INCLUSION OF INTERACTION AT THE LEVEL OF FIRST QUANTIZATION (ELECTROMAGNETIC CASE)

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Abstract

A nonlinear quantum mechanics is proposed with interaction equations maximally resembling those of classical field theory.

We have already proposed and argued for the inclusion of gravitational self-interaction into quantum mechanics (QM) [1,2,3], that is to the level of first quantization. However, the ideology and the way of realization of such the inclusion are not confined by the only gravitational theory, but can be applied to a theory of arbitrary fields. The object of the present study is the electromagnetic theory. We outline below some general arguments for our constructions.

Certainly, any nonstandard quantum theory of electromagnetic field must stand to the strict competition with the standard second quantized electromagnetic theory, i.e., quantum electrodynamics (QE), because the latter seems to be in a good agreement with experiments. Despite of the existence of ultraviolet divergences, it is a satisfactory enough theory due to its renormalizability. However, the appearance of divergences, although removable by renormalization, excites a great discomfort. The fact of renormalizability of QE is likely accidental. The existence of nonrenormalizable physical theories requires the revision of their foundations. Due to the unity of physics it must concern QE as well.

A common sense suggests that if two quantum-mechanical particles (i.e., particles described by wave functions) possess electric charges and magnetic momenta, they must interact with each other via electromagnetic field. This interaction should distort their wave functions. However, "the interaction of the two wave functions" without references to QE is nowhere considered by an appropriate way. The well-known problem of an electron in the electromagnetic field of an atom or a nucleus is formulated by such a way that this field is treated as a classical external field. In other words, since the linearity of QM prevents to include interaction in a self-consistent way, the many-particle problem of QM is not entirely resolved so far [4] (we propose below the way of its resolution). Thus, the necessity to include (self-)interaction to QM follows from QM itself.

QM remains up date an exceptional physical theory without (self-)interaction in contrast to classical and second-quantized theories because its linearity, expressed in the superposition principle, supports probability interpretation. However, the superposition principle seems as an only sufficient but not the necessary condition for the latter. Turning to the probability theory, remind that if p_1 and p_2 are the probabilities of the events E_1 and E_2 then the probability of the event $E_1 \cup E_2$ is equal to $p_1 + p_2$ for mutually exclusive E_1 and E_2 and the probability of the event $E_1 \cap E_2$ is equal to $p_1\, p_2$ for independent E_1 and E_2. In ordinary QM, these postulates are realizable in the cases of orthogonal states and of the many-particle problem, respectively, but they can remain true in QM with interaction (that is, without the superposition principle). It was shown that the orthogonality of stationary states may take place in the presence of self-interaction [2]. It is also easy to show that the second postulate survives in the framework of our many-particle problem with (self-)interaction.

There exists also the question of whether second quantization is necessary at all. Many effects which were primordially calculated in the framework of QE turned out to be possible to restore in the framework of nonlinear theory without second quantization. O.A.Barut with coauthors showed that the account of the self-interaction of an electron can give such the effects of second quantized theory as the Lamb shift [5,6], the anomalous magnetic moment [7], and the spontaneous emission [6,8]. We are not going to discuss their interpretation here, but recall that they used the set of the ordinary classical equations

$$[\gamma^\mu (i\hbar\, \partial_\mu - \frac{e}{c}\, A_\mu) - mc]\psi = 0, \tag{1}$$

$$\partial_\nu\, \partial^\nu A_\mu - \partial_\mu\, \partial^\nu A_\nu = 4\pi e\, \bar\psi\, \gamma_\mu\, \psi. \tag{2}$$

In the gauge $\partial_\nu\, A^\nu = 0$, the electromagnetic potential A_μ can be eliminated from Eq.(1) in order to obtain the nonlinear integro-differential equation which is sufficient for calculating mentioned effects:

$$[(i\hbar\, \gamma^\mu \partial_\mu - mc) + \frac{4e^2}{c} \int dy\, D(x-y)\, \bar\psi(y)\, \psi(y)]\psi(x) = 0 \tag{3}$$

with $D(x-y)$ chosen as the causal Green function.

