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Spatial–Frequency Analysis of the COBE Data and the CMB Anisotropy Spectrum

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Abstract—The problem of obtaining estimates for the cosmic microwave background (CMB) anisotropy spectrum when applying the Galaxy-removal procedure is discussed. A method of two-step spatial-frequency filtering that retains the orthogonality of spherical harmonics is proposed. This method is used to analyze the two-year COBE data. After the analysis of the difference maps containing only noise, the $l = 7$, 9, 13, 23, and 25 multipoles were excluded from the data under consideration because the noise power in these multipoles in the difference maps turned out to be statistically anomalous. This procedure is shown to introduce no significant systematic errors for the model noise-spectrum employed. Once the multipoles that are anomalous in terms of noise have been removed, the analysis yielded a power-law index of the CMB anisotropy power spectrum of $n = 1.84 \pm 0.29$ and a quadrupole mean of $Q_2 = 15.22 \pm 3.0$. It has been established that the COBE data are not described by the Harrison–Zel'dovich spectrum at 99% confidence. The need to increase the instrument sensitivity in order to more reliably separate the cosmological component from the microwave background is demonstrated.

1. INTRODUCTION

The large-scale cosmic microwave background (CMB) anisotropy carries information about the spectrum and amplitude of primordial metric perturbations, and its shape reflects the composition and evolution of the Universe. Analysis of the spatial CMB spectrum and its comparison with theoretical models for various scenarios of the evolution of the Universe provides a basis for deciphering this information.

Improvement in the sensitivity of CMB experiments has led to the detection of anisotropy in the microwave background. As a result, prerequisites for a more thorough study of the physical conditions accompanying the birth and evolution of the Universe have emerged. Having high sensitivity and several frequency channels, the COBE experiment (Smoot *et al.* 1992; Bennett *et al.* 1994) considerably extends our knowledge of the structure and spectrum of the microwave background. Unfortunately, the Galactic emission dominates the microwave background even at millimeter wavelengths. The power of the Galactic emission is several orders of magnitude greater than that expected from the cosmological component of the background.

This raises the problem of filtering this emission. Since the bulk of the Galactic emission comes from a rather narrow region along the Galactic plane, the principal method is to "cut out" this plane and study the properties of high-latitude regions (Strukov *et al.* 1991a, b; Smoot *et al.* 1992; Bennett *et al.* 1994; Gorski 1994; Gorski *et al.* 1994; Wright *et al.* 1994b). However, this widely accepted technique is quite adequate only in an analysis of variance of the data. In an analysis of the spatial spectrum, the orthogonality of spherical functions on the rest of the sphere is lost, and, as a consequence, the harmonics begin to influence each other. This problem can be partly resolved by introducing a new basis that is orthogonal only on part of the sphere (Gorski 1994) or orthogonal only to the monopole and dipole, whose removal on the incomplete sphere has the strongest effect on the spectrum (Wright *et al.* 1994b). The spectra obtained in this way may differ from the actual spectra (Bunn *et al.* 1994), leading to essentially different conclusions in the interpretation of the results.

With the advent of multi-frequency experiments, one would think that it has become possible to directly separate the blackbody CMB component from the frequency-dependent Galactic emission. However, this procedure cannot be implemented with the required accuracy due to variations in the spectral composition of the Galactic emission.

We propose a method of analysis of the spatial spectrum that gives the greatest resolution in the spatial harmonics without losing their mutual orthogonality.

The technique consists in CMB filtering after a spherical-harmonic expansion of the radio-brightness temperature map and involves two filtering steps. In the first step, spatial filtering is performed—some components related to the plane and center of the Galaxy are excluded from the overall spatial spectrum, resulting in a considerable reduction in the amplitude of the Galactic

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microwave emission. Only after applying this procedure, is a frequency "cleanup" made in the second step: several frequency channels are used to filter out the Galactic background depending on which Galactic-emission model is chosen. Nowhere do both these steps lead to the loss of orthogonality of the signal spherical harmonics. The principal difficulty of the technique lies in selecting an appropriate Galactic-emission model in the second step. However, the uncertainty arising from not knowing the true frequency model of the Galactic background is significantly reduced after the first step.

We used the freely available two-year COBE data for the entire sky obtained at frequencies of 31.5, 53, and 90 GHz as our input data. In addition, we employed the data from a 19.2-GHz balloon-borne survey kindly made available to us.

2. METHOD OF ANALYSIS

Reduction of actual observations is almost always accompanied by one or another type of data censoring—weighting, rejection of extreme values, or correcting for systematic errors, trends, etc. Selection of a censoring method is based on *a priori* information about the properties of the sample being analyzed and is aimed at improving the stability of the estimates obtained and bringing the statistical parameters of the actual data closer to the *a priori* models used.

In our case, the signal in the microwave map is the sum of the CMB proper, instrument noise, Galactic emission, and, possibly, some signal arising from uncorrected systematic errors. Since it is the cosmological signal that we are concerned with, the other components should be carefully estimated, considerably weakened, or completely eliminated. Ideally (in the absence of Galactic emission and systematic errors), this procedure cannot significantly affect estimates of the cosmological-signal parameters of interest.

