КОСМОЛОГИЧЕСКОЕ КРАСНОЕ СМЕЩЕНИЕ КАК ЭФФЕКТ УМЕНЬШЕНИЯ РАЗМЕРНОСТИ ВРЕМЕНИ В БОЛЬШИХ МАСШТАБАХ

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COSMOLOGICAL REDSHIFT AS AN EFFECT OF DECREASING TIME DIMENSIONALITY ON LARGE SCALES

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Abstract

In the framework of the concept of space and time dimensionalities which are noninteger and decrease with the growth of scales, we earlier proposed the model of eternal static closed 2-dimensional Universe which can resolve the problems of dark matter and of the age of the Universe in standard cosmology. In order to admit it to realize in the real Universe, we propose here a non-Doppler explanation of cosmological redshift, based on decreasing time dimensionality. We calculate the three cosmological tests: visual magnitude, angular distance and a number of sources versus redshift, following from our scheme.

The everyday experience tells us that our physical space has three dimensions and that our time has one dimension. However extrapolating these local facts to astronomical and cosmological scales may happen to be wrong, and we present below some arguments for our point of view based on the two unresolved problems of modern observational cosmology which are not explained theoretically in full. These are the problems of dark matter and of the age of the Universe which are closely connected with the notion of spatial dimensionality.

In dealing with far astronomical objects, we have to confine ourselves by the only dynamical notion of spatial dimensionality, i.e., by that following from the laws of gravitational interactions and light propagation. The dimensionality determined by such a way may be distinct from 3, noninteger and nonconstant in space and time. A logical way would be to infer the number of space dimensions from observations. Nevertheless, instead of doing by such a way, one prefers to prescribe to the whole Universe the laws which follow from the number 3 and then to save a situation. Probably, this just takes place for the mentioned problems, in particular, for the problem of dark matter: The dynamics of galaxies, clusters and superclusters of galaxies does not correspond to the amount of luminous matter there, so that one requires some hidden mass or dark matter to compensate this discrepancy. If one accepts that the law of gravitational force is between r^{-2} and r^{-1}, then the amount of luminous matter and its dynamics can certainly be brought in correlation. In terms of the dynamical dimensionality, this is equivalent to that the latter acquires some values between 3 and 2. Since it is known that dark matter is required in increasing amounts when passing to cosmological scales [1], we conclude that the dimensionality n of our physical space may decrease from 3 to 2 when passing to cosmological scales [2]. We mean that the spatial dimensionality may be noninteger and smoothly fall together with the growth of a relative distance between bodies. We consider the value 2 as a limit value which may be reached at cosmological scales.

The second cosmological problem is that the age of some galaxies and globular stellar clusters estimated from their chemical composition [3,4] is greater than a comparatively small age of the Universe calculated in the standard Friedmann-Robertson-Walker (FRW) cosmology. However this problem is also better resolved in less dimensions [2]: Accepting the dimensionality $n < 3$ with $m = 1$ in a FRW-like cosmological model gives a greater age of the Universe.

Earlier, we also considered the FRW-like model with the metric interval [5]

$$ds^2 = c^2(dt_1^2 + dt_2^2 + ... + dt_m^2) - a^2(t)\, dl^2_{(n,k)} \tag{1}$$

where t_A $(A = 1, 2, ..., m)$ is a Cartesian temporal coordinate and

$$t = (t_1^2 + t_2^2 + ... + t_m^2)^{1/2} \tag{2}$$

is a radial temporal coordinate in the flat m-dimensional temporal space; the scale factor a depends on the only t, the quantity $dl_{(n,k)}$ is a length element of the n-dimensional space with the constant Gauss curvature $(k = -1, 0, +1)$. Avoiding some details, we supplied our model with hydrodynamic matter (with an auxiliary tension in the temporal space) and derived the equations of gravitational field. After that we made the crucial step: we thought m and n as real constant (noninteger) parameters. (The self-consistent formulation of general relativity in terms of noninteger time and space dimensionlities was done in [6].) We studied the possibility of existing static solutions: We found that a physically admissible static solution exists in the only case $n = 2$ (which is an isolated case for the above equations) together with $k = +1$ and an arbitrary m. This solution describes eternally existing closed Universe where the world radius a is connected with the matter density ρ (whose physical dimensionality is ML^{-2}) and with the constant $\kappa_{(m,2)}$, which plays the role of the Einstein constant, by the simple relation $a \equiv a_0 = (\kappa_{(m,2)}\rho)^{-1/2}$. Equivalently, the above constant is expressible via the full mass of the Universe M: $\kappa_{(m,2)} = 4\pi/M$. The metric interval (1) is now

