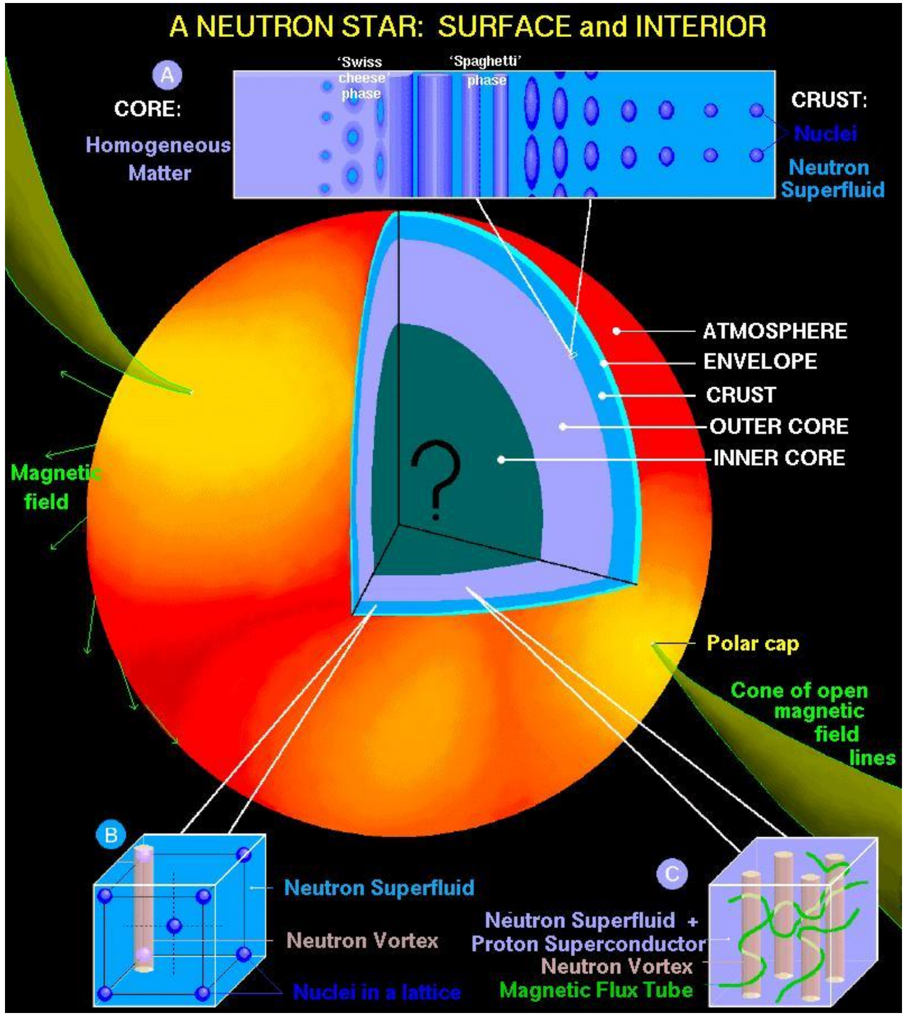