In the Galactic coordinate system, the Galactic emission is concentrated near the equator and center of the Galaxy; it is mainly described by a small number of spatial harmonics. Removing these harmonics (there are 23 of them in our case) substantially reduces the Galactic emission. At the same time, 649 harmonics remain in the spectrum of the cosmological signal after applying this procedure, quite sufficient to reliably estimate the parameters of the signal spectrum if the signal has a Gaussian distribution.

Parameters of the noise inherent in the data being analyzed must be known precisely or properly estimated if the signal parameters are to be correctly estimated. The presence of anomalies in the noise spectrum poses a separate problem. For a moderate signal-to-noise ratio, the presence of anomalies in the noise spectrum may have a significant effect on the estimated parameters. In addition, residual systematic errors may be responsible for anomalies in the noise spectrum. In this case, it may prove to be extremely difficult to construct a statistical model for the anomaly, sharply reducing the actual accuracy of signal estimation. If the number of anomalous harmonics is small compared to the total number of signal harmonics analyzed, the radical solution of the problem would be to exclude these anomalous harmonics from the analysis. In our case, five of 24 multipoles turned out to be anomalous. For homogeneous statistical data, removing harmonics that are anomalous in terms of noise is a fairly safe procedure. The reason is that the parameters describing a Gaussian cosmological signal in the presence of Gaussian noise are only slightly sensitive both to the very procedure of excluding some harmonics and to the rejection criterion for anomalous outliers (we considered outliers exceeding 3σ to be anomalous). However, if there are uncorrected systematic errors in the observational data, this same procedure substantially improves the homogeneity of the sample under study and raises the estimation efficiency. At the same time, if harmonic removal has a significant effect on the estimated parameters, this may serve as a strong argument for the hypothesis that the input data are inhomogeneous and, consequently, suggest the necessity of using a particular procedure to reduce this inhomogeneity.

It should be noted that using reduction techniques that violate the orthogonality of spatial harmonics leads to a distinct smoothing of the spectrum and may reduce the amplitude and wash out spectral anomalies. In this case, actual anomalies can be overlooked and, as a result, estimates of the signal parameters may be in error.

We made use of the fact that each frequency channel in the COBE experiment consisted of two identical receivers (channels A and B) (Wright *et al.* 1994a). Based on the COBE data, we constructed the sum map (half-sum of maps A and B) and the difference map (half-difference of maps A and B). In the sum map, the sum of the signal and noise was measured; in the difference map, there was no signal, and noise alone was measured.

Further, we excluded the monopole and dipole components, as well as the spectral components responsible for the Galactic emission, from the sum map. Next, the sum map was additionally "cleaned" of the Galactic emission. For this purpose, we used the 19- and 31-GHz maps to determine the spectral index of the Galactic emission and then subtracted the 31-GHz map from the 53-GHz map with a weighting factor corresponding to the derived spectral index.

Next, we assumed that the cosmological signal and instrument noise had a Gaussian distribution and that the shape of the noise spectrum was entirely determined by the number of measurements at each point in the sky. On this basis, we used the difference maps to calculate the parameters of the noise and identify its anomalous components. Further, we estimated the signal and its spectral parameters by means of Monte Carlo simulations and determined the confidence intervals of these estimates. The latter procedure was implemented both for the entire signal spectrum being analyzed using the procedure of rejecting spectral components exhibiting anomalous noise. In addition, we tested the stability of the obtained estimates to the procedure of rejecting the quadrupole component.

Signal Analysis on the Unit Sphere

A signal on the unit sphere $\Delta T(\theta, \varphi)$ can be expanded in spherical harmonics $Y_{l,m}(\theta, \varphi)$:

$$\Delta T(\theta, \varphi) = \sum a_l^m Y_{l,m}(\theta, \varphi);$$

the signal variance σ^2 on the unit sphere is then written as

$$\sigma^2 = \frac{1}{4\pi}\sum_l \sum_m (a_l^m)^2 = \sum_l \Delta T_l^2,$$

where ΔT_l^2 is the power of the lth spherical harmonic. In any real experiment, the signal under study $a_{l,\,\mathrm{signal}}^m$ is transformed by its transfer function W_l [in our subsequent analysis, we use the COBE antenna transfer function given in Wright *et al.* (1994a) for W_l], and some noise $a_{l,\,\mathrm{noise}}^m$ is added to the signal:

$$\Delta T(\theta, \varphi) = \sum (W_l a_{l,\,\mathrm{signal}}^m + a_{l,\,\mathrm{noise}}^m) Y_{l,m}(\theta, \varphi).$$

The cosmological signal on the unit sphere is a realization of a random process. In the standard cosmological scenario, inflation gives rise to adiabatic scalar and tensor Gaussian fields, i.e., the spectrum is assumed to be a realization of a Gaussian random process in which the harmonics are independent, $\langle a_l^m a_k^n \rangle \sim \delta(l, k)\delta(m, n)$. If the initial density-perturbation spectrum was $(\delta\rho/\rho)^2 = Ak^n$, then, taking the Sachs–Wolfe effect for the ensemble average into account, the low-order multipoles in the expansion $\Delta T(\theta, \varphi)$ take the form (Bond and Efstathiou 1987)

$$\langle (a_l^m)^2 \rangle \sim \frac{\Gamma(l + (n-1)/2)\Gamma((9-n)/2)}{\Gamma(l + (5-n)/2)\Gamma((3+n)/2)}. \quad (1)$$

