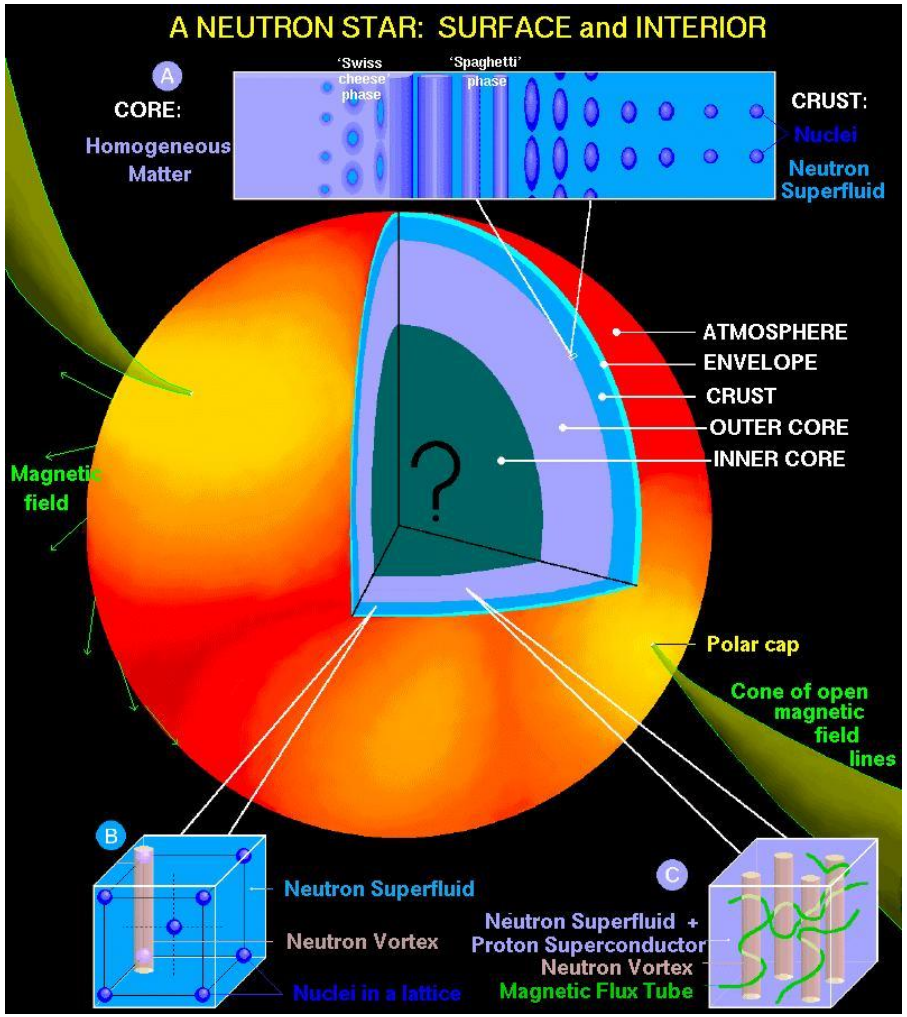

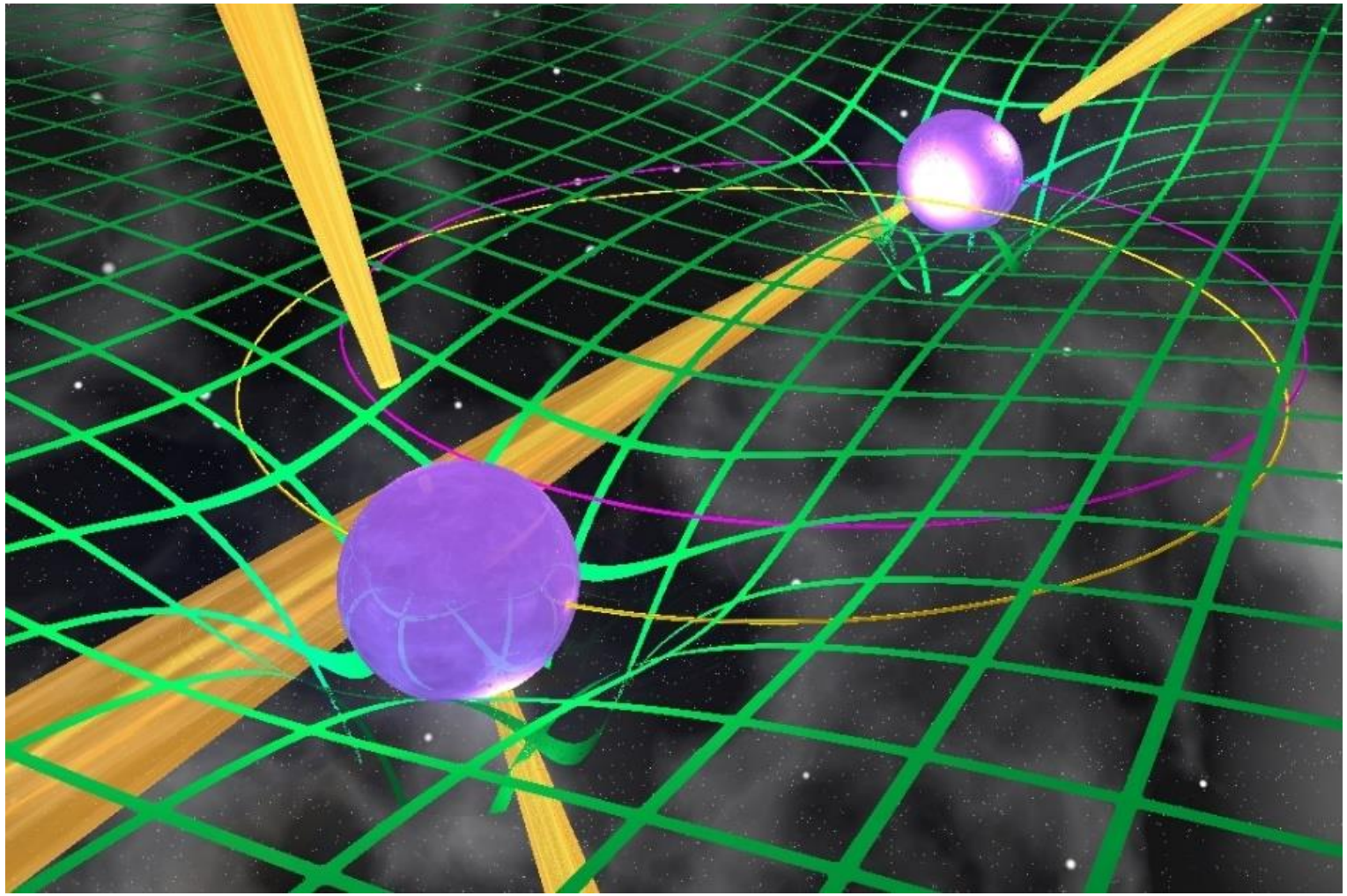

Современная феноменология нейтронных звезд и черных дыр.

Семинар 2

15.02.2021

1. Yp-e Λ-Э.

$$P = K\rho^{\gamma} \qquad \gamma = 1 + \frac{1}{n}$$

$$\frac{dm}{dr} = 4\pi r^2 \rho \qquad \frac{1}{\rho}\frac{dP}{dr} = -\frac{GM}{r^2}$$

$$\frac{d}{dr}\left(\frac{1}{\rho}\frac{dP}{dr}\right) = \frac{2GM}{r^3} - \frac{G}{r^2}\frac{dM}{dr} = -\frac{2}{\rho r}\frac{dP}{dr} - 4\pi G\rho$$

$\times r^2$

$$\Rightarrow \quad r^2 \frac{d}{dr}\left(\frac{1}{\rho}\frac{dP}{dr}\right) + \frac{2r}{\rho}\frac{dP}{dr} = \frac{d}{dr}\left(\frac{r^2}{\rho}\frac{dP}{dr}\right) = -4\pi G\rho r^2$$

$\div r^2$

$$\Rightarrow \quad \frac{1}{r^2}\frac{d}{dr}\left(\frac{r^2}{\rho}\frac{dP}{dr}\right) = -4\pi G\rho$$

$$P = K\rho_c^{1+\frac{1}{n}}\Theta^{n+1} \qquad \rho = \rho_c\Theta^n$$

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2 K\rho_c^{\frac{1}{n}}(n+1)\frac{d\Theta}{dr}\right) = -4\pi G\rho_c\Theta^n$$

$$r = a\xi \qquad a^2 = (n+1)K\rho_c^{1/n-1}$$

$$\boxed{\frac{1}{\xi^2}\frac{d}{d\xi}\left(\xi^2\frac{d\Theta}{d\xi}\right) = -\Theta^n}$$

