{+0.0087}_{-0.0088}$	$1.4216^{+0.0089}_{-0.0090}$
σ_1 (keV) (fixed)	0.08	0.08
τ_1	0.098	0.10
F_x (pn) ^c	$2.084^{+0.010}_{-0.011}$	$2.078^{+0.010}_{-0.012}$
χ^2_ν (DoF)	1.39(283)	1.32(276)

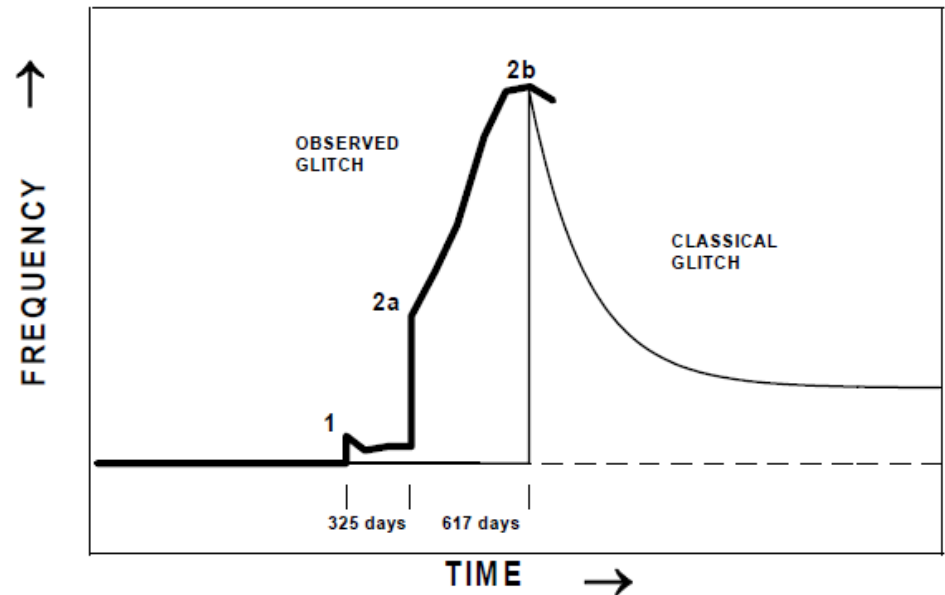


Glitch Parameters	
Epoch (MJD)	57295 ^b
Δf	$(1.23 \pm 0.19) \times 10^{-8} \text{ s}^{-1}$
$\Delta f/f_{\text{pred}}$	$(5.22 \pm 0.80) \times 10^{-9}$
$\Delta \dot{f}$	$(-1.58 \pm 0.31) \times 10^{-16} \text{ s}^{-2}$
$\Delta \dot{f}/\dot{f}_{\text{pred}}$	1.27 ± 0.25

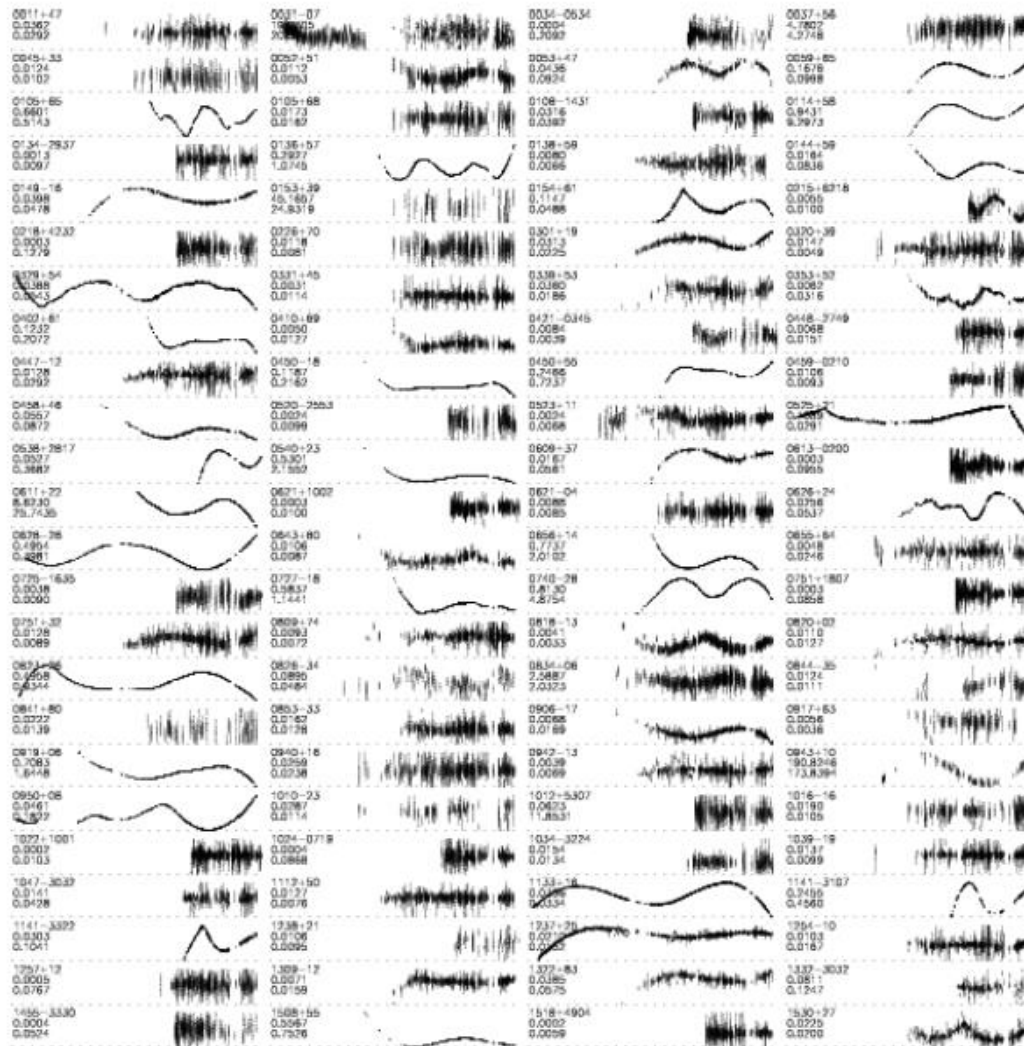
Slow glitches



Below: a slow glitch by PSR B1822-09 (Shabanova 1998)

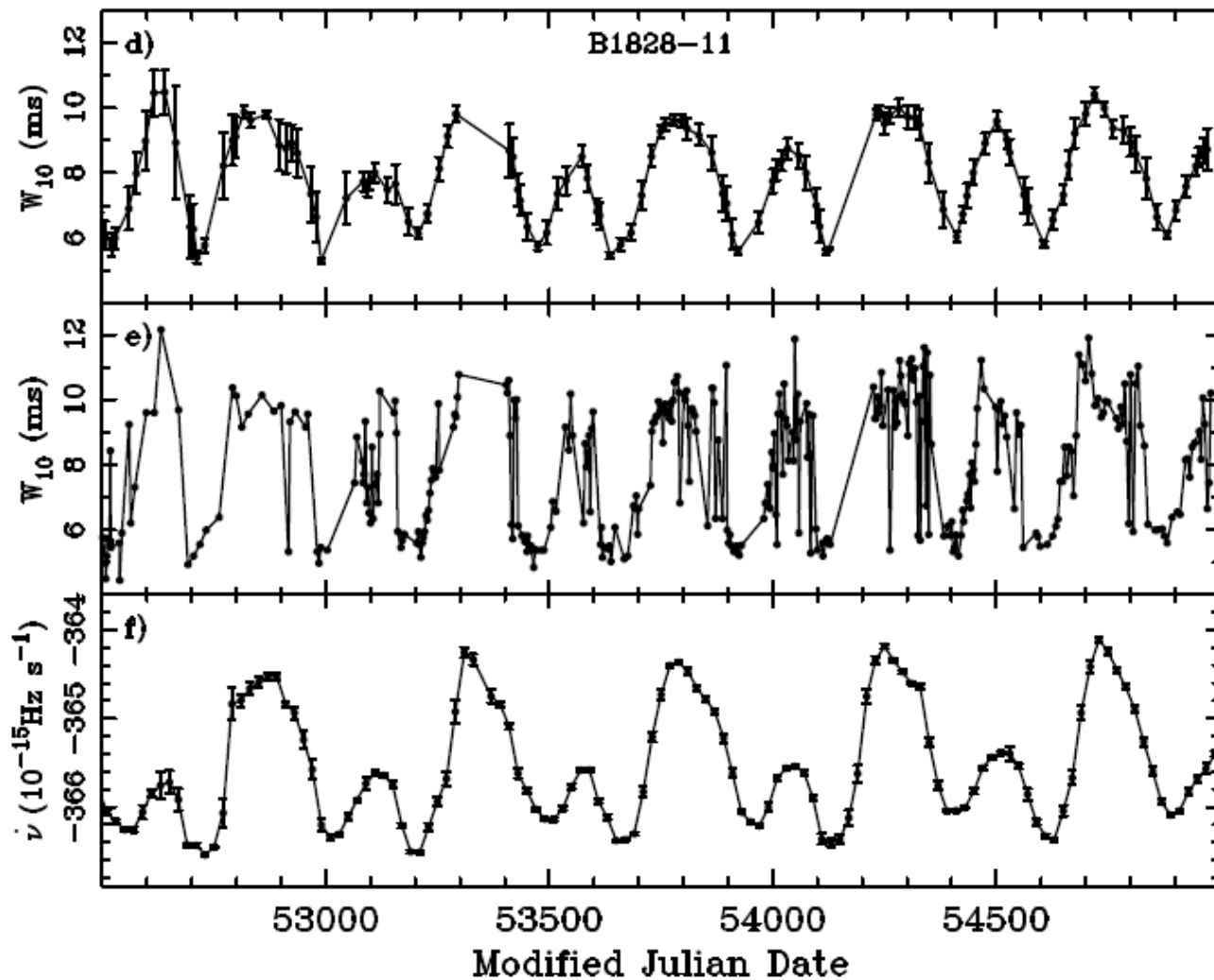


Timing irregularities



Analysis demonstrates different type of irregularities including quasi-periodic.

Possible explanation?