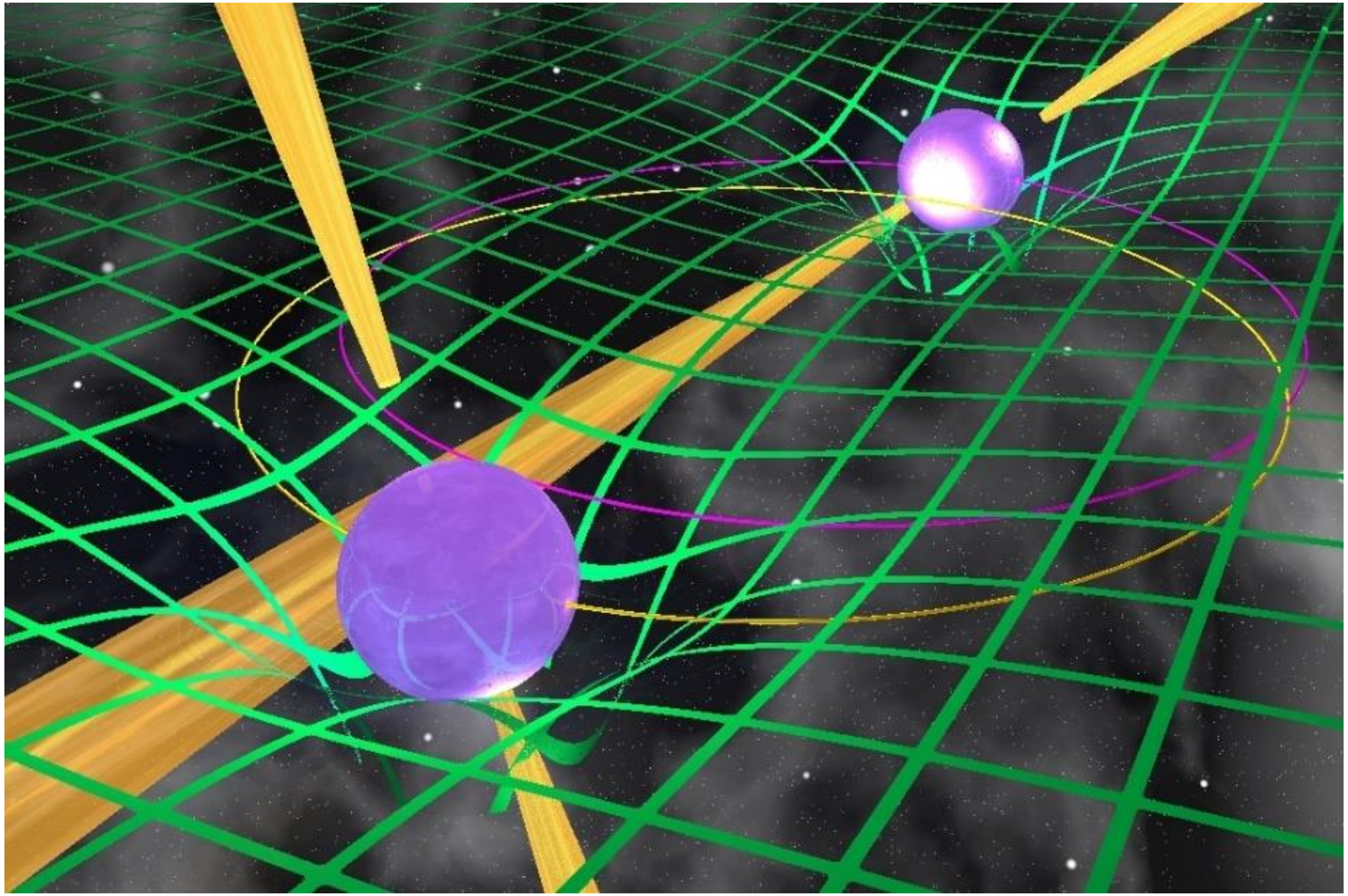