Allowance for other physical factors (evolution effects, acoustic oscillations, the Silk effect, etc.) results in a more complex shape of the cosmological spectrum (White *et al.* 1994 and references therein). However, in what follows we will approximate the observed spectrum by formula (1). For a quantitative description of the spectrum, we will use two parameters: the spectral index n and the quadrupole rms $Q_{rms}^2 = \langle \Delta T_2^2 \rangle$ (this latter parameter will serve as the amplitude).

The correlation function of the signal on the unit sphere $C(\beta) = \langle (\Delta T(\mathbf{q}_1))(\Delta T(\mathbf{q}_2)) \rangle$, $\mathbf{q}_1\mathbf{q}_2 = \cos(\beta)$ can be expressed in terms of spherical harmonics as follows:

$$C(\beta) = \frac{1}{4\pi}\sum P_l(\cos\beta)\sum_m (a_l^m)^2.$$

Using the spherical-harmonic expansion, we can write the cross-correlation coefficient between two maps (e.g., at 31.5 and 53 Ghz) as

$$\rho(\mathrm{Map1}, \mathrm{Map2}) = \frac{\sum_{l,m} a_l^m(1) a_l^m(2)}{\sqrt{\sum_{l,m} (a_l^m(1))^2 \sum_{l,m} (a_l^m(2))^2}}.$$

Analysis of Spectrum Parameters

Models of cosmological scenarios predict the average parameters of the CMB fluctuation spectrum, whereas experimenters have to make do with a single realization of such a scenario. Furthermore, noise is always present in actual data. In view of this, the problem of estimating the spectrum parameters arises. This problem has already been discussed by Bunn *et al.* (1994) and Sazhin *et al.* (1995). Here, we give only a brief outline of this subject as applied to estimation of not only the signal but also the noise parameters.

Estimates obtained by the maximum-likelihood method are known to be the most efficient. The problem of estimating a signal spectrum with a specified shape can be used to illustrate this fact.

Let the spectrum be described by $\langle (a_l^m)^2 \rangle = Q^2 F_{lm}$. For spectral amplitude Q and specified shape of the spectrum, the likelihood of our realizing such a spectrum can then be expressed in the form of the likehood function as

$$f(a_{lm}, Q | F_l) = \prod_{lm} \frac{1}{\sqrt{2\pi Q^2 F_{lm}}} \exp\left(-\frac{a_{lm}^2}{2Q^2 F_{lm}}\right).$$

The maximum-likelihood estimate Q_{ML} can then be obtained by solving the equation maximizing the likelihood function:

$$\frac{\partial \ln f(Q | F_{lm})}{\partial Q^2} = 0,$$

$$Q_{ML}^2 = \frac{\sum (a_{lm}^2 / F_{lm})}{M}, \quad M = \sum_{l,m} 1,$$

where M is the number of harmonics used in the analysis. Obviously, this quantity has a χ_M^2 distribution with M degrees of freedom and $Q_{ML}^2 \sim \chi_M^2$. At the same time, the amplitude estimate based on the signal variance σ^2 on the unit sphere is

$$Q^2_{\rm power} = \frac{\sum \Delta T_l^2}{\sum \langle \Delta T_l^2 \rangle | Q = 1} = \frac{\sigma^2}{\sigma^2_{\rm th} | Q = 1} \sim \chi^2_{N_{\rm eff}}.$$

Clearly, $N_{\rm eff} = \frac{(\sum_{lm} \Delta T_l^2)^2}{\sum_{lm} (\Delta T_l^2)^2} \le M$; thus, for the Harrison–Zel'dovich spectrum, $N_{\rm eff} \approx 100$ and $M \approx 700$ for $l_{\rm max} = 25$, giving an accuracy of the maximum-likelihood and total-power estimates of about 5% and 14%, respectively.

The above approach can be used to determine the parameters of the signal spatial spectrum in the presence of noise. Maximum-likelihood estimation then reduces to finding the minimum of the functional

$$\sum_{l,m} \ln(W_l^2 \langle (a_l(Q,n))^2 \rangle + \langle (n^m_{l,\rm noise})^2 \rangle) + \frac{(b_l^m)^2}{W_l^2 \langle (a_l(Q,n))^2 \rangle + \langle (a^m_{l,\rm noise})^2 \rangle} \quad (2)$$

with respect to Q and n for the specified noise spectrum and the measured spectrum of the signal and noise realization. Here, $\langle (a_l(Q, n))^2 \rangle$ is a model of the cosmological-signal spectrum depending on the sought-after parameters and given in the form (1), $(b_l^m)^2$ is the measured spectrum defined below, and $\langle (a^m_{l,\rm noise})^2 \rangle$ is a model of the noise spectrum [see formula (3) below] whose amplitude is found in the same way from the difference map. Of course, we must allow for the effects of Galactic-emission "cleaning" and convertion to a thermodynamic scale on all input quantities. Summation is performed over the multipoles chosen for the analysis.