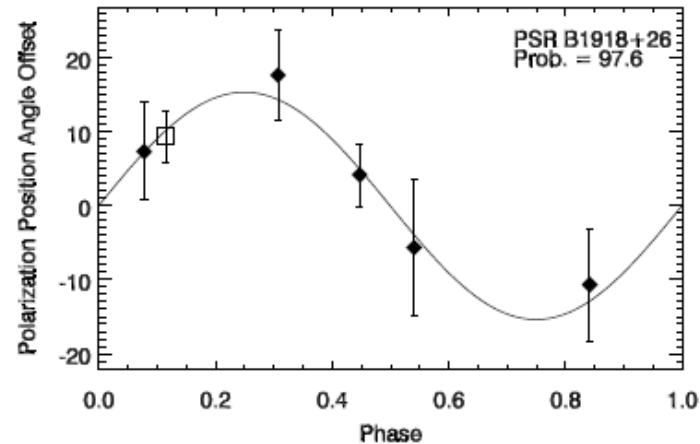
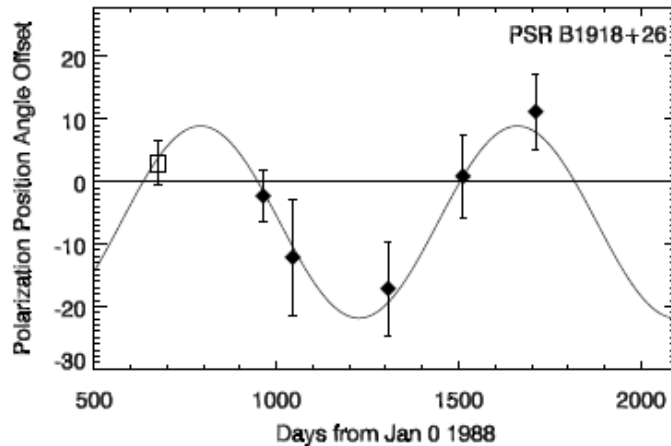
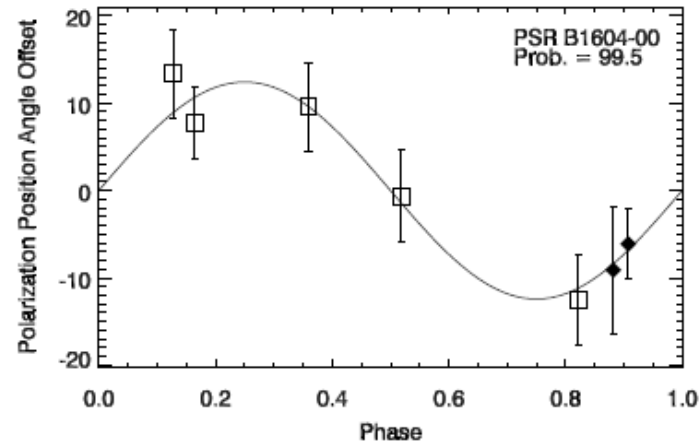
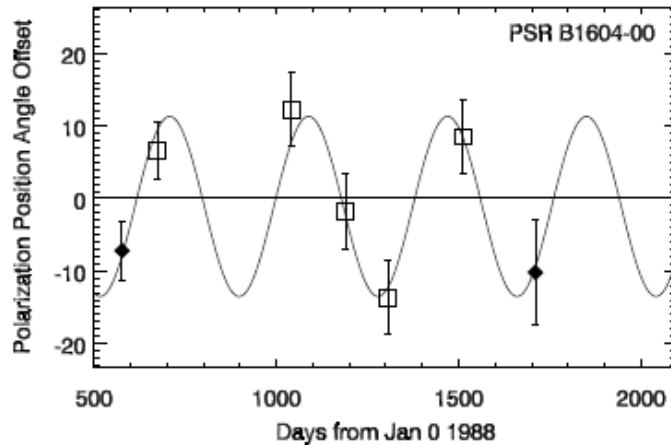


Magnetospheric effect?

Two stages characterized by particular pulse profile and spin-down rate.

Switching between these states happens rapidly.

Polarization angle variations



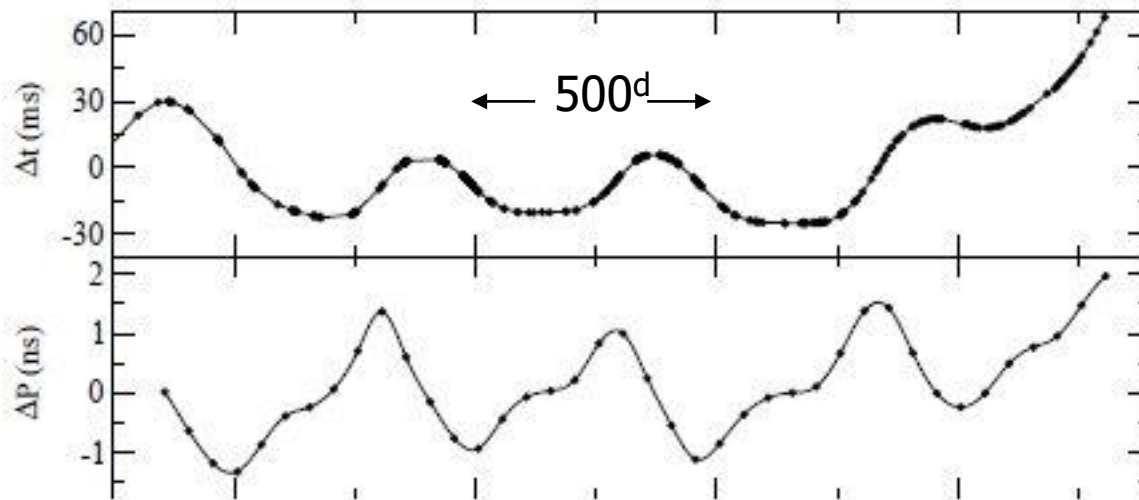
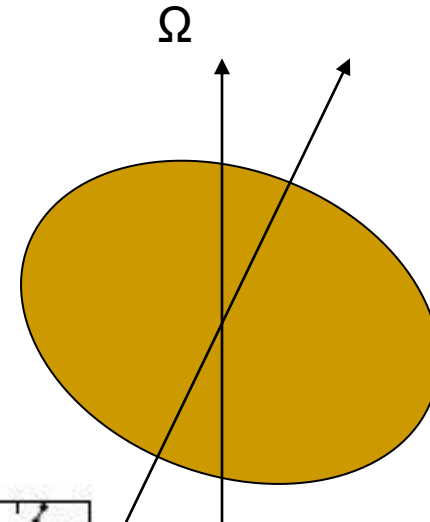
Such variations could be caused by precession

Precession in NSs

$$P_{\text{prec}} = P/\epsilon,$$

ϵ -oblateness: $\epsilon \sim 10^{-8}$ \Rightarrow $P_{\text{prec}} \sim \text{year}$

(More complicated models are developed, too.
See Akgun, Link, Wasserman, 2005)



Time of arrival
and period residuals
for PSR B1828-11.
Wobbling angle is $\sim 3-5^\circ$

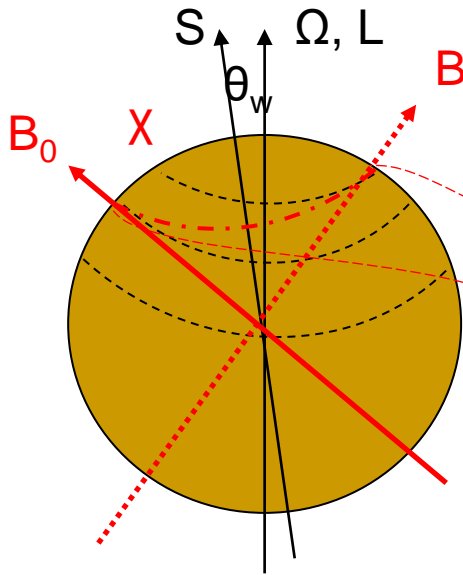
But why among ~ 1500
there are just 1-2
candidates... ?

New analysis confirms that PSR 1826-11 can have precession (1510.03579).
Still, it is difficult to bring it in correspondence with glitches from this PSR (1610.03509).

Precession (nutation)

If we consider the free precession,
then we have a superposition of two motions:

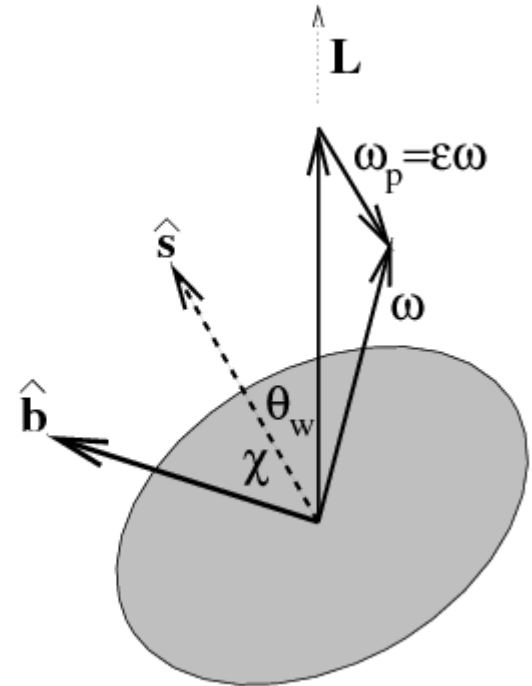
1. Rapid ($\sim\Omega$) rotation around total angular momentum axis – L
2. Slow (Ω_p) retrograde rotation around the symmetry axis (s)