Современная феноменология нейтронных звезд и черных дыр.

<u>Семинар 2</u>

14.02.2022

① Ферми

$$1fm = \frac{\hbar}{m_\pi \cdot c} \simeq 10^{-13} cm \qquad m_\pi = 135 MeV/c^2$$

$$\rho_N \sim n_N \cdot m_N = \left[\frac{4}{3}\pi(1fm)^3\right]^{-1} \cdot m_N \simeq 3 \cdot 10^{14} g/cm^3$$

1511.08813

② Сфера Ферми и импульс Ферми

$\Delta p \Delta x \simeq \hbar \Rightarrow \Delta p_x \Delta x \, \Delta p_y \Delta y \, \Delta p_z \Delta z = \hbar^3 / g_s$

$S = \frac{1}{2} \quad g_s = 2$

$dN = \frac{g_s}{\hbar^3} \cdot V \cdot 4\pi p^2 dp \qquad p < |p| < p + dp$

$N = \frac{g_s}{\hbar^3} V \cdot \frac{4}{3} \pi p_F^3 \qquad p_F^3 = \left(\frac{N}{V}\right) \frac{\hbar^3}{g_s} \frac{3}{4\pi}$, $\frac{N}{V} = n$

$p_F = \hbar (3\pi^2 n)^{1/3}$

$d = n^{-1/3}$

~~$p_F = \hbar / \left(d (3\pi^2)^{1/3}\right)$~~

$p_F = \hbar (3\pi^2)^{1/3} \cdot n^{1/3} =$

$= \hbar (3\pi^2)^{1/3} / d$

3 $$dE = T\,dS - P\,dV + \mu\,dN \qquad T = S = \varnothing$$

$$\varepsilon = E/N \qquad P = -\left(\frac{\partial E}{\partial V}\right)_N = -\left(\frac{\partial (E/N)}{\partial (V/N)}\right)_N =$$

$$= n^2 \frac{d}{dn}\left(\frac{\varepsilon}{n}\right)$$

$$\hat{\varepsilon}(p) = \sqrt{(mc^2)^2 + (pc)^2}$$ – эн. д. рел.ч. p_F

$$E = \int \hat{\varepsilon}(p)\,dN = \frac{g_s}{h^3} V 4\pi \int_0^{p_F} \hat{\varepsilon}(p)\,p^2\,dp \qquad (*)$$

а) решаем (*) для нерел. случ. $p \ll mc$

$$\tilde{\varepsilon}(p) = mc^2 + p^2/2m$$

$$E = N\left(mc^2 + \frac{3}{5}\frac{p_F^2}{2m}\right)$$

$$\mathcal{E} = \frac{E}{V} = mc^2 \cdot n + \frac{3^{5/3}\pi^{4/3}}{10}\frac{\hbar^2}{m} n^{5/3}$$

$$P = n^2 \frac{d}{dn}\left(\frac{\mathcal{E}}{n}\right) = \frac{(3\pi^2)^{2/3}\hbar^2}{5m} n^{5/3} \Rightarrow P \sim \rho^{5/3}$$

$$\gamma = 5/3$$

б) Решаем (*) для $p \gg mc$

$\tilde{\varepsilon}(p) = p \cdot c$ $\qquad$ $\frac{E}{N} = \frac{3}{4} p_F \cdot c$

$$\varepsilon = \frac{3}{4} (3\pi^2)^{1/3} \hbar c n^{4/3}$$

$$P = \frac{\varepsilon}{3} = \frac{1}{4} (3\pi^2)^{1/3} \hbar c n^{4/3} \Rightarrow P \sim \rho^{4/3} \quad \gamma = 4/3$$

Переход от а) к б) $p \sim mc$

$$\underline{\underline{n}} = \left(\frac{8\pi}{3}\right) \lambda^{-3} \qquad \lambda = \frac{h}{mc}$$

e^- ! $n \sim 10^{30}$ см$^{-3}$

p^+ ! $n \sim 10^{40}$ см$^{-3}$

4) Нулевая темп.

$kT \ll E_{кин}$
$\sim mc^2$

WD(e^-): $kT \ll m_e c^2 \sim 0{,}5\,MeV$ — $\underline{\underline{10^{10}K}}$

NS(p,n): $kT \ll m_N \cdot c^2 \sim 1\,GeV$ — $\underline{\underline{10^{13}K}}$

5) WD

NR: $\frac{3}{5}\frac{p^2}{2m}$

$$E_K^{NR} = \frac{3}{5}\frac{\hbar^2}{2m_e}\left(3\pi^2 n_e\right)^{2/3} \sim n_e^{2/3}$$