$$ds^2 = c^2(dt_1^2 + dt_2^2 + ... + dt_m^2) - a_0^2(dr^2 + \sin^2 r\, d\phi^2) \tag{3}$$

where r is a dimensionless radius (in the sense of its physical dimensionality) which corresponds to the latitude angular coordinate on the surface of a 2-dimensional sphere, $0 \leq r \leq \pi$, and ϕ is the polar angular coordinate.

Such the solution seems to be very attractive psychologically. At least, it resolves the mentioned two problem. However, an assumption that it is realized in our Universe entails another great problem: We should give alternative (non-Doppler) explanation of cosmological redshift, and this is just the purpose of the present paper. We propose an explanation under an assumption that time itself "vanishes" on cosmological scales, or in other (mathematical) words, that the dimensionality of time is also noninteger and may decrease from 1 to 0 when passing to cosmological distances. Certainly, we have no adequate mathematical apparatus to realize our speculations in a strict formalism. We shall try to make impossible things starting from usual ones. It is handy for us to account our construction as a consequence of **seven** conceptual steps which will be introduced not at once but during the whole text.

Thus, we start with the flat m-dimensional temporal space (m-space) with some integer $m \geq 1$. It is not matter for us the number n of spatial coordinates as yet. In Eq.(1), we have made an assumption **(step** 1^***)** that the same velocity of light c corresponds to every time direction. We also make an important assumption **(step** 2^***)** that every physical body and/or every physically connected system of bodies have its own time directions stochastically distributed in the m-space. Further, let us consider two physical systems at rest in the (n-dimensional) space points 1 and 2 (independently of their spatial separation) in which time "flows" along the directions $\vec{t_1}$ and $\vec{t_2}$, respectively. Due to the postulated flatness of the temporal space, there are no problems in the parallel transfer of the vector $\vec{t_1}$ to the point 2 or *vice versa* of that of $\vec{t_2}$ to the point 1. Then, for an observer in the point 1, the rate of physical processes occurring in $\vec{t_2}$ will be determined by the projection of $\vec{t_2}$ onto $\vec{t_1}$, or in other words, by the scalar product of $\vec{t_1}$ and $\vec{t_2}$. The same is true for an observer in the point 2.

Let T be a period of some process which is the same for both the observers. Then each observer will fix the projected period $T_{pr} = T\cos\theta$ for the process in the other system where θ is an angle between $\vec{t_1}$ and $\vec{t_2}$. Moreover, this situation is symmetric for both the observers "living" in $\vec{t_1}$ and $\vec{t_2}$. Let $\lambda = cT$ be the wave length of light emission common for both the observers. Then, the projected wave length is $\lambda_{pr} = \lambda\cos\theta$. In the case $m > 1$, λ_{pr} is always LESS than λ: Thus light radiation will be perceived as more blue. If one imagine the situation when $\theta = \pi/2$, we shall fix in one moment of time ALL the history of the other system. If the latter represent the above light emission, we shall fix a flash with zero wave length (meaning infinite energy). Now, we make an additional assumption **(step** 3^***)** that the m-space is elliptic, i.e., that with identified points which are opposed relatively to the zero moment of time [zero value of the coordinate (2): $t = 0$], in order to avoid negative projections: Time should not admit inverse consequences of events.