We see two requirements to a theory to construct. The first is the invariance under the rescaling transformations:

(one-particle wave function) $\rightarrow a$ *(one-particle wave function)*

with a an arbitrary (not unimodular) complex number. This supports probability interpretation: after any non-evolutionary changes in a physical system the modulus of a norm square can be reduced to unity (the so-called reduction of a wave packet). The second is as minimal deviations from classical field theory (i.e., from Eqs. (1) - (3)) as possible. These requirements can be achieved when, unlike classical ones, interaction equations will have sources which will be ordinary bilinear currents divided on the conserved norm square; the latter being, on the one hand, a functional and, on the other hand, merely a number. To this purpose we construct below some exotic variational principles. Our only justification is that equations are always more fundamental quantities than any variational principles. These principles incorporate those of extremality for the Rayleigh functionals [1,2] and those of averaged extremality [2]. If we interpret such a theory by the quantum-mechanical way, the interaction field, in our case electromagnetic, becomes nonclassical, i.e., acquires stochastic features: In [1], we paid much attention to such the interpretation of the gravitational field.

Thus, consider the functional bilinear in spinorial functions:

$$K(t) = c \int d^3x\, \bar\psi \left[\gamma^\mu \left(i\hbar \frac{\partial}{\partial x^\mu} - \frac{e}{c}\, A_\mu\right) - mc\right] \psi \equiv \int d^3x\, \psi^* \left(i\hbar \frac{\partial}{\partial t} - \hat H\right) \psi \tag{4}$$

where

$$\hat H = c\vec\alpha \left(-i\hbar \frac{\partial}{\partial \vec x} - \frac{e}{c}\, \vec A\right) + \beta mc^2 + e\Phi.$$

Here and below, all conventions and notations are taken from [9], although with restored dimensional coefficients. If the arguments of a function are not indicated, they are implied to be t and $\vec{x}$. The first expression in (4) represents the 4-dimensional form of K, $\bar{\psi}$ and ψ are Dirac conjugate spinors. We shall use below the second expression in (4) which is the $(3+1)$-split form of K, ψ^* and ψ are complex conjugate spinors, A_μ is decomposed onto the electric potential Φ and the magnetic potential $\vec{A}$: $A_\mu \rightarrow (\Phi, -\vec{A})$, however the matrices β and $\vec{\alpha}$ are not fixed, i.e., spinor representation is arbitrary. Spacetime is implied to be Minkowskian, but our constructions can be easily generalized to a curved spacetime. Once $3+1$ splitting is given, the fields Φ and $\vec{A}$ are determined uniquely and cannot be mixed by means of admissible remaining 3-space transformations.

The quantity (4) can be considered a Lagrangian function, so that its integration over time performs a standard spinorial action in the presence of electric and magnetic fields. The extremum of such an action with respect to ψ^* gives the equation of motion (the $(3+1)$-split form of (1))

$$\frac{\delta}{\delta\psi^*(t,\vec{x})}\int dt\, K(t) = 0 = \left[i\hbar\frac{\partial}{\partial t} + c\vec{\alpha}\left(i\hbar\frac{\partial}{\partial\vec{x}} + \frac{e}{c}\,\vec{A}\right) - \beta mc^2 - e\Phi\right]\psi. \tag{5}$$

Note that the expression

$$K(t) - K^*(t) = i\hbar\,\frac{\partial}{\partial t}\int d^3x\; \psi^*\psi \tag{6}$$

follows from (4) and leads, in turn, to the conservation of the Schrodinger norm square

$$N^2 = \int d^3x\; \psi^*(t,\vec{x})\,\psi(t,\vec{x}) = Const \tag{7}$$

when Eq.(5) and its complex conjugate are fulfilled.