п-ть $\tilde{E}_K^{NR} = \dfrac{3^{5/3}\pi^{4/3}\hbar^2}{10\,m}\, n^{5/3}$ $\qquad E_K^{NR} = \dfrac{\tilde{E}_K^{NR}}{n}$

Кул. эн. взаим. e^- с ядром

$$E_{e-nuc} \sim -Ze^2\left(\frac{n_e}{Z}\right)^{1/3} = -Z^{2/3} e^2 n_e^{1/3} \sim n_e^{1/3}$$

$$E_K^{UR} = \frac{3}{4}\hbar c\left(3\pi^2 n_e\right)^{1/3} \sim n_e^{1/3}$$

$$\frac{E_{e-nuc}}{E_K^{UR}} \sim 10^{-2}\left(\frac{Z}{6}\right)^{2/3}$$

$E_{e-nuc} \ll E_K^{UR}$

$$P = \begin{cases} \frac{(3\pi^2)^{2/3}}{5} \frac{\hbar^2}{m_e} \left(\frac{Z}{A} \frac{\rho}{m_\alpha}\right)^{5/3} & NR \\ \frac{(3\pi^2)^{1/3}}{4} \hbar c \left(\frac{Z}{A} \frac{\rho}{m_\alpha}\right)^{4/3} & UR \end{cases}$$

$$\rho = \frac{n_e}{Z} A \cdot m_\alpha$$

$$E_{TOTAL} = \underbrace{N_e \frac{3}{5} \frac{p_{Fe}^2}{2 m_e}}_{NR} - \frac{3}{5} \frac{G M^2}{R} \sim \qquad N_e = \frac{Z M}{A m_N}$$

$$\sim \frac{3}{10} \left(\frac{9\pi}{4}\right)^{2/3} \frac{\hbar^2}{m_e R^2} \left(\frac{Z}{A} \frac{M}{m_N}\right)^{5/3} - \frac{3}{5} \frac{G M^2}{R}$$

$$\frac{6}{10} \left(\frac{9\pi}{4}\right)^{2/3} \frac{\hbar^2}{m_e R^2} \left(\frac{Z}{A} \frac{M}{m_N}\right)^{5/3} = \frac{3}{5} \frac{G M^2}{R}$$

$$G M^2 = \left(\frac{9\pi}{4}\right)^{2/3} \frac{\hbar^2}{m_e R} \left(\frac{Z}{A} \frac{M}{m_N}\right)^{5/3}$$

$$R_{WD} = \left(\frac{9\pi}{4}\right)^{2/3} \frac{\hbar^2}{m_e} \frac{1}{G M^2} \left(\frac{Z M}{A m_N}\right)^{5/3} \sim M^{-1/3}$$

$$R_{WD} = 7000 \text{ km} \left(\frac{2Z}{A}\right)^{5/3} \left(\frac{M}{M_\odot}\right)^{-1/3}$$

UR:

$$E_{total} = N_e \cdot \frac{3}{4} p_F \cdot c - \frac{3}{5} \frac{G M^2}{R}$$

$$\frac{3}{4} \left(\frac{9\pi}{4}\right)^{1/3} \frac{\hbar c}{R} \left(\frac{Z}{A} \frac{M}{m_N}\right)^{4/3} = \frac{3}{5} \frac{G M^2}{R}$$

$$M_{ch} \sim \frac{15}{16} (5\pi)^{1/2} \left(\frac{Z}{A \cdot m_N}\right)^2 \cdot m_{pl}^3 \sim 1.7 \left(\frac{2Z}{A}\right)^2 M_\odot$$

$$m_{pl} = \left(\frac{\hbar c}{G}\right)^{1/2} \qquad m_{pl}^4 \qquad \left(\frac{\hbar c}{G}\right)^{3/2}$$

6 Ур-е Л-Э.

$$\frac{dm}{dr} = 4\pi r^2 \rho$$

$$\frac{1}{\rho}\frac{dP}{dr} = -\frac{Gm}{r^2}$$

$$\frac{d}{dr}\left(\frac{1}{\rho}\frac{dP}{dr}\right) = \frac{2Gm}{r^3} - \frac{G}{r^2}\frac{dm}{dr} = -\frac{2}{\rho r}\frac{dP}{dr} - 4\pi G\rho$$