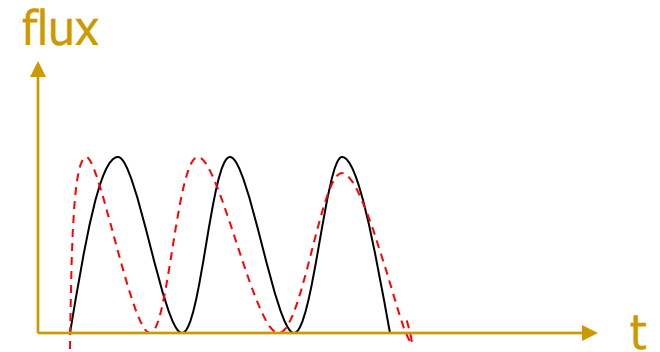
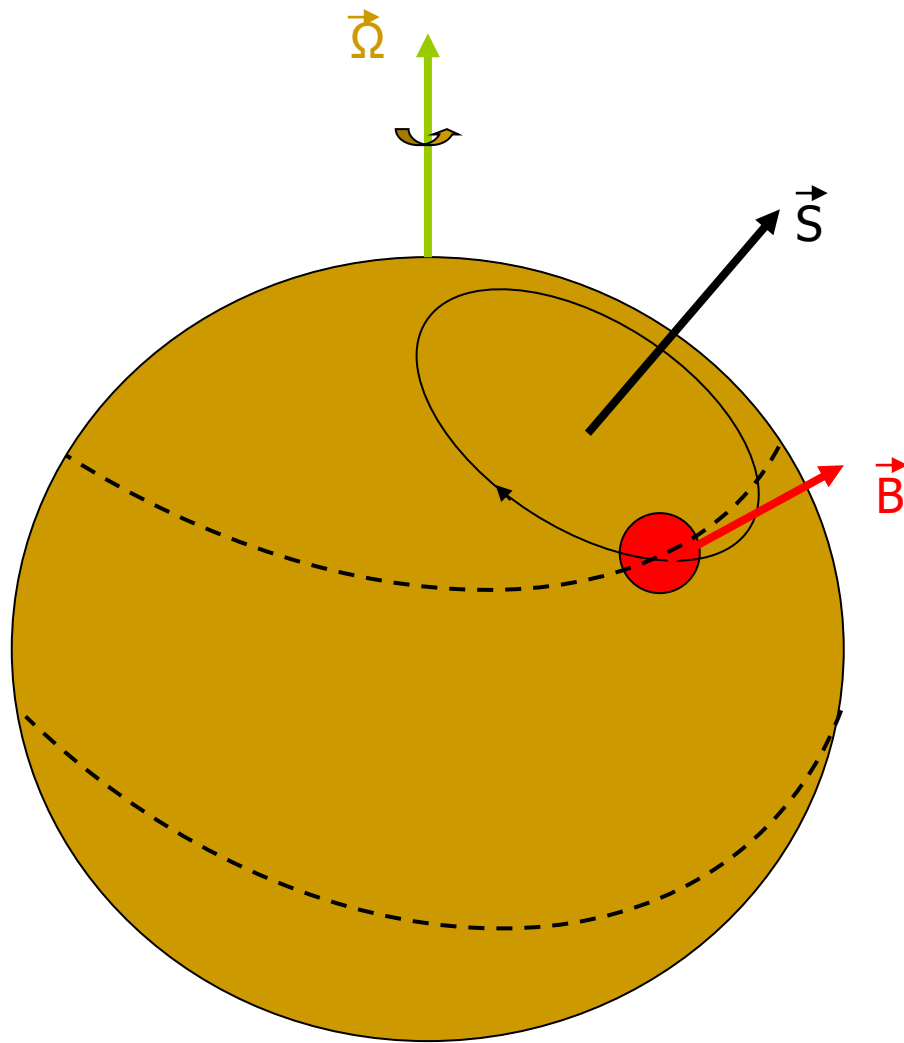
$$\Delta\varphi=\varphi_{\max}-\varphi_{\min}=(\chi+\theta_w)-(\chi-\theta_w)=2\theta_w$$

Beam width variation

Θ_w – is small
 Ω and L are very close

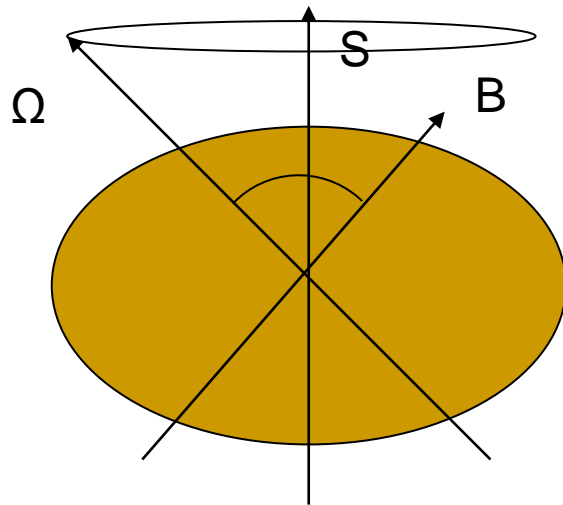


A toy model



This is a picture seen by an external observer.

In the coordinate frame of the body

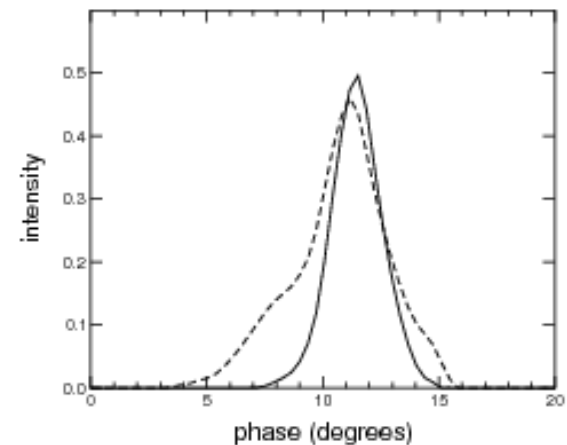
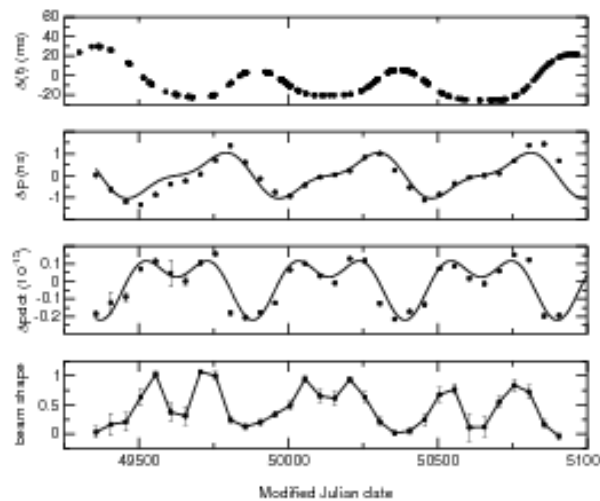


In this system the rotation axis is rotating around the symmetry axis. So, it is clear that the angle between spin axis and the magnetic axis changes.

This results in an additional effect in timing: Now the spin-down rate changes with the period of precession.

$$\frac{1}{2} I_1 \frac{d\omega^2}{dt} \simeq \boldsymbol{\omega} \cdot \mathbf{N},$$

$$\mathbf{N} = \frac{2\omega^2}{3c^3} (\boldsymbol{\omega} \times \mathbf{m}) \times \mathbf{m},$$

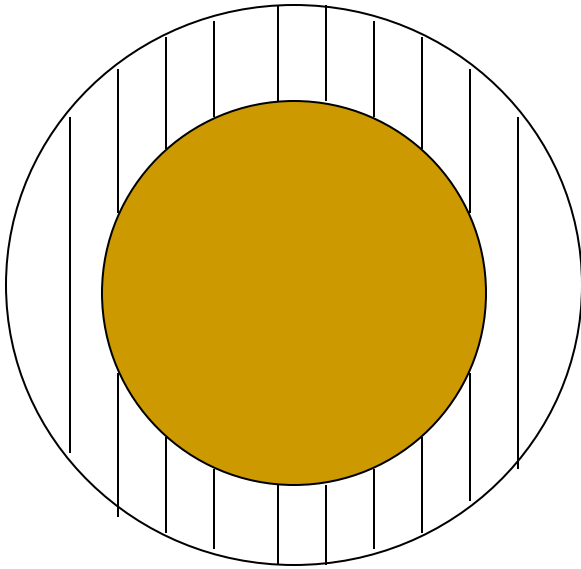


Complications ...

A neutron star is not a solid body ...

At least crust contains superfluid neutron vortices.

They are responsible for $I_p \sim 0.01$ of the total moment of inertia.



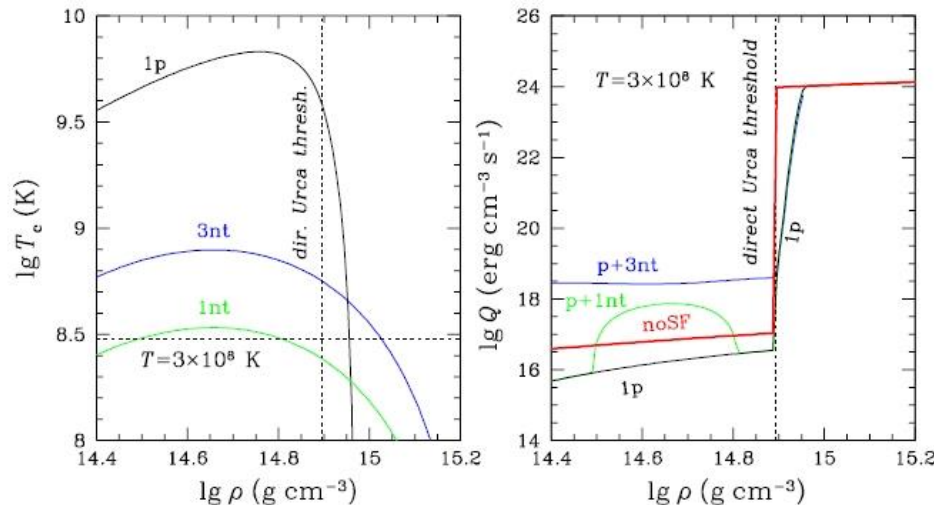
There are several effects related to vortices.

Neutron vortices can interact with the crust.
So-called “pinning” can happen.

The vortex array works as a gyroscope.
If vortices are absolutely pinned to the crust
then $\omega_{\text{prec}} = (I_p/I)\Omega \sim 10^{-2}\Omega$ (Shaham, 1977).
But due to finite temperature the pinning is not
that strong, and precession is possible
(Alpar, Ogelman, 1987).

Superfluidity in NSs

50 years ago it was proposed (Migdal, 1959) that neutrons in NS interiors can be *superfluid*.