The necessary condition for such estimates is a knowledge of the measured signal and noise spectra.

Analysis of Noise in the COBE Data

A large number of papers are devoted to the analysis of noise in the COBE data. Bennett *et al.* (1994) and Lineweaver *et al.* (1994) have discussed the correlation properties of the noise and have shown that a correlation at a level of 0.45% is introduced by the process of restoring the sky radio-brightness temperature map from differential observations between points separated by 60° (the angle between the COBE horns), and the noise may be considered to be virtually uncorrelated. The noise power can be estimated from the difference maps.

In the first approximation, the noise may be assumed to be white. However, the accuracy of the actual measurements varies over the entire sphere. The orbital configuration of the COBE satellite was such that the points near the poles of the ecliptic were observed longer than the remaining points. Let us consider methods of allowing for unequal accuracy of the measurements.

Let the noise at each point on the unit sphere be Gaussian with zero mean and variance $\sigma^2(\theta, \varphi) = \sigma_0^2 / N(\theta, \varphi)$, where $N(\theta, \varphi)$ is the number of measurements at the point with coordinates θ and φ. We may then propose the following model for the sky noise spectrum.

If there is a realization of white noise $G = \sum n_l^m Y_l^m$, $\langle n_l^m n_{l'}^{m'} \rangle = 0$, $\langle (n_l^m)^2 \rangle = n^2 = \text{const}$, then the true noise on the sphere is the product of $G(\theta, \varphi)$ and $1/\sqrt{N(\theta, \varphi)}$:

$$R(\theta, \varphi) = 1/\sqrt{N(\theta, \varphi)} G(\theta, \varphi) = w(\theta, \varphi) G(\theta, \varphi).$$

Following Peebles (1980), we write $w(\Omega) Y_l^m(\Omega) = \sum_{l'm'} w_{ll'}^{mm'} Y_{l'}^{m'}(\Omega)$, where

$$w_{ll'}^{mm'} = \int d\Omega w(\Omega) Y_{l'}^{m'}(\Omega) Y_l^m(\Omega),$$

and for the coefficients of the spherical-harmonic expansion of the true noise function $R = \sum a^m_{l,\rm noise} Y_l^m$ we have $a^m_{l,\rm noise} = \sum_{l'm'} w_{ll'}^{mm'} n_{l'}^{m'}$. Next, noting that the harmonics of white noise are uncorrelated, we can write the covariance for $a^m_{l,\rm noise}$ as

$$\langle a^m_{l,\rm noise} a^{m*}_{l*,\rm noise} \rangle = \sum_{l'm'} \sum_{l''m''} w_{ll'}^{mm'} w_{l*l''}^{m*m''} \langle n_{l'}^{m'} n_{l''}^{m''} \rangle = n^2 \sum_{l'm'} w_{ll'}^{mm'} w_{l*l'}^{m*m'}.$$

The noise power spectrum then finally takes the form

$$\langle |a^m_{l,\rm noise}|^2 \rangle = n^2 \sum_{l'm'} |w_{ll'}^{mm'}|^2 = n^2 \int d\Omega w^2(\Omega) |Y_l^m(\Omega)|^2. \quad (3)$$

Since we use arbitrary weights and not only unit or zero weights, this expression differs from the result of Peebles in that it contains the squared weighting factor.

Next, based on the difference maps, we can determine the amplitude of the noise spectrum thus specified by the maximum-likelihood method.

The availability of a model of the noise power spectrum makes it possible to analyze the noise spectrum measured from the difference map. Such an analysis shows that the noise spectrum is generally well described both by model (3) and by the white-noise model, which is the result of a fairly good coverage of the sky by COBE. However, the same analysis indicates that individual noise harmonics exhibit an excess power in both models. Thus, for example, the power in the noise spectrum of the 31-GHz maps for $l = 13$ is greater than the value expected from (3), in which the expected variation of the corresponding multipole σ_l was calculated on the basis of the model and the number of degrees of freedom of this component, by $3.4\sigma_{l=13}$. In what follows, we excluded this and similar anomalous components from our analysis, i.e., we censored the spectral harmonics.

The $l = 25$ components of the noise spectrum exhibit an anomalously high anticorrelation between the 31- and 53-GHz maps: after subtracting the Galactic emission, the noise power exceeded the expected value by $3.3\sigma_{l=25}$. The $l = 25$ components were also excluded from the analysis.

We detected considerable deviations from the specified spectrum at 53 GHz. In view of this, we excluded those multipoles which showed departures from the specified spectrum by more than $3\sigma_l$ and then determined the noise parameters from the remaining components; this procedure was repeated until the spectrum was free of such outliers. Thus, we successively removed the harmonics with $l = 7$, 23, and 9, which exhibited deviations of $3.11\sigma_{l=7}$, $2.63\sigma_{l=23}$, and $2.31\sigma_{l=9}$ in the input spectrum.

As a result, we analyzed the components from $l_{min} = 2$ to $l_{max} = 22$ with the $l = 7$, 9, and 13 components excluded.