For further considerations, we need to learn how one can average λ_{pr} over solid angles in the temporal space. In the case of integer m directions we can write the known formula for the volume element of the flat m-space in the spherical set of coordinates:

$$dV^{(m)} = ds^{(m)}\,\rho^{m-1}d\rho$$

where $ds^{(m)}$ is the element of the m-dimensional solid angle, and ρ is a radial coordinate. (Spherical sets of coordinates for multidimensional spaces were considered in nonlinear sigma-models, see, e.g., [7]). Let us choose the set of angle coordinates by such a way that the time vector of some fixed observer is directed to the "North Pole" of the m-dimensional sphere from where one count out the (main) latitude angle θ which changes from 0 to π. This angle is just the angle between the observer and a projected direction.

There always exists the relation

$$ds^{(m)} = \sin^{m-2}\theta \, d\theta \, ds^{(m-1)} \tag{4}$$

where $ds^{(m-1)}$ corresponds to the element of the $(m-1)$-dimensional solid angle which includes another plane angles. For example,

$$ds^{(4)} = \sin^2\theta \, d\theta \, \sin\xi \, d\xi \, d\phi,$$

$$ds^{(3)} = \sin\theta \, d\theta \, d\phi,$$

$$ds^{(2)} = d\theta$$

where ξ and ϕ are some relevant angles. In the only latter case θ changes from 0 to 2π because it is the angle in the plane polar set of coordinates.

The averaged over all the angles λ-projection, $\bar{\lambda}_{pr}$, can be expressed as follows

$$\bar{\lambda}_{pr} = \left(\int ds^{(m)} \right)^{-1} \int ds^{(m)} \lambda_{pr} \cos\theta =$$

$$2\lambda \left(\int ds^{(m)} \right)^{-1} \int_0^{\pi/2} d\theta \, \cos\theta \sin^{m-2}\theta \int ds^{(m-1)} \tag{5}$$

where we recall the fact that in the elliptic m-space we should average over the half value of the θ-interval: $0 \leq \theta \leq \pi/2$, and where Eq.(1) was used in the second equality. The integral over $ds^{(m)}$ represents a surface of the unit m-dimensional sphere, it can be expressed via the gamma-function as follows

$$\int ds^{(m)} = \frac{2\pi^{m/2}}{\Gamma(m/2)}. \tag{6}$$

Using (6), the expression (5) can be calculated immediately

$$\bar{\lambda}_{pr} = \lambda \, \frac{\Gamma(m/2)}{\sqrt{\pi}\Gamma((m+1)/2)}. \tag{7}$$

The formula (7) can be checked by the simple expressions for $m = 4, 3, 2, 1$:

$$\bar{\lambda}_{pr}^{(4)} = \frac{4}{3\pi}\lambda, \quad \bar{\lambda}_{pr}^{(3)} = \frac{1}{2}\lambda, \quad \bar{\lambda}_{pr}^{(2)} = \frac{2}{\pi}\lambda, \quad \bar{\lambda}_{pr}^{(1)} = \lambda.$$

Eq.(7) is suitable not only for integer m, that suggests us to make the conceptual step **(step** 4^***)** that the dimensionality m can acquire real (noninteger) values $m \geq 1$. We also define the square of dispersion of λ_{pr} about $\bar{\lambda}_{pr}$ as usual

$$(\Delta\lambda)^2 = \overline{(\lambda_{pr} - \bar{\lambda}_{pr})^2} = \overline{\lambda_{pr}^2} - (\bar{\lambda}_{pr})^2. \tag{8}$$

Taking an analogy with Eq.(5), the simple expression for $\overline{\lambda_{pr}{}^2}$ can be obtained

$$\overline{\lambda_{pr}{}^2} = 2\lambda^2 \left(\int ds^{(m)} \right)^{-1} \int_0^{\pi/2} d\theta \cos^2\theta \sin^{m-2}\theta \int ds^{(m-1)} = \frac{1}{m}\,\lambda^2.$$

Whence, (8) is as follows

$$(\Delta\lambda)^2 = \lambda^2 \left(\frac{1}{m} - \frac{\Gamma(m/2)}{\sqrt{\pi}\Gamma((m+1)/2)} \right).$$

After that we make the next step (**step** 5^*): due to the fact that the obtained formulae have no singularities for $0 < m < 1$, we now assume that m can acquire all positive real values: $m > 0$. It is easily seen that if $m > 1$, then $\bar{\lambda}_{pr} < \lambda$ and $(\Delta\lambda)$ is a real number, and if $0 < m < 1$, then $\bar{\lambda}_{pr} > \lambda$ and $(\Delta\lambda)$ is a pure imaginary number. In the latter case, it cannot serve as a measure of dispersion, however the next our step (**step** 6^*) is that we assume $\sqrt{|(\Delta\lambda)^2|}$ to somewhat characterize the distribution of λ_{pr} as dispersion.