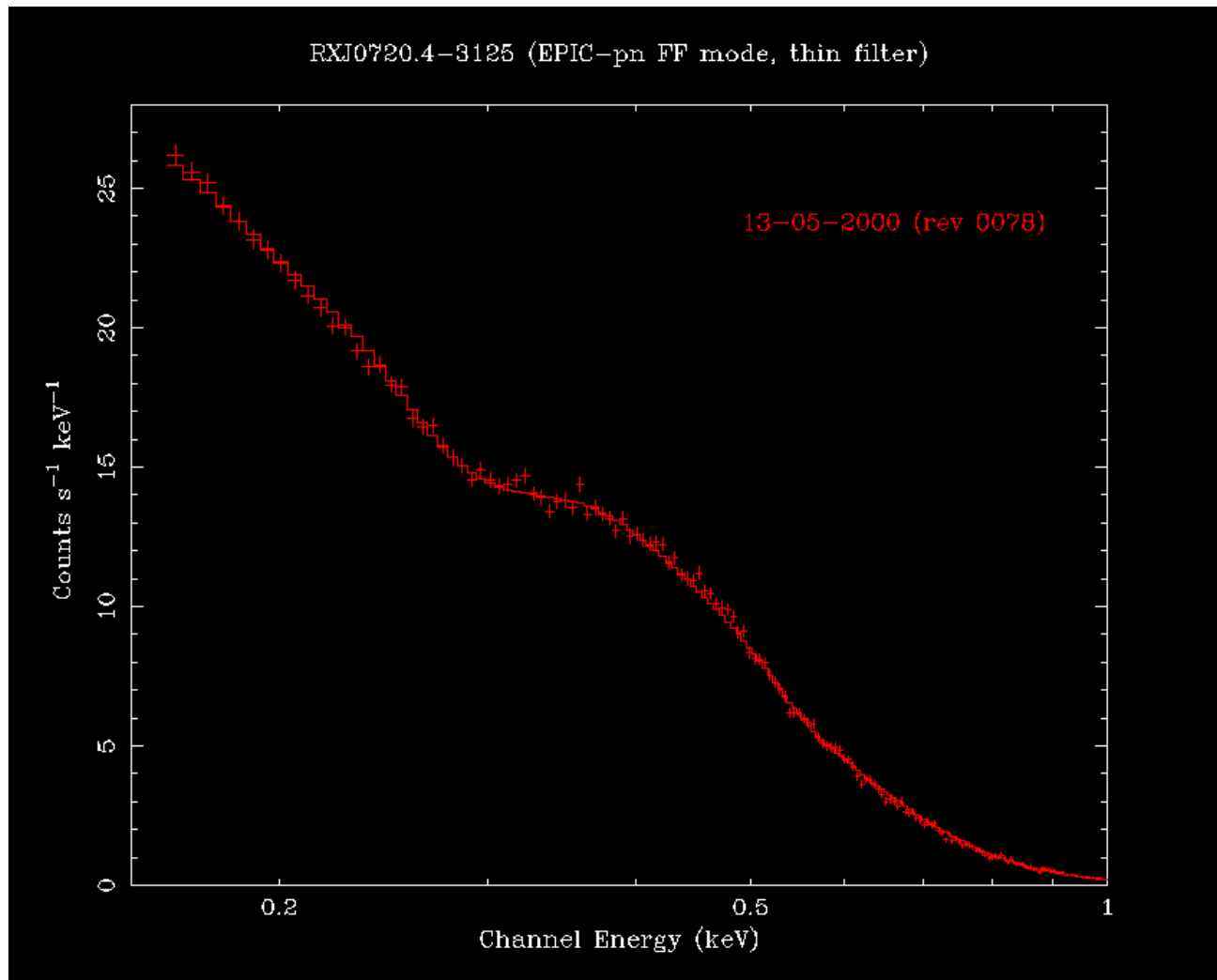
Various baryons in neutron star matter can be in *superfluid* state produced by Cooper pairing of baryons due to an attractive component of baryon-baryon interaction.

Now it is assumed that

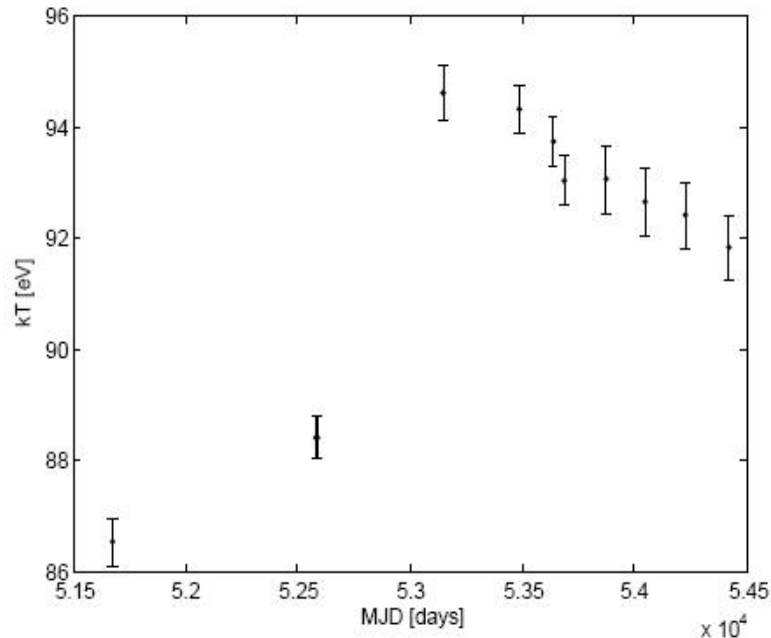
- neutrons are superfluid in the crust (singlet)
- protons are superfluid in the core (singlet)
- neutrons can also be superfluid in the core (triplet)

Onsager and Feynman revealed that rotating superfluids were threaded by an array of quantized vortex lines.

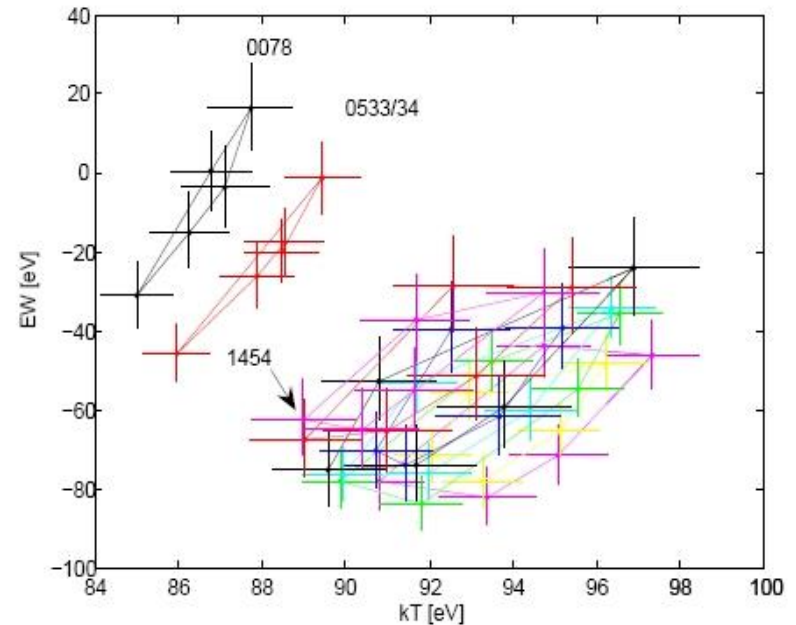
Peculiar behavior of RX J0720



RX J0720.4-3125 as a variable source

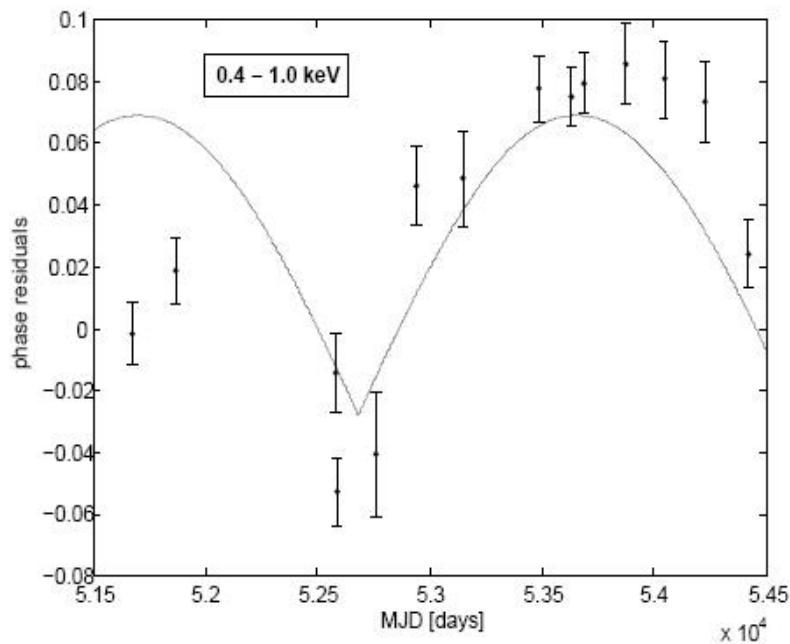


Long term phase averaged spectrum variations

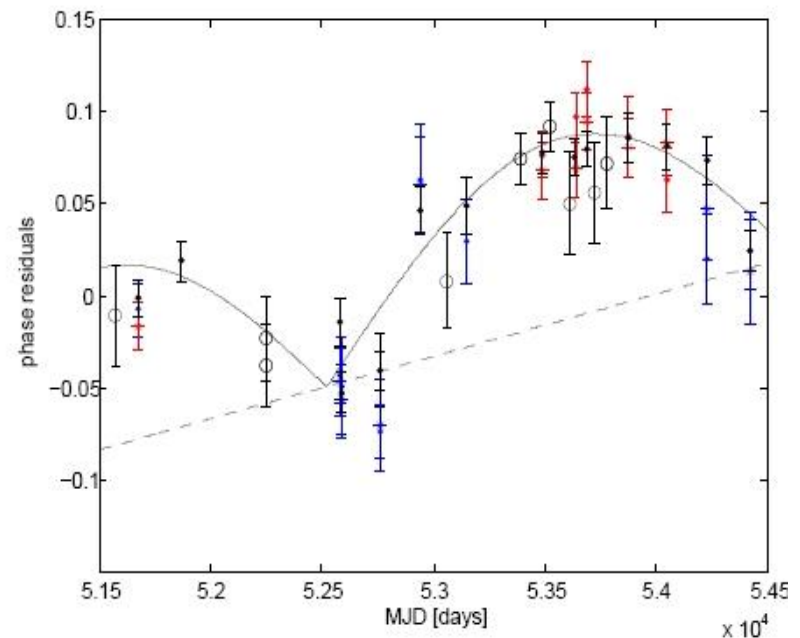


Phase dependent variations during different observations.

~ 10 years period: precession???



10.711 \pm 0.058 yrs



However, the situation is not clear.
New results and a different timing solution.
The estimate of the period of precession
slightly changed down to ~ 7 years.