(2) 1511.08813

1) Ферми $\quad 1\,fm = \frac{\hbar}{m_\pi c} \simeq 10^{-13}\,cm \qquad m_\pi \sim 135\,\frac{MeV}{c^2}$

$$\rho_N \sim n_N \cdot m_N = \left[\frac{4}{3}\pi (1fm)^3\right]^{-1} \cdot m_N \simeq 3\cdot 10^{14}\,g/cm^3$$

2) Сфера Ферми

$$\Delta p \cdot \Delta x \sim \hbar \quad \Rightarrow \quad \Delta p_x \Delta x \Delta p_y \Delta y \Delta p_z \Delta z \sim \hbar^3/g_s$$

$g_s = 2 \qquad s = \pm 1/2$

$$dN = \frac{g_s}{h^3} \cdot V \cdot 4\pi p^2 dp \qquad p < p < p + dp$$

$$N = \frac{g_s}{h^3} V \cdot \frac{4\pi}{3} p_F^3 \qquad p_F = \left(\frac{3h^3 N}{4\pi g_s V}\right)^{1/3} = \hbar\left(3\pi^2 n\right)^{1/3} \sim n^{1/3}$$

$$p_F = \hbar / \left(d/(3\pi^2)^{1/3}\right) \qquad d \simeq n^{-1/3}$$ — расст. между частицам

5) $dE = TdS - PdV + \mu dN$ $\quad$ E – полная энергия.

$T = S = 0$ $\quad$ $\varepsilon \equiv E/V$ $\quad$ $dN = 0$ $\quad$ $N = n \cdot V$

$$P = -\left(\frac{\partial E}{\partial V}\right)_N = -\left(\frac{\partial (E/N)}{\partial (V/N)}\right)_N = n^2 \frac{d}{dn}\left(\frac{\varepsilon}{n}\right)$$

$$E = \int_{p<p_F} \hat{\varepsilon}(p)\, dN = \frac{g_s}{h^3} 4\pi V \int_0^{p_F} \hat{\varepsilon}(p) p^2 dp \qquad (*)$$

$$\hat{\varepsilon}(p) = \sqrt{(mc^2)^2 + (pc)^2}$$ – эн. ч. частицы

а. $p \ll mc$ $\quad$ б. $p \gg mc$

a. Не релят.

$$\tilde{\varepsilon}(p) = mc^2 + p^2/2m$$

$$E = N\left(mc^2 + \underbrace{\frac{3}{5}\frac{p_F^2}{2m}}_{\text{кин. зн.}}\right)$$

$$\varepsilon = \frac{E}{V} = mc^2 \cdot n + \frac{3^{5/3}\pi^{4/3}}{10}\frac{\hbar^2}{m} n^{5/3}$$

$$P \sim \rho^{5/3} \quad (!) \quad \gamma = 5/3$$

$$P \sim \frac{1}{m}$$

8. Penet.

$$\tilde{\varepsilon}(p) = pc \qquad \frac{E}{N} = \frac{3}{4} p_F \cdot c$$

$$\varepsilon = \frac{3}{4}\left(3\pi^2\right)^{1/3} \hbar c\, n^{4/3}$$

$$P = \frac{\varepsilon}{3} = \frac{1}{4}\left(3\pi^2\right)^{1/3} \hbar c\, \underline{n^{4/3}}$$

$$P \sim \rho^{4/3} \qquad \gamma = 4/3 \quad (!)$$

$$a \to \delta: \quad p \sim mc \qquad \lambda = \frac{h}{mc}$$

$$n = \left(\frac{8\pi}{3}\right)\lambda^{-3}$$

$$e^-: \quad n \sim 10^{30}\ \mathrm{cm}^{-3}$$

$$p, n: \quad n \sim 10^{40}\ \mathrm{cm}^{-3}$$

4. Нулевая темп.