Now, we rewrite the pure electromagnetic Lagrangian density in the $(3+1)$-split terms

$$L^{e-m} = -\frac{1}{16\pi c}F_{\mu\nu}F^{\mu\nu} = -\frac{1}{8\pi c}\left[\left(\vec{\nabla}\Phi + \frac{1}{c}\frac{\partial}{\partial t}\vec{A}\right)^2 + (\vec{\nabla}\times\vec{A})^2\right].$$

and recall the way of inclusion of interaction habitually used in the classical field theory. More precisely, this way is accompanied by the interpretation of entering fields as classical ones. It consists of the extension of derivatives in accordance with a gauge group (that is already done) and implies the construction of the action in terms of group invariants. This construction does not require $3+1$ splitting, but we present it in the $(3+1)$-split form in order to compare it with further constructions. Thus, the total Lagrangian function is

$$\begin{aligned}\mathrm{L}^{tot}(t) &= c\int d^3x\; L^{e-m}(t,\vec{x}) - K(t)\\ &= -\frac{1}{8\pi}\int d^3x\;\left[\left(\vec{\nabla}\Phi + \frac{1}{c}\frac{\partial}{\partial t}\vec{A}\right)^2 + (\vec{\nabla}\times\vec{A})^2\right]\\ &\quad - \int d^3x\; \psi^*\left[i\hbar\frac{\partial}{\partial t} + c\vec{\alpha}\left(i\hbar\frac{\partial}{\partial\vec{x}} + \frac{e}{c}\,\vec{A}\right) - \beta mc^2 - e\Phi\right]\psi\end{aligned} \tag{8}$$

and the action is

$$J^{tot} = \int dt\; \mathrm{L}^{tot}(t). \tag{9}$$

As it was mentioned, the extremum of (9) with respect to ψ^* gives Eq.(5), whereas the extrema of (9) with respect to Φ and $\vec{A}$ give the (3+1)-split form of the Maxwell equations (2)

$$-\nabla^2\Phi - \frac{1}{c}\frac{\partial}{\partial t}(\vec{\nabla}\cdot\vec{A}) = 4\pi\rho, \tag{10a}$$

$$\frac{1}{c^2}\left(\frac{\partial}{\partial t}\right)^2 \vec{A} - \nabla^2\vec{A} + \vec{\nabla}(\vec{\nabla}\cdot\vec{A}) + \frac{1}{c}\frac{\partial}{\partial t}\vec{\nabla}\Phi = \frac{4\pi}{c}\vec{j} \tag{10b}$$

with

$$\rho = e\,\psi^*\psi, \qquad \vec{j} = ce\,\psi^*\vec{\alpha}\psi$$

the charge density and the current density, respectively.

Obviously, the set of Eqs.(5) and (10) is invariant under the gauge transformations

$$\Phi \to \Phi + \frac{1}{c}\frac{\partial}{\partial t}\Lambda, \qquad \vec{A} \to \vec{A} - \vec{\nabla}\Lambda, \tag{11a}$$

$$\psi \to \exp\left(-i\,\frac{e}{\hbar c}\,\Lambda\right)\psi, \qquad \psi^* \to \exp\left(+i\,\frac{e}{\hbar c}\,\Lambda\right)\psi^*. \tag{11b}$$

Now, we propose another way of inclusion of interaction which permits us to maintain the probability interpretation. The constructed theory will have the meaning of a one-particle nonlinear QM. Instead of the functional (4), we consider another functional

$$\mathrm{K}(t) = \hbar\int d^3x\,\psi^{*A}\,\mathrm{R}_A{}^B\psi_B - \int d^3x\,\psi^{*A}\hat{H}_A{}^B\psi_B \tag{12}$$

where we restore the bispinor indices with the aid of capital Latin letters: The covariant index denotes a spinor column and the contravariant index denotes a spinor row. We have introduced some new bispinor matrix $\mathrm{R}_A{}^B$ depending on time and space coordinates. First of all, we note that the quantity

$$\mathrm{Tr}\int dt'd^3x\,\frac{\delta\mathrm{K}(t')}{\delta\mathrm{R}(t',\vec{x})} \equiv I_A{}^B\int d^3x\,\frac{\delta\mathrm{K}(t')}{\delta\mathrm{R}_A{}^B(t',\vec{x})} = \int d^3x\,\psi^{*A}(t,\vec{x})\,\psi_A(t,\vec{x}) \tag{13}$$

with $I_A{}^B$ the unit matrix has the form of the norm square (7). Unlike quantum-mechanical theories with higher time derivatives [1,2], the functional (13) is positive definite and is equal to zero in the only case $\psi \equiv 0$. Thus, we shall always consider (13) a nonzero quantity.

