# Internal structure of Neutron Stars

# Artistic view





# Astronomy meets QCD



arXiv: 0808.1279

# Hydrostatic equilibrium for a star



For NSs we can take T=0 and neglect the third equation

For a NS effects of GR are also important.

$$r_g = \frac{2GM}{c^2} \approx 2.95 \frac{M}{M_{SUN}} \text{ km}$$

 $M/R \sim 0.15 (M/M_{\odot})(R/10 \text{ km})^{-1}$ J/M ~ 0.25 (1 ms/P) (M/M<sub> $\odot$ </sub>)(R/10km)<sup>2</sup>

#### Lane-Emden equation. Polytrops. $P = K\rho^{\gamma}, \quad K, \gamma = \text{const}, \quad \gamma = 1 + \frac{1}{-1}$ $\frac{dP}{dr} = -\frac{Gm\rho}{r^2} = g\rho, \qquad g = -\frac{Gm}{r^2} = -\frac{d\varphi}{dr}$ $\frac{dP}{dr} = -\rho \ \frac{d\varphi}{dr}, \qquad \Delta \varphi = 4\pi G\rho$ $\rho = \rho_c \Theta^n$ , $\Theta = 1$ при r = 0 $P = K \rho_c^{1+1/n} \Theta^{1+n}, \quad \frac{dP}{dr} = (n+1) K \rho_c^{1+1/n} \Theta^n \frac{d\Theta}{dr}$ $\frac{d\varphi}{dr} = -(n+1)K\rho_c^{1/n} \frac{d\Theta}{dr}$ $\Delta \Theta = -\frac{4\pi G \rho_c^{1-1/n}}{(n+1)K} \ \Theta^n$ $\Theta = \Theta(\xi)$ $0 \leq \xi \leq \xi_1$ $\xi = r/a, \quad a^2 = (n+1)K\rho_c^{1/n-1}/(4\pi G)$ $\Theta(0) = 1, \quad \Theta'(0) = 0$ $\frac{1}{\xi^2} \frac{d}{d\xi} \xi^2 \frac{d}{d\xi} \Theta = -\Theta^n$ $\Theta(\xi_1) = 0$

# Properties of polytropic stars

#### **Analytic solutions:**

$$n = 0 \qquad \Theta = 1 - \frac{\xi^2}{6} \qquad \xi_1 = \sqrt{6}$$
$$n = 1 \qquad \Theta = \frac{\sin \xi}{\xi} \qquad \xi_1 = \pi$$
$$n = 5 \qquad \Theta = \frac{1}{\sqrt{1 + \xi^2 / 3}} \qquad \xi_1 = \infty$$

$$M = 4\pi \int_{0}^{R} dr \, r^{2} \rho = 4\pi \rho_{c} a^{3} \xi_{1}^{2} |\Theta'(\xi_{1})|$$
$$\frac{\rho_{c}}{\rho} = \frac{4\pi R^{3} \rho_{c}}{3M} = \frac{\xi_{1}}{3|\Theta'(\xi_{1})|}$$

	n	0	1	1.5	2	3	
	$\xi_1$	2.449	3.142	3.654	4.353	6.897	
$M \sim \rho_c^{(3-n)/(2n)}$	$ \Theta'_1 $	0.7789	0.3183	0.2033	0.1272	0.04243	
<b>P</b> $(1-n)/(2n)$	$\rho_c/\overline{\rho}$	1	3.290	5.991	11.41	54.04	
$\mathbf{K} \sim \boldsymbol{\rho}_c$	$n=0$ $M \sim R^3$						

$$n = 0 \qquad M \sim R^{3}$$

$$n = 1 \qquad M \sim \rho_{c} \qquad R = \text{const}$$

$$n = 1.5 \qquad M \sim \sqrt{\rho_{c}} \sim R^{-3}$$

$$n = 3 \qquad M = \text{const} \quad R \sim \rho_{c}^{-1/3}$$

$$R \sim \rho_c^{(1-n)/(2n)}$$

$$M \sim R^{(3-n)/(1-n)}$$

#### Useful equations White dwarfs

- 1. Non-relativistic electrons  $\gamma=5/3$ , K=(3<sup>2/3</sup>  $\pi^{4/3}/5$ ) ( $\hbar^2/m_e m_u^{5/3} \mu_e^{5/3}$ );  $\mu_e$ -mean molecular weight per one electron K=1.0036 10<sup>13</sup>  $\mu_e^{-5/3}$  (CGS)
- 2. Relativistic electrons  $\gamma = 4/3, K = (3^{1/3} \pi^{2/3} / 4) (\hbar c/m_u^{4/3} \mu_e^{4/3});$  N K=1.2435 10<sup>15</sup>  $\mu_e^{-4/3}$  (CGS)