For a Gaussian distribution of the harmonic amplitude, such a censoring procedure affects the spectrum parameters only slightly. We used numerical simulations, the results of which are presented in Section 3, to study this problem.

In our analysis of the signal-spectrum parameters, we neglected the correlation between components of the noise spectrum and used only the shape of the power spectrum. In addition, further simulations show that the results remain unaffected when one goes to purely white noise.

Selection of a Galactic-Emission Model

Our approach to isolating the cosmological component from the microwave background is based on an attempt to reduce the effect of the lack of exact information about the frequency dependence of the Galactic emission by means of spatial filtering of the data. The widely accepted Galactic-emission model for the frequency range analyzed assumes the presence of a synchrotron component with a frequency dependence of the form $T = T_0\nu^\alpha$ c $\alpha = -3 \pm 0.2$ and ionized-hydrogen emission with $\alpha = -2.1$ (Bennett *et al.* 1992). One may expect spatial variations in the spectral index α if there are regions along the line of sight with different emission mechanisms and arbitrary relations between the components.

Table 1. Comparison of the effects of spatial filtering (i.e., excluding the components with $m = 0$ for $l = 2k$ and $m = 1$ for $l = 2k$ from $l = 2$ to $l = 25$ inclusive) on the signal amplitude for various frequencies

Frequency, GHz	Spatial filtering	$\sigma_{(A+B)/2}$, μm	$\sigma_{(A-B)/2}$, μm	σ_{Sky}, μm
31.5	no	641.	95.92	634.70
	yes	254.79	94.52	239.83
53.	no	191.64	33.04	188.80
	yes	83.32	32.20	76.86
Frequency "cleaning" $\alpha = -2.15$	yes	50.78	45.44	22.67

Note: The last row gives the signal amplitude after applying spatial-frequency filtering assuming that the spectral index of the Galactic emission corresponds to that of ionized-hydrogen plasma. $\sigma_{(A+B)/2}$ is the amplitude measured from the half-sum of two maps (i.e., signal plus noise), $\sigma_{(A-B)/2}$ is the noise amplitude measured from the half-difference of two maps, and σ_{Sky} is an estimate for the signal amplitude on the sky.

Let us try to roughly estimate the contribution of such variations for some effective value α_0. Let the true value of the spectral index be $\alpha = \alpha_0 + \Delta\alpha$. An attempt to remove the frequency-dependent part from the data for two frequency channels T_1 and T_2 then yields

$$\delta T = T_1\left(\frac{\nu_2}{\nu_1}\right)^{\alpha_0+\Delta\alpha} - T_1\left(\frac{\nu_2}{\nu_1}\right)^{\alpha_0}$$

$$\approx T_2\left[\left(\frac{\nu_2}{\nu_1}\right)^{\Delta\alpha} - 1\right] \approx T_2\Delta\alpha\ln\left(\frac{\nu_2}{\nu_1}\right).$$

Thus, the residual-signal power associated with variations in the spectral index is $\sigma^2_{var} = \Delta\alpha^2(\ln\nu_2/\nu_1)^2\sigma^2_{map2}$. For the frequencies 31.5 and 53 GHz, the residual rms is less than $0.52\Delta\alpha\sigma_{map2}$, i.e., the smaller the Galactic-emission contribution to the high-frequency map, the smaller the residual rms.

It is rather obvious that the bulk of the Galactic emission comes from regions near the center and plane of the Galaxy, implying that the components with $m = 0$ for even multipoles (corresponding to the Galactic plane) and with $m = 1$ for odd multipoles (corresponding to the Galactic center) account for the bulk of the power in the spatial spectrum defined in the Galactic coordinate system. Consequently, their removal sharply reduces the effect of variations in α on the isolation of

Table 2. Mean spectral indices and their variations as determined in various frequency ranges for spherical harmonics

Frequencies, GHz	α_5	$\Delta\alpha_5$	$\Delta\alpha_N$	α_W	$\Delta\alpha_W$
19.2–31.5	−2.13	0.36	0.38	−2.12	0.69
31.5–53.0	−2.27	0.28	0.36	−2.19	0.76

Note: The frequency bands in which the spectral index α was analyzed are in the first column, the spectral index α_5 derived from the most significant components in the spectrum with a signal-to-noise ratio greater than 5 is in the second column, its variations $\Delta\alpha_5$ are in the third column, the variation $\Delta\alpha_N$ in the constant spectral index expected from noise analysis is in the fourth column, the weighted mean spectral index α_W for all harmonics with $m \neq 0$ for $l = 2k$ and $m \neq 1$ for $l = 2k + 1$ is in the fifth column, and its variations $\Delta\alpha_W$ are in the sixth column.

the cosmological component. At the same time, this procedure does not violate multipole orthogonality.

Table 1 gives some data obtained in the analysis of the corresponding spectra for $l_{\min} = 2$ and $l_{\max} = 25$.