RX J0720.4-3125: timing residuals

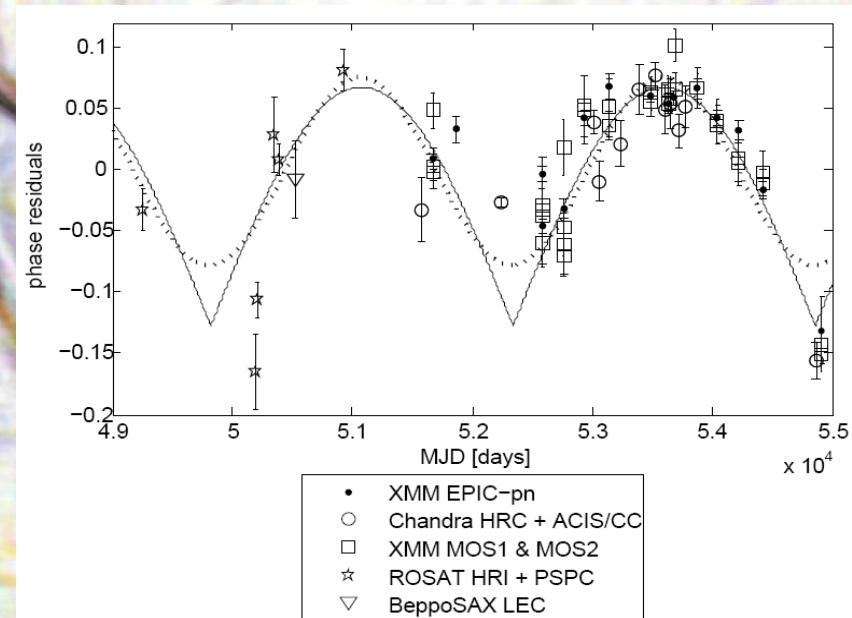
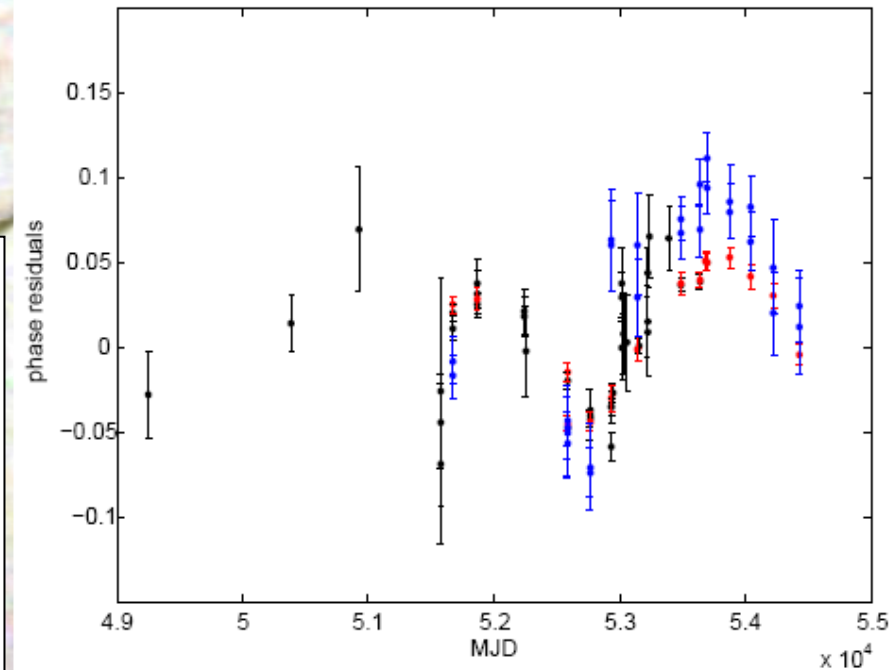
-for $P(t_0)$ and dP/dt : phase coherent timing
-in Kaplan & van Kerkwijk (2005) and
van Kerkwijk 2007, without energy
restriction

-now: restricting to the hard band
(except for ROSAT and Chandra/HRC)
+five new XMM-Newton
+two new Chandra/HRC
observations

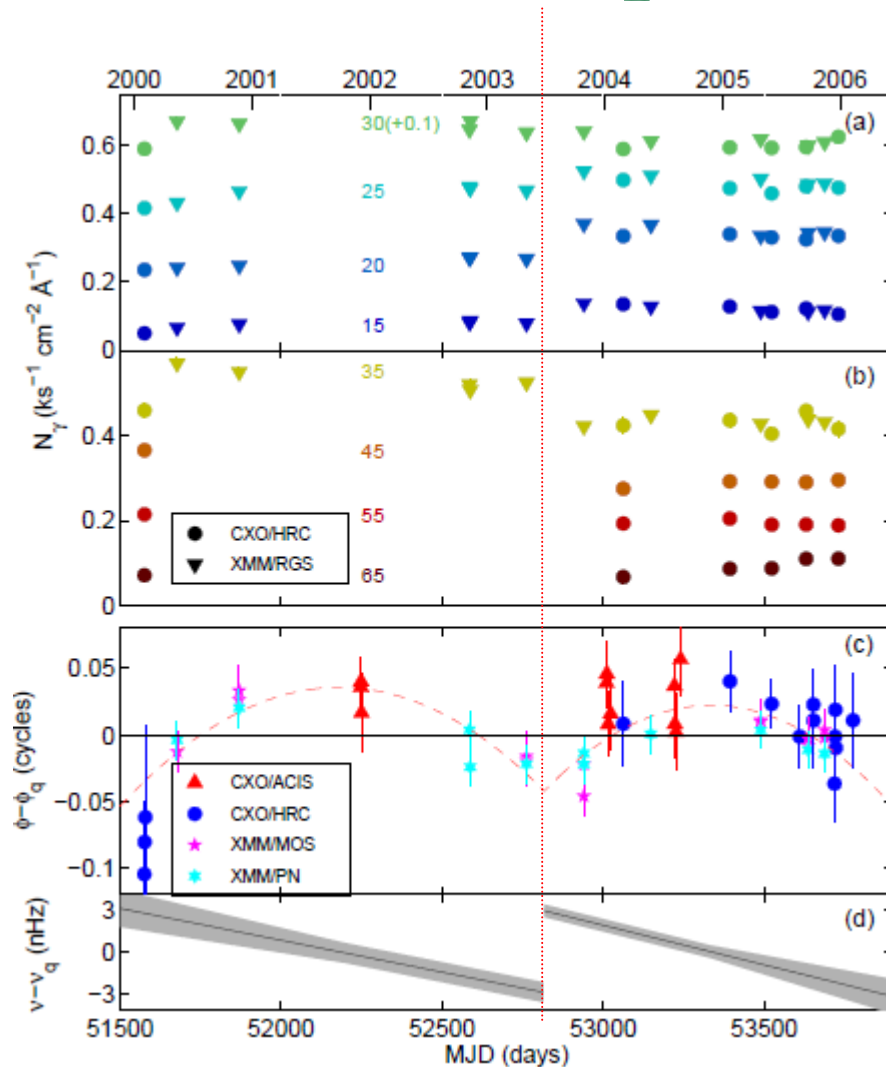
$P(t_0)=8.3911132650(91)\text{s}$
 $dP/dt=6.9742(19) \cdot 10^{-14} \text{ s/s}$

-long term period: $(6.91 \pm 0.17) \text{ yrs}$
Haberl (2007): $(7.70 \pm 0.60) \text{ yrs}$
for two hot spots: $\text{abs}(\text{sine})$
with 13-15.5yrs period

The slide from a talk by
Markus Hohle (Jena observatory).



Another interpretation: glitch + ?



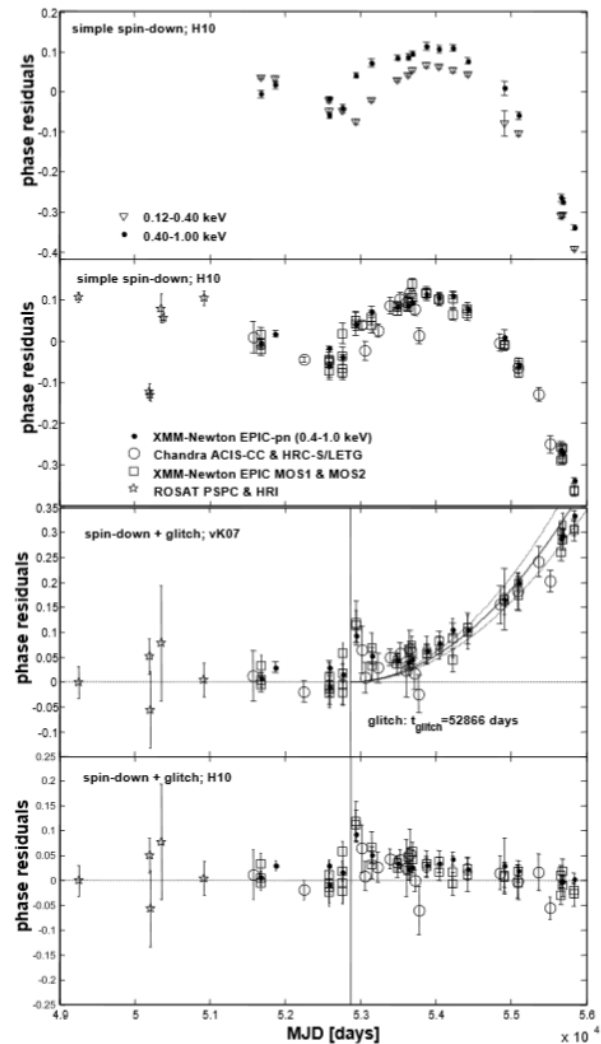
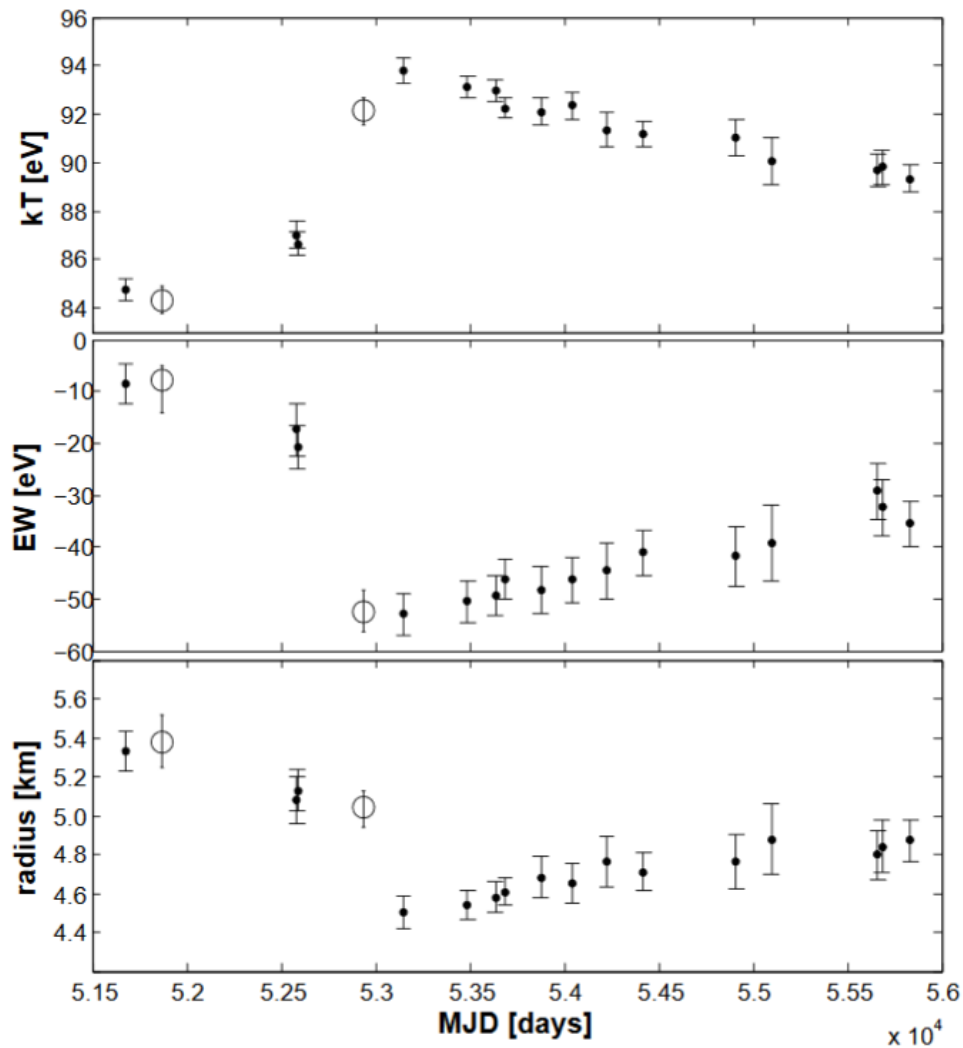
TIMING SOLUTIONS FOR RX J0720.4-3125