#### **Neutron stars**

- 1. Non-relativistic neutrons  $\gamma$ =5/3, K=(3<sup>2/3</sup>  $\pi$ <sup>4/3</sup>/5) ( $\hbar$ <sup>2</sup>/m<sub>n</sub><sup>8/3</sup>); K=5.3802 10<sup>9</sup> (CGS)
- Relativistic neutrons γ=4/3, K=(3<sup>1/3</sup> π<sup>2/3</sup> /4) (ħc/m<sub>n</sub><sup>4/3</sup>); K=1.2293 10<sup>15</sup> (CGS)

#### Neutron stars

#### Superdense matter and superstrong magnetic fields



Proto-neutron stars



Mass fraction of nuclei in the nuclear chart for matter at T = 1 MeV,  $n_B = 10^{-3}$  fm<sup>-3</sup>, and  $Y_P = 0.4$  (proton fraction). Different colors indicate mass fraction in Log<sub>10</sub> scale.

1202.5791

Proto-NS EoS must be well-matched with NS EoS, see 1207.2184

# EoS for core-collapse, proto-NS and NS-NS mergers

	Core-collapse	<b>Proto-neutron</b>	Mergers of compact	
	supernovae	stars	binary stars	
$n/n_s$	$10^{-8}$ - $10$	$10^{-8}$ - 10	$10^{-8}$ - 10	
$T({ m MeV})$	0 - 30	0 - 50	0 - 100	
$Y_e$	0.35 - 0.45	0.01 - 0.3	0.01 - 0.6	
$S(k_B)$	0.5 - 10	0 - 10	0 - 100	

Wide ranges of parameters

# Astrophysical point of view

Astrophysical appearence of NSs is mainly determined by:

- Spin
- Magnetic field
- Temperature
- Velocity
- Environment



The first four are related to the NS structure!

# Equator and radius

#### $ds^2 = c^2 dt^2 e^{2\Phi} - e^{2\lambda} dr^2 - r^2 [d\theta^2 + sin^2\theta d\phi^2]$

In flat space  $\Phi(r)$  and  $\lambda(r)$  are equal to zero.

### Gravitational redshift





It is useful to use m(r) – gravitational mass inside r – instead of  $\lambda(r)$ 

#### Outside of the star



# Bounding energy



If you drop a kilo on a NS, then you increase its mass for < kilo

M<sub>acc</sub> is shown with color

 $M_{acc} = \Delta M_G + \Delta BE/c^2 = \Delta M_B$ BE- binding energy BE=(M<sub>B</sub>-M<sub>G</sub>)c<sup>2</sup>

# Binding energy vs. mass



## Gravitational mass vs. baryonic



# TOV equation

$$R_{ik} - \frac{1}{2} g_{ik} R = \frac{8\pi G}{c^4} T_{ik}$$

(1) 
$$\frac{dP}{dr} = -\frac{G\rho m}{r^2} \left(1 + \frac{P}{\rho c^2}\right) \left(1 + \frac{4\pi r^3 P}{mc^2}\right) \left(1 - \frac{2Gm}{rc^2}\right)^{-1}$$
  
(2) 
$$\frac{dm}{dr} = 4\pi r^2 \rho$$
  
(3) 
$$\frac{d\Phi}{dr} = -\frac{1}{\rho c^2} \frac{dP}{dr} \left(1 + \frac{P}{\rho c^2}\right)^{-1}$$
  
(4) 
$$P = P(\rho)$$
  
Tolman (1939)  
Oppenheimer-  
Volkoff (1939)





#### Neutron star interiors



Radius: 10 km Mass: 1-2 solar Density: above the nuclear Strong magnetic fields



# Different phases



## Neutron star crust



Many contributions to the book are available in the arXiv.