Let the functional (10) vanish or, in other words, let the integral matrix equation

$$\mathrm{K}(t) = \hbar\int d^3x\,\psi^{*A}\,\mathrm{R}_A{}^B\psi_B - \int d^3x\,\psi^{*A}\hat{H}_A{}^B\psi_B = 0 \tag{14}$$

be fulfilled and the matrix $\mathrm{R}_A{}^B$ be its some solution. The matrix $\mathrm{R}_A{}^B$ is now a nonlocal functional of the fields ψ^*, ψ, Φ and $\vec{A}$. However we stress that it is not necessary to know its explicit form. Instead of the Lagrangian function (6), we take the following nonlocal matrix function

$$\mathrm{L}_A{}^B(t,\vec{x}) = c\,I_A{}^B\int d^3x'\,L^{e-m}(t,\vec{x}') - \hbar\,\mathrm{R}_A{}^B(t,\vec{x}). \tag{15}$$

Now, we make some statements.

1°. *The simultaneous fulfillment of the conditions*

$$\left(\int d^3x''\, \psi^{*D}(t,\vec{x}'')\, \psi_D(t,\vec{x}'')\right)^{-1} \int dt'\, d^3x'\, \psi^{*B}(t',\vec{x}')\, \frac{\delta L_B{}^C(t',\vec{x}')}{\delta\psi^{*A}(t,\vec{x})}\, \psi_C(t',\vec{x}') = 0 \tag{16}$$

and

$$i\frac{\partial}{\partial t}\psi_A = R_A{}^B\psi_B \tag{17}$$

is equivalent to Eq.(5).

The proof of 1° becomes evident if one considers the following identity in terms of functional derivatives

$$\frac{\delta}{\delta\psi^{*A}(t,\vec{x})}\int dt'\, \mathrm{K}(t') + \int dt'' dt' d^3x'\, \frac{\delta \mathrm{K}(t'')}{\delta \mathrm{R}_C{}^B(t',\vec{x}')}\frac{\delta \mathrm{R}_C{}^B(t',\vec{x}')}{\delta\psi^{*A}(t,\vec{x})} \equiv 0. \tag{18}$$

Indeed, the quantity ψ^{*A} enters the functional $\mathrm{R}_A{}^B$ only, therefore, the differentiation of (15) with respect to ψ^{*A} involves the only second term of (15). The nominator on the right-hand side of (16) is just the second term of (18), while the first term of (18) gives Eq.(5) under the condition (17).

The condition (16) may be called the smoothed extremum condition of the functional (15) with respect to ψ^{*A}. It worth noting that we smooth not an action but the Lagrangian function because, in (16), smoothing functions enter the sign of integration over t' which usually performs the action.

2°. *When fulfilling (14) and the condition (17), the functional (13) is conserved, i.e., the equality (7) holds.*

This is true due to the condition (17), Eq.(14) and the relation (6). The condition (16) is not necessary here.

3°. *The fulfillment of the condition*

$$\left(\int d^3x''\, \psi^{*D}(t,\vec{x}'')\, \psi_D(t,\vec{x}'')\right)^{-1} \int dt'\, d^3x'\, \psi^{*B}(t',\vec{x}')\, \frac{\delta L_B{}^C(t',\vec{x}')}{\delta[\Phi;\vec{A}](t,\vec{x})}\, \psi_C(t',\vec{x}') = 0$$

simultaneously with the conditions (16) and (17) of 1° *is equivalent to the set of equations*

$$-\nabla^2\Phi - \frac{1}{c}\frac{\partial}{\partial t}(\vec{\nabla}\cdot\vec{A}) = 4\pi e\left(\int d^3x'\, \psi^*(t,\vec{x}')\, \psi(t,\vec{x}')\right)^{-1}\psi^*\psi, \tag{19a}$$