$kT \ll E_{kin}$ ← Паули

$\approx mc^2$

WD (e^-) : $kT \ll m_e c^2 \sim 0{,}5\ MeV$

$T \ll 10^{10} K$

NS (p, n) : $kT \ll m_N c^2 \sim 1\ GeV$

$T \ll 10^{13} K$

5. <u>WD</u>

$$p \sim n_e^{5/3} \longrightarrow \varepsilon = \frac{E}{V}$$

$$\varepsilon_K^{NR} = \frac{3}{5}\frac{\hbar^2}{2m_e}\left(3\pi^2 n_e\right)^{2/3} \sim n_e^{2/3}$$

Кулон. энерг. взаим. с яд.

$$\varepsilon_{e\text{-}nuc} \sim -Ze^2\left(\frac{n_e}{Z}\right)^{1/3} = -Z^{2/3}e^2 n_e^{1/3} \sim n_e^{1/3}$$

$$\varepsilon_K^{UR} = \frac{3}{4}\hbar c\left(3\pi^2 n_e\right)^{1/3} \sim n_e^{1/3} \qquad \frac{\varepsilon_{e\text{-}nuc}}{\varepsilon_K^{UR}} \sim 10^{-2}\left(\frac{Z}{6}\right)^{2/3} \ll 1$$

Давл. – e^-, на – Nb – ядрами

$$P = \begin{cases} \dfrac{(3\pi^2)^{2/3}}{5}\dfrac{\hbar^2}{m_e}\left(\dfrac{Z}{A}\dfrac{\rho}{m_N}\right)^{5/3} & NR \\[2ex] \dfrac{(3\pi^2)^{1/3}}{4}\hbar c\left(\dfrac{Z}{A}\dfrac{\rho}{m_N}\right)^{4/3} & UR \end{cases}$$

$$E_{TOTAL} = N_e \frac{3}{5} \frac{p_{Fe}^2}{2m_e} - \frac{3}{5} \frac{GM^2}{R} = \qquad M = \rho \frac{4}{3}\pi R^3$$

$$= \frac{3}{10} \left(\frac{9\pi}{4}\right)^{2/3} \frac{\hbar^2}{m_e R^2} \left(\frac{ZM}{A \cdot m_N}\right)^{5/3} - \frac{3}{5} \frac{GM^2}{R}$$

$$N_e = \frac{Z}{A} \frac{M}{m_N}$$

Вирал: $$R_{WD} \simeq \left(\frac{9\pi}{4}\right)^{2/3} \left(\frac{Z}{A}\right)^{5/3} \frac{\hbar^2}{G m_e m_N^{5/3} M^{1/3}}$$

$$= 0{,}7 \cdot 10^4 \text{ км} \left(\frac{2Z}{A}\right)^{5/3} \left(\frac{M}{M_\odot}\right)^{-1/3} \sim M^{-1/3}$$

В рел. $$E = N_e \frac{3}{4} p_{Fe} \cdot c - \frac{3}{5} \frac{GM^2}{R}$$

$$\cancel{\frac{M^{4/3}}{R}} \text{ vs. } \cancel{\frac{M^2}{R}}$$

$$M_{ch} = \frac{15}{16} (5\pi)^{1/2} \left(\frac{Z}{A \cdot m_N}\right)^2 \cdot m_p^3 \qquad m_p = \left(\frac{\hbar c}{G}\right)^{1/2}$$

$$\sim 1{,}7 \left(\frac{2Z}{A}\right)^2 M_\odot$$

6). Политропа

$$\frac{dP}{dr} = -\frac{Gm}{r^2}\rho \qquad \frac{dm}{dr} = 4\pi\rho r^2 \qquad P \sim \rho^{1+\frac{1}{n}}$$

$$\hat{P} = P/P_c \qquad \hat{\rho} = \rho/\rho_c \qquad \hat{P}(0) = \hat{\rho}(0) = 1$$

$$\hat{m} = m/\bar{m} \qquad \hat{r} = r/\bar{r} \qquad \bar{r} = \left(\frac{P_c}{4\pi G\rho_c^2}\right)^{1/2} = \left(\frac{K\rho_c^{\gamma-2}}{4\pi G}\right)^{1/2}$$