Mechanical properties of crusts are continuously discussed, see 1208.3258, 1808.06415, 2010.08398. About mountains see 2009.12794

## EoS for the crust



It is necessary to match the EoS for the crust and the core accurately, see 2106.12845

# Element composition of the crust



# Inner crust properties



See new results in 2001.09739

# Accreted crust

It is interesting that the crust formed by accreted matter differs from the crust formed from catalyzed matter. The former is thicker.



1104.0385

#### See new results in 1910.03932

# Composition of accreted crust



2001.09739

A more detailed treatment of the crust is proposed in 2004.04195.

# Crust and limiting rotation

Model	$\sigma$	$f_{\rm in}^{1.4}$ (Hz)	$f_{\mathrm{fin}}^{1.4}$ (Hz)	$f_{\rm in}^{1.8}$ (Hz)	$f_{\mathrm{fin}}^{1.8}$ (Hz)
HLPSStiff	0.05	0	326	35	368
	0.10	136	479	236	569
IU-FSU	0.05	349	515	909	1022
	0.10	781	947	1875	1988
IU-FSUmax	0.05	35	358	374	586
	0.10	232	555	854	1066

The maximum initial rotational frequency of a neutron star whose crust will never fail from slowing down, assuming a breaking strain of  $\sigma$ = 0.1.



Failure of the crust can be the reason of the limiting frequency. Spinning-up of a NS due to accretion can result in crust failure.

Then the shape of the star is deformed, it gains ellipticity.

So, GWs are emitted which slow down the compact object.

(about limits based on EoS see 1805.11277)

# Configurations



A RNS code is developed and made available to the public by Sterligioulas and Friedman ApJ 444, 306 (1995) http://www.gravity.phys.uwm.edu/rns/

NS mass vs. central density (Weber et al. arXiv: 0705.2708)

Stable configurations for neutron stars and hybrid stars (astro-ph/0611595).



## EoS



(Weber et al. ArXiv: 0705.2708)

## Mass-radius

Mass and radius are marcoscopical potentially measured parameters. Thus, it is important to formulate EoS in terms of these two parameters. Mass-radius relations for CSs with possible phase transition to deconfined quark matter.



About hyperon stars see a review in 1002.1658. About strange stars and some other exotic options – 1002.1793

astro-ph/0611595

# Mass-radius relation



#### Main features

- Max. mass
- Diff. branches (quark and normal)
- Stiff and soft EoS
- Small differences for realistic parameters
- Softening of an EoS with growing mass

Rotation is neglected here.

Obviously, rotation results in:

- larger max. mass
- larger equatorial radius
   Spin-down can result in phase transition, as well as spin-up (due to accreted mass), see 1109.1179



Haensel, Zdunik astro-ph/0610549

## Sound velocity and M-R relation







# Moment of inertia measurements to constrain EoS



Precise (+/-10%) measurements of a moment of inertia can significantly constrain the range of allowed radii for a give EoS.



# Theory vs. observations



# Au-Au collisions





# Experimental results and comparison



1 Mev/fm<sup>3</sup> = 1.6 10<sup>32</sup> Pa

**GSI-SIS** and AGS data

Danielewicz et al. nucl-th/0208016

New heavy-ion data and discussion: 1211.0427

Also laboratory measurements of lead nuclei radius can be important, see 1202.5701. See a recent brief review in 2110.02616



See 1803.01836

# Phase diagram

Phase diagram for isospin symmetry using the most favorable hybrid EoS studied in astro-ph/0611595.



(astro-ph/0611595)

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Particle fractions
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Effective chiral model of Hanauske et al. (2000)

Relativistic mean-field model TM1 of Sugahara & Toki (1971)



(Yakovlev)

Яковлев и др. УФН 1999

# Quark stars





#### 1210.1910

See also 1112.6430

## Formation of quark stars



1109.0539

as in SNIa. Neutrino signal due to

12 14

0.6 5.0 1.0

(b)  $t = 0.7 \,\mathrm{ms}$ 

(d)  $t = 4.0 \,\mathrm{ms}$ 

Turbulent deflagration,

conversion of a NS into a quark star was calculated in 1304.6884

Hybrid stars



Stars with quark cores are reviewed in 1904.05471

See also 1302.4732, 1903.08963, 1903.09121

# Massive hybrid stars

Stars with quark cores can be massive, and so this hypothesis is compatible with existence of pulsars with M>2 Msolar



1304.6907 New solutions also allow such objects, see 1903.08963, 1903.09121





New exotic solution.