$$\frac{1}{c^2}\left(\frac{\partial}{\partial t}\right)^2\vec{A} - \nabla^2\vec{A} + \vec{\nabla}(\vec{\nabla}\cdot\vec{A}) + \frac{1}{c}\frac{\partial}{\partial t}\vec{\nabla}\Phi = 4\pi e\left(\int d^3x'\, \psi^*(t,\vec{x}')\, \psi(t,\vec{x}')\right)^{-1}\psi^*\vec{\alpha}\psi \tag{19b}$$

where, for shortness, the square brackets indicate that one quantity from the two: Φ *or* $\vec{A}$*, should be taken.*

The validity of the right hand sides of (19) follows from functional identities of the type (18) with ψ^{*A} replaced by Φ or $\vec{A}$. To verify the validity of the left-hand sides of (19), we write the corresponding term

$$\left(\int d^3x'''\, \psi^{*D}(t,\vec{x}''')\, \psi_D(t,\vec{x}''')\right)^{-1}\int dt'\, \left[\int d^3x'\, \psi^{*B}(t',\vec{x}')\psi_B(t',\vec{x}')\right]\times$$

$$\frac{\delta}{\delta[\Phi;\vec{A}](t,\vec{x})}\int d^3x'' L^{e-m}(t',\vec{x}'') = \int dt'\, d^3x'\, \frac{\delta L^{e-m}(t',\vec{x}')}{\delta[\Phi;\vec{A}](t,\vec{x})}$$

where we merely combine the quantities depending on x' and use 2°.

Certainly, Eqs.(19) are rescaling invariant and also invariant under the transformations (11).

Thus, we call the statements 1° and 3° the principles of smoothed extremality. We have satisfied the two requirements formulated above. However, this is done at the expense of losing the three important properties of the standard procedure: the principles of extremality, the non-privileged position of the time coordinate and the unambiguous way of constructing the Lagrangian function. The latter means that since R is a complex functional, we have the two possibilities: to choose R or R* in the functional (13). If we chose R*, then an equation complex conjugate to (5) would be obtained as a smoothed extremum condition of R* with respect to ψ, whereas Eqs.(19) are insensitive to the above choice. That is why our variational principles are "one-sided" because only one equation from the two complex conjugate ones supplies one of the suitable functionals R or R* with extremal values.

Emphasize that the condition (17) is necessary to compensate introduced superfluous quantity R, it is a natural generalization of a similar condition in a stationary case (see below). It is sufficient to use (17) in the only smoothed extrema in order to impose less restrictions to the functional R.

Now, we demonstrate how the variational principles formulated above transfer to the Rayleigh variational principles in the stationary case. The stationary condition is

$$i\frac{\partial}{\partial t}\psi_A = \lambda\psi_A \tag{20}$$

leading to

$$\psi_A = \exp(-i\lambda t)\,\tilde{\psi}_A(\vec{x})$$

where $\tilde{\psi}_A$ depends only on $\vec{x}$, such a solution can exist in the case when the potentials Φ and $\vec{A}$ depend only on $\vec{x}$ as well. In this case, we should search for the solution to Eq.(14) in the form

$$\mathrm{R}_A{}^B = I_A{}^B\,R \tag{21}$$

where the functional R does not depend on time and space coordinates. Then, the substitution of (21) into (14) gives the Rayleigh functional (see [1,2] and references therein).

$$R = \left(\hbar\int d^3x'\,\tilde{\psi}^{*C}(\vec{x}')\,\tilde{\psi}_C(\vec{x}')\right)^{-1}\left(\int d^3x\,\tilde{\psi}^{*A}(\vec{x})\,\hat{H}_A{}^B(\vec{x})\,\tilde{\psi}_B(\vec{x})\right).$$