$$\bar{m} = \frac{(P_c/G)^{3/2}}{(4\pi^2)^{1/2}\rho_c^2} = \frac{(K/G)^{3/2}}{(4\pi)^{1/2}}\rho_c^{\frac{3\gamma-4}{2}}$$

$$\frac{d\hat{P}}{d\hat{r}} = -\hat{m}\frac{\hat{\rho}^{1/\gamma}}{\hat{r}^2} \qquad \frac{d\hat{m}}{d\hat{r}} = \hat{P}^{1/\gamma}\cdot\hat{r}^2$$

$$M \sim \bar{m} \sim \rho_c^{\frac{3\gamma-4}{2}} \qquad R \sim \bar{r} \sim \rho_c^{\frac{\gamma-2}{2}} \sim M^{\frac{\gamma-2}{3\gamma-4}}$$

Для $\gamma = 5/3$ $\quad R \sim M^{-1/3}$

7) ОУ

$$n \to p + e^- + \bar{\nu}_e$$
$$p + e^- \to n + \nu_e$$

$$Q = (m_n - m_p - m_e)c^2 = 1{,}5\, m_e c^2$$

β-равновесие

$$\mu_n = \mu_p + \mu_e$$

$$\mu_i = \left(m_i^2 c^4 + p_{F_i}^2 c^2\right)^{1/2} \qquad p_{F_i} \sim n_i^{1/3}$$

Е. $p_{F_e} \gg m_e c$ и $p_{F_n}, p_{F_p} \ll m_N c \Rightarrow p_{F_p} \ll m_N \cdot c$

$$\frac{p_{F_n}^2}{2m_N} = \frac{p_{F_p}^2}{2m_N} + p_{F_e} \cdot c \qquad \frac{p_{F_p}^2}{2m_N} \ll p_{F_e} \cdot c$$

$$Y \equiv \frac{n_p}{n_n} = \frac{n_e}{n_n} \sim \frac{n_N}{n_0} \qquad n_0 = \left(\frac{64\pi}{3}\right)\lambda_N^{-3}, \quad \lambda_N = \frac{h}{m_N c}$$

Е. n и p не рел., то $Y \ll 1$

$\mu_n = \mu_p + \mu_e$

Pen.

$\mu_i = p_{Fi} \cdot c \qquad p_F \sim n^{1/3}$

$p_{Fp} = p_{Fe} \Rightarrow \mu_p = \mu_e$

$\mu_n = 2\mu_p$

$p_{Fn} = 2 p_p$

$n_n^{1/3} = 2 n_p^{1/3}$

$\frac{n_p}{n_n} = \frac{1}{8}$

$\mathcal{E}.\ \mu_e > m_\mu \cdot c^2, \qquad 105{,}7\ \mathrm{MeV}$

то нозл. μ.

Нерел. нейтр. $A = Z = 1$ $\qquad R_{WD} = 4{,}5 \left(\frac{Z}{A}\right)^{5/3} \frac{\hbar^2}{G m_e \cdot m_N^{5/3}} \frac{1}{M^{1/3}}$

$$R_{NS} = 4{,}5 \frac{\hbar^2}{G M_N^{8/3}} \frac{1}{M^{1/3}} = 15 \left(\frac{M_\odot}{M}\right)^{1/3} \text{км}$$

а. Кан Mch. $\qquad M_{NS\,max} = M_{OV} = 3 \frac{m_p^3}{m_N^2} = 5{,}7\, M_\odot$

5. $R_{NS} = \frac{2GM}{c^2}$ $\qquad M_{OV} \sim 3 M_\odot$

6. $M \sim \frac{4\pi}{3} R^3 \left(\frac{\varepsilon}{c^2}\right) \Rightarrow R = \ldots$

$R = \beta R_{BH}$ $\qquad \left[\frac{3}{4\pi} M \left(\frac{c^2}{\varepsilon}\right)\right]^{1/3} = 2\beta \frac{GM}{c^2}$

$$M_{OV} = \left(\frac{3}{32\pi\beta^3}\right)^{1/2} \frac{c^4}{G^{3/2} \varepsilon^{1/2}} \qquad \varepsilon \sim \frac{m_N \cdot c^2}{\lambda_{\bar{N}}^3} \Rightarrow M_{OV} \simeq 2 \cdot \left(\frac{2}{\beta}\right)^{3/2} M_\odot$$

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