$(\times r^2)$

$$r^2 \frac{d}{dr}\left(\frac{1}{\rho}\frac{dP}{dr}\right) + \frac{2r}{\rho}\frac{dP}{dr} = -4\pi G\rho r^2$$

$$\frac{d}{dr}\left[\frac{r^2}{\rho}\frac{dP}{dr}\right] = -4\pi G\rho r^2$$

$(\div r^2)$

$$\frac{1}{r^2}\frac{d}{dr}\left[\frac{r^2}{\rho}\frac{dP}{dr}\right] = -4\pi G\rho$$

$$\frac{1}{r^2}\frac{d}{dr}\left[\frac{r^2}{\rho}\frac{dP}{dr}\right] = -4\pi G\rho \qquad \gamma = 1+\frac{1}{n}$$

$$P = K\rho_c^{1+\frac{1}{n}}\,\Theta^{n+1}$$

$$\rho = \rho_c\Theta^n$$

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2K\rho_c^{1/n}(n+1)\frac{d\Theta}{dr}\right) = -4\pi G\rho_c\Theta^n$$

$$r = a\xi \qquad a^2 = (n+1)K\rho_c^{\frac{1}{n}-1}$$

$$\frac{1}{\xi^2}\frac{d}{d\xi}\left(\xi^2\frac{d\Theta}{d\xi}\right) = -\Theta^n$$

$$\frac{dP}{dr} = -\frac{G m \rho}{r^2} \qquad \frac{dm}{dr} = 4\pi \rho r^2$$

$$P \sim \rho^{1+\frac{1}{n}} \qquad \hat{P} = P/P_c \qquad \hat{\rho} = \rho/\rho_c$$

$$\hat{P}(0) = \hat{\rho}(0) = 1$$

$$\hat{m} = m/\bar{m} \qquad \hat{r} = r/\bar{r}$$

$$\bar{m} = \frac{(P_c/G)^{3/2}}{(4\pi)^{1/2}\rho_c^2} = \frac{(K/G)^{3/2}}{(4\pi)^{1/2}}\rho_c^{\frac{3\gamma-4}{2}}$$

$$\bar{r} = \left(\frac{P_c}{4\pi G \rho_c^2}\right)^{1/2} = \left(\frac{K\rho_c^{\gamma-2}}{4\pi G}\right)^{1/2}$$

$$\frac{d\hat{p}}{d\hat{r}} = -\frac{\hat{m}\,\hat{p}^{1/\gamma}}{\hat{r}^2} \qquad \frac{d\hat{m}}{d\hat{r}} = \hat{p}^{1/\gamma}\hat{r}^2$$

$$M \sim \bar{m} \sim \rho_c^{\frac{3\gamma-4}{2}} \qquad R \sim \bar{r} \sim \rho_c^{\frac{\gamma-2}{2}} \sim M^{\frac{\gamma-2}{3\gamma-4}}$$

$\gamma = 5/3$: $R \sim M^{-1/3}$ $\qquad R_{WD} = 8700\,\mathrm{km}\left(\frac{M}{M_\odot}\right)^{-1/3}$

$\gamma = 4/3$: nuca. $\quad M_{ch} = \ldots = 1.4\left(\frac{2Z}{A}\right)^2 M_\odot$

-7) NS

$n \to p + e^- + \bar{\nu}_e$ β-распад.