Using the formula (without taking sum on the repeated indices)

$$\frac{\delta R}{\delta\psi^{*A}(t,\vec{x})} = {}_{(3)}\frac{\delta R}{\delta\tilde{\psi}^{*A}(\vec{x})}\left(\frac{d\psi^{*A}(t,\vec{x})}{d\tilde{\psi}^{*A}(\vec{x})}\right)^{-1} = {}_{(3)}\frac{\delta R}{\delta\tilde{\psi}^{*A}(\vec{x})}\;\exp(i\lambda t),$$

where the index ${}_{(3)}$ denotes "3-dimensional" functional derivatives [2], the variational principles of smoothed extremality can be converted into principles of extremality of the scalar (not matrix) functional

$$\mathrm{L} = c\int d^3x' L^{e-m}(x') - \hbar\,R,$$

that is

$${}_{(3)}\frac{\delta \mathrm{L}}{\delta\tilde{\psi}^{*A}(\vec{x})} = {}_{(3)}\frac{\delta R}{\delta\tilde{\psi}^{*A}(\vec{x})} \equiv \left(\hbar\int d^3x'\,\tilde{\psi}^{*C}(\vec{x}')\,\tilde{\psi}_C(\vec{x}')\right)^{-1}\left(\hat{H}_A{}^B(\vec{x}) - \hbar R\right)\tilde{\psi}_B(\vec{x}) = 0, \tag{22}$$

$$ {}_{(3)}\frac{\delta \mathrm{L}}{\delta[\Phi;\vec{A}](\vec{x})} \equiv {}_{(3)}\frac{\delta}{\delta[\Phi;\vec{A}](\vec{x})}\left(c\int d^3x' L^{e-m}(\vec{x}') - \hbar\, R\right) = 0. \tag{23} $$

The extremum condition (22) is equivalent to the eigenvalue problem: the set of extremal values of $\hbar R$ coincides with an energy eigenvalue set. Strictly speaking, this is true for discrete eigenvalues only. The condition (17) is now

$$ i\frac{\partial}{\partial t}\psi_A = R\,\psi_A. \tag{24} $$

Eq.(24) represents the equation to find the function $\psi_A(t,\vec{x})$ and coincides with (20) in the extrema of R. It is easy to verify by immediate differentiation that the extremum conditions (23) give the set of stationary Maxwell equations

$$ -\nabla^2\Phi = 4\pi e\left(\int d^3x'\,\tilde{\psi}^*(\vec{x}')\,\tilde{\psi}(\vec{x}')\right)^{-1}\tilde{\psi}^*\tilde{\psi}, $$

$$ -\nabla^2\vec{A} + \vec{\nabla}(\vec{\nabla}\cdot\vec{A}) = 4\pi e\left(\int d^3x'\,\tilde{\psi}^*(\vec{x}')\,\tilde{\psi}(\vec{x}')\right)^{-1}\tilde{\psi}^*\vec{\alpha}\tilde{\psi}. $$

Note that the norm square functional is here trivially independent of time.

Turn now to the many-particle problem. For the sake of simplicity, consider the two-particle problem for the particles of the spin 1/2. Adding scalar particles may be performed following [3]. Let ξ_{A_1} and η_{A_2} be the two bispinors which describe the particles with the masses m_1 and m_2 and the charges e_1 and e_2, respectively, embedded in common electric and magnetic fields, Φ and $\vec{A}$. As the wave function of our system, we consider both the function with noncoinciding time arguments

$$ \Psi_{A_1A_2}(t_1,t_2,\vec{x}_1,\vec{x}_2) = \xi_{A_1}(t_1,\vec{x}_1)\,\eta_{A_2}(t_2,\vec{x}_2) \tag{25} $$

and the habitually used function with coinciding time arguments

$$ \mathit{\Psi}_{A_1A_2}(t,\vec{x}_1,\vec{x}_2) \equiv \Psi_{A_1A_2}(t,t,\vec{x}_1,\vec{x}_2) = \xi_{A_1}(t,\vec{x}_1)\,\eta_{A_2}(t,\vec{x}_2). \tag{26} $$