Now we should recall the definition of the redshift z as the ratio of the difference between an observed wave length and a perceived one to the perceived one. In our terms,

$$z = \frac{\lambda_{pr}}{\lambda} - 1. \tag{9}$$

Eq.(9) demonstrates that z is uniquely determined by λ_{pr}/λ and does not depend on λ itself. This fact is very important for our interpretation, the same takes place for the Doppler effect and for observational cosmological redshift: Due to this reason the latter is just explained by the former.

Further, we can calculate the value of (9) averaged over all the angles in the above m-space:

$$\bar{z} = \frac{\Gamma(m/2)}{\sqrt{\pi}\Gamma((m+1)/2)} - 1. \tag{10}$$

Clearly, $\bar{z} < 0$ (blue shift) for $m > 1$, $\bar{z} > 0$ (redshift) for $m < 1$ and $\bar{z} = 0$ (zero shift) for $m = 1$. Moreover, we can see that $\bar{z} \to \infty$ then $m \to 0$.

By a similar way, denote the dispersion of z, $(\Delta z)^2$ as

$$(\Delta z)^2 = |\overline{(z-\bar{z})^2}| = \frac{|(\Delta\lambda)^2|}{\lambda^2} = \left| \frac{1}{m} - \frac{\Gamma^2(m/2)}{\pi\Gamma^2((m+1)/2)} \right|. \tag{11}$$

Thus, we have derived the expressions for $\bar{z}$ and $(\Delta z)^2$ under the assumption of constant everywhere $m > 0$. Now, the next conceptual (**step** 7^*) is to assume that m is a smooth function of the coordinate r [see (3)]: $m = m(r)$. More precisely, we admit m to fall with the growth of r. Let, for small r, m be approximately described as follows

$$m = 1 - \alpha r + \beta r^2 \tag{12}$$

where

$$0 < \alpha \equiv -\left.\frac{dm}{dr}\right|_{r=0} \ll 1, \qquad \beta \equiv \left.\frac{1}{2}\frac{d^2m}{dr^2}\right|_{r=0}, \tag{13}$$

and $|\beta|$ is of the order of α^2.

Now, we can calculate how $\bar{z}$ changes with the coordinate r. In substituting (12) into (10), we find in the two first orders in r

$$\begin{aligned} \bar{z} &= (\ln)2\,\alpha r + \left[\frac{1}{2}\left(\ln^2 2 + \frac{\pi^2}{12}\right)\alpha^2 - (\ln 2)\,\beta\right] r^2 \\ &\approx 0.693\alpha r + (0.651\alpha^2 - 0.693\beta) r^2. \end{aligned} \tag{14}$$

By a similar way, in substituting (12) into (11),

$$(\Delta z)^2 = \left| (2\ln 2 - 1)\,\alpha r - \left[(2\ln 2 - 1)\,\beta + \left(1 - 2\ln^2 2 - \frac{\pi^2}{12}\right)\alpha^2\right] r^2 \right|$$

$$\approx |0.386\alpha r + (0.783\alpha^2 - 0.386\beta)r^2|. \tag{15}$$

It is easy to calculate from (14) and (15) that, at least for small r

$$\Delta z \approx 0.746\, \bar{z} < \bar{z}.$$

Further, we want to calculate the classical cosmological tests, i.e., the theoretical dependencies of visual magnitude, angular distance and a number of sources versus red-shift. The above quantities change with r, so that in our scheme we can obtain required dependencies on $\bar{z}$ if we express inversely r via $\bar{z}$ from (14):

$$r = \frac{\bar{z}}{\alpha \ln 2}\left[1 - \left(\frac{1}{2} + \frac{\pi^2}{24 \ln^2 2} - \frac{1}{\ln 2}\frac{\beta}{\alpha^2}\right)\bar{z}\right]. \tag{16}$$