$Q = (m_n - m_p - m_e)c^2 = 1.5 \cdot m_e c^2$

$p + e^- \to n + \nu_e$

$\mu_n = \mu_p + \mu_e$ $\mu_i = \sqrt{(m_i c^2)^2 + (p_{F_i} c)^2}$

a) $p_{F_e} \gg m_e c^2$ $p_{F_n}, p_{F_p} \ll m_N c$

$$\frac{p_{F_n}^2}{2m_N} = \frac{p_{F_p}^2}{2m_N} + p_{F_e} \cdot c \qquad \frac{p_{F_p}^2}{2m_N} \ll p_{F_e} \cdot c$$

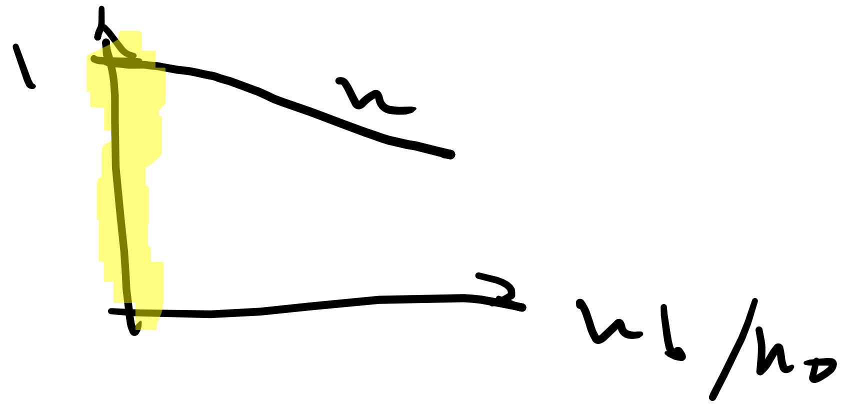
$p_{F_i} = \hbar (3\pi^2 n_i)^{1/3}$ $n_p = n_e \Rightarrow p_{F_p} = p_{F_e}$

$$Y \equiv \frac{n_p}{n_n} = \frac{n_e}{n_n} \sim \frac{n_N}{n_0}$$

небл. стаб. ядро

реакт.

$$n_0 = \frac{64\pi}{3}\lambda_N^{-3}, \quad \lambda_N = \frac{h}{m_N c}$$



Для перел. небл. $Y \ll 1$

б) e, p, n – рел.

$$p_{F_N} \cdot c = p_{F_p} \cdot c + p_{F_e} \cdot c$$
$$= 2 p_{F_p} \cdot c$$

$$p_{F_p} = p_{F_e}$$

$$p_{F_N} = 2\, p_{F_p}$$

$$\cancel{\hbar}(3\pi^2 n_n)^{1/3} = 2\cancel{\hbar}(3\pi^2 n_p)^{1/3}$$

$$n_n = 8\, n_p$$

$\mu_e > \underbrace{m_\mu \cdot c^2}$ 105,7 MeV

Перел. нейтр. $A = Z = 1$

$$R_{NS} = 4{,}5 \frac{\hbar^2}{G m_N^{8/3} M^{1/3}} = 15 \left(\frac{M}{M_\odot}\right)^{-1/3} \text{km}$$

a) $M_{NS,max} = 3 \frac{m_{pl}^3}{m_n^2} = 5{,}7 M_\odot$

b) $R_{NS} \sim M^{-1/3}$

$R_{NS,min} = R(M_{NS,max})$

$R_{NS,min} = R_{Sh} = \frac{2GM}{c^2}$

$M_{NS,max} = 3 M_\odot \sim \frac{3\pi^{1/2}}{2^{7/4}} \frac{m_{pl}^3}{m_n^2}$

c) $M \simeq \frac{4\pi}{3} R^3 \left(\bar{\varepsilon}/c^2\right)$

$R = \beta R_{Sh} = \frac{2\beta G M}{c^2}$

$M_{NS,max} = \left(\frac{3}{32\pi\beta^3}\right)^{1/2} \frac{c^4}{G^{3/2}\bar{\varepsilon}^{1/2}}$

$\bar{\varepsilon} \sim \frac{m_n c^2}{\lambda_n^3}$

$M_{NS,max} = 2\left(\frac{2}{\beta}\right)^{3/2} M_\odot$