The generalization of the functional (4) can be made using the function (25):

$$ \begin{aligned} K(t_1,t_2) = \int d^3x_1 d^3x_2\, \Psi^{*A_1A_2}(t_1,t_2,\vec{x}_1,\vec{x}_2)\left[i\hbar\, I_{A_1}{}^{B_1}\frac{\partial}{\partial t_1} - (\hat{H}_1)_{A_1}{}^{B_1}(t_1,\vec{x}_1)\right]\times \\ \left[i\hbar\, I_{A_2}{}^{B_2}\frac{\partial}{\partial t_2} - (\hat{H}_2)_{A_2}{}^{B_2}(t_2,\vec{x}_2)\right]\Psi_{B_1B_2}(t_1,t_2,\vec{x}_1,\vec{x}_2) = K(t_1)\,K(t_2) \end{aligned} \tag{27} $$

where $\hat{H}_1$ and $\hat{H}_2$ represent the same Hamiltonian as in Eq.(4) but with m replaced by m_1 and m_2 and e replaced by e_1 and e_2, respectively, and with the same Φ and $\vec{A}$. Then, equating the functional derivative of (27) with respect to $\Psi^{*A_1A_2}(t_1,t_2,\vec{x}_1,\vec{x}_2)$ to zero leads to the equation

$$ \left[i\hbar\, I_{A_1}{}^{B_1}\frac{\partial}{\partial t_1} - (\hat{H}_1)_{A_1}{}^{B_1}(t_1,\vec{x}_1)\right]\left[i\hbar\, I_{A_2}{}^{B_2}\frac{\partial}{\partial t_2} - (\hat{H}_2)_{A_2}{}^{B_2}(t_2,\vec{x}_2)\right]\Psi_{B_1B_2}(t_1,t_2,\vec{x}_1,\vec{x}_2) = 0 $$

which decomposes to the set of equations (where, by intention, we write the same time arguments)

$$ \left[i\hbar\, I_{A_1}{}^{B_1}\frac{\partial}{\partial t} - (\hat{H}_1)_{A_1}{}^{B_1}(t,\vec{x}_1)\right]\xi_{B_1}(t,\vec{x}_1) = 0, \tag{28a} $$

$$\left[i\hbar\, I_{A_2}{}^{B_2}\,\frac{\partial}{\partial t} - (\hat{H}_2)_{A_2}{}^{B_2}(t,\vec{x}_2)\right]\eta_{B_2}(t,\vec{x}_2) = 0. \tag{28b}$$

After contracting (28a) with ξ_{A_1} and (28b) with η_{A_2} and summarizing, the set (28) is equivalent to the following single equation for the function (26)

$$\left[i\hbar\, I_{A_1}{}^{B_1}\, I_{A_2}{}^{B_2}\,\frac{\partial}{\partial t} - I_{A_2}{}^{B_2}\,(\hat{H}_1)_{A_1}{}^{B_1}(t,\vec{x}_1) - I_{A_1}{}^{B_1}\,(\hat{H}_2)_{A_2}{}^{B_2}(t,\vec{x}_2)\right]\Psi_{B_1B_2}(t,\vec{x}_1,\vec{x}_2) = 0.$$

However, this is only true for the case of equations which are of the first order in the time derivative and is not true for equations with higher time derivatives.

As to the inclusion of interaction, the functional (27) is not fit for this purpose. Any attempt to include an interaction term into such the Lagrangian function meets with a failure. This is because of the two difficulties: to bring in correlation the dimensionalities of matter and interaction terms in a total Lagrangian function and to get a sum of sources in interaction equations. The generalization of our scheme seems more advantageous.

Taking the functionals R_1 and R_2 the solutions to the equations of the type (12)

$$\mathrm{K}_1(t,(\mathrm{R}_1)_{A_1}{}^{B_1}) = 0,$$

$$\mathrm{K}_2(t,(\mathrm{R}_2)_{A_2}{}^{B_2}) = 0,$$

we consider the nonlocal matrix Lagrangian function

$$\mathrm{L}_{A_1A_2}{}^{B_1B_2}(t,\vec{x}_1,\vec{x}_2) = c\,I_{A_1}{}^{B_1}\,I_{A_2}{}^{B_2}\int d^3x'\,L^{e-m}(t,\vec{x}') - \hbar[(\mathrm{R}_1)_{A_1}{}^{B_1}(t,\vec{x}_1) + (\mathrm{R}_2)_{A_2}{}^{B_2}(t,\vec{x}_2)].$$

Then, the following statements take place.