Quantity	Excl. <i>ROSAT</i>	All Data
Spindown only		
t_0 (MJD)	53010.2635646(7)	53010.2635626(6)
ν (Hz)	0.11917366979(12)	0.11917366954(11)
$\dot{\nu}$ (Hz s $^{-1}$)	$-9.74(4) \times 10^{-16}$	$-9.88(13) \times 10^{-16}$
TOA rms (s)	0.26	0.29
χ^2/dof	77.6/46=1.69	150.8/49=3.08
Spin-down + Glitch		
t_0 (MJD)	53010.2635686(10)	53010.2635667(10)
ν (Hz)	0.1191736716(9)	0.1191736716(9)
$\dot{\nu}$ (10^{-15} Hz s $^{-1}$)	-1.04(3)	-1.04(3)
t_g (MJD)	52817(61)	52866(73)
$\Delta\nu$ (nHz)	5.7(17)	4.1(12)
$\Delta\dot{\nu}$ (10^{-17} Hz s $^{-1}$)	-1(4)	-4(3)
TOA rms (s)	0.15	0.24
χ^2/dof	37.0/43=0.86	45.1/46 = 0.98

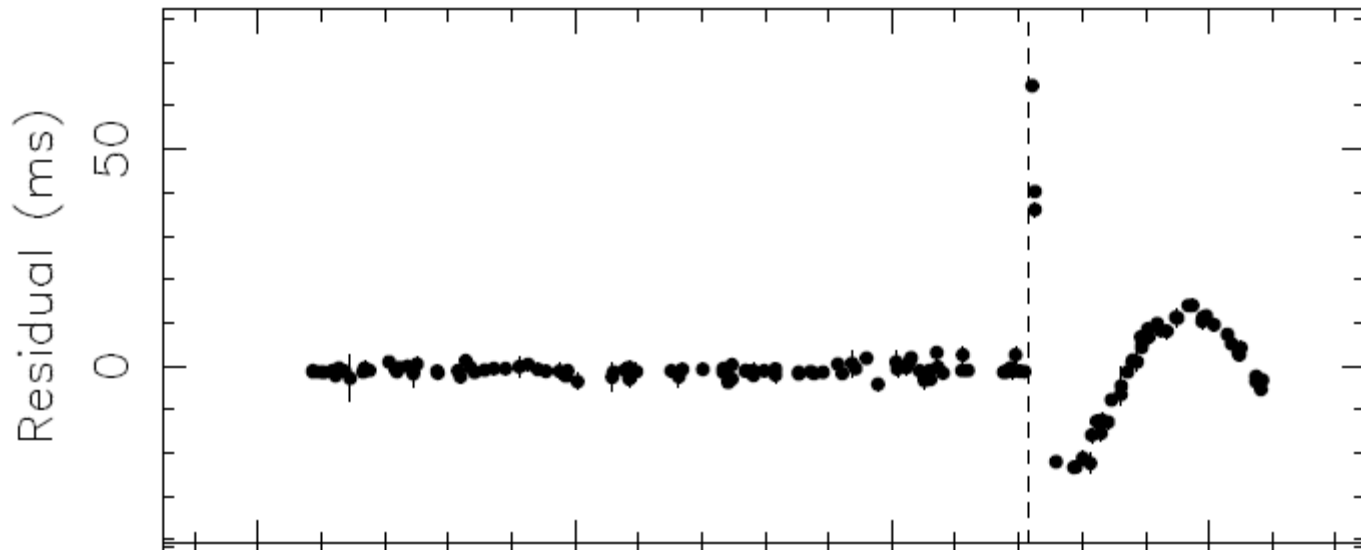
NOTE. — The parameters determine the cycle count plus phase via $\phi(t) = \nu(t-t_0) + \frac{1}{2}\dot{\nu}(t-t_0)^2 + \Delta\phi_g(t)$, where $\Delta\phi_g(t) = -\Delta\nu(t-t_g) - \frac{1}{2}\Delta\dot{\nu}(t-t_g)^2$ for $t < t_g$ in the glitch model and zero otherwise. For all fits, a 0.11 s systematic uncertainty has been added in quadrature to the times of arrival (TOAs), and the uncertainties quoted are twice the formal 1σ values.

Van Kerkwijk et al. astro-ph/0703326

RX J0720.4-3125: a glitch



Glitch+? in a PSR



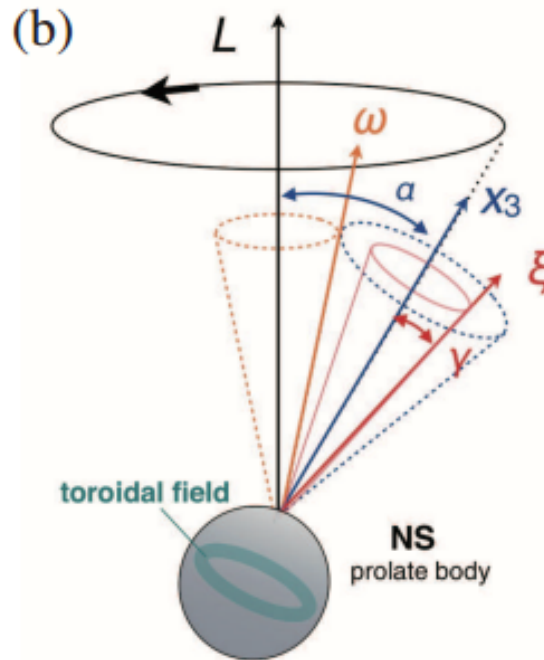
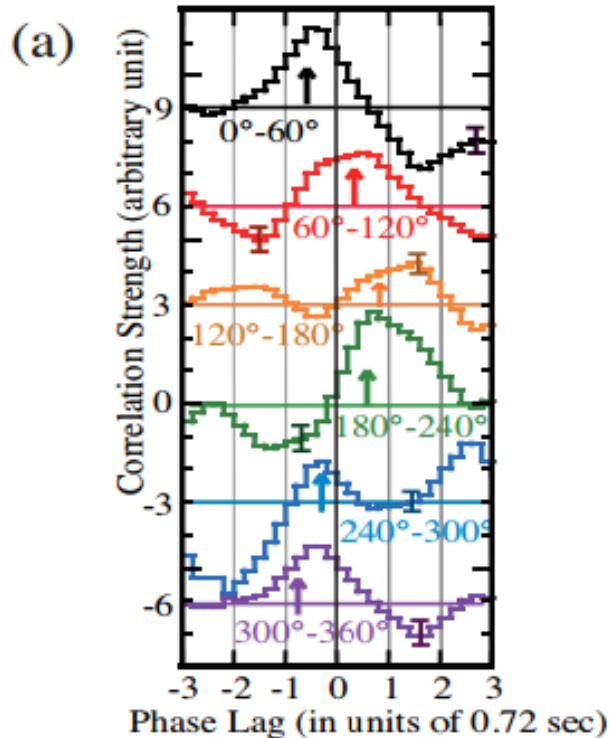
PSR B2334+61

arXiv: 1007.1143

Precession after a glitch was proposed as possible feature due to Tkachenko waves excitation (arXiv: [0808.3040](#)).

Precession as a viable mechanism for long-term modulation was recently discussed in details in 1107.3503.

Free precession of a magnetar?