In addition, the last row gives data on the difference between the 53-GHz channel and the 31-GHz channel scaled with the spectral index $\alpha = -2.15$ after applying the spatial filter. We performed our calculations for the antenna temperatures without first converting them to thermodynamic temperatures. The derived value of $\sigma_{\rm Sky}$ can be accounted for both by the presence of a cosmological signal and by variations in the spectral index within $\Delta\alpha \approx 0.5$.

It is very unfortunate that the existing data give no way of estimating these variations with sufficient accuracy, and a many-fold improvement in measurement sensitivity is required to draw more definitive conclusions.

Based on the freely available high-frequency surveys, we made an attempt at least to estimate variations in the spectral index. To this end, we used both the COBE data and the 19.2-GHz survey (Boughn *et al.* 1992). The spectrum of the 19.2-GHz map was first reduced to the COBE beam (Wright *et al.* 1994a). Then, we used the most significant components of the expansion in spherical multipoles to analyze their frequency dependence, together with the accuracy of determination of this dependence from the estimated noise spectrum.

For the selected signal-to-noise ratio of 5, the data are presented in Table 2, which gives, respectively, the frequency bands in which the spectral index α was analyzed, the spectral index α_5 as derived from the most significant components of the spectrum, its variations $\Delta\alpha_5$, and the variation of the constant spectral index $\Delta\alpha_N$ expected from the noise analysis. Also given in the table are weighted mean spectral indices α_W derived from all harmonics for which $m \neq 0$ for $l = 2k$ and $m \neq 1$ for $l = 2k + 1$, and their variations $\Delta\alpha_W$.

The spectral index α_{19-31} was found to be systematically lower in absolute value than α_{31-53}. The contribution of the spectral components for a signal-to-noise ratio of 5 to the total power was more than 40% at 53 GHz. Only for one spectral harmonic did we find the spectral index corresponding to the synchrotron component of the Galactic emission expected at these frequencies. This harmonic turned out to be $a_{2.2}$, and the spectral index is $\alpha_{19-31} = -2.86 \pm 0.15$ and $\alpha_{31-53} = -2.92 \pm 0.22$, respectively.

Analysis of the correlation between different maps provides an estimate of the mean spectral index lying within the range $\alpha = -(2.2–2.3)$.

Thus, we may assume that the spectral index is constant and that its measured variations are caused by instrument noise.

We can draw the following conclusion from our analysis: for the most significant components of the Galactic-signal spectrum (except for the component $a_{2.2}$), the Galactic-emission model can be treated as a one-component model (with emission from ionized hydrogen), and the variations in $\Delta\alpha$ may be taken to be zero.

We can now define the measured spectrum itself $(b_l^m)^2$ appearing in expression (2) as

$$b_l^m = \left(k(53) - k(31)\left(\frac{53}{31.5}\right)^{-2.15}\right)^{-1}$$
$$\times\left(a_l^m(53\text{ GHz}) - a_l^m(31.5\text{ GHz})\left(\frac{53}{31.5}\right)^{-2.15}\right),$$

where $k(31)$=1./1.025724 and $k(53)$=1./1.074197 are the coefficients of conversion from thermodynamic to antenna temperatures for 31.5 and 53 GHz; a_l^m (31.5 Ghz) and a_l^m (53 GHz) are the coefficients of the multipole expansion of the 31.5- and 53-GHz sky maps, respectively.

Unfortunately, the experimental sensitivity is not high enough, and the accuracy of the estimates obtained is governed by instrument noise. We hope that the planned Relict-2 cosmic experiment, which is expected to improve the sensitivity by one order of magnitude, will make it possible to perform a fairly reliable analysis of variations in the Galactic-emission

Table 3. Results of spectral analysis after spatial-frequency filtering of the Galactic emission

no.	Analysis of quadrupole	σ^2_{A+B}, μm^2	σ^2_{A-B}, μm^2	σ^2_{Sky}, μm^2	Q_2, μm	n	M
1	yes	$(70.28)^2$	$(56.76)^2$	$(41.44)^2 \pm (14.68)^2$	15.22 ± 2.9	1.84 ± 0.29	446
2	no	$(68.92)^2$	$(56.47)^2$	$(39.51)^2 \pm (14.65)^2$	15.55 ± 3.8	1.81 ± 0.37	442
3	yes	$(82.91)^2$	$(74.19)^2$	$(37.03)^2 \pm (17.48)^2$	18.03	1.31	649
4	no				20.3	1.12	645

Note: The effects of removing the quadrupole components on the final result are presented. The results obtained after excluding the anomalous harmonics are compared. The first row corresponds to analysis of the spectrum from $l = 2$ to $l = 22$ with the anomalous harmonics excluded. The second row gives the same result but with the quadrupole excluded. The third row gives results for the case where all harmonics from $l = 2$ to $l = 25$ are used. The fourth row shows the effects of excluding the quadrupole from the analysis. σ^2_{A+B} is the total measured power in the given spectral window, σ^2_{A-B} is an estimate of the noise power in the same window obtained from the spectrum of the difference map, σ^2_{Sky} is an estimate of the cosmological-signal power ($\sigma^2_{Sky} = \sigma^2_{A+B} - \sigma^2_{A-B}$), Q_2 and n are estimates of the parameters of a spectrum of form (1), and M is the total number of harmonics included in the analysis. We assume that $m \neq 0$ for $l = 2k$ and $m \neq 1$ for $l = 2k + 1$ throughout.

spectral index and to definitively isolate the cosmological signal from the Galactic emission.