It is not clear if it can be applied to any known type of sources.

## NS interiors: resume



(Weber et al. ArXiv: 0705.2708)

### Maximum mass

Maximum mass of NSs depends on the EoS, however, it is possible to make calculations on the base of some fundamental assumptions.



# NS-NS coalescence and EoS



GW signals for different EoS for two non-spinning 1.4 solar mass NSs. Vertical dashed line correspond to 1kHz frequency. Differences are due to different tidal response.

1912.01461, see 1907.08534 on methods of such studies and a review in 2103.16371.

# Constraints on the M-R plane



1907.08534, about physics of mergers see 2002.03863

# Calculations based on recent data on NS-NS coalescence

What uniform rotation can give:  $M_{\text{max}} = (1.20^{+0.02}_{-0.02}) M_{\text{TOV}}$  independently of the EOS



1711.00314

 $M_{TOV}$ =2-2.3 solar masses

# Another constraint from GW170817

 $M_{\rm NSNS} \approx 2.74 \lesssim M_{\rm thresh} \approx \alpha M_{\rm max}^{\rm sph}$ . As there was no prompt collapse

Here  $\alpha \approx 1.3 - 1.7$  is the ratio of the HMNS threshold mass limit to the NS spherical maximum mass as gleaned from multiple numerical experiments of merging NSNSs

 $M_{\rm NSNS} \approx 2.74 \gtrsim M_{\rm max}^{\rm sup} \approx \beta M_{\rm max}^{\rm sph},$ 

where  $\beta \approx 1.2$  is the ratio of the uniformly rotating supramassive NS limit to the nonrotating spherical maximum

$$\begin{split} M_{\max}^{\rm sph} &= 4.8 \left( \frac{2 \times 10^{14} \text{ gr/cm}^3}{\rho_m/c^2} \right)^{1/2} M_{\odot} , \quad \text{-TOV limit from causality} \\ M_{\max}^{\rm sup} &= 6.1 \left( \frac{2 \times 10^{14} \text{ gr/cm}^3}{\rho_m/c^2} \right)^{1/2} M_{\odot} , \qquad 2.74/\alpha \lesssim M_{\max}^{\rm sph} \lesssim 2.74/\beta \\ \beta \approx 1.27. \qquad \qquad M_{\max}^{\rm sph} \lesssim 2.16. \quad \beta \approx 1.27. \\ M_{\max}^{\rm sph} \lesssim 2.28. \quad \beta = 1.2 \end{split}$$

1711.00473, see a review on the methods in 1904.04233 and 1912.01461



# Papers to read

- 1. astro-ph/0405262 Lattimer, Prakash "Physics of neutron stars"
- 2. 0705.2708 Weber et al. "Neutron stars interiors and equation of state ..."
- 3. physics/0503245 Baym, Lamb "Neutron stars"
- 4. 0901.4475 Piekarewicz "Nuclear physics of neutron stars" (first part)
- 5. 2105.03747 Burgio et al. "Neutron Stars and the Nuclear Equation of State"
- 6. 1512.07820 Lattimer, Prakash "The EoS of hot dense matter ...."
- 7. 1001.3294 Schmitt "Dense matter in compact stars A pedagogical introduction "
- 8. 1303.4662 Hebeler et al. "EoS and NS properties vs. nuclear phys. and observation"
- 9. 1210.1910 Weber et al. Structure of quark star
- 10. 1302.1928 Stone "High density matter"
- 11. 1707.04966 Baym et al. "From hadrons to quarks in neutron stars: a review"
- 12. 1804.03020. Burgio, Fantina "Nuclear EoS for Compact Stars and Supernovae"
- 13. 1803.01836 Blaschke, Chamel. "Phases of dense matter in compact stars"
- 14. 1904.05471 Alford et al. "Signatures of quark matter..."
- 15. 1904.08907 Coleman Miller et al. "Constraining the EoS ...."
- 16. 1912.11876 Pethick "Dense matter and neutron stars"
- 17. 2202.04117 Younes, Coleman Miller, Yagi "GW and X-ray Probes of the NS EoS"

#### + the book by Haensel, Yakovlev, Potekhin

# Lectures on the Web

Lectures can be found at my homepage:

http://xray.sai.msu.ru/~polar/html/presentations.html