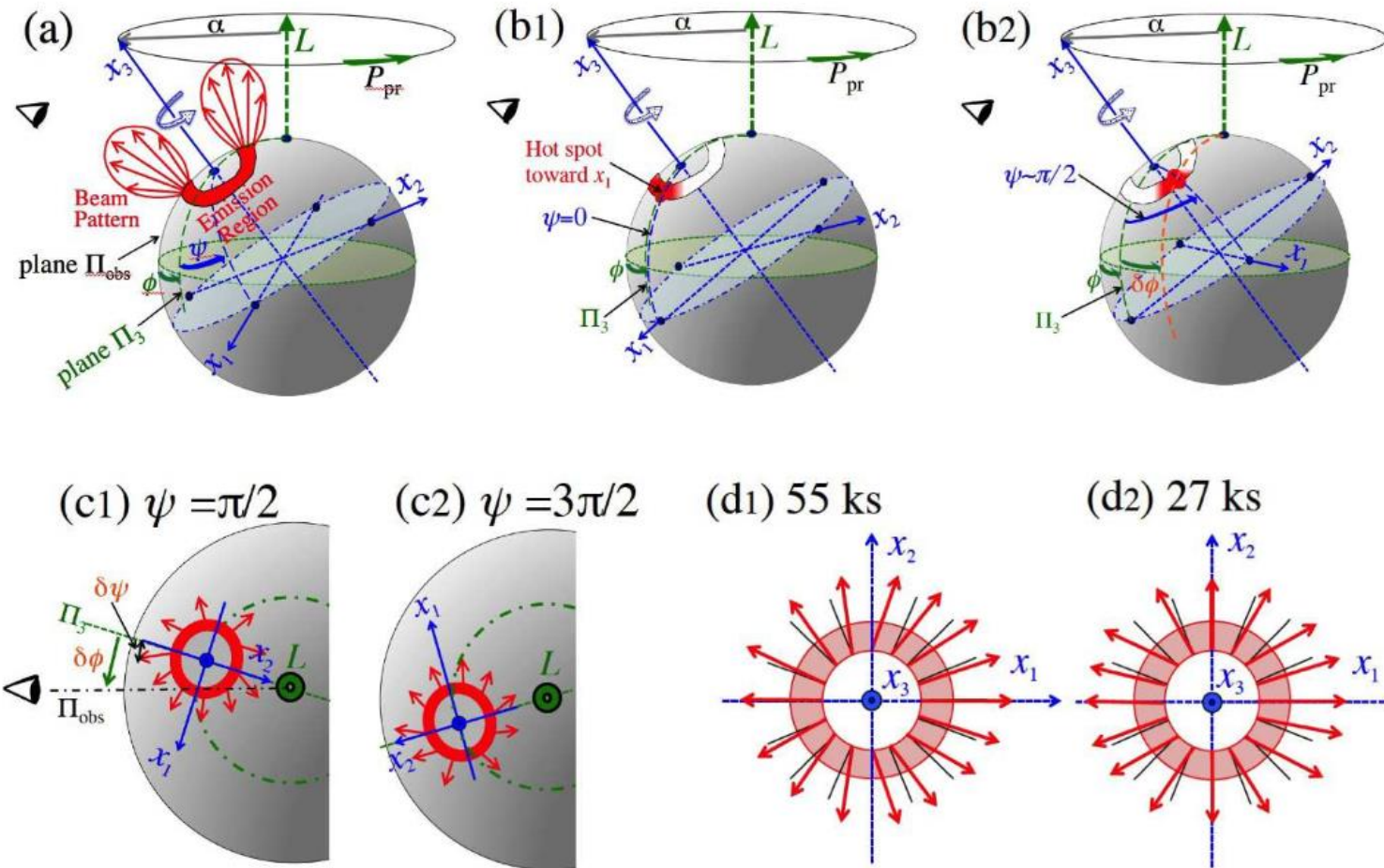


The authors observe modulation of the pulse profile with a period ~ 15 hours. If it is interpreted by a free precession, then the NS is significantly deformed which can be due to strong toroidal field. This field might be $\sim 10^{16}$ G.

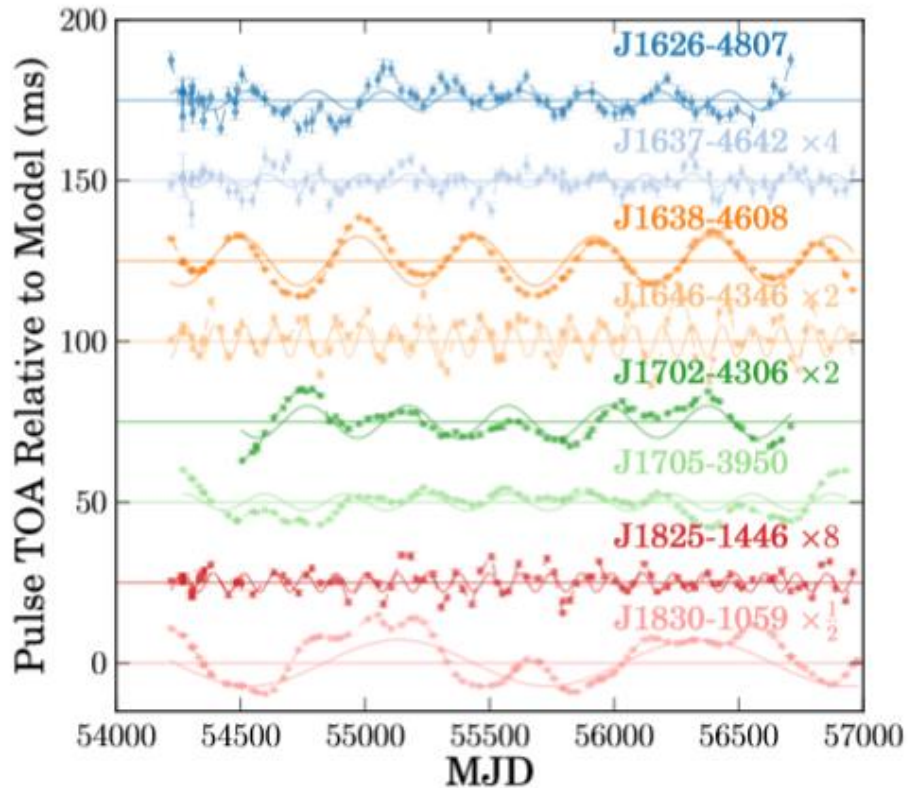
See new results and analysis in 1810.11147

More X-ray data confirms modulation

4U 0142+61

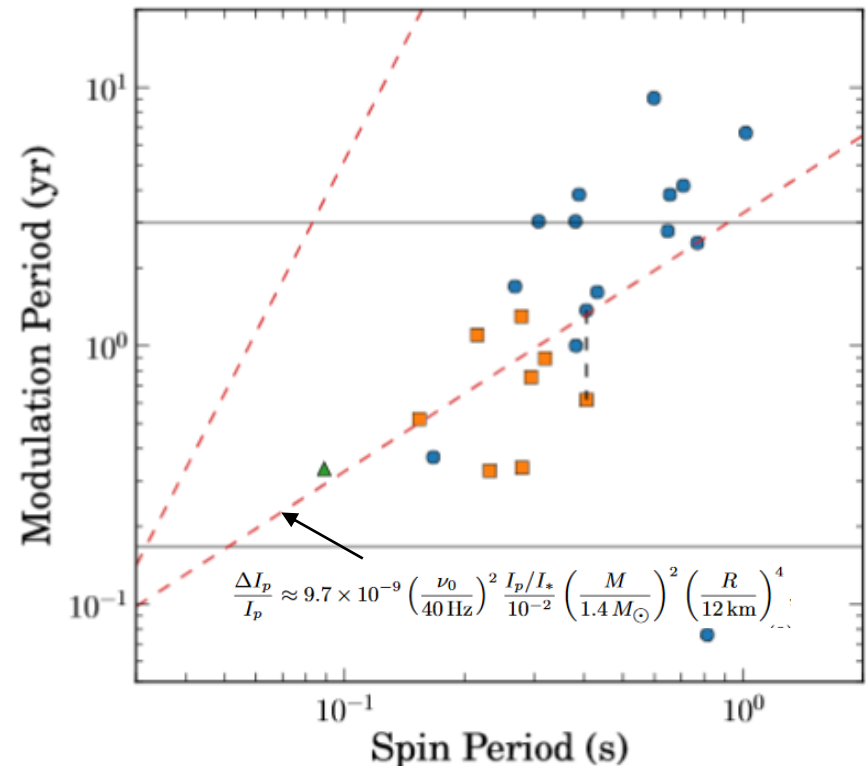


New precession candidates among PSRs



Correlations of the modulation period with spin period, characteristic age and spin-down power.

Periodic modulations which can be interpreted as free precession.



Conclusion

Many observed phenomena are related to internal dynamics of NSs.

- Glitches
- Precession

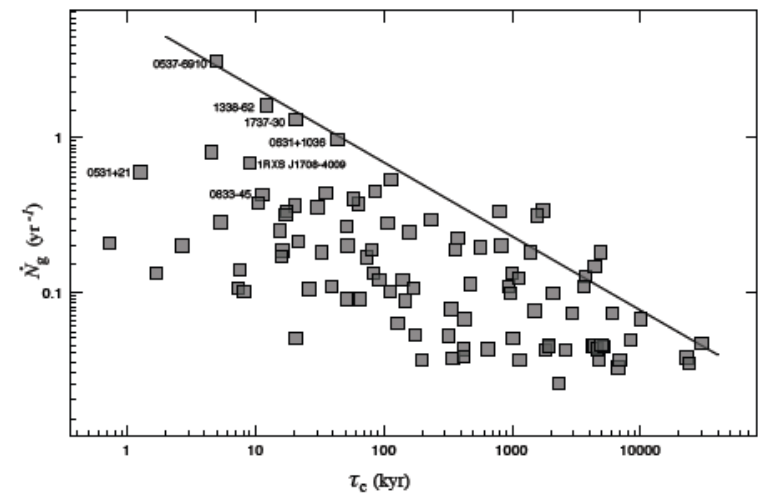
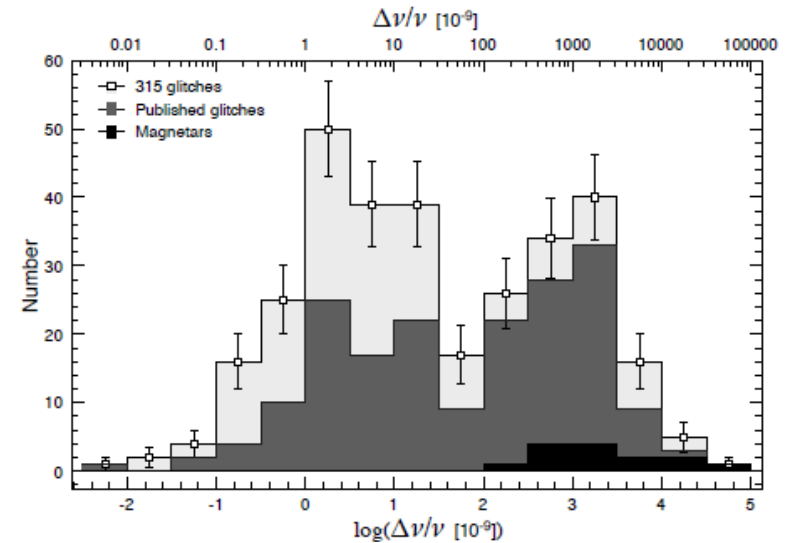
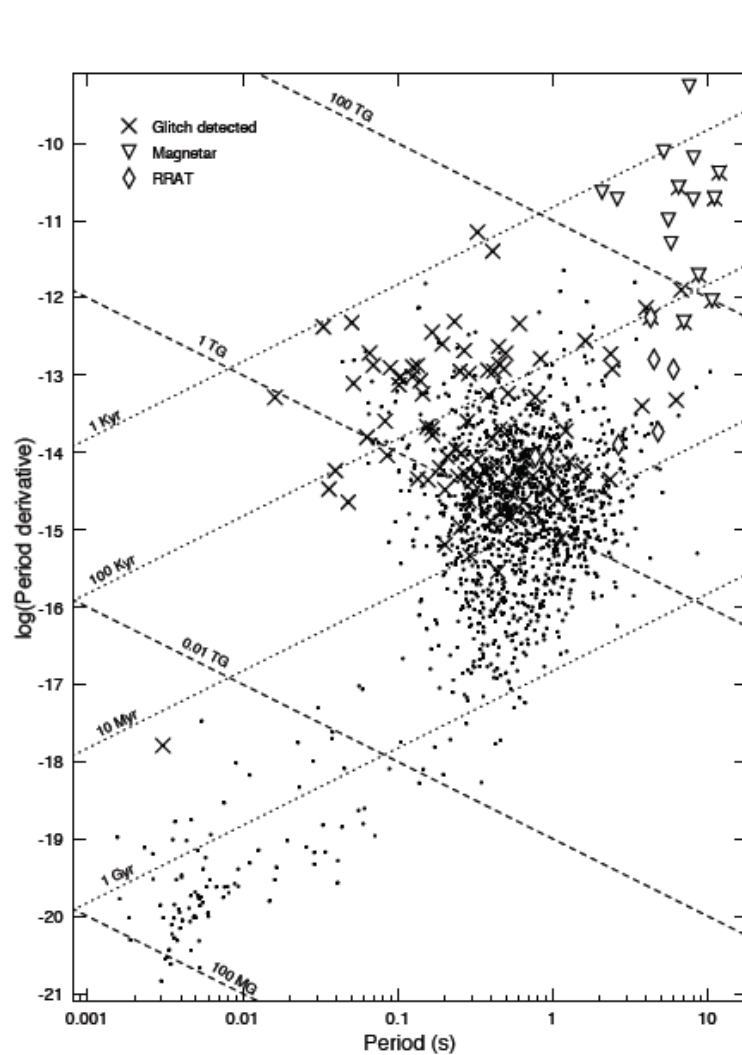
Glitches are related to the existence of some reservoir for angular momentum. Most probably, it is a layer of superfluid neutrons in the inner crust.