3. RESULTS OF ANALYSIS

Our analysis provides estimates of the spatial-spectrum parameters assuming a one-component model of the Galactic emission. We obtained

$$Q_2 = 15.22 \pm 3.0 \ \mu K$$

for the quadrupole and

$$n = 1.84 \pm 0.29$$

for the power-law index for $l_{min} = 2$, $l_{max} = 22$, $m \neq 0$ for $l = 2k$ and $m \neq 1$ for $l = 2k + 1$, and, considering the peculiarities of the noise spectrum, we removed the $l \neq 7$, 9, and 13 harmonics.

We used Monte Carlo simulations to determine the accuracy of the derived parameters and have given rms errors for the estimates. Our simulations involved the construction of difference and sum maps and a determination of the noise parameters in each realization. The estimates include not only the effects of noise and its variations but also cosmic variance also taken into account in the simulations.

In the spatial-frequency window chosen, the measured power was $(70.28 \ \mu K)^2$, the noise estimate was $(56.76 \ \mu K)^2$, and the signal estimate on the unit sphere was $(41.44 \ \mu K)^2 \pm (14.68 \ \mu K)^2$ for the number of analyzed harmonics equal to $M = 446$.

We estimated the probability that the measured spectrum matched the Harrison–Zel'dovich spectrum for a trivial case with the Doppler peak excluded to be $P(H - Z) \leq 1\%$. The inferences are all based on 1000 realizations of experimental models.

Table 3 summarizes the results of our analysis. Also given in the table are the results of our analysis of all harmonics (including the harmonics with anomalous noise and in the window with $l_{min} = 2$ and $l_{max} = 25$, $m \neq 0$ for $l = 2k$ and $m \neq 1$ for $l = 2k + 1$) and with the quadrupole component excluded.

We studied the effect of removing individual noise harmonics separately. In the case where the outliers in the noise spectrum are caused by systematic errors, the removal procedure does not bias the estimate, and results only in an increase in the scatter of the estimates as compared to the uncensored sample. Our simulations demonstrate the validity of this statement; the additional variations in the estimates of Q_2 and n are 0.9 and 0.11, respectively.

At the same time, if the outliers are stochastic in nature (i.e., we are dealing with a rather unlikely noise realization), a bias in the estimate of the noise amplitude may arise and, as a consequence, a bias in the estimates of the signal parameters. We simulated this case as well. Of the noise-spectrum realizations obtained from the difference map, we considered those in which the power of individual multipole harmonics exceeded a given level (specifically, we chose the 2.3σ level); we also simulated the measured signal and noise spectrum. Then we estimated the signal parameters from the input spectrum and from the (censored) spectrum with the outliers and, correspondingly, the biases in the estimates eliminated.

Five hundred realizations of the spectra with outliers (of a total number of 1228 realizations) yielded the following values: $\langle n_{cenzor} - n_{full} \rangle = 0.07 \pm 0.08$ and $\langle Q_{cenzor} - Q_{full} \rangle = -0.4 \pm 0.83$. It should be noted that these estimates are upper bounds because we removed the spectral components from the input data that were subsequently used in some linear combination and thus were additionally normalized, generally reducing the effect of rejecting the outliers.

Our analysis of the COBE data (see Table 3) shows that excluding the anomalous harmonics significantly affects n. For random Gaussian noise, the probability that such a large difference will arise is less than 0.02%. Applying the procedure of removing anomalous

Table 4. Analysis of the parameters of the CMB anisotropy spatial spectrum obtained by various authors from the two-year COBE data

Author	n	Q_{rms-PS}
Quadrupole included		
Bennett *et al.* (1994)	$1.42^{+0.49}_{-0.55}$	$12.8^{+5.2}_{-3.3}$
Gorski *et al.* (1994)	$1.22^{+0.43}_{-0.52}$	$17.0^{+7.5}_{-5.2}$
Wright *et al.* (1994b)	$1.39^{+0.34}_{-0.39}$	
This study	1.84 ± 0.29	15.22 ± 2.9
Quadrupole excluded		
Bennett *et al.* (1994)	$1.11^{+0.60}_{-0.55}$	$15.8^{+7.5}_{-5.2}$
Gorski *et al.* (1994)	$1.02^{+0.53}_{-0.59}$	$20.0^{+10.5}_{-6.5}$
Wright *et al.* (1994b), $l = 3$–30	$1.25^{+0.40}_{-0.45}$	
Wright *et al.* (1994b), $l = 3$–19	$1.46^{+0.39}_{-0.44}$	
This study	1.81 ± 0.37	15.55 ± 3.8

Note: Our result is given for comparison. The effects of removing the quadrupole on the final result are shown.

harmonics to the COBE data leads to a sharp reduction in the dependence of our results on whether or not the quadrupole component is included in the analysis. These all may serve as circumstantial evidence for the necessity of the proposed data censoring.