Now, we can immediately come to the derivation of the tests.

i) *Visual magnitude vs. redshift.* This test is sufficiently complete because it unevitably includes the change of spatial dimensionality along light propagation. We can hardly describe any smooth change of n, but we assumed [2] that n changes by leap on some relative distance R_0 between any two bodies from 3 to $n < 3$. Here, we think $n = 2$, that is why our derivation is quite rough. The standard definition of the distance modulus $\tilde{m} - \tilde{M}$ is as follows

$$\tilde{m} - \tilde{M} = 2.5\,(\lg E^{(3)}_{std} - \lg E^{(3)}) \tag{17}$$

where $\tilde{m}$ is a visual magnitude, $\tilde{M}$ is an absolute visual magnitude, $E^{(3)}$ is an intensity which is perceived by a usual observer in the 3-dimensional space and

$$E^{(3)}_{std} = \frac{L}{4\pi \cdot 10^2} \tag{18}$$

is the standard intensity. Taking into account the above change of dimensionality and assuming that $R_0/a_0 r \ll 1$,

$$E^{(3)} = \frac{2}{\pi}\, E^{(2)}_{pr}\, \frac{1}{R_0}\left(1 + \frac{1}{24}\frac{R_0^2}{a_0 r^2}\right) \tag{19}$$

where $E^{(2)}_{pr}$ is in turn the intensity in the 2-dimensional space projected onto the time direction of the observer:

$$E^{(2)}_{pr} = \frac{1}{2\pi}\frac{L_{pr}}{a_0\, r} \tag{20}$$

with L_{pr} a projected luminosity.

In collecting together (17), (18), (19) and (20), we obtain

$$\tilde{m} - \tilde{M} = 5\lg(a_0\, r) - 5 - \frac{5}{2}\lg\frac{L_{pr}}{L} + \frac{5}{2}\lg\frac{R_0}{a_0 r} - \frac{5\,\lg e}{48}\left(\frac{R_0}{a_0 r}\right)^2 - \frac{5}{2}\lg\frac{4}{\pi}. \tag{21}$$

One remains to find the ratio L_{pr}/L. Luminosity is in essence energy emitted in the unit time, and energy itself is proportional to a frequency of light quanta, therefore

$$\frac{L_{pr}}{L} = \left(\frac{\lambda_{pr}}{\lambda}\right)^{-2} = \frac{1}{(1+z)^2}. \tag{22}$$

After that we substitute (16) and (22) into (21), expand the term $5\lg(1+z) \approx 5\, z\, \lg e$ following from (22) and replace roughly z by $\bar{z}$, then we finally obtain the desired expression

$$\tilde{m}(\bar{z}) - \tilde{M} = 5\lg\frac{a_0\,\bar{z}}{\alpha} + \frac{5\,\lg e}{2}\left(\frac{3}{2} - \frac{\pi^2}{24\ln^2 2} + \frac{1}{\ln 2}\frac{\beta}{\alpha^2}\right)\bar{z} +$$

$$\frac{5}{2}\,\lg\frac{R_0\alpha}{a_0\bar z} - 5 - \frac{5\,\lg e\,\ln^2 2}{48}\left(\frac{R_0\alpha}{a_0\bar z}\right)^2 + \frac{5}{2}\,\lg\frac{\pi}{4\,\ln 2} \tag{23}$$

$$\approx 5\,\lg\frac{a_0\,\bar z}{\alpha} + \left(0.699 + 1.567\,\frac{\beta}{\alpha^2}\right)\bar z + \frac{5}{2}\,\lg\frac{R_0\alpha}{a_0\bar z} - 0.0217\,\left(\frac{R_0\alpha}{a_0\bar z}\right)^2 - 4.864.$$

ii) *Angular distance vs. redshift.* The second test is expressed as usual as the ratio of the linear size (l) and angular size (θ) of an extended source:

$$\tilde R = \frac{l}{\theta}.$$

This quantity is just the distance along the curved 2-dimensional sphere. In the case of the closed static Universe, we can write

$$\tilde R = a_0\,\sin\,r \approx a_0\left(r - \frac{1}{3!}\,r^3 + ...\right).$$

However, due to that we confine ourselves by the second order in z, it is enough to take

$$\tilde R(\bar z) = a_0\,r = \frac{1}{\ln 2}\,\frac{a_0\,\bar z}{\alpha}\left[1 - \left(\frac{1}{2} + \frac{\pi^2}{24\,\ln^2 2} - \frac{1}{\ln 2}\,\frac{\beta}{\alpha^2}\right)\bar z\right]$$

$$\approx 1.443\,\frac{a_0\,\bar z}{\alpha}\left[1 + \left(1.443\,\frac{\beta}{\alpha^2} - 1.356\right)\bar z\right]. \tag{24}$$

iii) *The number of sources vs. redshift.* This test is as usual presented in the differential form. In the 2-dimensional space, the differential of number N (dN) of the sources of a required sort at the differential of $\bar z$ ($d\bar z$) is

$$dN = 2\pi\nu_0\,a_0^2\,r(\bar z)\,\frac{dr}{d\bar z}\,d\bar z \tag{25}$$

where ν_0 is the 2-dimensional density of the above sources at $z = 0$. Using (16), Eq. (25) acquires the form

$$dN(\bar z) = \frac{2\pi}{\ln^2 2}\,\frac{\nu_0\,a_0^2}{\alpha}\,\bar z\left[1 - \left(\frac{3}{2} + \frac{\pi^2}{8\,\ln^2 2} - \frac{3}{\ln 2}\,\frac{\beta}{\alpha^2}\right)\bar z\right]d\bar z$$

$$\approx 12.97\,\frac{\nu_0\,a_0^2}{\alpha}\,\bar z\left[1 + \left(4.328\,\frac{\beta}{\alpha^2} - 4.068\right)\bar z\right]d\bar z. \tag{26}$$

Thus, the obtained dependencies (23), (25) and (26) can be treated as the "statistical" ones, in other words, they are true as averaged. In accepting our scheme, if we compare them with the same observed dependencies we can reinterpret some observable quantities. For example, the second test, in its standard form for the case $m = 1$ and an arbitrary n, looks like this [2]:

$$\tilde R(z) = \frac{cz}{H_0}\left[1 - \left(\frac{3}{2} + \frac{q_0}{2}\right)z\right]$$

where H_0 is the modern value of the Hubble parameter and q_0 is the modern value of the deceleration parameter. If H_0 and q_0 are obtained from an observed dependence $\tilde R(z)$, then we can correspond to them the quantities α and β

$$\alpha = \frac{1}{\ln 2}\,\frac{a_0 H_0}{c} \approx 1.443\,\frac{a_0 H_0}{c},$$

$$\beta = -\left(\frac{1}{\ln 2} - \frac{\pi^2}{24\ln^3 2} + \frac{1}{2\ln 2}q_0\right)\left(\frac{a_0 H_0}{c}\right)^2 \approx -(0.208 + 0.721 q_0)\left(\frac{a_0 H_0}{c}\right)^2 .$$

Thus, real values of α and β can be estimated if one knows a_0.

The present paper has a preliminary character, and we hope to give elsewhere an account in more details and to pay more attention for interpretation.

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References

[1] M.Davis *et al.*, *Astroph. J.* **238** (1980), L113.

[2] A.D.Popova, *Astron. and Astroph. Trans.* **5** (1994), 31; to appear *ibid.*

[3] A.G.Brusual, *Astron. J.* **270** (1983), 105.

[4] T.S.van Olbada *et al.*, *Monthly Notes* **196** (1981), 823.

[5] A.D.Popova and A.N.Kulik, to appear in *Astron. and Astroph. Trans.*

[6] V.Yu.Koloskov, *Nuov. Cim.* **107B** (1992), 1051.

[7] G.Ghika and N. Visinescu, *Nuovo Cimento* **B59** (1980), 59;