Some glitches of magnetars can be related to a different process.

Main papers

- Link et al. [astro-ph/0001245](#) Glitches
 - Link [astro-ph/0211182](#) Precession
 - Jones, Andersson [astro-ph/0011063](#) Precession
 - Dib et al. arXiv: 0706.4156 AXP glitches
 - Haskell, Melatos arXiv: 1502.07062 Big review
 - Haskell, Sedrakian arXiv: 1709.10340 Big review on superfluidity
 - Fuentes et al. arXiv: 1710.00952 Glitch statistics
 - Manchester arXiv: [1801.04332](#) Brief review on glitches
-

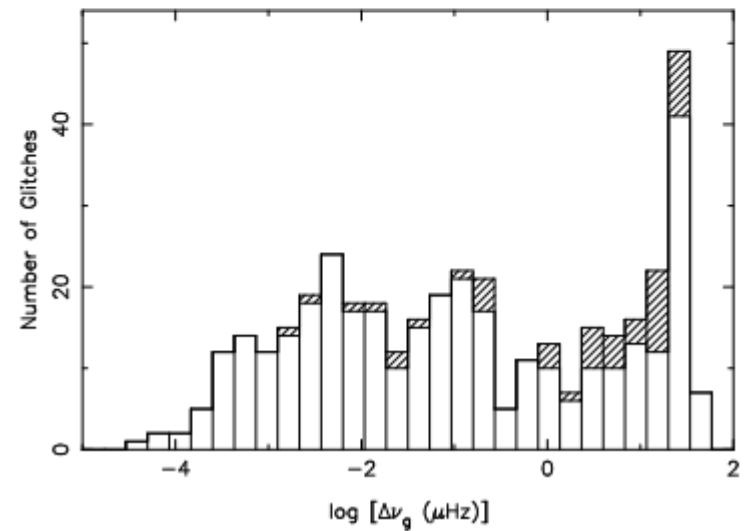
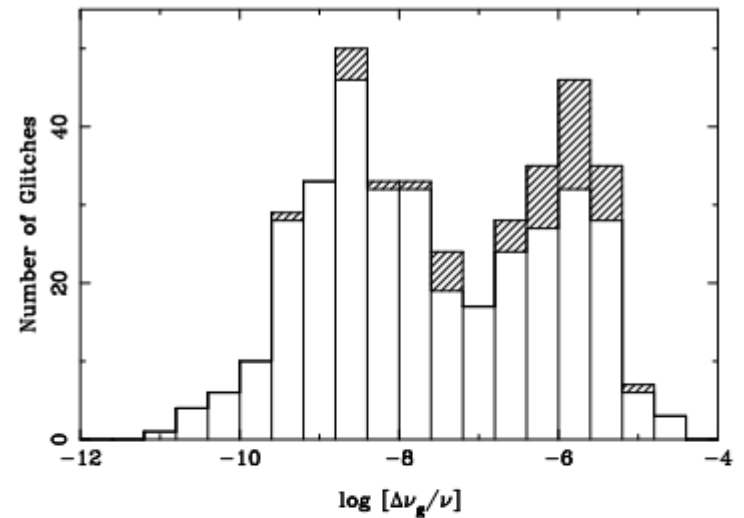
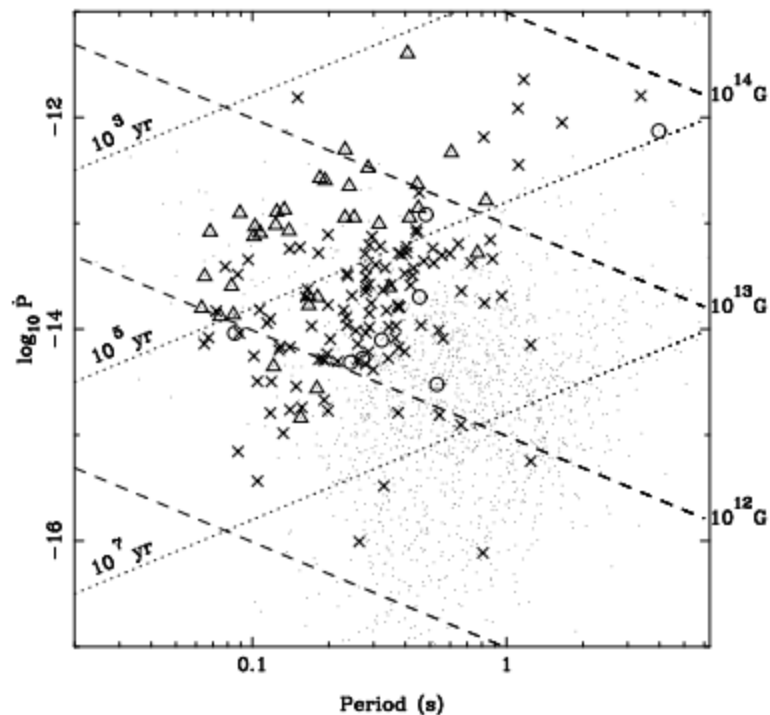
Many-many glitches ...



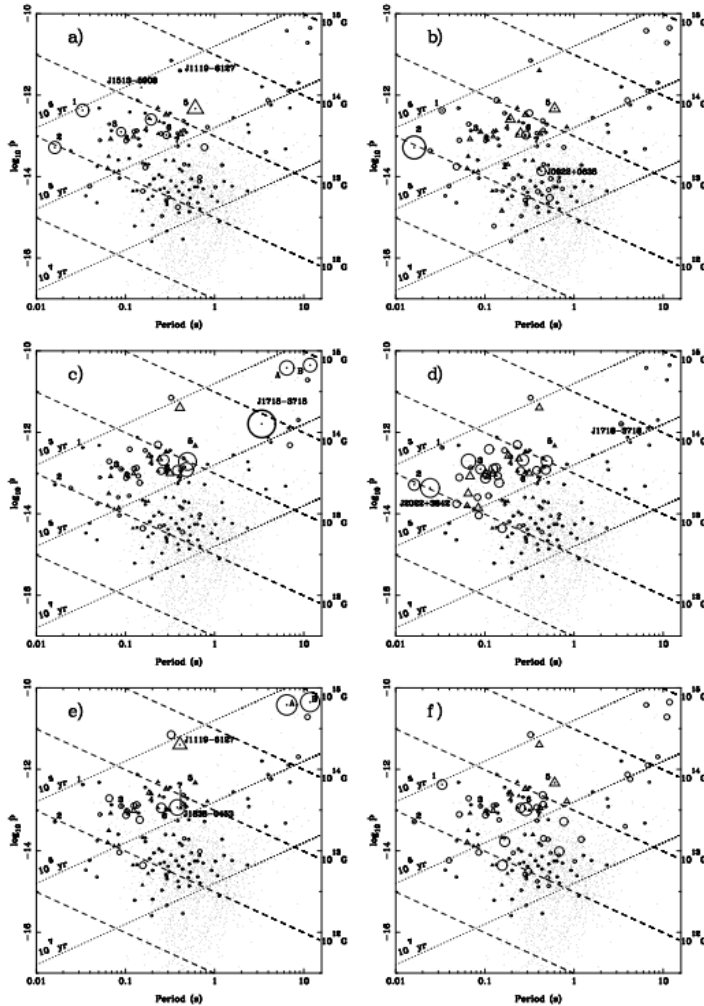
1102.1743

315 glitches in 102 PSRs

107 new glitches in 36 pulsars



P–Pdot diagrams for glitch-related quantities



a) number of detected glitches; b) average number of glitches per year; c) maximum fractional glitch size; d) maximum glitch size; e) rms fractional glitch size; and f) rms fractional size normalised by the mean. A circle indicates the parameter was obtained from the ATNF Pulsar Catalogue glitch table, whereas a triangle symbol indicates a parameter from this work. In the various plots, the seven pulsars exhibiting ten or more glitches are marked: 1 – PSR B0531+21 (Crab pulsar); 2 – PSR J0537–6910; 3 – PSR B0833–45 (Vela pulsar); 4 – PSR J1341–6220; 5 – PSR J1740–3015; 6 – PSR J0631+1036; 7 – PSR J1801–2304; and two magnetars: A – PSR J1048–5937 (1E 1048.1–5937) and B – PSR J1841–0456 (1E 1841–045).

Modeling glitches

Mean field approach to describe vortex dynamics