Based on the derived parameters, we can compute the expected signal in an experiment similar to the one that was carried out in Tenerife (Hancock *et al.* 1994). Taking the antenna beam to be 5°.5, the antenna separation to be 8°.1, and a three-point scheme of measurements, i.e., $\Delta T = T_0 - (T_1 + T_2)/2$, we obtain the following values for spectra of form (1) with our parameters Q_2 and n:

$\sigma_{\text{Tenerif}} = 54.82$ μK after performing spatial filtering with the anomalous components excluded; the same analysis but with the anomalous components included yields two estimates:

$\sigma_{\text{Tenerif}} = 37.22$ μK with the quadrupole included and

$\sigma_{\text{Tenerif}} = 34.47$ μK with the quadrupole excluded.

We did not bin the data in 4° regions in our analysis, which may have somewhat reduced our values. Such a binning procedure was employed in the Tenerife experiment, yielding the following results (Hancock *et al.* 1994):

$\sigma_s = 49 \pm 10$ μK at 33 GHz, and the sum of the 15- and 31 GHz channels gives $\sigma_s = 42 \pm 9$ μm. Thus, the cosmological-signal spectrum with our parameters is in good agreement with an experiment that is sensitive to higher-order spatial harmonics than in the COBE experiment.

4. CONCLUSIONS AND IMPLICATIONS

The problem of eliminating systematic effects arises in virtually any experiment. In CMB anisotropy experiments, such an effect is primarily the Galactic emission. Conventional techniques for suppressing this emission (Galactic-plane cut) necessitate a separate consideration of the procedure of removing the monopole and dipole components from the microwave background, which, in turn, introduce new systematic errors but at a lower level. Introducing an orthogonal basis, together with spatial-frequency filtering of the Galactic emission, allows these types of systematic effects to be eliminated. At the same time, it becomes possible to analyze and remove finer effects that are responsible for uncertainties in the spectrum parameters. Thus, for example, assuming that the instrument noise is Gaussian, data censoring cannot result in a statistically significant difference between the spectrum parameters derived from the full and censored samples. Such a difference may arise either from uncorrected residual systematic effects or from a deviation of the distribution function from a Gaussian. Even assuming a Gaussian distribution function, we have to exclude those spectral components that exhibit anomalous behavior in the noise spectrum.

Our parameters of the cosmological-signal spectrum differ somewhat from those derived in previous studies that used the same data for analysis.

This difference is more likely a result of removing harmonics exhibiting anomalous noise than of spatial-frequency filtering of the Galactic emission. The harmonic exclusion became possible because the spherical harmonics retain their orthogonality.

In all probability, the sensitivity of the results to the quadrupole component noted earlier by a number of authors (see Table 4) is also a result of the influence of anomalous noise components.

The results obtained by different authors from analyses of the two-year COBE data are summarized in Table 4. In addition, we may recall a comparison of the Tenerife experiment with the first-year COBE data given by Hancock *et al.* (1994), in which it was found that $n = 1.7$.

Thus, our value of n is slightly greater than the estimate obtained by other authors and is more accurate.

It should be noted that nobody appears to have analyzed the spectral properties of the noise maps in detail before. Gorski *et al.* (1994) used only the mean noise parameters assuming their distribution to be Gaussian in order to obtain estimates. For their estimates, Wright *et al.* (1994b) used the spectrum of the difference maps, but the basis that they introduced was not quite orthogonal. In their analysis, Bennett *et al.* (1994) considered the correlation function on part of the unit sphere. In the latter two cases, the orthogonality of the basis functions is lost, and the power is a mean of adjacent harmonics, making it impossible to analyze the noise spectrum in detail.

Our next result that differs from the previously published results is the rejection of the trivial $n = 1$ Harrison–Zel'dovich spectrum with 99% confidence.

As a consequence, models with a large λ term turn out to be highly unlikely, while open models with $\Omega < 1$ prove to be more probable if we assume that the initial density perturbations are governed by the power law-index $n = 1$ (Kamionkowski and Spergel 1994). Our result also lends support to the existence of baryonic entropy models (Sugiyama and Silk 1994). At the same time, models with a more complex inflation potential giving $n > 1$ on the scales under study should not be ruled out (Starobinsky 1992; White *et al.* 1994 and references therein).

It should be emphasized, however, that the accuracy of spectrum-parameter determination is not yet high enough draw more definitive conclusions. Besides, we can see that the results we have obtained depend heavily on the particular estimation method used and on the accuracy of the noise determination. Unfortunately, even the four-year COBE observations of the entire sky, which, we hope, will soon be available, will not improve the situation drastically. First, the sensitivity needs to be increased by at least one order of magnitude, as will be achieved in the planned Relict-2 experiment. It will then be possible to achieve an accuracy of estimation of the power-law index n on the order of 5–7%; such accuracy will be governed by the cosmic variance alone rather than by instrument noise (Sazhin *et al.* 1995). Second, the angular resolution of CMB experiments needs to be improved with a simultaneous increase in the degree of sky coverage (Scott *et al.* 1994). Since the current intermediate- and small-scale studies provide information about only a few points in the sky, their results are highly susceptible to variations in the cosmological spectrum.

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