4°. *The simultaneous fulfillment of the conditions*

$$\begin{gathered}\left[\int d^3x_1''\,d^3x_2''\,\Psi^{*D_1D_2}(t,\vec{x}_1'',\vec{x}_2'')\,\Psi_{D_1D_2}(t,\vec{x}_1'',\vec{x}_2'')\right]^{-1}\times\\ \int dt'\,d^3x_1'\,d^3x_2'\,\Psi^{*B_1B_2}(t',\vec{x}_1',\vec{x}_2')\,\frac{\delta L_{B_1B_2}{}^{C_1C_2}(t',\vec{x}')}{\delta[\xi^{*A_1};\eta^{*A_2}](t,\vec{x})}\,\Psi_{C_1C_2}(t',\vec{x}_1',\vec{x}_2') = 0\end{gathered} \tag{29}$$

and

$$i\frac{\partial}{\partial t}\,\xi_{A_1}(t,\vec{x}_1) = (R_1)_{A_1}{}^{B_1}\,\xi_{B_1}(t,\vec{x}_1), \tag{30a}$$

$$i\frac{\partial}{\partial t}\,\eta_{A_2}(t,\vec{x}_2) = (R_2)_{A_2}{}^{B_2}\,\eta_{B_2}(t,\vec{x}_2) \tag{30b}$$

is equivalent to Eqs.(28) with coinciding space arguments: $\vec{x}_1 = \vec{x}_2 = \vec{x}$.

5°. *The fulfillment of the conditions*

$$\begin{gathered}\left[\int d^3x_1''\,d^3x_2''\,\Psi^{*D_1D_2}(t,\vec{x}_1'',\vec{x}_2'')\,\Psi_{D_1D_2}(t,\vec{x}_1'',\vec{x}_2'')\right]^{-1}\times\\ \int dt'\,d^3x_1'\,d^3x_2'\,\Psi^{*B_1B_2}(t',\vec{x}_1',\vec{x}_2')\,\frac{\delta L_{B_1B_2}{}^{C_1C_2}(t',\vec{x}')}{\delta[\Phi;\vec{A}](t,\vec{x})}\,\Psi_{C_1C_2}(t',\vec{x}_1',\vec{x}_2') = 0\end{gathered} \tag{31}$$

simultaneously with the conditions (29) and (30) of 3° *is equivalent to the set of equations*

$$-\nabla^2\Phi - \frac{1}{c}\frac{\partial}{\partial t}\vec{\nabla}\cdot\vec{A} = 4\pi\left[e_1\left(\int d^3x'\,\xi^*(t,\vec{x}')\,\xi(t,\vec{x}')\right)^{-1}\xi^*\xi + e_2\left(\int d^3x'\,\eta^*(t,\vec{x}')\,\eta(t,\vec{x}')\right)^{-1}\eta^*\eta\right],$$

$$\frac{1}{c^2}\left(\frac{\partial}{\partial t}\right)^2\vec{A}-\nabla^2\vec{A}+\vec{\nabla}(\vec{\nabla}\cdot\vec{A})+\frac{1}{c}\frac{\partial}{\partial t}\vec{\nabla}\Phi$$
$$=4\pi\left[e_1\left(\int d^3x'\,\xi^*(t,\vec{x}')\,\xi(t,\vec{x}')\right)^{-1}\xi^*\vec{\alpha}\xi+e_2\left(\int d^3x'\,\eta^*(t,\vec{x}')\,\eta(t,\vec{x}')\right)^{-1}\eta^*\vec{\alpha}\eta\right].$$

The proofs of these statements are trivial and repeat those of statements 1° and 2°. We emphasize that interaction occur in the same (single) spacetime labelled with the coordinates t and $\vec{x}$. Note that the denominators in (29) and (31) represent a two-particle norm square for the function (26) which is conserved as before. In conclusion, we add that our self-consistent two-particle problem can be reduced to that of the standard QM using some contradictory assumptions.

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